On the O(d,d) structure of non-abelian T-duality, generalised fluxes and Yang-Baxter deformations

based on arXiv:1803.03971 with Dieter Lüst

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motivation

- for abelian T-duality
  - duality group $\mathrm{O}(d, d)$
    Buscher rules, B-shifts, β-shifts,...
  - duality covariant frameworks (DFT, gen. geometry)
  - connecting geometric and non-geometric backgrounds

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$\rightarrow$ similar things for non-abelian T-dualities (NATD)?

- deformations of integrable string σ-model
  - *homogeneous* Yang-Baxter deformationen
    $\Leftrightarrow$ non-abelian T-duality? In which sense?
  - $\eta \Leftrightarrow \lambda$?

overview

1. Poisson-Lie T-duality

2. Non-abelian T-duality group

3. Yang-Baxter deformations as β-shifts

Lie bialgebras

- $(\mathfrak{d}, \mathfrak{g}, \mathfrak{g}^\star)$ - Manin triple decomposition
  - $2d$-dimensional Lie algebra $\mathfrak{d}$ with $\mathrm{O}(d, d)$-metric
  - two complementary Lagrangian subalgebras $\mathfrak{g}$, $\mathfrak{g}^\star$

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- examples:
  - semi-abelian bialgebra: $\mathfrak{g} \oplus \mathfrak{u}(1)^d$
  - each solution to the (m)cYBe: $g^\star$ with $\overline{f}_c{}^{ab} = f^{[a}{}_{cd} r^{b]d}$

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- Manin triple $(\mathfrak{d}, \mathfrak{g}, \mathfrak{g}^\star) \leftrightarrow$ Poisson bivector on $\mathcal{G}$

$$\Pi^{ab}(g) = \bar{f}_c{}^{ab} x^c - \frac{1}{2} \bar{f}_c{}^{k[a} f^{b]}{}_{dk} x^c x^d + \dots , \quad \text{for } g = \exp(x^a t_a) \in \mathcal{G},$$

Poisson-Lie T-duality

[Klimcik, Severa '95; Sfetsos, von Unge, Hlavaty, Hull/Reid-Edwards, ...]

- 'doubled' string σ-model
  input: Lie bialgebra $\mathfrak{d}$, $\mathcal{H}(G_0, B_0) \to$ dynamics via $\mathfrak{d} = \mathfrak{d}^+ \perp \mathfrak{d}^-$

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- so far: Buscher-like duality $\mathfrak{g} \oplus \mathfrak{g}^\star \leftrightarrow \mathfrak{g}^\star \oplus \mathfrak{g}$

The non-abelian T-duality group

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  - $(\mathfrak{d}, \mathfrak{g}', \mathfrak{g}'^\star) = (\mathfrak{d}, \varphi(\mathfrak{g}), \varphi(\mathfrak{g}^\star))$,
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  - insights into group via standard $\mathrm{O}(d, d)$ subgroups

example: non-abelian T-duality B-shifts 1

- B-shift as $\mathrm{O}(d,d)$-transformation: $\varphi_B = \begin{pmatrix} \mathbb{1} & \sigma \\ 0 & \mathbb{1} \end{pmatrix}$

$$t_a \; , \; \bar{t}^a \quad \overset{\varphi \in O(d,d)}{\longrightarrow} \quad t'_a = t_a + \sigma_{ab}\bar{t}^b, \;\; \bar{t}'^a = \bar{t}^a$$

imposing closure

$$[t'_a, t'_b] = F^c{}_{ab} t'_c + H_{abc}\bar{t}'^c$$

$$\text{with} \quad F^c{}_{ab} = f^c{}_{ab} + \sigma_{k[a}\bar{f}_{b]}{}^{kc}$$

$$\text{and} \quad H_{abc} = \underbrace{\sigma_{[a|d}\sigma_{|b|e}\bar{f}_{|c]}{}^{de}}_{cYBe} - \underbrace{\sigma_{k[a}f^k{}_{bc]}}_{2-cocycle} \overset{!}{=} 0,$$

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  - $\mathfrak{g}^\star$ is abelian: $\rightarrow$ $\sigma_{ab}(g^{-1}\mathrm{d}g)^a \wedge (g^{-1}\mathrm{d}g)^b$ closed

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    gauge transformation of the **H**-flux:

$$S \propto \int \mathrm{d}^2\sigma (g^{-1}\partial_+ g)^a \left[G_0 + B_0 + \sigma\right]_{ab} (g^{-1}\partial_- g)^b,$$

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  - generic case: B-shifts connect groups $\mathcal{G}$, s.t. original Poisson bivector is still Poisson

$$S_B \propto \int \mathrm{d}^2\sigma (g'^{-1}\partial_+ g')^a \left[\frac{1}{\frac{1}{G_0+B_0+\sigma} + \Pi'(g')}\right]_{ab} (g'^{-1}\partial_- g')^b,$$

generalised fluxes of Poisson-Lie σ-model

[Shelton et. al 05; Grana et al. 09; Blumenhagen et al. 12]

- open string variables

$$g + \beta = \frac{1}{G_0 + B_0} + \Pi(g) = g + \underbrace{\beta_0 + \Pi(g)}_{\beta}$$

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- *claim:* generalised fluxes from β

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- *side note:* this would imply that standard NA T-duals should be viewed as $\mathbf{Q}$-flux backgrounds.

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- Yang-Baxter (or η) deformations of PCM [Klimcik 09]
  generalisations to coset and GS σ-models exist [Arutyunov, Borsato, Delduc, Hoare, Magro, Tseytlin, van Tongeren, Vicedo, Wulff, ...]

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  - a possible generalised flux interpretation of the Poisson-Lie type σ-models
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Thank you for your attention!

backup

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  - B-shifts - shifts of B by a constant skewsymmetric σ
  - β-shifts - shifts of β by a constant skewsymmetric r
- different parameterisation of $\mathcal{H}$: $g + \beta = \frac{1}{G+B}$
  - g^{-1}: open string metric
  - β^{MN} open string non-commutativity parameters, conjugate (dual) B-field

non-abelian T-duality β-shifts 1

- β-shift as $\mathrm{O}(d,d)$-transformation: $\varphi_\beta = \begin{pmatrix} \mathbb{1} & 0 \\ r & \mathbb{1} \end{pmatrix}$
  - imposing closure condition

$$[\bar{t}'^a, \bar{t}'^b] = \overline{F}_c{}^{ab} \bar{t}'^c + R^{abc} t'_c$$

$$\text{with} \quad \overline{F}_c{}^{ab} = \overline{f}_c{}^{ab} + r^{k(a} f^{b)}{}_{kc}$$

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  - generic case:

    β-shifts connect different Poisson structures to the group $\mathcal{G}$

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definitions and Bianchi identities

- definitions in a non-holonomic basis $t_a = e_a{}^i \partial_i \equiv$ "∂_a"

$$
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\mathbf{H}_{abc} &= \partial_{(a} B_{bc)} + f^d{}_{(ab} B_{c)d} \\
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- fluxes not independent - Bianchi identities

$$
\begin{aligned}
&0 = \mathbf{H}_{k(ab} \mathbf{f}^k{}_{cd)}, \qquad 0 = \mathbf{f}^a{}_{k(b} \mathbf{f}^k{}_{cd)} + \mathbf{H}_{k(bc} \mathbf{Q}_{d)}{}^{ak} \\
&0 = \mathbf{R}^{kab} \mathbf{H}_{kcd} + \mathbf{Q}_k{}^{ab} \mathbf{f}^c{}_{cd} - \mathbf{f}^{(a}{}_{k(c} \mathbf{Q}_{d)}{}^{b)k} \\
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- in general: B-shifts gauge transformations of **H**-flux, β-shifts for **R**-flux

bi-Yang-Baxter deformations

- more general: integrable 2-parameter deformation [Klimcik 14]

$$S = \frac{1}{2} \int \mathrm{d}^2\sigma (g^{-1}\partial_+ g)^a \kappa_{ac} \left(\frac{1}{1 - \xi R - \eta R_g} \right)^c{}_b (g^{-1}\partial_- g)^b .$$

- non-geometric fluxes:

$$\begin{aligned} \beta^{ab} &= (\xi + \eta) r^{ab} - \eta \Pi^{ab}(g), \\ \mathbf{Q}_c{}^{ab} &= -\xi f^{(a}{}_{cd} r^{b)d} \\ \mathbf{R}^{abc} &= -(\xi + \eta)^2 r^{(a|m} r^{|b|n} f^{|c)}{}_{mn} . \end{aligned}$$

bi-Yang-Baxter deformations

- more general: integrable 2-parameter deformation [Klimcik 14]

$$S = \frac{1}{2}\int \mathrm{d}^2\sigma (g^{-1}\partial_+ g)^a \kappa_{ac} \left(\frac{1}{1-\xi R - \eta R_g}\right)^c{}_b (g^{-1}\partial_- g)^b.$$

- non-geometric fluxes:

$$\begin{aligned} \beta^{ab} &= (\xi+\eta) r^{ab} - \eta \Pi^{ab}(g), \\ \mathbf{Q}_c{}^{ab} &= -\xi f^{(a}{}_{cd} r^{b)d} \\ \mathbf{R}^{abc} &= -(\xi+\eta)^2 r^{(a|m} r^{|b|n} f^{|c)}{}_{mn}. \end{aligned}$$

- for $\xi = -\eta$: **R**-flux free model, not related to PCM by β-shift