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The Yang-Baxter Wess-Zumino model and Poisson-Lie T-duality

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Based on
"Classical and Quantum Aspects of Yang-Baxter Wess-Zumino Models";
SD, S. Driezen, A. Sevrin, D.C. Thompson

JHEP 1803 (2018) 041, arXiv:1711.00084.

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Deform known integrable model whilst preserving integrability
$\rightarrow$ understand 'integrability'

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The Yang-Baxter Wess-Zumino model:

$$\mathcal{S} = -\frac{1}{2\pi}\int d\sigma d\tau \langle g^{-1}\partial_+ g, \left(\alpha 1 + \beta\mathcal{R} + \gamma\mathcal{R}^2\right) g^{-1}\partial_- g\rangle + \frac{k}{24\pi}\int_{M_3}\langle \bar{g}^{-1}d\bar{g}, [\bar{g}^{-1}d\bar{g}, \bar{g}^{-1}d\bar{g}]\rangle$$

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Integrable model (PCM) + deforming using $\mathcal{R}$

+ WZ term

with $\mathcal{R}$ solving the modified classical Yang-Baxter equation

$$[\mathcal{R}x, \mathcal{R}y) - \mathcal{R}([\mathcal{R}x, y] + [x, \mathcal{R}y]) = [x, y] \quad \text{and} \quad \mathcal{R}^3 = -\mathcal{R} \quad \forall x, y \in \mathfrak{g}.$$

YB-WZ model extends η-model

Reduces to an η-model:

$$\mathcal{S} = -\frac{1}{2\pi\tau(1+\eta^2)} \int d\sigma d\tau \langle g^{-1}\partial_+ g, (1+\eta\mathcal{R}+\eta^2\mathcal{R}^2) g^{-1}\partial_- g\rangle$$

Integrable model (PCM) + deforming using $\mathcal{R}$

by taking

$$k = 0\,, \quad \alpha = 1/\tau\,, \quad \beta = \frac{\eta}{\tau(1+\eta^2)}\,, \quad \gamma = \frac{\eta^2}{\tau(1+\eta^2)}\,.$$

What do we learn from the YB-WZ model

The YB-WZ model combines remarkably properties:

$$\mathcal{S} = -\frac{1}{2\pi}\int d\sigma d\tau \langle g^{-1}\partial_+ g, (\alpha 1 + \beta\mathcal{R} + \gamma\mathcal{R}^2) g^{-1}\partial_- g\rangle + \frac{k}{24\pi}\int_{M_3}\langle \bar{g}^{-1}d\bar{g}, [\bar{g}^{-1}d\bar{g}, \bar{g}^{-1}d\bar{g}]\rangle$$

- Determines an integrable system!

  extending the η-deformation
- For any group G, at one-loop RG remains a consistent sigma-model

  relates to class. intb and quantum group structure
- Currents determine a quantum group

  that is preserved by the RG flow
- Features very simple Poisson-Lie T-duality

(Quantum ?) integrability

- **Integrability ?**

  Eoms = flat Lax connection $\rightarrow$ ∞ tower of conserved charges

- **Yes**, flatness ensured by algebraic constraint

  " integrable locus "

  $$\beta^2 = \frac{\gamma}{\alpha}\left(\alpha^2 - \alpha\gamma - k^2\right)$$

  [Delduc, Magro, Vicedo]

- **Surprising**

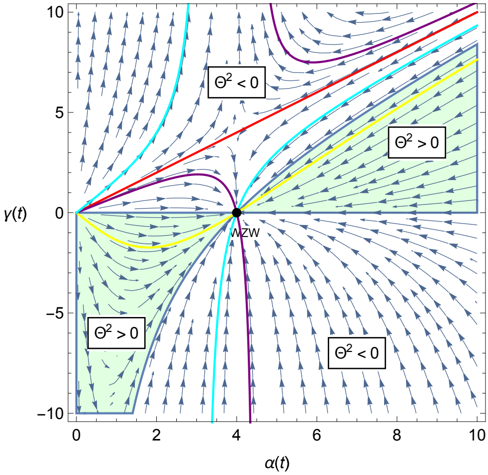
  classical integrability condition preserved by RG flow

  classical property $\longleftrightarrow$ *quantum* phenomenon

  [SD, Driezen, Sevrin, Thompson]

YB-WZ: RG flow preserves the σ-model structure

Example for $G = SU(3)$ and $k = 4$



- Green regions are physical
- Flows to the WZW-model in the IR
- Trajectories labeled by an RG-invariant

$$\Theta^2 = \frac{\alpha(\alpha(\alpha-\gamma)-k^2)}{\gamma}$$

[SD, Driezen, Sevrin, Thompson]

Quantum group structure

Integrable models closely related to quantum groups

Deformation enhanced symmetry group of the PCM to a quantum group

[Kawaguchi, Yoshida]

$$G_L \times G_R \quad \longrightarrow \quad \mathcal{Y}(\mathfrak{g}) \times \mathcal{Y}(\mathfrak{g})$$

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$$G_L \times G_R \quad \longrightarrow \quad \mathcal{Y}(\mathfrak{g}) \times \mathcal{Y}(\mathfrak{g}) \quad \xrightarrow{\text{deformation}} \quad \mathcal{Y}(\mathfrak{g}) \times \mathcal{U}_q(\hat{\mathfrak{g}})$$

Non-local charges $\mathfrak{Q}^H, \mathfrak{Q}^+, \mathfrak{Q}^-$:

$$\{\mathfrak{Q}^+, \mathfrak{Q}^-\} = i\frac{q^{\mathfrak{Q}^H} - q^{-\mathfrak{Q}^H}}{q - q^{-1}} \xrightarrow{q\to 1} \mathfrak{Q}^H\,, \quad \{\mathfrak{Q}^\pm, \mathfrak{Q}^H\} = \pm i\mathfrak{Q}^\pm\ .$$

The quantum parameter q is an RG invariant!

$$q = \exp\left[\frac{8\pi\Theta}{\Theta^2 + k^2}\right] \qquad \text{where} \qquad \Theta^2 = \frac{\alpha(\alpha(\alpha-\gamma) - k^2}{\gamma}\,.$$

$\mathcal{E}$-models of Poisson-Lie T-duals

The basic ingredients

First order formulation of Poisson-Lie T-duality [Klimčik, Ševera]

$\mathscr{E}$ – model: $\left\{ \begin{array}{ccc} (\mathfrak{d}, (\cdot,\cdot)_{\mathfrak{d}}, \mathfrak{h}_1 \subset \mathfrak{d}) & \text{and} & \mathscr{E} \\ \textit{algebra and subalgebra} & & \textit{an operator} \end{array} \right\}$

$\downarrow$

First order Hamiltonian

$H_{\mathscr{E}}$ on $D = H_1 \oplus H_2$

$\mathcal{E}$-models of Poisson-Lie T-duals

$\mathscr{E}$ – model: $(\mathfrak{d}, (\,\cdot\,,\,\cdot\,)_{\mathfrak{d}}, \mathscr{E}, \mathfrak{h})$

First order
Hamiltonian $H_{\mathscr{E}}$ on

$$D = H_1 \oplus H_2$$

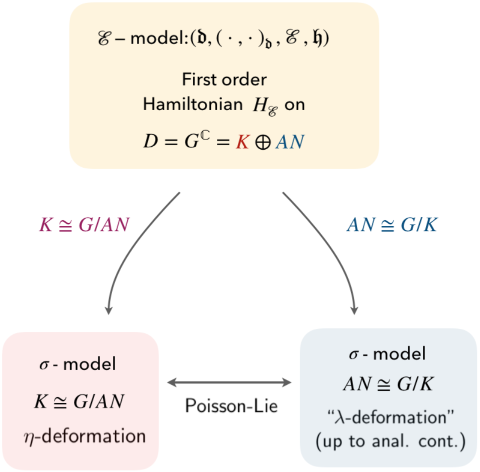
$\mathscr{P} : D/H_1$ $\qquad$ $\widetilde{\mathscr{P}} : D/H_2$

$$S = S^{\mathfrak{d}}_{\text{WZW}} + k \int \mathrm{d}^2\sigma \left(\mathscr{P} L_+, L_-\right)_{\mathfrak{d}} \quad \xleftrightarrow{\text{Poisson-Lie dual}} \quad S = S^{\mathfrak{d}}_{\text{WZW}} + k \int \mathrm{d}^2\sigma \left(\widetilde{\mathscr{P}} L_+, L_-\right)_{\mathfrak{d}}$$

where $\quad L_{\pm} = g^{-1}\partial_{\pm} g\,, \quad g \in D$

$\mathcal{E}$-models of Poisson-Lie T-duals

"Famous" Poisson-Lie T-duals: η & λ



[Hoare, Klimcik, Seibold, Sfetsos, Siampos, Thompson, Tseytlin]

Poisson-Lie applied to YB-WZ model

$\mathscr{E}$ – model: $(\mathfrak{d}, (\cdot\,,\cdot)_{\mathfrak{d}}, \mathscr{E}, \mathfrak{h})$

First order Hamiltonian $H_{\mathscr{E}}$ on

$$D = G^{\mathbb{C}} = H_\rho \cdot \widetilde{H}_\rho$$

$$\mathscr{P} : G \cong G^{\mathbb{C}}/H_\rho$$

$$S = -\int \mathrm{d}^2\sigma \, \langle L_+ , \left(\alpha + \beta\mathscr{R} + \gamma\mathscr{R}^2\right) L_- \rangle + S_{\text{WZW},k}$$

where $L_\pm = g^{-1}\partial_\pm g\,, \quad g \in D$

YB-WZ: surprisingly simple Poisson-Lie duality

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The YB-WZ model on G

$$S = -\int \mathrm{d}^2\sigma \, \langle L_+ , \left(\alpha + \beta \mathscr{R} + \gamma \mathscr{R}^2\right) L_- \rangle + S_{\text{WZW},k}$$

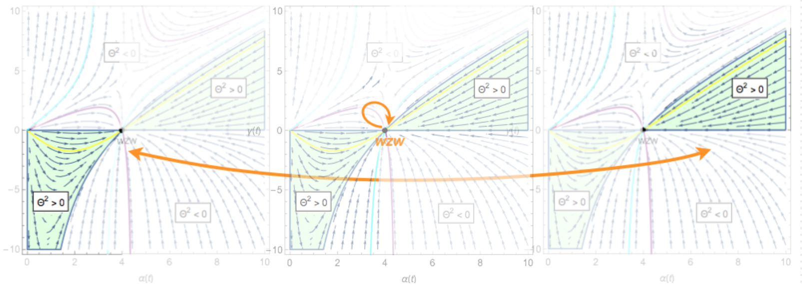
$\longleftrightarrow$ Poisson-Lie dual

The dual of YB-WZ model on G

$$S = -\int \mathrm{d}^2\sigma \, \langle L_+ , \left(\frac{k^2}{\alpha} + (-\beta)\mathscr{R} + \left(-\frac{\beta}{\gamma}\right)\mathscr{R}^2\right) L_- \rangle + S_{\text{WZW},k}$$

[Klimcik; SD, Driezen, Sevrin, Thompson]

YB-WZ: surprisingly simple Poisson-Lie duality



PL duality on YB-WZ

$$\begin{cases} \alpha \\ \beta \\ \gamma \end{cases} \mapsto \begin{cases} k^2/\alpha \\ -\beta \\ -\beta^2/\gamma \end{cases}$$

- Resembles simple radial 'inversion'
- Remains on the integrable locus !

  Canonical transformation [Sfetsos, Klimčik, Hlavatý]
- Swaps the physical regions of the RG flow
- The RG fixed pt WZW also self dual under PL
- Quantum group parameter q is invariant

Conclusions

Summary

- The YB-WZ model is an integrable extension of the η-model
- The one-loop RG flow shows

  classical integrability $\longleftrightarrow$ the quantum RG flow
- Displays a remarkably simple behaviour under PL duality

Future directions

- Understand in the PL manifest DFT framework [Haßler's talk]
- Interpretation of the dual as a λ-model