Dynamics of branes in DFT

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based on works with
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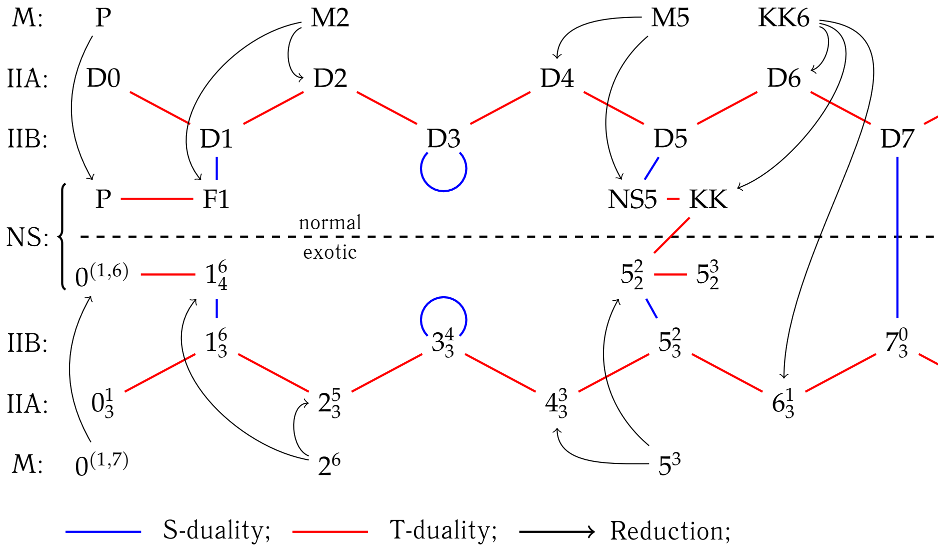
Dualities
Corfu, 2018

Web of (some) branes



Summary

T-duality orbits

$$\mathrm{NS5}(5_2^0) - \mathrm{KK5}(5_2^1) - \mathrm{Q}(5_2^2) - \mathrm{R}(5_2^3) - \mathrm{R}'(5_2^4)$$

$$\mathrm{D0} - \mathrm{D1} - \mathrm{D2} - \mathrm{D3} - \mathrm{D4} - \mathrm{D5} - \mathrm{D6} - \mathrm{D7} - \mathrm{D8} - \mathrm{D9}$$

- Effective actions, which depending on orientation project down to actions for normal branes.
- Probably some non-geometric effects for D-branes
- Wess-Zumino terms for $\alpha = -3, 4$ branes (linear)

T-duality orbit of NS branes

	0	1	2	3	4	5	6	7	8	9
NS5 :	×	×	×	×	×	×	•	•	•	•
KK5 :	×	×	×	×	×	×	•	•	•	⊙
Q :	×	×	×	×	×	×	•	•	⊙	⊙
R :	×	×	×	×	×	×	•	⊙	⊙	⊙
R′ :	×	×	×	×	×	×	⊙	⊙	⊙	⊙

NS5: world-volume (1–5), transverse (6–9)

KK5: world-volume (1–5), transverse (6–8), special (9)

T_9: NS5 ↔ KK5

T_8: KK5 ↔ Q

(1)

[deBoer, Shigemori]

T-duality orbit of D-branes

$$
\begin{array}{rcccccccccc}
 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
\mathrm{D0}: & \times & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
\mathrm{D1}: & \times & \times & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
\mathrm{D2}: & \times & \times & \times & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
\mathrm{D3}: & \times & \times & \times & \times & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
\mathrm{D4}: & \times & \times & \times & \times & \times & \bullet & \bullet & \bullet & \bullet & \bullet \\
\mathrm{D5}: & \times & \times & \times & \times & \times & \times & \bullet & \bullet & \bullet & \bullet \\
\mathrm{D6}: & \times & \times & \times & \times & \times & \times & \times & \bullet & \bullet & \bullet \\
\mathrm{D7}: & \times & \times & \times & \times & \times & \times & \times & \times & \bullet & \bullet \\
\mathrm{D8}: & \times & \times & \times & \times & \times & \times & \times & \times & \times & \bullet \\
\mathrm{D9}: & \times & \times & \times & \times & \times & \times & \times & \times & \times & \times
\end{array}
\tag{2}
$$

! T-duality changes dimension of a D-brane

Potentials

- Dp-brane: (p+1)-form $C_{(p+1)}$

Potentials

- D*p*-brane: $(p+1)$-form $C_{(p+1)}$
- NS5-brane — magnetic dual of the string, $B_{\mu_1\dots\mu_6}$

$$
\begin{array}{ccccc}
B_{\mu\nu} & \to & H_{\mu\nu\rho} = 3\partial_{[\mu}B_{\nu\rho]} & \to & *_{10}H_{(3)} \\
 & & & & || \\
B_{\mu_1\dots\mu_6} & \to & H_{\mu_1\dots\mu_7} = 7\partial_{[\mu_1}B_{\mu_2\dots\mu_7]} & \leftarrow & H_{(7)}
\end{array}
\tag{3}
$$

- KK5-monopole — magnetic dual of the graviton, $A_{\mu_1\dots\mu_7}{}^{\nu}$

$$
\begin{array}{ccccc}
A_m{}^z = e_m{}^z & \to & f_{mn}{}^z = 2e_{\bar z}{}^z\partial_{[m}e_{n]}{}^{\bar z} & \to & *_{10}f_{(2)}{}^z \\
 & & & & || \\
A_{m_1\dots m_6 z}{}^z & \to & f_{m_1\dots m_7 z}{}^z = 8\partial_{[m_1}A_{m_2\dots m_7 z]}{}^z & \leftarrow & f_{(8)}{}^z
\end{array}
\tag{4}
$$

Exotic potentials

- 5^2_2-brane, $Q_a{}^{bc}$-flux, interacts with $B_{\mu_1\dots\mu_8}{}^{\nu_1\nu_2}$

$$\begin{array}{ccccc} \beta^{34} & \to & Q_a{}^{34} = \partial_a \beta^{34} & \to & *_{10}\, Q_{(1)}{}^{34} \\ & & & & || \\ B_{\alpha_1\dots\alpha_6 34}{}^{34} & \leftarrow & Q_{\alpha_1\dots\alpha_7 34}{}^{34} = 9\partial_{[\alpha_1} B_{\alpha_2\dots\alpha_7 34]}{}^{34} & \leftarrow & Q_{(9)}{}^{34} \end{array} \tag{5}$$

- 5^3_2-brane, R^{abc}-flux, interacts with $B_{\mu_1\dots\mu_9}{}^{\nu_1\nu_2\nu_3}$

$$\begin{array}{ccccc} \beta^{34} & \to & R^{234} = \tilde{\partial}^2 \beta^{34} & \to & *_{10}\, R_{(0)}{}^{234} \\ & & & & || \\ B_{\alpha_1\dots\alpha_6 234}{}^{234} & \leftarrow & R_{\alpha_1\dots\alpha_7 234}{}^{234} = 10\partial_{[\alpha_1} B_{\alpha_2\dots\alpha_7 234]}{}^{234} & \leftarrow & R_{(10)}{}^{234} \end{array} \tag{6}$$

In general 5^p_2-brane interacts with $B_{(6+p,p)}$-potential

Bergshoeff, Kleinschmidt, Ricconi

The problem

- Construct a T-covariant action reproducing the background of the DFT monopole when coupled to DFT
- Write an invariant Wess-Zumino term

DFT monopole

- A solution of DFT defined by the harmonic function

$$H(z, y^1, y^2, y^3) = 1 + \frac{h}{z^2 + \delta_{ij} y^i y^j} \tag{7}$$

- and identification between the doubled coordinates $\mathbb{X}^M = (x^z, x^i, x^a, \tilde{x}_z, \tilde{x}_i, \tilde{x}_a)$ and parameters $(z, y^i, \tilde{z}, \tilde{y}_i)$

$$\begin{aligned}
(x^z, x^1, x^2, x^3) &= (z, y^1, y^2, y^3), && \text{NS5-brane, } 5_2^0\\
(x^z, x^1, x^2, x^3) &= (\tilde{z}, y^1, y^2, y^3), && \text{KK5-brane, } 5_2^1\\
(x^z, x^1, x^2, x^3) &= (\tilde{z}, \tilde{y}_1, y^2, y^3), && \text{Q-brane, } 5_2^2\\
(x^z, x^1, x^2, x^3) &= (\tilde{z}, \tilde{y}_1, \tilde{y}_2, y^3), && \text{R-brane, } 5_2^3\\
(x^z, x^1, x^2, x^3) &= (\tilde{z}, \tilde{y}_1, \tilde{y}_2, \tilde{y}_3), && \text{R'-brane, } 5_2^4
\end{aligned} \tag{8}$$

- Non-geometric backgrounds depend on dual coordinates. The same for xFT (recall Ray's talk)

[Berman, Rudolph, Bakhmatov, Kleinschmidt, EtM, Otsuki]

Embedding of the brane

$$\begin{array}{cccccc|cccccccc} x^0 & x^1 & x^2 & x^3 & x^4 & x^5 & y^1 & y^2 & y^3 & y^4 & \tilde{y}_1 & \tilde{y}_2 & \tilde{y}_3 & \tilde{y}_4 \\ \hline \times & \times & \times & \times & \times & \times & & & & & & & & \end{array} \tag{9}$$

$\underbrace{\qquad\qquad}_{\text{world-volume}}$ $\underbrace{\qquad\qquad}_{\text{transverse directions}}$

Coordinates of the space-time = scalar fields on the brane

$\Downarrow$

no issues with doubled geometry, section condition etc.

- doubled number of scalar fields on the brane
- projection condition which leaves only half of them (to keep SUSY)

The action

$$S_{DBI} = \int d^6\xi e^{-2d} \sqrt{|h_{ab}|} \sqrt{1 + e^{2d}|h_{ab}|^{-1/2}(\bar{\lambda}_{brane}\mathcal{C})^2} \times$$
$$\times \sqrt{-\left| g_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu + \mathcal{H}_{MN}\hat{D}_\alpha Y^M \hat{D}_\beta Y^N - \frac{e^d(\det h)^{-1/4}\bar{\lambda}_{brane}\mathcal{G}_{\alpha\beta}}{\sqrt{1+e^{2d}(\det h)^{-1/2}(\bar{\lambda}_{brane}\mathcal{C})^2}} \right|}, \tag{10}$$

where

$$\mathcal{G}_{\alpha\beta} = 2\partial_{[\alpha}\tilde{c}_{\beta]} + \widetilde{\mathcal{C}}_{\alpha\beta} \tag{11}$$

is a worldvolume field strength with the following pullback of RR fields:

$$\widetilde{\mathcal{C}}_{\alpha\beta} = \left(\mathcal{C}_{\mu\nu} - (B_{\mu\nu} + \frac{1}{2}A_\mu{}^M A_\nu{}^N \Gamma_{MN})\mathcal{C} + \sqrt{2}A_\mu{}^M \Gamma_M \mathcal{C}_\nu\right)\partial_{[\alpha}X^\mu \partial_{\beta]}X^\nu$$
$$+ \sqrt{2}\Gamma_M \left(\mathcal{C}_\mu - \frac{1}{\sqrt{2}}A_\mu{}^N \Gamma_N \mathcal{C}\right)\partial_{[\alpha}X^\mu \hat{D}_{\beta]}Y^M - \frac{1}{2}\Gamma_{MN}\mathcal{C}\hat{D}_{[\alpha}Y^M \hat{D}_{\beta]}Y^N. \tag{12}$$

The action

Keeping only NS-NS fields $\mathbb{X}^M = (x^m, \tilde{x}_m)$:

$$S^{(NS)}_{DBI} = \int d^6\xi e^{-2d}\sqrt{\det(h_{ab})} \times \sqrt{-\det\left(g_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu + \Pi_{MN}\, D_\alpha \mathbb{X}^M D_\beta \mathbb{X}^N\right)}, \tag{13}$$

Kaluza-Klein-like projections

$$\begin{aligned} D_\alpha \mathbb{X}^M &= \partial_\alpha \mathbb{X}^M + \partial_\alpha X^\mu A_\mu{}^M \\ \Pi_{MN} &= \mathcal{H}_{MN} - h^{ab}\mathcal{H}_{MP}\mathcal{H}_{NQ}k_a^P k_b^Q, \\ h_{ab} &= \mathcal{H}_{MN}k_a^M k_b^M. \end{aligned} \tag{14}$$

The section constraint (orientation):

$$\eta_{MN}k_a^M k_b^N = 0, \tag{15}$$

Allowed number of non-zero Killing vectors is half of the total.

Invariant Wess-Zumino term

- The exotic potentials $B_{(6+p,p)}$ are all combined into

$$D_{\mu_1\dots\mu_6,MNKL}, \quad \mathbb{X}^M = (x^a, \tilde{x}_a), \quad a \in 1,2,3,4 \tag{16}$$

- The invariant charge

$$T^{MNKL} = k_a{}^M k_b{}^N k_c{}^K k_d{}^L \varepsilon^{abcd} \tag{17}$$

- The invariant Wess-Zumino term

$$S_{WZ} = \int d^6\xi\, T^{MNKL} D_{(6)MNKL} \tag{18}$$

T-duality frames

$$\eta_{MN} k_a^M k_b^N = 0, \quad \& \quad T^{MNKL} \neq 0 \tag{19}$$

Killing vectors $k_a^M = (k_a^m ; \tilde{k}_{am})$ satisfying the constraint define the embedding

	world-volume						transverse directions							
	x^0	x^1	x^2	x^3	x^4	x^5	y^1	y^2	y^3	y^4	$\tilde{y}_1$	$\tilde{y}_2$	$\tilde{y}_3$	$\tilde{y}_4$
NS5	×	×	×	×	×	×	•	•	•	•	k	k	k	k
KK5	×	×	×	×	×	×	•	•	•	k	k	k	k	•
Q	×	×	×	×	×	×	•	•	k	k	k	k	•	•
R	×	×	×	×	×	×	•	k	k	k	k	•	•	•
R′	×	×	×	×	×	×	k	k	k	k	•	•	•	•

(20)

• — localization direction
k — Killing direction

Invariant Wess-Zumino term

All couplings to the exotic potential appear from the single term

$$\begin{aligned}
D_{\mu_1\dots\mu_6}{}^{1234} &= B_{\mu_1\dots\mu_6}, \\
D_{\mu_1\dots\mu_6,1}{}^{234} &= A_{\ \mu_1\dots\mu_6 1}{}^{1}, \\
D_{\mu_1\dots\mu_6,12}{}^{34} &= B_{\mu_1\dots\mu_6 12}{}^{12}, \\
D_{\mu_1\dots\mu_6,123}{}^{4} &= B_{\mu_1\dots\mu_6 123}{}^{123}, \\
D_{\mu_1\dots\mu_6,1234} &= B_{\mu_1\dots\mu_6 1234}{}^{1234},
\end{aligned} \tag{21}$$

To gauge covariantize the world-volume WZ expression one adds w-v gauge fields Bergshoeff, Riccioni

D-brane covariant potentials

- D-brane potentials $C_{(p+1)}$ can be combined:

$$|\chi\rangle = \sum_{p=0}^{10} C_{m_1\dots m_p}\,\Gamma^{m_1\cdots m_p}|0\rangle \tag{22}$$

with $\{\Gamma_M,\Gamma_N\} = 2\eta_{MN}$; vacuum: $\Gamma_m|0\rangle = 0$

- Invariant interaction

$$S_{wz} = \int d^{10}\xi\ \langle Q|\Gamma_{M_1\dots M_{10}}|\chi\rangle dX^{M_1}\wedge\cdots\wedge dX^{M_{10}}\,. \tag{23}$$

- The vectors $k_\alpha^M = (k_\alpha^m, \tilde{k}_{\alpha m})$ and the charge must satisfy

$$\begin{aligned}\eta_{MN}k_\alpha{}^M k_\beta{}^N &= 0,\\ \Gamma_M k_\alpha^M|Q_{p+1}\rangle &= 0.\end{aligned} \tag{24}$$

Dynamics

DBI action written in the same variables

$$S_D = \int d^{10}\xi e^{-d} \det |h_{\alpha\beta}|^{\frac{1}{4}} \sqrt{-\det \left(\mathcal{H}_{MN} \hat{\partial}_a X^M \hat{\partial}_b X^N + \dots\right)}, \qquad (25)$$

where one needs the projected derivatives

$$\begin{aligned} \hat{\partial}_a X^M &= \partial_a X^M - (h^{-1})^{\alpha\beta} k_\alpha{}^M k_\beta{}^N \mathcal{H}_{NK} \partial_a X^K, \\ h_{\alpha\beta} &= k_\alpha{}^M k_\beta{}^N \mathcal{H}_{MN} \qquad (26) \\ \eta_{MN} k_a^M k_b^N &= 0. \end{aligned}$$

Other approaches: Asakawa, Sasa, Watamura, Albertsson, Dai, Kao, Lin

Embedding of branes

	0	1	2	3	4	5	6	7	8	9	$\tilde{1}$	$\tilde{2}$	$\tilde{3}$	$\tilde{4}$	$\tilde{5}$	$\tilde{6}$	$\tilde{7}$	$\tilde{8}$	$\tilde{9}$
D0	k	•	•	•	•	•	•	•	•	•	k	k	k	k	k	k	k	k	k
D1	k	k	•	•	•	•	•	•	•	•	•	k	k	k	k	k	k	k	k
D2	k	k	k	•	•	•	•	•	•	•	•	•	k	k	k	k	k	k	k
D3	k	k	k	k	•	•	•	•	•	•	•	•	•	k	k	k	k	k	k
D4	k	k	k	k	k	•	•	•	•	•	•	•	•	•	k	k	k	k	k
D5	k	k	k	k	k	k	•	•	•	•	•	•	•	•	•	k	k	k	k
D6	k	k	k	k	k	k	k	•	•	•	•	•	•	•	•	•	k	k	k
D7	k	k	k	k	k	k	k	k	•	•	•	•	•	•	•	•	•	k	k
D8	k	k	k	k	k	k	k	k	k	•	•	•	•	•	•	•	•	•	k
D9	k	k	k	k	k	k	k	k	k	k	•	•	•	•	•	•	•	•	•

! Depending on the choice of **k**'s one gets different D-branes

!? D-branes can localize in dual space

Higher exotic branes: $T \sim {g_s}^{-3}$

D7

$$
\begin{array}{ccccccccccccccc}
0_3^{(0,7)} & — & 1_3^{(0,6)} & — & 2_3^{(0,5)} & — & 3_3^{(0,4)} & — & 4_3^{(0,3)} & — & 5_3^{(0,2)} & — & 6_3^{(0,1)} & — & 7_3^{(0,0)} \\
| & & | & & | & & | & & | & & | & & | & & | \\
0_3^{(1,7)} & — & 1_3^{(1,6)} & — & 2_3^{(1,5)} & — & 3_3^{(1,4)} & — & 4_3^{(1,3)} & — & 5_3^{(1,2)} & — & 6_3^{(1,1)} & — & 7_3^{(1,0)} \\
| & & | & & | & & | & & | & & | & & | & & | \\
0_3^{(2,7)} & — & 1_3^{(2,6)} & — & 2_3^{(2,5)} & — & 3_3^{(2,4)} & — & 4_3^{(2,3)} & — & 5_3^{(2,2)} & — & 6_3^{(2,1)} & — & 7_3^{(2,0)}
\end{array}
$$

These interact with exotic potentials of the type

$$
\begin{aligned}
E_{(p,p+q,p+q+r)} \quad &\Longleftrightarrow \quad E_{\underbrace{m_1\dots m_p}, \underbrace{n_1\dots n_q}, \underbrace{k_1\dots k_r}} \\
\text{e.g. } 4_3^{(1,3)} \quad &\Longleftrightarrow \quad E_{m_1\dots m_5 uxyz, uxyz, u}
\end{aligned} \tag{27}
$$

Higher exotic branes: $T \sim g_s{}^{-3}$

D7

$$0_3^{(0,7)} - 1_3^{(0,6)} - 2_3^{(0,5)} - 3_3^{(0,4)} - 4_3^{(0,3)} - 5_3^{(0,2)} - 6_3^{(0,1)} - 7_3^{(0,0)}$$

$$0_3^{(1,7)} - 1_3^{(1,6)} - 2_3^{(1,5)} - 3_3^{(1,4)} - 4_3^{(1,3)} - 5_3^{(1,2)} - 6_3^{(1,1)} - 7_3^{(1,0)}$$

$$0_3^{(2,7)} - 1_3^{(2,6)} - 2_3^{(2,5)} - 3_3^{(2,4)} - 4_3^{(2,3)} - 5_3^{(2,2)} - 6_3^{(2,1)} - 7_3^{(2,0)}$$

T-duality covariant potential is $|E^{MN}\rangle$

$$|E^{m_1 m_2}\rangle = \varepsilon^{m_1 \dots m_{10}} \sum_p E_{m_3 \dots m_{10}, n_1 \dots n_p} \Gamma^{n_1 \dots n_p} |0\rangle$$

$$|E^{m_1}{}_q\rangle = \varepsilon^{m_1 \dots m_{10}} \sum_p E_{m_2 \dots m_{10}, n_1 \dots n_p, q} \Gamma^{n_1 \dots n_p} |0\rangle$$

$$|E_{q_1 q_2}\rangle = \varepsilon^{m_1 \dots m_{10}} \sum_p E_{m_1 \dots m_{10}, n_1 \dots n_p, q_1 q_2} \Gamma^{n_1 \dots n_p} |0\rangle \quad .$$

Higher exotic branes: $T \sim {g_s}^{-3}$

D7

$$
\begin{array}{ccccccccccccccc}
0_3^{(0,7)} & — & 1_3^{(0,6)} & — & 2_3^{(0,5)} & — & 3_3^{(0,4)} & — & 4_3^{(0,3)} & — & 5_3^{(0,2)} & — & 6_3^{(0,1)} & — & 7_3^{(0,0)} \\
| & & | & & | & & | & & | & & | & & | & & | \\
0_3^{(1,7)} & — & 1_3^{(1,6)} & — & 2_3^{(1,5)} & — & 3_3^{(1,4)} & — & 4_3^{(1,3)} & — & 5_3^{(1,2)} & — & 6_3^{(1,1)} & — & 7_3^{(1,0)} \\
| & & | & & | & & | & & | & & | & & | & & | \\
0_3^{(2,7)} & — & 1_3^{(2,6)} & — & 2_3^{(2,5)} & — & 3_3^{(2,4)} & — & 4_3^{(2,3)} & — & 5_3^{(2,2)} & — & 6_3^{(2,1)} & — & 7_3^{(2,0)}
\end{array}
$$

The covariant Wess-Zumino term

$$\int d^8\xi \, \langle Q_{M_1 \dots M_{10}} | E^{M_1 M_2} \rangle \hat{d}X^{M_3} \wedge \cdots \wedge \hat{d}X^{M_{10}} \, . \tag{27}$$

Higher exotic branes: $T \sim {g_s}^{-3}$

$$
\begin{array}{ccccccccccccccc}
 & & & & & & & & & & & & & & \mathrm{D7} \\
 & & & & & & & & & & & & & & | \\
0_3^{(0,7)} & — & 1_3^{(0,6)} & — & 2_3^{(0,5)} & — & 3_3^{(0,4)} & — & 4_3^{(0,3)} & — & 5_3^{(0,2)} & — & 6_3^{(0,1)} & — & 7_3^{(0,0)} \\
| & & | & & | & & | & & | & & | & & | & & | \\
0_3^{(1,7)} & — & 1_3^{(1,6)} & — & 2_3^{(1,5)} & — & 3_3^{(1,4)} & — & 4_3^{(1,3)} & — & 5_3^{(1,2)} & — & 6_3^{(1,1)} & — & 7_3^{(1,0)} \\
| & & | & & | & & | & & | & & | & & | & & | \\
0_3^{(2,7)} & — & 1_3^{(2,6)} & — & 2_3^{(2,5)} & — & 3_3^{(2,4)} & — & 4_3^{(2,3)} & — & 5_3^{(2,2)} & — & 6_3^{(2,1)} & — & 7_3^{(2,0)}
\end{array}
$$

More interesting is the structure

$$
\begin{aligned}
|E^{m_1 m_2}\rangle &= \varepsilon^{m_1 \dots m_{10}} \sum_p E_{m_3 \dots m_{10}, n_1 \dots n_p} \Gamma^{n_1 \dots n_p} |0\rangle \\
|E^{m_1}{}_q\rangle &= \varepsilon^{m_1 \dots m_{10}} \sum_p E_{m_2 \dots m_{10}, n_1 \dots n_p, q} \Gamma^{n_1 \dots n_p} |0\rangle \\
|E_{q_1 q_2}\rangle &= \varepsilon^{m_1 \dots m_{10}} \sum_p E_{m_1 \dots m_{10}, n_1 \dots n_p, q_1 q_2} \Gamma^{n_1 \dots n_p} |0\rangle \quad .
\end{aligned}
$$

The message

- One is able to construct a single action for several branes, related by T-duality
- For D-branes this suggests localization in dual space
- Structure of T-dualities between branes repeats when going down in powers of g_s.

Discussion

- Prove microscopically, that D-branes (do not) localize in the dual space, calculate instanton corrections
- Field theories on worldvolume (especially for D-branes)
- Generalize stuff for exceptional field theories and U-dualities

What's the use of all that?

- Tadpole cancellation conditions for flux compactifications (Bianchi identities), support for internal space
- String behavior on such backgrounds: non-commutativity and non-associativity
- Little string theories from NS five-branes
- Speculations around AdS/CFT correspondence

Thank you for your attention!