Scalar-Gauss-Bonnet Theories: Evasion of No-Hair Theorems and Novel Black-Hole Solutions

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Outline

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- The synergy between the Ricci and Gauss-Bonnet terms
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Based on 1711.03390 [hep-th] (PRL, 2018)
and 1711.07431 [hep-th] (PRD, 2018)
in collaboration with G. Antoniou (Univ. of Minnesota)
and A. Bakopoulos (Univ. of Ioannina)

Introduction

Einstein's theory of General Relativity is a very good theory of Gravity: mathematically beautiful and experimentally tested

But it is not a perfect theory (if there is such a thing...)....

- The Standard Cosmological Model has a number of open problems: the nature of dark matter and dark energy, the coincidence problem, the spacetime singularities, the right model for inflation...
- On the gravity side, GR predicts the existence of only three families of black-hole solutions (information loss problem)
- Unification with the other forces seems unlikely within the GR (GR is based on tensors and is not renormalizable)

Perhaps, all these accumulated problems point to the need for changing the theoretical framework?

Introduction

A generalised theory of gravity could have the form

$$S = \int d^4x \sqrt{-g} \left[f(R, R_{\mu\nu}, R_{\mu\nu\rho\sigma}, \Phi_i) + \mathcal{L}_X(\Phi_i) \right]$$

- as part of the string effective action at low energies
- as part of a Lovelock effective theory in four dimensions
- as part of a modified scalar-tensor (Horndeski or DHOST) theory

Are there, then, many novel black-hole solutions beyond the limits of GR? Are the old GR solutions not valid any more?

- The old 'No-Hair Theorem' (Bekenstein, 1972; Teitelboim, 1972) excluded such solutions in minimally-coupled scalar-tensor theories:

  "There are no static black-hole solutions with scalar hair"

Introduction

- This was evaded for black-hole solutions with a Yang-Mills (Volkov & Galtsov, 1989, Bizon, 1990; Greene et al, 1993; Maeda et al, 1994) or Skyrme field (Luckock & Moss, 1986; Droz et al, 1994) or for a conformally-coupled scalar field (Bekenstein, 1974)
- The "novel No-Hair Theorem" (Bekenstein, 1995) applied in non-minimally-coupled scalar fields, and was extended to general scalar-tensor theories (Sotiriou & Faraoni, 2012; Hui & Nicolis, 2013)
- These were again evaded in the case of dilatonic BHs (Kanti et al, 1996), coloured BHs (Torii et al, 1997; Kanti et al, 1997), rotating BHs (Kleihaus et al, 2011; Pani et al, 2011) and the shift-symmetric Galileon BHs (Babichev & Charmousis, 2014; Sotiriou & Zhou, 2014)

In fact, a small number of novel black-hole solutions have been found... Can we extend this 'elite' group of theories that evade the no-hair theorems and lead to novel black holes?

The Einstein-Scalar-Gauss-Bonnet Theory

We considered the following generalised theory of gravity

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{16\pi G} - \frac{1}{2}\, \partial_\mu \phi\, \partial^\mu \phi + f(\phi)\, R^2_{GB} \right],$$

with $f(\phi)$ a coupling function between a scalar field ϕ and the GB term

$$R^2_{GB} = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2$$

This theory has a number of attractive points:

- It contains a quadratic gravitational term (next important term in strong-curvature regimes) but leads to field equations with up to 2nd-order derivatives, and with no Ostrogradski instabilities
- It is a very "rich" theory: for $f(\phi) \sim \ln\left[2e^\phi \eta^4(ie^\phi)\right]$, cosmological singularity-free solutions (Antoniadis, Rizos & Tamvakis, 1994) emerge; for $f(\phi) \sim e^\phi$ we get the Dilatonic Black Holes (Kanti et al, 1996; 1998) and wormhole solutions (Kanti, Kleihaus & Kunz, 2011), whereas for $f(\phi) \sim \phi$, the shift-symmetric Galileon, static black holes arise (Sotiriou & Zhou, 2014)

Novel Einstein-Scalar-GB Black-Hole Solutions

• The Basic Question: For what forms of the coupling function $f(\phi)$ can one get a static, spherically-symmetric black-hole solution?

Keeping therefore the form of $f(\phi)$ arbitrary, we assume the line-element

$$ds^2 = -e^{A(r)}dt^2 + e^{B(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

while the equations of motion read

$$\nabla^2\phi + \dot{f}(\phi)R^2_{GB} = 0\,, \qquad R_{\mu\nu} - \frac{1}{2}\,g_{\mu\nu}\,R = T_{\mu\nu}$$

where

$$T_{\mu\nu} = -\frac{1}{4}g_{\mu\nu}(\partial\phi)^2 + \frac{1}{2}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}(g_{\rho\mu}g_{\lambda\nu} + g_{\lambda\mu}g_{\rho\nu})\eta^{\kappa\lambda\alpha\beta}\tilde{R}^{\rho\gamma}{}_{\alpha\beta}\nabla_\gamma\partial_\kappa f\,,$$

$$\Downarrow$$

$$\boxed{A'' = \frac{P}{S}\,, \qquad \phi'' = \frac{Q}{S}\,, \qquad P, Q, S = g(r, \phi, \phi', A')}$$

Novel Einstein-Scalar-GB Black-Hole Solutions

For the existence of a regular black-hole horizon we demand that

$$e^{A(r)} \to 0, \quad e^{-B(r)} \to 0, \quad \phi(r) \to \phi_h$$

Demanding that ϕ'' is also finite at the horizon r_h, we find the constraint

$$\phi'_h = \frac{r_h}{4\dot{f}_h}\left(-1 \pm \sqrt{1 - \frac{96\dot{f}_h^2}{r_h^4}}\right), \qquad \dot{f}_h^2 < \frac{r_h^4}{96}$$

Using the constraint on ϕ'_h in the equation for A'', we find

$$A'' = -A'^2 + ... \Rightarrow A' = (r - r_h)^{-1} + \mathcal{O}(1)$$

that upon integration leads to the complete solution near the horizon

$$e^A = a_1(r - r_h) + \dots, \qquad e^{-B} = b_1(r - r_h) + \dots,$$

$$\phi = \phi_h + \phi'_h(r - r_h) + \phi''_h(r - r_h)^2 + \dots$$

Novel Einstein-Scalar-GB Black-Hole Solutions

At large distances from the horizon, we assume a power series expansion in $1/r$, and by substituting in the equations of motion, we find

$$e^A = 1 - \frac{2M}{r} + \frac{MD^2}{12r^3} + \frac{24MD\dot{f} + M^2D^2}{6r^4} + \ldots$$

$$e^B = 1 + \frac{2M}{r} + \frac{16M^2 - D^2}{4r^2} + \frac{32M^3 - 5MD^2}{4r^3} + \mathcal{O}\left(\frac{1}{r^4}\right) + \ldots$$

$$\phi = \phi_\infty + \frac{D}{r} + \frac{MD}{r^2} + \frac{32M^2D - D^3}{24r^3} + \frac{12M^3D - 24M^2\dot{f} - MD^3}{6r^4} + \ldots$$

It is in order $\mathcal{O}(1/r^4)$ that the explicit form of the coupling function $f(\phi)$ first makes its appearance

Thus, a general coupling function f does not interfere with the existence of an asymptotically-flat limit for the spacetime

Novel Einstein-Scalar-GB Black-Hole Solutions

Can we smoothly connect these two asymptotic solutions? Bekenstein's *Novel No-Hair* theorem (1995) said no, because:

- "at radial infinity: T_r^r is positive and decreasing"

Indeed, even in the presence of the GB term: $T_r^r \simeq \phi'^2/4 \simeq D^2/4r^4 + ...$

- "near the BH horizon: T_r^r is negative and increasing"

If true, the smooth connection of the two demands an extremum - this is *excluded* by the positivity of energy in ordinary scalar-tensor theories

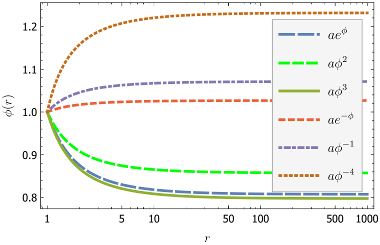
However, in the Einstein-scalar-Gauss-Bonnet theories with general f, the second clause is not true. Instead, we find that

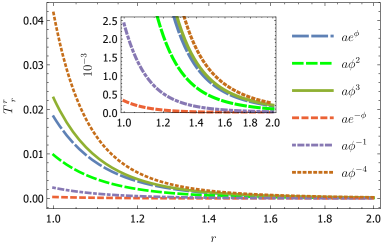
$$sign(T_r^r)_h = -sign(\dot{f}_h \phi'_h) = 1 \mp \sqrt{1 - 96\dot{f}^2/r_h^4}) > 0$$

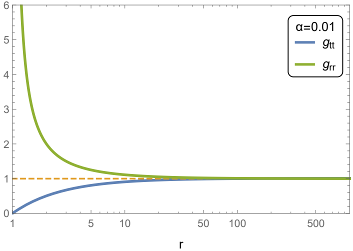
The regularity of the horizon automatically guarantees the positivity of T_r^r

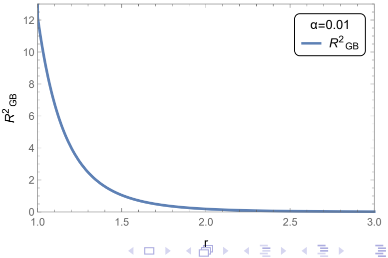
Novel Einstein-Scalar-GB Black-Hole Solutions

(Antoniou, Bakopoulos & Kanti, 1711.03390, 1711.07431)

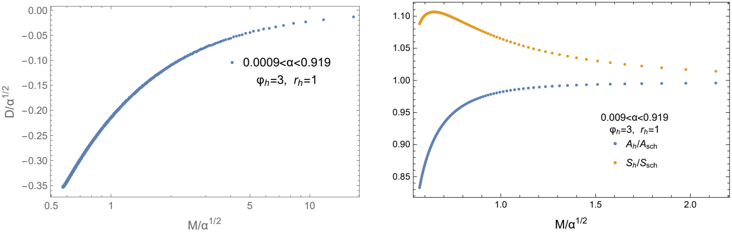








Novel Einstein-Scalar-GB Black-Hole Solutions



- The scalar charge is a "secondary" conserved quantity
- In the limit of large mass, all GB black holes reduce to the Schwarzschild solution
- The entropy of the GB black holes may at times exceed that of the Schwarzschild solution (shown that of $f(\phi) = a/\phi$)
- All GB black holes are smaller than the corresponding Schwarzschild solution and have a minimum mass

Doneva & Yazadjiev, 1711.01187 ($f = 1 - e^{-\phi^2}$), Silva et al, 1711.02080 ($f = \phi^2$)

The Einstein-Scalar-Gauss-Bonnet Theory with Λ

We extend the previous theory by adding a cosmological constant

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{16\pi G} - \frac{1}{2} \partial_\mu \phi \, \partial^\mu \phi + f(\phi) \, R^2_{GB} - \Lambda \right],$$

In this case, the field equations remain unchanged apart from the shift

$$T_{\mu\nu} \to T_{\mu\nu} - \Lambda \, g_{\mu\nu}$$

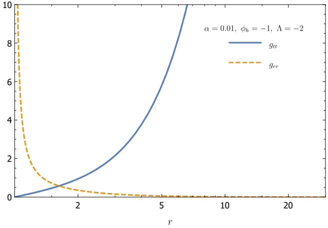
Despite the minimal change, the situation differs a lot from the previous one:

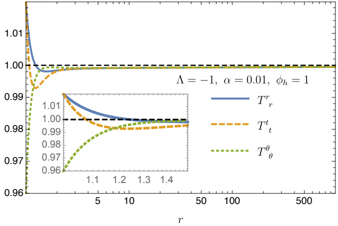
- The spacetime is not asymptotically-flat but is expected to reduce to a (Anti)-de Sitter background (for a "localised" $T^{(\phi)}_{\mu\nu}$ ...)
- At infinity, ϕ has a power-series form for small Λ, but goes like $\phi(r) \sim \ln(r)$, for large Λ
- A regular black-hole horizon emerges provided that

$$\phi'_h = \frac{16\Lambda r_h \dot{f}^2 \, (\Lambda r_h^2 - 3) + \Lambda r_h^5 - r_h^3 \mp \sqrt{R}}{4\dot{f}[r_h^2 - \Lambda(r_h^4 - \dot{f}^2)]}$$

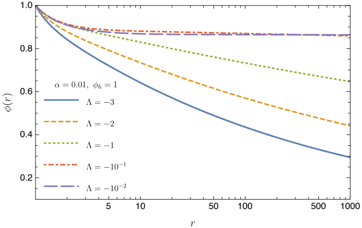
Novel Einstein-Scalar-GB Black-Hole Solutions

For $\Lambda < 0$, we find complete BH, asymptotic Anti-de Sitter solutions:





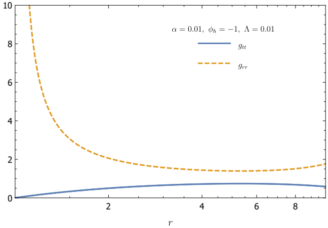
$$[\, f(\phi) = \alpha\, \phi^2 \,]$$

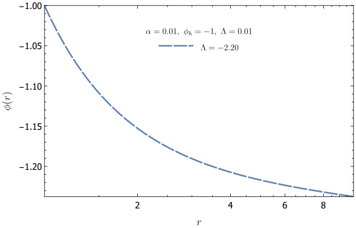


We found similar solutions for $f(\phi) = e^{\pm\phi}, \phi^{\pm 2n}, \phi^{\pm(2n+1)}, \ln\phi, \ldots$

Novel Einstein-Scalar-GB Black-Hole Solutions

For $\Lambda > 0$, though, no complete BH, asymptotic de Sitter solution was found...





It is difficult to get information from Bekenstein's no-hair theorem. Now:

$$T^r{}_r = -\frac{2e^{-B}}{r^2} A'\phi'\dot{f} - \Lambda + \mathcal{O}(r - r_h)$$

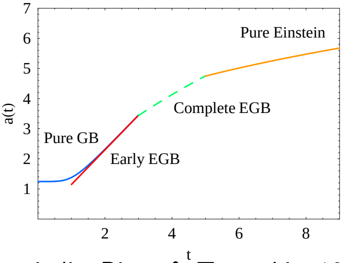
Therefore, for $\Lambda < 0$, $T^r{}_r$ gets a positive contribution and moves away for the "range of validity" of Bekenstein's no-hair theorem, while for $\Lambda > 0$, $T^r{}_r$ decreases and BH solutions quite likely are forbidden

Synergy between Ricci and GB terms

In the asymptotically-flat case, the quadratic GB term is negligible at large distances but is very important near the horizon – in fact, the smaller the BH, the larger the curvature, and the more important the GB term is

Can we claim that there is a class of BH solutions that may be attributed almost solely to the GB term?

In a cosmological set-up, we have shown that, in the early universe, as $t \to 0$, R becomes negligible, R^2_{GB} dominates, and the field eqns may then be analytically solved yielding a class of singularity-free solutions



(Kanti, Gannouji & Dadhich, 2015; Antoniadis, Rizos & Tamvakis, 1994)

Synergy between Ricci and GB terms

At first sight, it looks as the GB term may support a non-trivial scalar field, and that in turn keeps the GB term in the theory

However, we have failed to find "pure" GB black-hole solutions. If we ignore the Ricci term in the field eqns, assume again a regular scalar field and a vanishing g_{tt}, at some particular value r_h, we obtain

$$e^B \simeq 3 + \mathcal{O}(r - r_h) \,, \qquad (\times)$$

Alternatively, demanding that $e^B \to \infty$, as $r \to r_h$, we get

$$e^{-B} = 2 \ln(r/r_h) \,, \qquad \phi'' \to \infty \,, \qquad (\times)$$

Therefore, the emergence of regular BH solutions with scalar hair demands the synergy between the Ricci and GB terms

The regime of dominance of the GB term lies perhaps beyond the minimum-mass limit (no such "barrier" exists in the cosmological set-up)

Conclusions

- The Generalised Theories of Gravity may be the way forward in gravitational physics
- The Einstein-scalar-Gauss-Bonnet theory is a particular type of such a theory and has been intensively studied for decades
- Our work has shown that, under a mild constraint related to the regularity of the horizon, an asymptotically-flat BH solution with scalar hair always emerges for any Einstein-scalar-GB theory
- The emergence of solutions with an (Anti)-de Sitter asymptotic behaviour is currently under way – a positive cosmological constant apparently pushes the potential solutions back into the regime of no-hair theorem