Localization of the standard model via Higgs mechanism and a finite electroweak monopole from non-compact extra dimensions

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Workshop on the standard model and beyond

based on the paper arXiv:1802.06649 (PTEP '18)

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Why does the SM need Higgs?

Because it is the chiral gauge theory.

It cannot have mass terms of $\left\{\begin{array}{l}\text{fermions}\\ \text{gauge bosons}\end{array}\right.$.

But, $\left\{\begin{array}{c}\text{quarks and leptons}\\ \text{weak bosons}\end{array}\right\}$ are massive in reality.

▼

The Higgs field, without violating gauge principle, provides a simple mechanism for

① SSB of EW gauge symmetry (weak boson mass)

② generating fermion masses

Are there only two roles of the Higgs field?

For the SM in 4 dim., answer is YES.

However, ...

What I would like to tell you today is

the third important role of the Higgs field

would exist, when we consider the SM in a brane world scenario.

③ The Higgs field localizes the SM gauge fields on a brane via the Higgs mechanism.

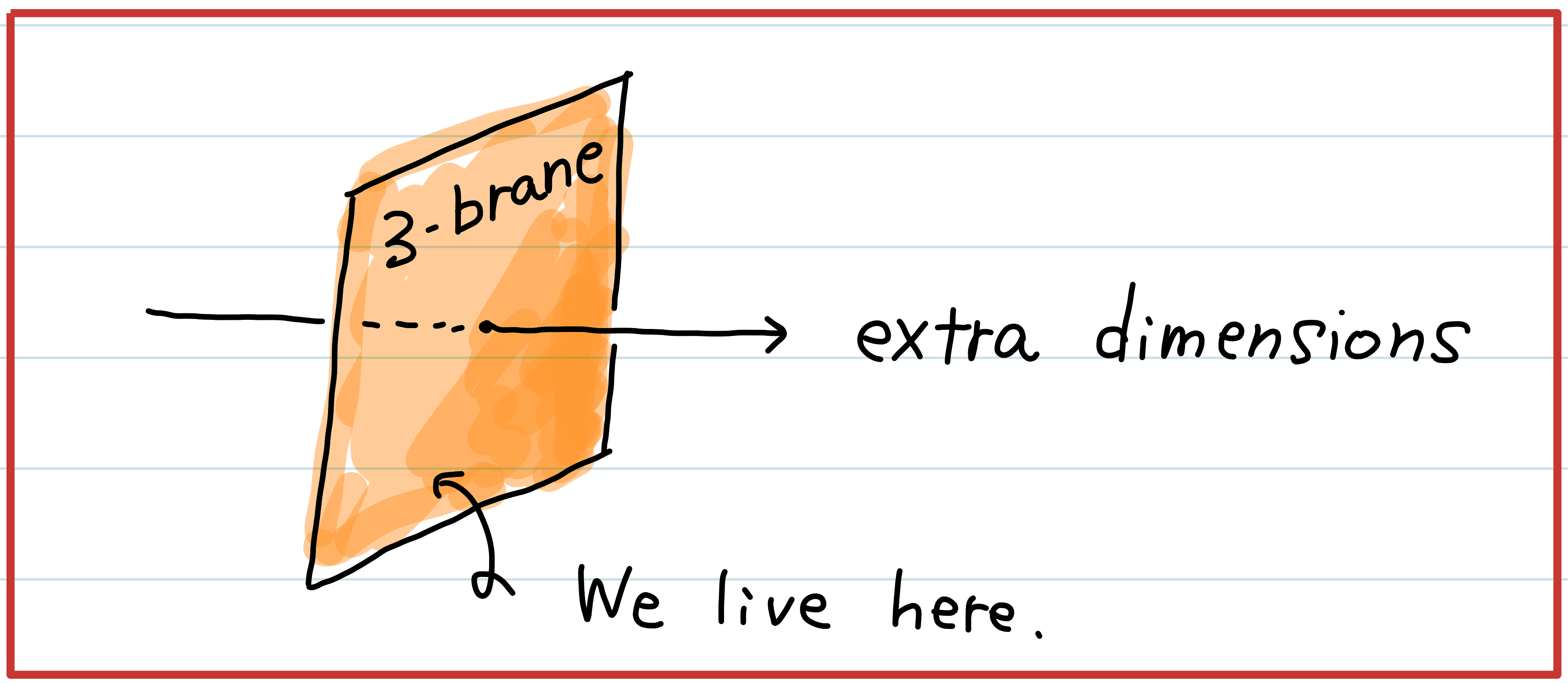
1. Introduction

* Beyond the SM $\left\{ \begin{array}{l} \text{string theory} \\ \text{supersymmetry} \\ \text{extra dimensions} \\ \vdots \end{array} \right.$

aiming a more satisfactory theory than the SM.

* Brane world scenario is one of the BSMs.
(ADD, RS ...)

Typical image of brane world



Thousands of brane world models have been studied, and each has some benefits.

Popular set up common to many models

Extra dimensions are commpact manifold / orbifold.

4 dim. and extra dim. are distinguished from the beginning. It sounds artifitial.

A 3-brane exists and all the SM particles are localized. These are set up of models:

$$\mathcal{L} = \mathcal{L}_{bulk} + \mathcal{L}_{brane}\, \delta(y)$$

How is the brane born? Is it stable?
Why are the SM particles localized?

A natural question:

Is it possible to constract a brane-world model which does not need those assumptions?

Yes! It can be realized by a topological soliton!

To be concrete, today I will consider 5 dimensions:

$$\mathbb{R}^{4,1}$$

No distinctions between all the spacial coordinates.

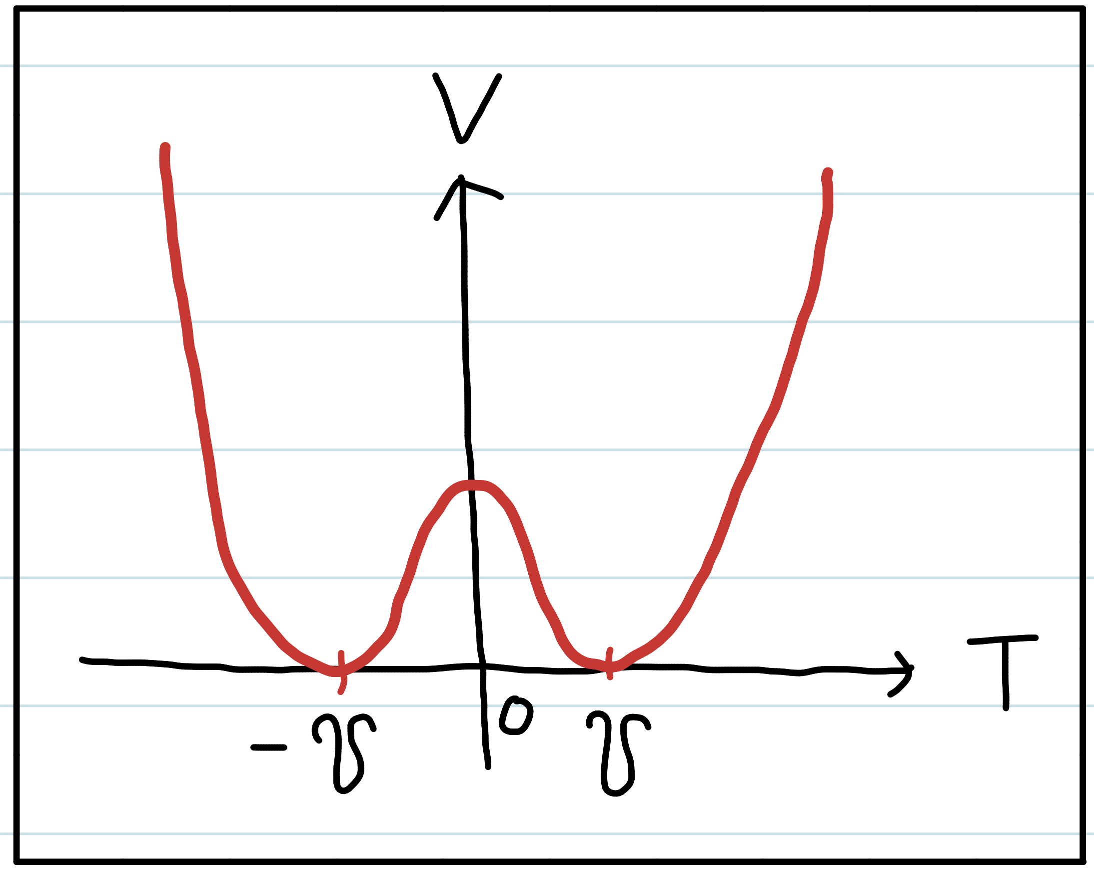
1 extra dimension will be dynamically separated from 4D as a consequence of an SSB.

2. Domain wall as 3 brane (Review)

$\boxed{\text{model}}$ T : a real scalar field

$$V = \frac{\lambda^2}{4}\left(T^2 - v^2\right)^2$$

$$\mathbb{Z}_2 : T \to -T$$



$\boxed{\text{vacuum}}$ 2 discrete vacua $T = \pm v$

$\downarrow$

SSB of $\mathbb{Z}_2$ symmetry

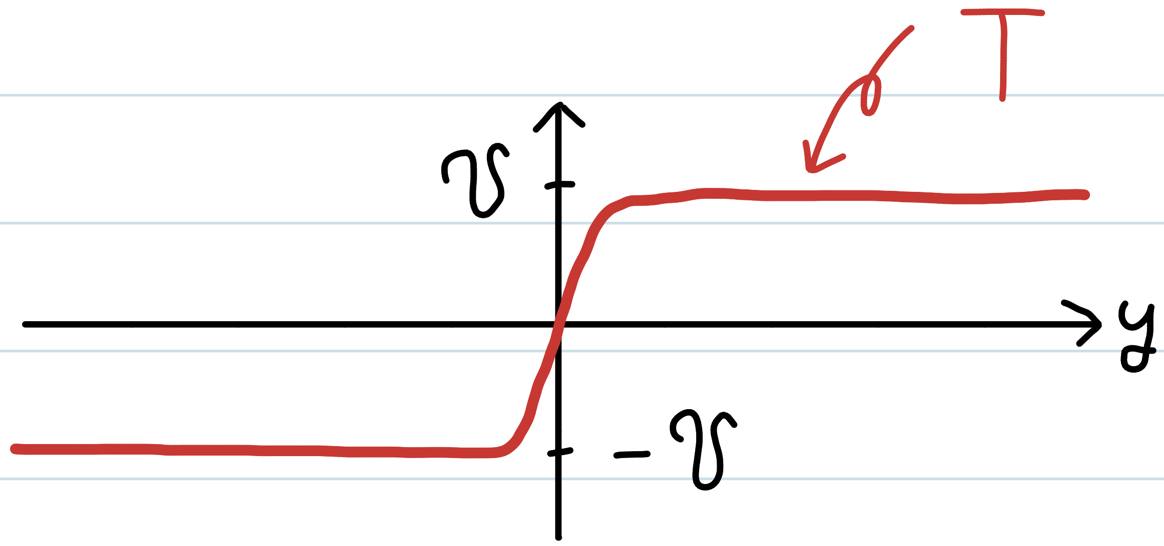
topology of vacua

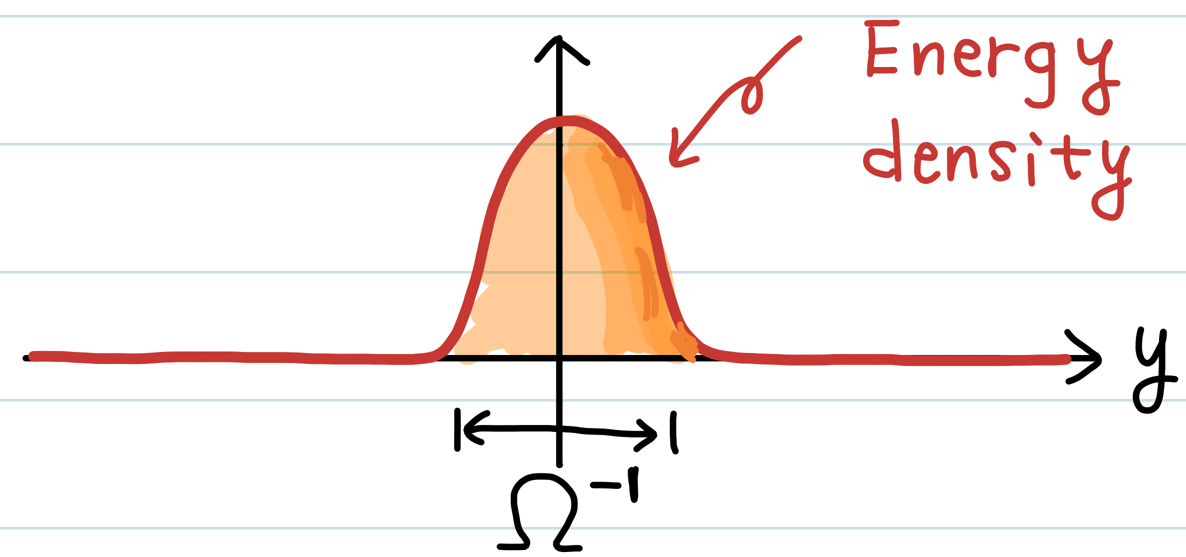
$$\pi_0(\mathcal{M}_{vac}) = \text{disconnected 2 points}$$

Topologically stable domain wall

domain wall solution

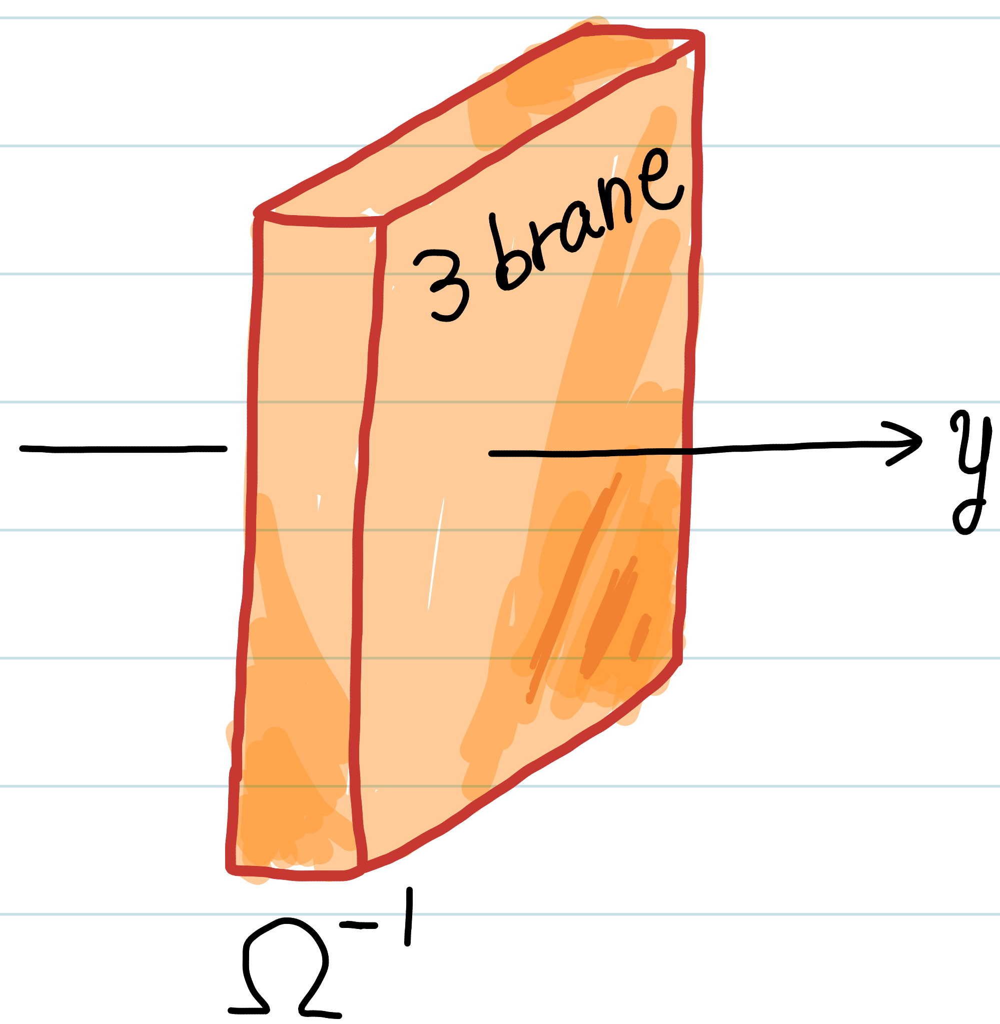
$$T = v \tanh \Omega y , \quad \Omega \equiv \frac{\sqrt{\lambda v}}{2}$$





Dynamical compactification

As a consequence of $\mathbb{Z}_2$ SSB, topologically stable domain wall is dynamically generated in $\mathbb{R}^{4,1}$!! (Dvali - Shifman NPB 97)



Domain wall fermions

* Chirality doesn't exist in 5 dimensions

  Ψ : 4 component Dirac fermion

* If a Yukawa interaction exists,

$$\mathcal{L}_F = i\bar{\Psi}\Gamma^M\partial_M\Psi - \eta T\bar{\Psi}\Psi$$

  and T enjoys domain wall solution,
  a massless chiral fermion appears.
  topological state by Jackiw-Rebbi mechanism (PRD '76)

* Rubakov-Shaposhnikov (PLB '83) identified the chiral fermions to the SM matters on the domain wall.

KK spectrum of fermions

Dirac Eq. $\left(i \Gamma^M \partial_M - \eta T_{DW} \right) \Psi = 0$

	5D		4D
	Γ^μ	$=$	γ^μ
	Γ^4	$=$	$i\gamma_5$

$\Longrightarrow \quad \Psi = \Psi_L + \Psi_R$

$\left(\gamma_5 \Psi_L = - \Psi_L , \; \gamma_5 \Psi_R = \Psi_R \right)$

$$\begin{cases} \left(\Box_4 + Q Q^\dagger \right) \Psi_L = 0 \\ \left(\Box_4 + Q^\dagger Q \right) \Psi_R = 0 \end{cases}$$

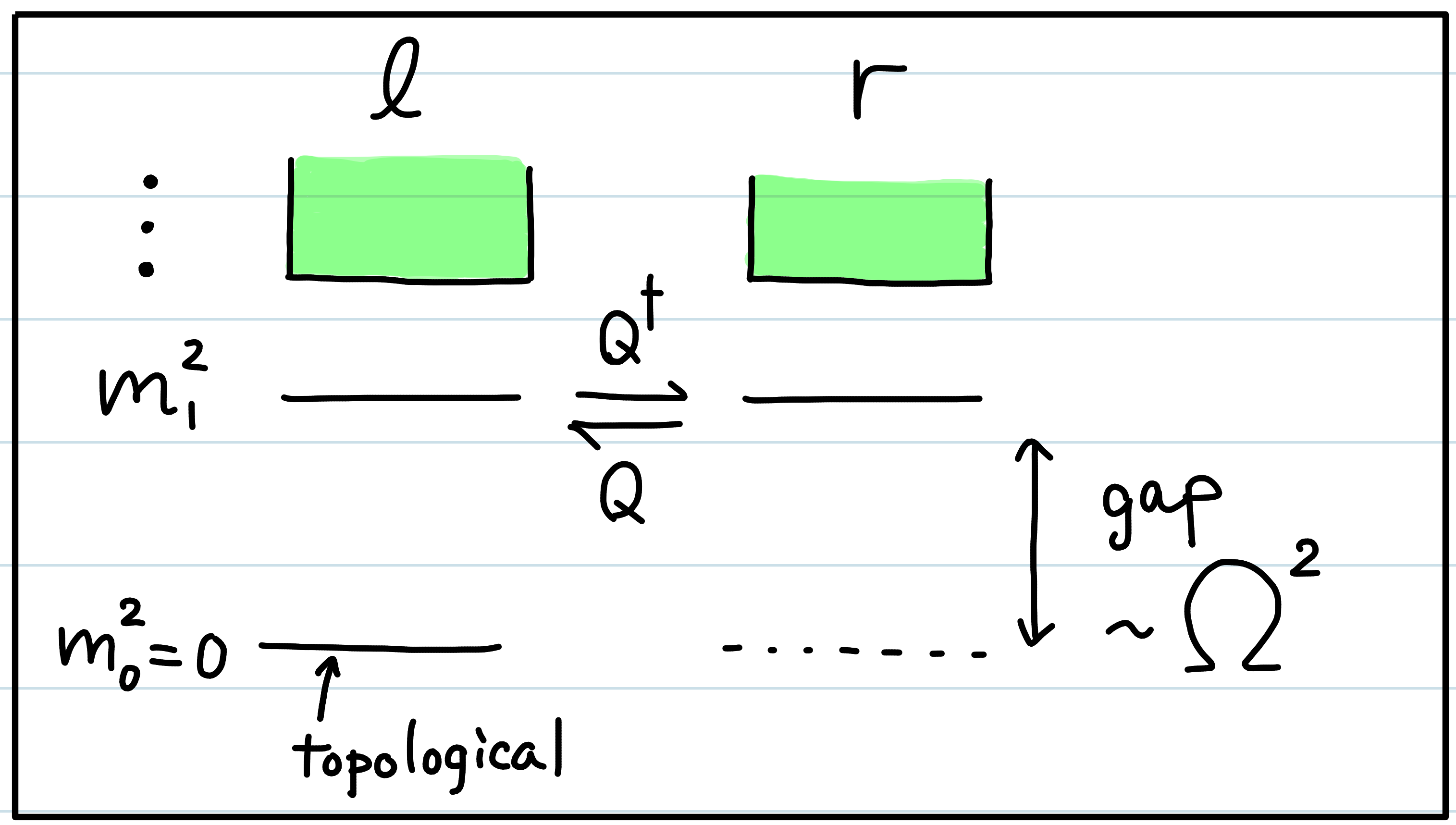
w/ $\quad Q = -\partial_y + \eta T_{DW}$

$\quad\; Q^\dagger = \partial_y + \eta T_{DW}$

$$Q Q^\dagger \, l_n = m_n^2 \, l_n$$

|| SUSY QM

$$Q^\dagger Q \, r_n = m_n^2 \, r_n$$



3. Localization of gauge fields

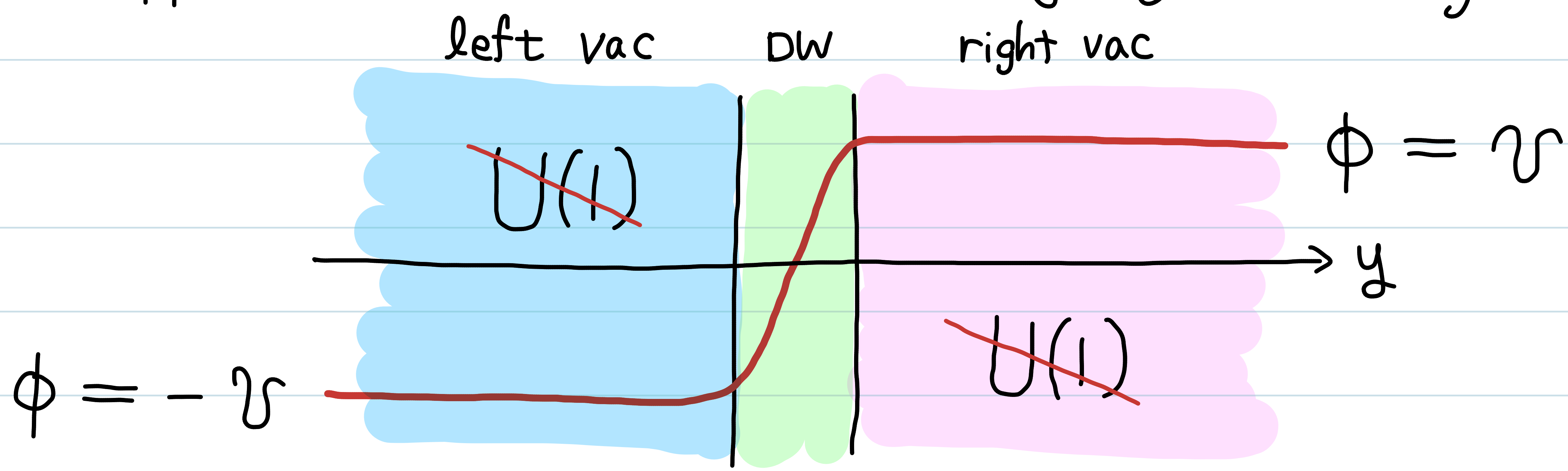
We need massless non-Abelian gauge fields localized on the domain wall.

→ It is not easy task.

Why is it so difficult ?

Because, DW interpolates vacua with $\phi \neq 0$.

Suppose ϕ has a $U(1)$ gauge charge.

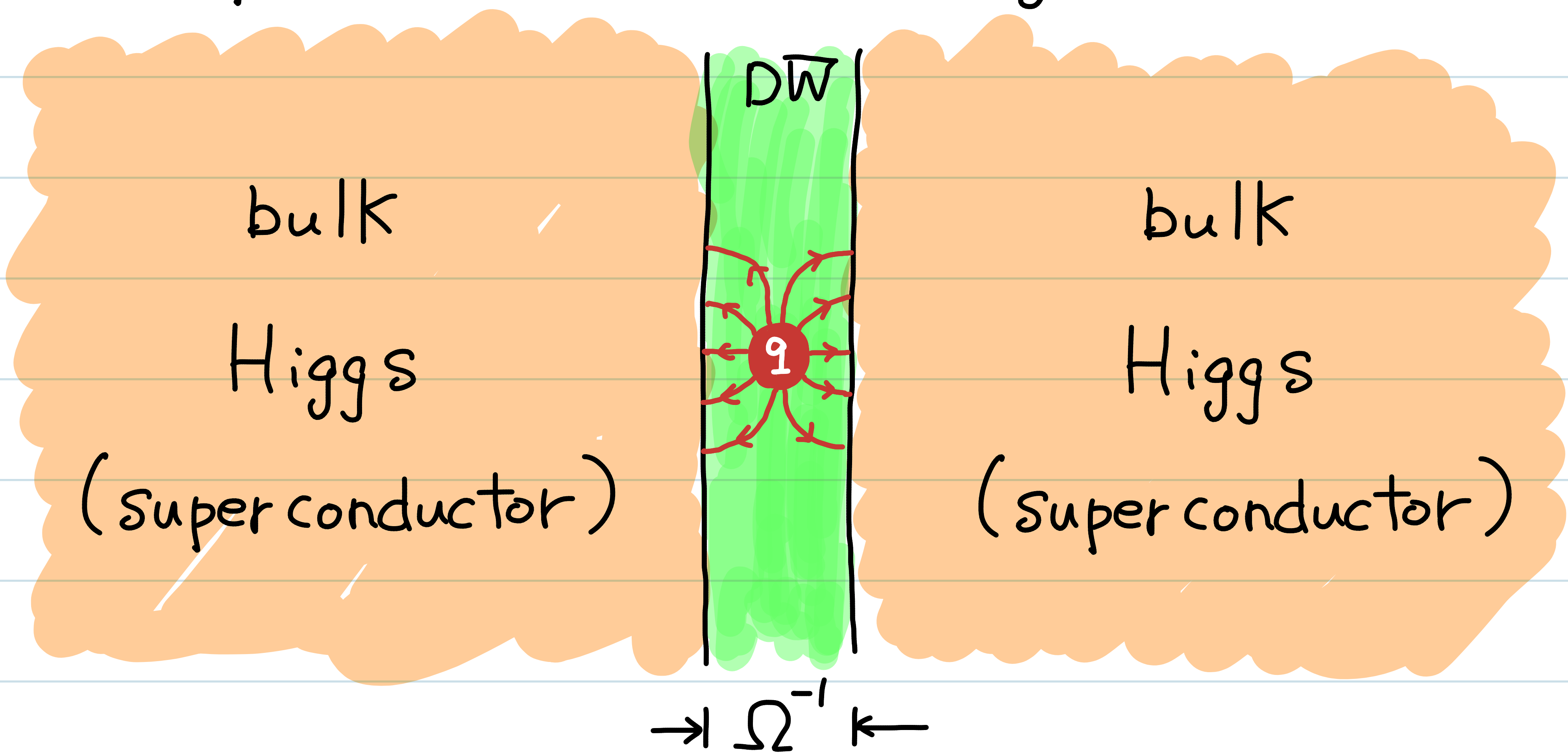


Inside DW, ϕ goes across $\phi = 0$ where U(1) gauge symmetry is unbroken.

A massless gauge field is likely to localize.

But it is not true.

Let us put an electric charge inside DW :



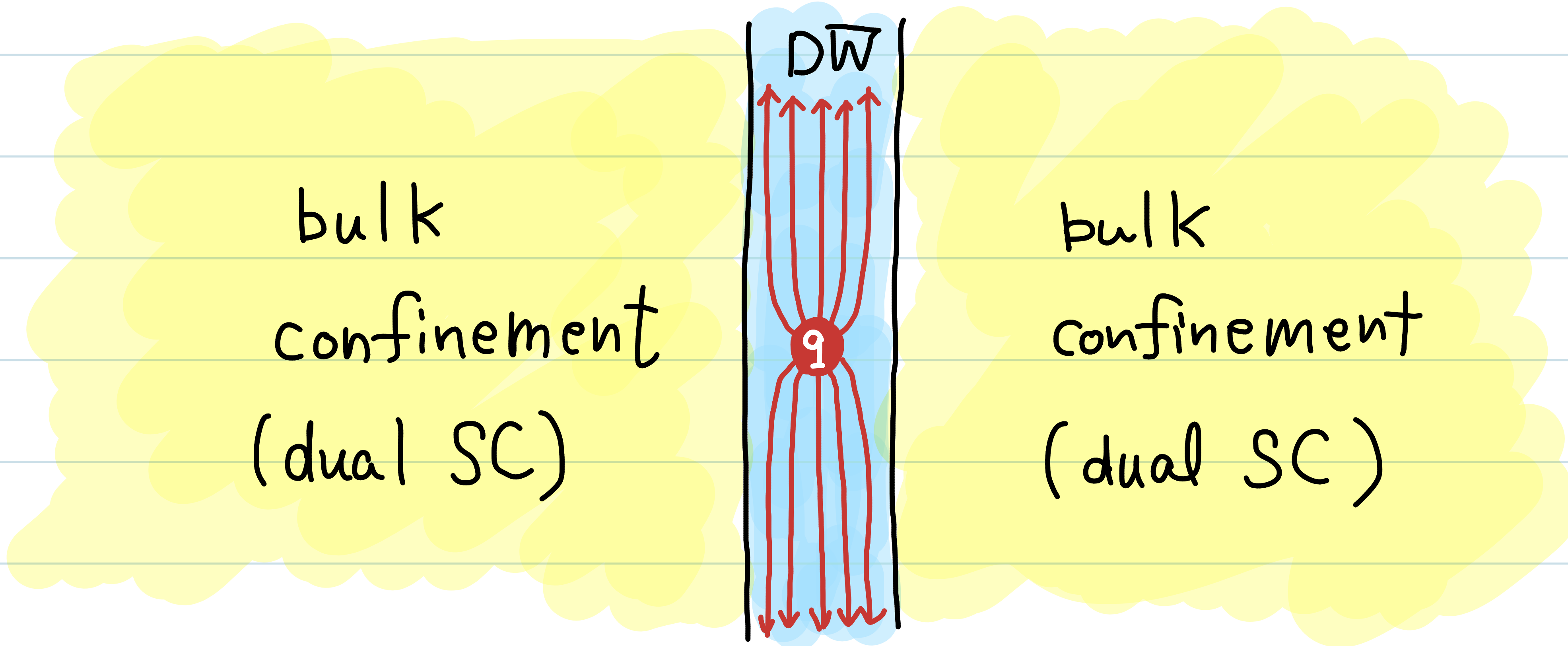
bulk = superconductor : electric flux cannot infinitely extend.

Screening mass $m_\gamma \sim \Omega$

Dvali-Shifman mechanism (PLB '97)

Domain walls in strongly coupled theory

$\boxed{\text{IDEA : Higgs} \longrightarrow \text{confining}}$



dual Meissner effect

Electric flux is expelled by bulk

Electric flux is confined inside DW.

Localization of massless gauge fields!!

However ...

- @ strongly coupled theory $\rightarrow$ NOT easy
- @ confinement in higher dim. $\rightarrow$ unclear

Possible attitudes

- ☆ assuming confinement occurs.
- ☆ inventing a model mimiking DS mechanism.

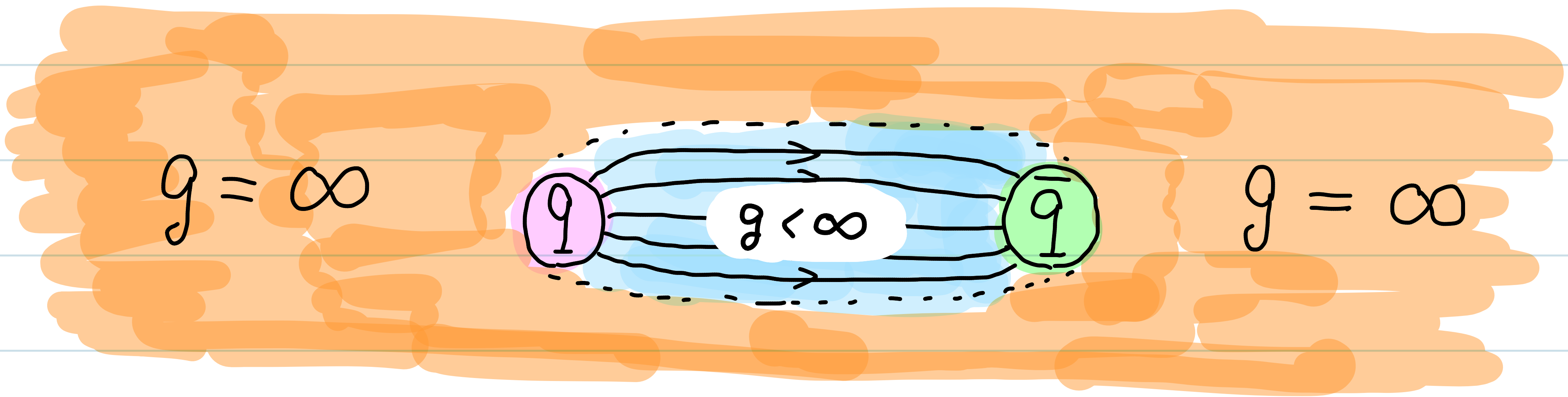
Necessary condition for DS mechanism

- * gauge symmetry is unbroken in bulk
- * gauge interaction is strong in bulk

A phenomenological model of confinement

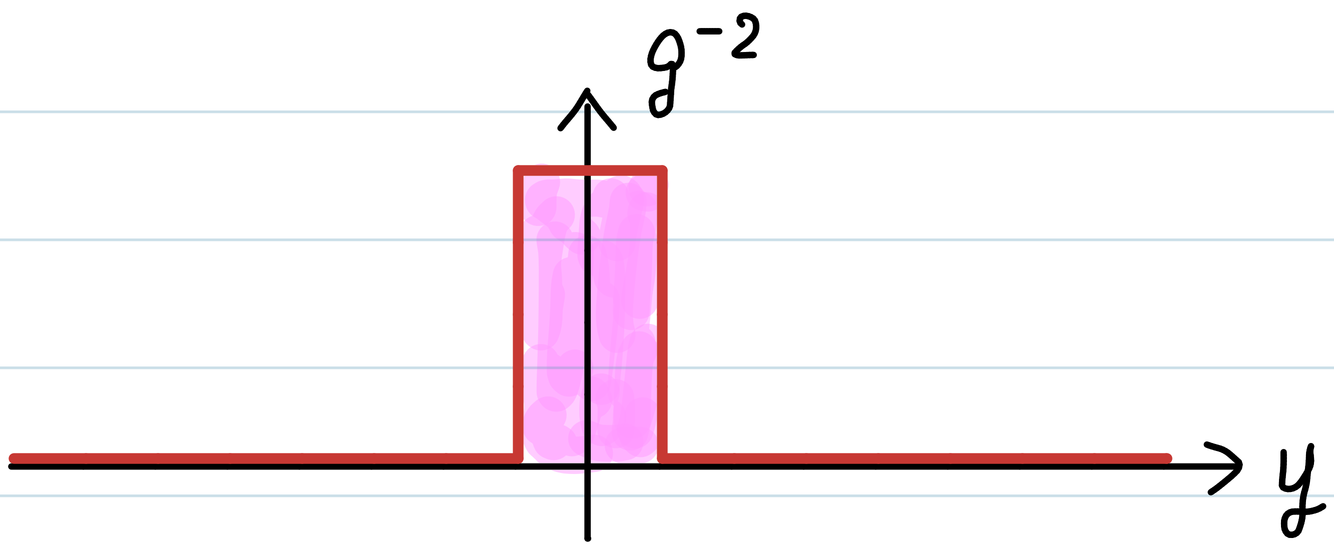
$$\mathcal{L} = -\frac{1}{4\,g(x)^2} F^a_{\mu\nu} F^{a\mu\nu}$$

[Kogut - Susskind PRD '74
Friedberg - Lee PRD '77
Fukuda PLB '78, PLA '09]

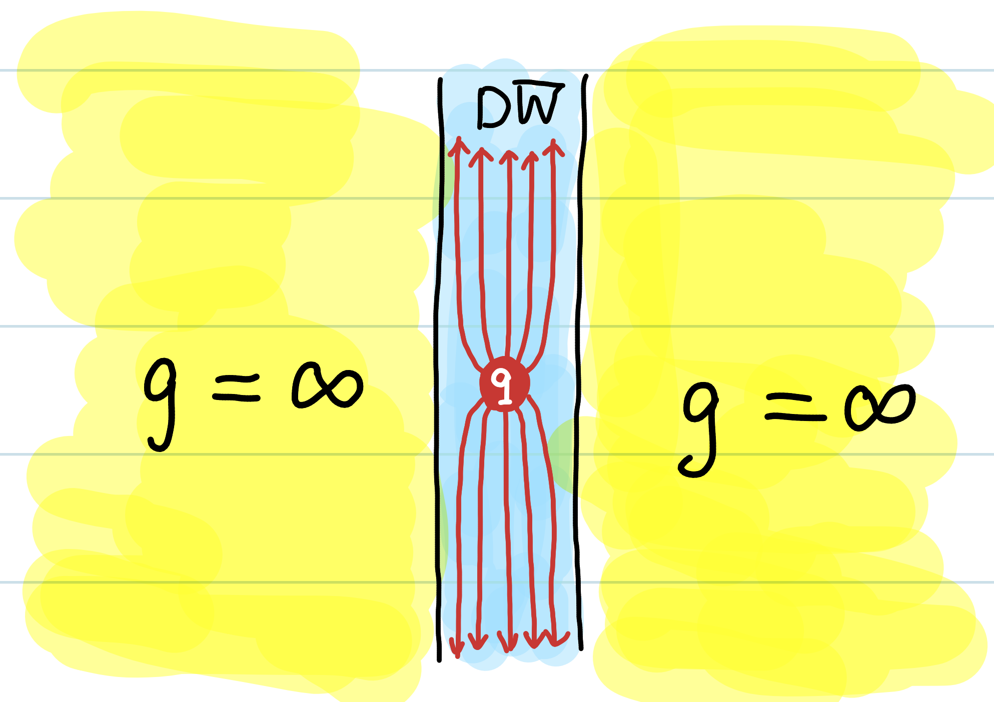


Borrowing the idea for our purpose, one can consider

$$\mathcal{L} = -\frac{1}{4\,g(y)^2} F^a_{MN} F^{aMN}$$



$\Rightarrow$



But, y-dependent gauge coupling $g(y)$ manifestly violates Lorentz symmetry in 5 dimensions.

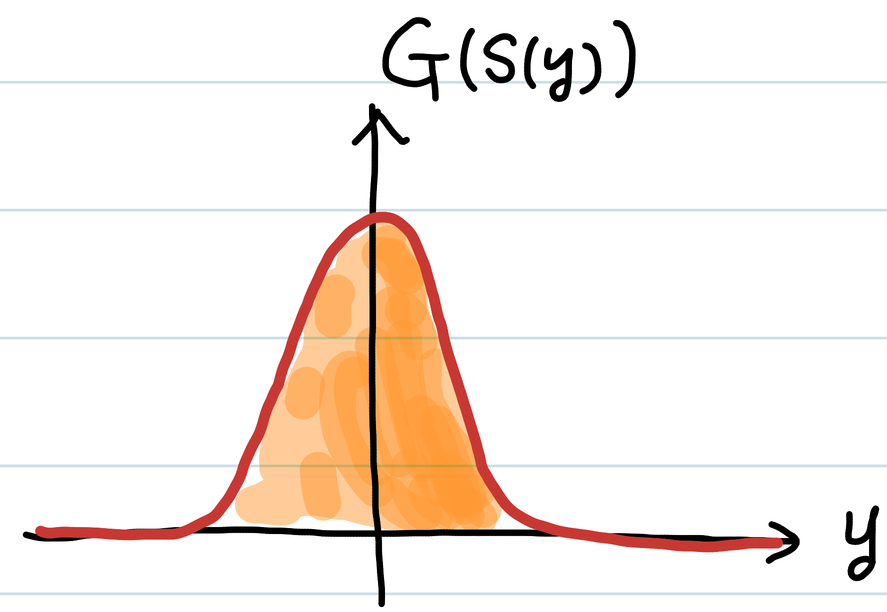
$\longrightarrow$ The fifth dimension is distinguished from the beginning. 

$$\boxed{\text{We would like to put all spacial coordinates on equal footing at the Lagrangian level.}}$$

A possible wayout is

$$\mathcal{L} = - G(\phi) F^a_{MN} F^{aMN}$$

$$\text{E.O.M.} \quad \phi = \phi(y) \longrightarrow G(\phi(y))$$



[Dubouski-Rubakov '00, Ohta-Sakai '10, Chumbes-Hoff da Silva-Hott '11, Arai-Blaschke-ME-Sakai '13, '14, '17, '18, Okada-Raut-Villalba '17, '18]

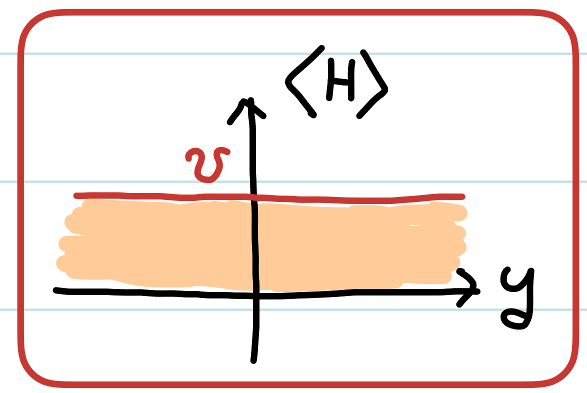
4. Localization of gauge fields via Higgs mechanism

So far, I explained fermions & gauge fields.

Next, about Higgs.

Higgs field must condense for EW ~~symmetry~~.

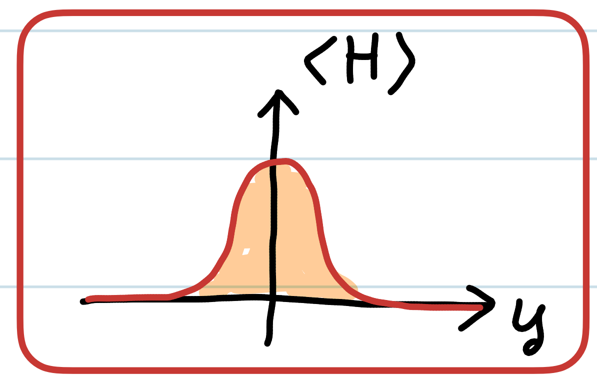
Where is EW symmetry broken?



Is it whole over space?

or

Is it only inside domain wall?



gauge field localization ⟶ bulk ≠ Higgs phase

Domain wall induces Higgs condensation.

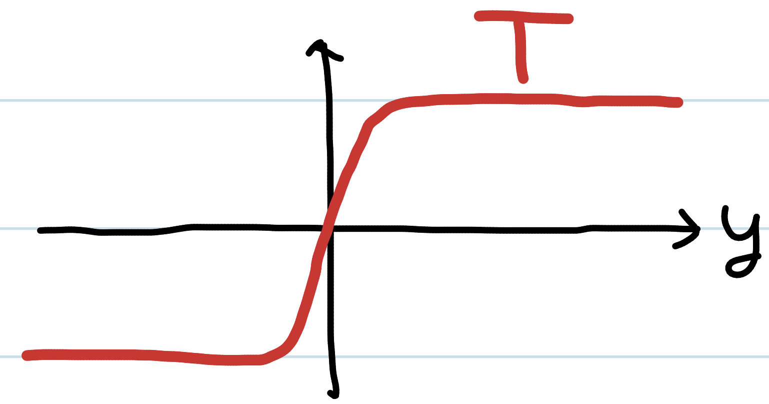
$$V_{5D}(\mathcal{H},T) = \Omega^2 |\mathcal{H}|^2 + \lambda^2 \left(|\mathcal{H}|^2 + T^2 - v^2 \right)^2$$

$\mathcal{H}$: SM Higgs doublet

T : real scalar field responsible for DW

Vacua : $\quad T = \underbrace{\pm v}_{\text{domain wall}}, \quad \underbrace{\mathcal{H} = 0}_{\text{EW symmetry unbroken}}$

$$T = v \tanh ky$$

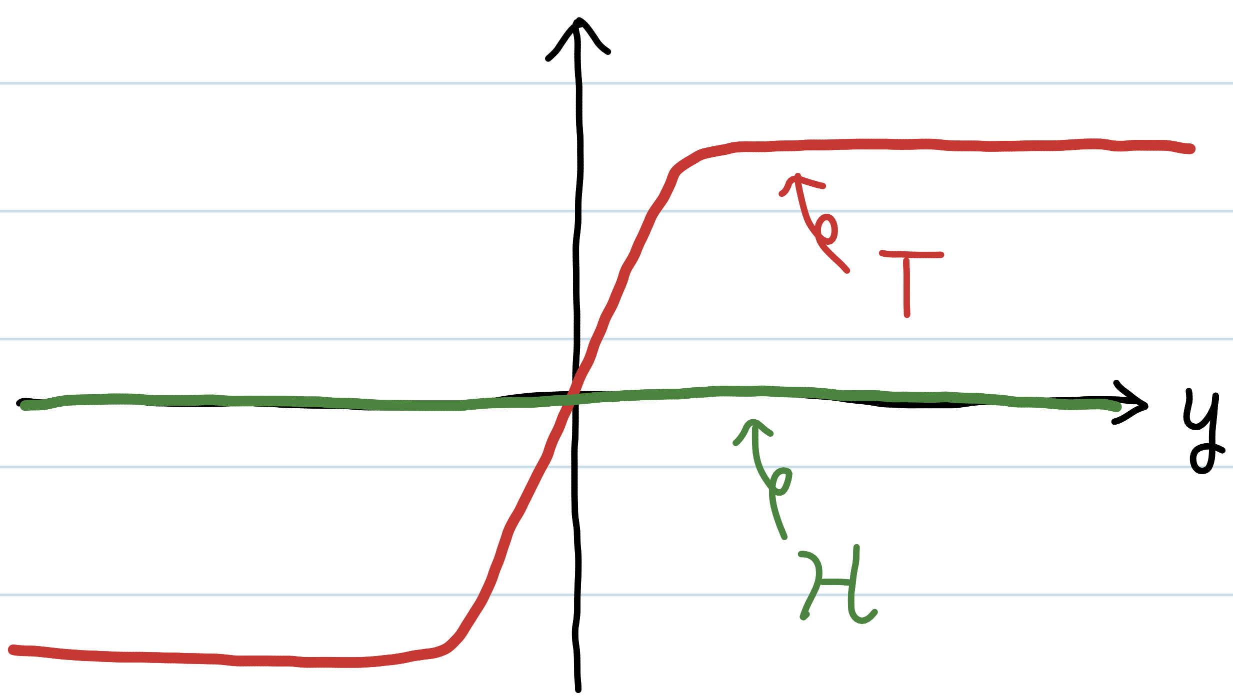


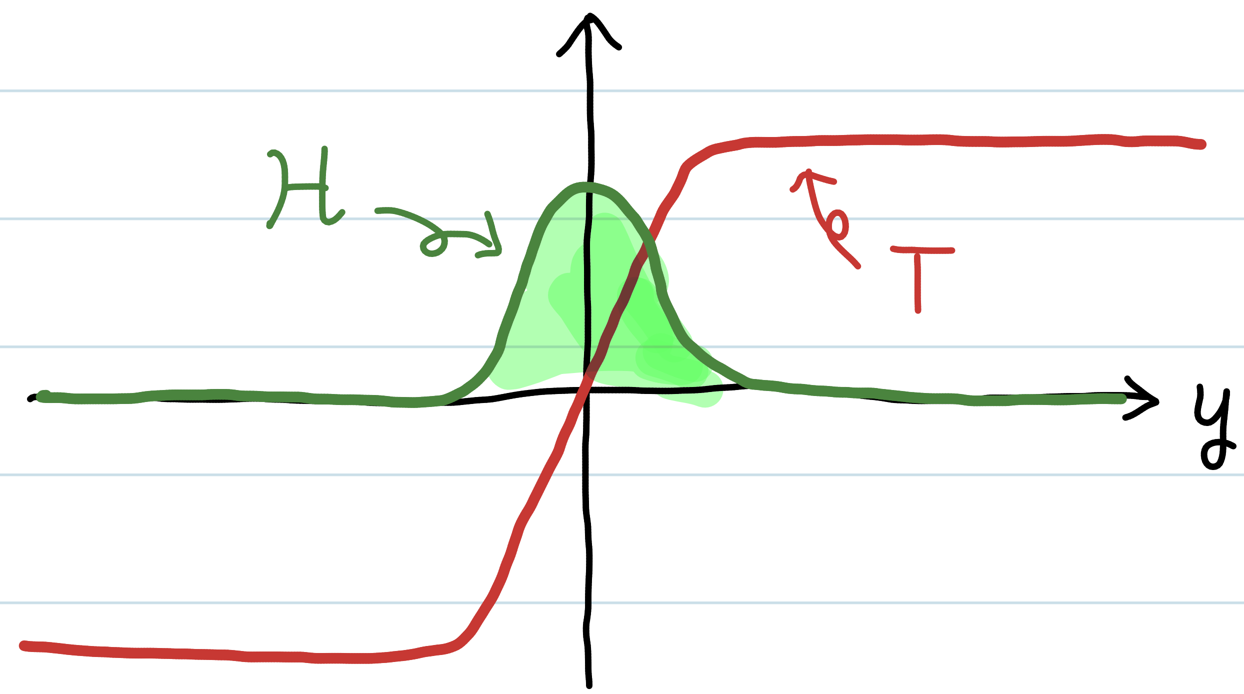
Inside DW, $T \simeq 0$.

$$V(\mathcal{H}, T=0) = \lambda^2 |\mathcal{H}|^4 + \big(\underbrace{\Omega^2 - 2v^2\lambda^2}_{\text{can be negative}}\big)|\mathcal{H}|^2 + \cdots$$

Threshold is $\boxed{\bar{v}^2 = 0}$ with $\bar{v}^2 \equiv v^2 - \dfrac{\Omega^2}{\lambda^2}$

$\bar{v}^2 < 0$	$\bar{v}^2 > 0$
$T = v \tanh \lambda v y$	$T = v \tanh \Omega y$
$\mathcal{H} = 0$	$\mathcal{H} = \bar{v} \operatorname{sech} \Omega y$





Our interest

$$\boxed{\begin{array}{ccc} \mathcal{H} = 0 & \xrightarrow{\text{domain wall}} & \mathcal{H} \neq 0 \\ \text{in vacua} & & \text{in domain wall} \end{array}}$$

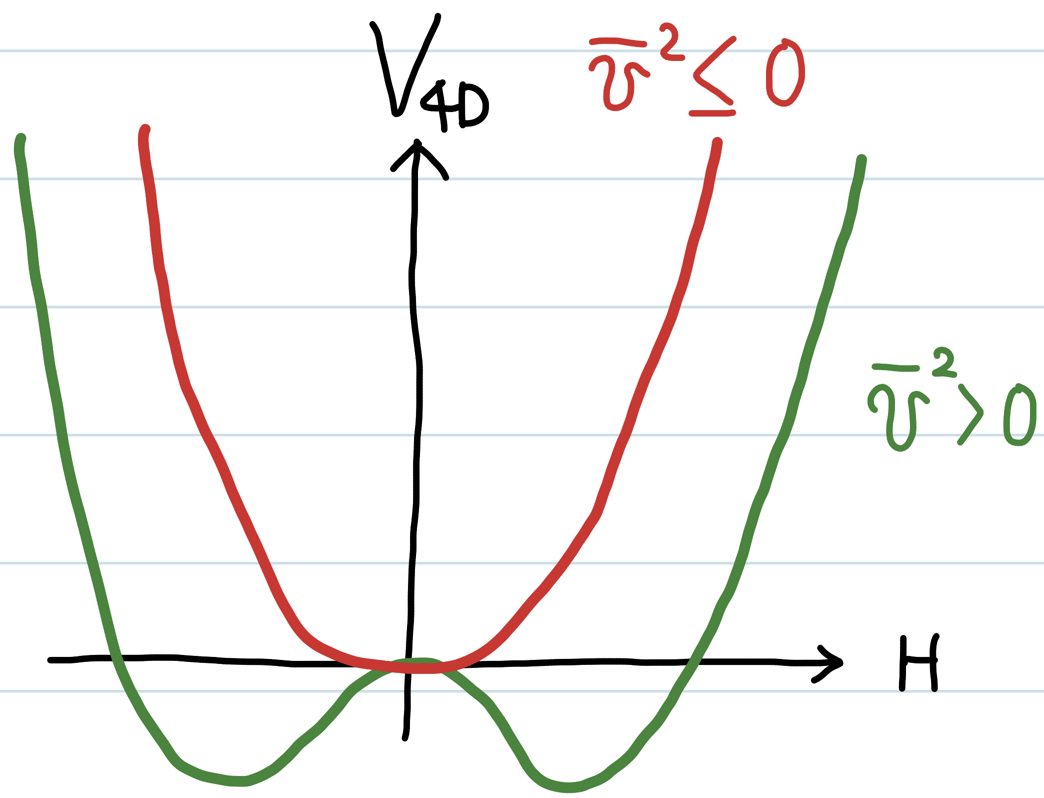
Higgs mechanism inside domain wall

$$\mathcal{H}(x^\mu, y) = \left(\sqrt{\frac{\Omega}{2}}\,\mathrm{sech}\,\Omega y\right) H(x^\mu) + \text{heavy modes}$$

5D Higgs — normalized wave function — 4D Higgs — $\left(\text{mass} \sim \mathcal{O}(\Omega)\right)$

$$V_{4D}(H) \equiv \int_{-\infty}^{\infty} dy \left\{ -\mathcal{L}_{5D} \right\}_{T = v\tanh\Omega y,\ \mathcal{H} = \sqrt{\frac{\Omega}{2}} H\,\mathrm{sech}\,\Omega y}$$

$$= -\frac{4}{3}\lambda^2 \bar{v}^2 |H|^2 + \frac{2}{3}\lambda^2 \Omega |H|^4$$



$\bar{v}^2 > 0 \rightarrow$ Higgs condenses

$$\boxed{\begin{aligned} \langle H \rangle &= \sqrt{\frac{2}{\Omega}}\,\bar{v} \equiv \frac{v_R}{\sqrt{2}} \\ m_H^2 &= \frac{8}{3}\lambda^2 \bar{v}^2 \end{aligned}}$$

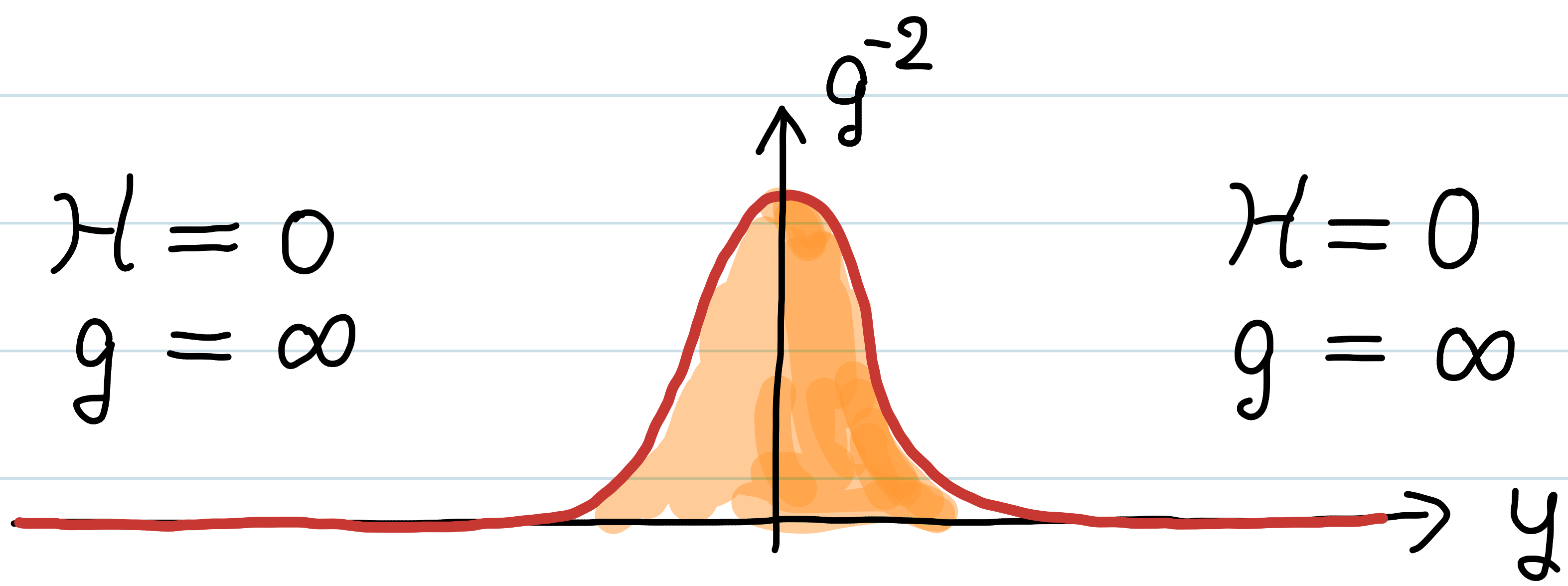
The central idea of our work is

$\boxed{\text{making use of the local Higgs condensation to confine the SM gauge fields into DW.}}$

$\boxed{\text{example}}$

$$\mathcal{L} = -\underbrace{\frac{|\mathcal{H}|^2}{4\mu^2}}\, \mathcal{F}^a_{MN}\, \mathcal{F}^{aMN} \qquad \left(\begin{array}{l}\text{Lorentz inv.} \\ \text{gauge inv.}\end{array}\right)$$

$$\hookrightarrow g^{-2} = \frac{|\mathcal{H}|^2}{\mu^2} \xrightarrow[\text{DW}]{} \frac{\bar{v}^2}{\mu^2}\,\mathrm{sech}^2\,\Omega y$$



Our story

$\boxed{\text{model}}$ The SM fields + T scalar field } in flat noncompact 5 dims.

$\boxed{\text{Vacua}}$ $(T, H) = (\pm v, 0)$ { $\mathbb{Z}_2$ SSB; $SU(3)\times SU(2)\times U(1)$ is unbroken

$\mathbb{Z}_2$ SSB

↓

dynamical compactification by domain wall

↳ { chiral fermions; Higgs condensation → ① EW symmetry SSB, ② fermion mass, ③ localization of gauge fields

The 3rd fundamental role of Higgs field !!

KK spectrum of gauge fields

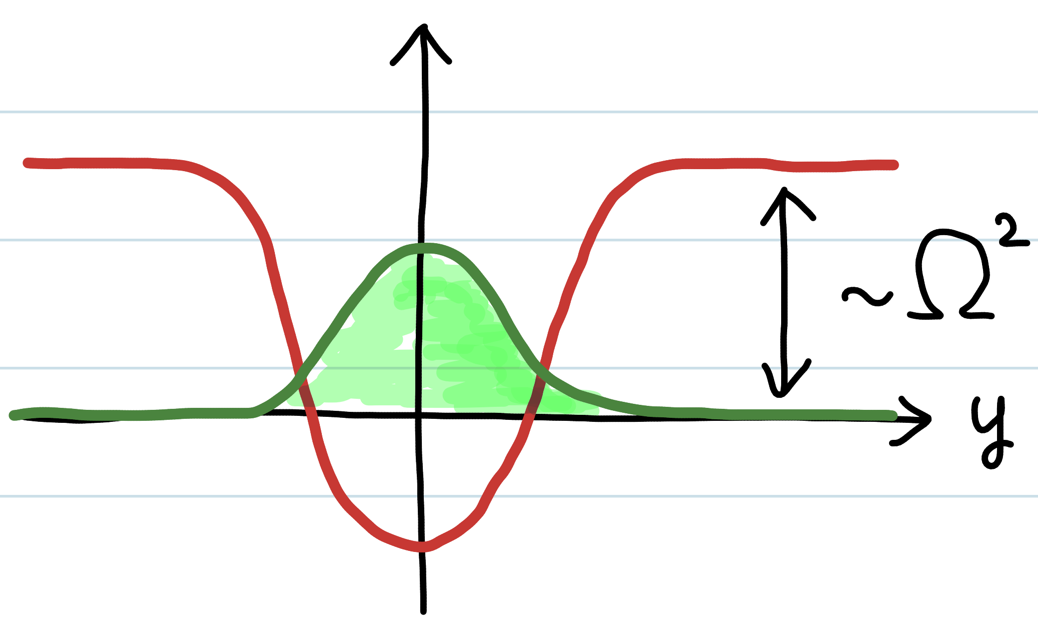
$$\mathcal{L} = -\beta(H)^2 \, \mathcal{F}^a_{MN} \mathcal{F}^{aMN}, \quad \beta = \frac{\mathcal{H}}{2\mu}$$

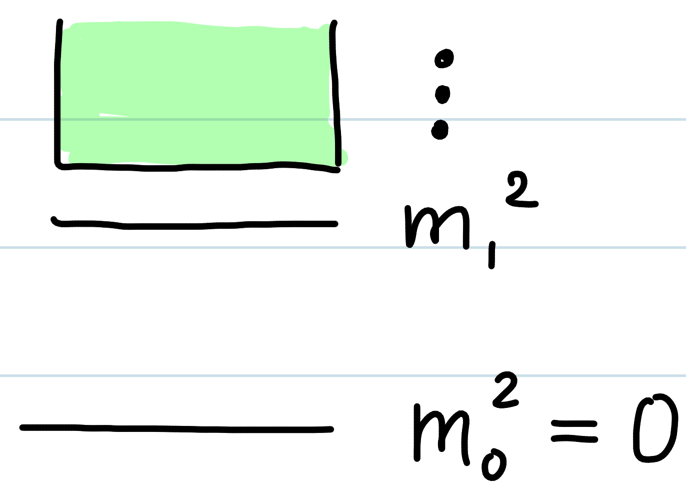
$$\left(\mathcal{F}_{MN} = \partial_M \mathcal{A}_N - \partial_N \mathcal{A}_M + i\,[\mathcal{A}_M, \mathcal{A}_N] \right)$$

For convenience, define canonical field $A_\mu \equiv 2\beta \mathcal{A}_\mu$

If H is neutral, KK spectrum reads, $A_\mu = \sum_n a_n(y) A^{(n)}_\mu(x)$

$$D^\dagger D a_n = m_n^2 a_n \quad \left(D = -\partial_y + \frac{\beta'}{\beta}, \quad D^\dagger = \partial_y + \frac{\beta'}{\beta} \right)$$





zero mode : $\left(-\partial_y + \frac{\beta'}{\beta} \right) a_0 = 0 \longrightarrow a_0 \propto \beta$

$$\boxed{\beta = \frac{\mathcal{H}}{2\mu} = \frac{\bar{v}}{2\mu}\,\mathrm{sech}\,\Omega y \longrightarrow a_0 = \sqrt{\frac{\Omega}{2}}\,\mathrm{sech}\,\Omega y \quad \text{(normalized wave func.)}}$$

Plugging this into $\mathcal{A}_\mu = \frac{1}{2\beta} A_\mu$, we find

$$\mathcal{A}^a_\mu = \underbrace{\frac{a_0(y)}{2\beta(y)}}_{\text{Constant}} A^{(0)a}_\mu (x^\mu) + \text{heavy modes}$$

Constant $\longrightarrow \frac{\mu\sqrt{\Omega}}{\bar{v}\sqrt{2}} \equiv e_4$ ◂ 4D gauge coupling

$$\mathcal{F}_{\mu\nu}(x^\mu, y) = \frac{a_0}{2\beta} \left(\partial_\mu A^{(0)}_\nu - \partial_\nu A^{(0)}_\mu \right) + i \left(\frac{a_0}{2\beta} \right)^2 [A^{(0)}_\mu, A^{(0)}_\nu] + \cdots$$

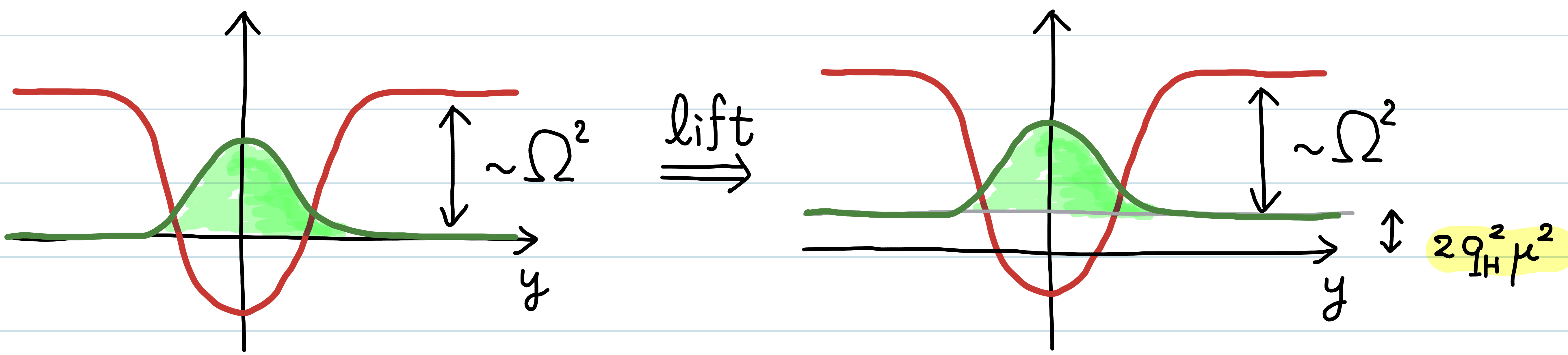
$$= \frac{a_0}{2\beta} \Big\{ \underbrace{\partial_\mu A^{(0)}_\nu - \partial_\nu A^{(0)}_\mu + i e_4 [A^{(0)}_\mu, A^{(0)}_\nu]}_{\equiv F^{(0)}_{\mu\nu}(x^\mu)} \Big\} + \cdots$$

When $\mathcal{H}$ is charged as $D_M \mathcal{H} = (\partial_M + i q_H A_M)\mathcal{H}$

neutral $\quad D^\dagger D a_n = m_n^2 a_n$

$\downarrow$

charged $\quad (D^\dagger D + 2 q_H^2 \mu^2) a_n = \tilde{m}_n^2 a_n$



$$\boxed{\text{All the KK masses are shifted by } \sqrt{2}\, q_H \mu}$$

$$\mathcal{L}_{5D} \supset -\frac{|\mathcal{H}|^2}{4\mu^2} F^a_{MN} F^{aMN} + \left|(\partial_M + i q_H A_M)\mathcal{H}\right|^2$$
$$- \Omega^2 |\mathcal{H}|^2 - \lambda^2 \left(|\mathcal{H}|^2 + T^2 - v^2\right)^2$$

Ω : 5 dim mass scale $\gg M_{EW}$

μ, λ, v can be set to realize experimental values

$$v_h = \frac{2}{\sqrt{\Omega}} \bar{v}, \quad m_H^2 = \frac{8}{3} \lambda^2 \bar{v}^2, \quad m_A^2 = 2 q_H^2 \mu^2$$

Relation between v_h, e_4, m_A is same as usual Higgs mechanism

$$e_4 = \frac{\mu\sqrt{\Omega}}{\sqrt{2}\,\bar{v}}, \quad v_h = \frac{2}{\sqrt{\Omega}} \bar{v}, \quad m_A = \sqrt{2}\, q_H \mu$$

$$\boxed{m_A = q_H e_4 v_h}$$

A simplest 5D model for the SM is

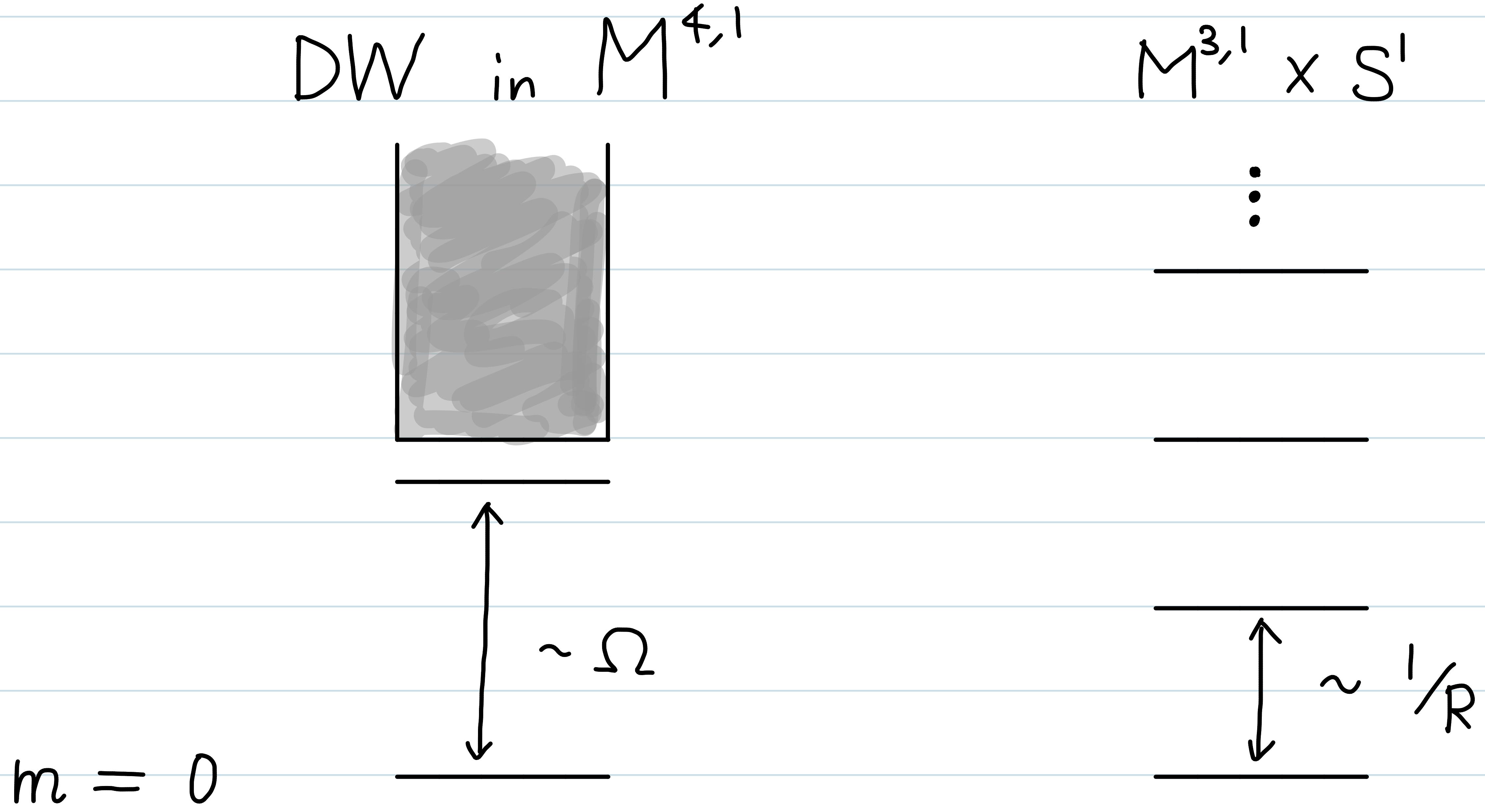
$$\mathcal{L}_{5D} \supset -\frac{|\mathcal{H}|^2}{4\tilde{\mu}^2} \overset{SU(3)}{\mathcal{G}^a_{MN}\mathcal{G}^{aMN}} - \frac{|\mathcal{H}|^2}{4\mu^2}\left(\overset{SU(2)}{\mathcal{W}^i_{MN}\mathcal{W}^{iMN}} + \overset{U(1)}{\mathcal{B}_{MN}\mathcal{B}^{MN}} \right)$$

$$+ \left| \left(\partial_M + \frac{i}{2} g \mathcal{W}_M + \frac{i}{2} g' \mathcal{B}_M \right) \mathcal{H} \right|^2$$

$\mathcal{G}^a_{MN}$, A_M ... massless, Z_M, $W^\pm_M$ ... massive

5. Four footprints of the domain wall

① KK modes



A few discrete DW states
+
continuum bulk states

Infinite discrete
bulk states

② DW spontaneously breaks translational symmetry

$\downarrow$

a massless NG boson

$\boxed{\text{Moduli approximation}}$

(Manton '82)

$$\begin{cases} T = v \tanh\left(\Omega y - \frac{1}{f_Y} Y(x^\mu)\right) \\ \mathcal{H} = \sqrt{\frac{\Omega}{2}}\, H(x^\mu)\, \mathrm{sech}\left(\Omega y - \frac{1}{f_Y} Y(x^\mu)\right) \end{cases}$$

moduli field

Direct interaction between the SM particles and NG

$$\mathcal{L}_{NG} = \frac{1}{2} \partial_\mu Y \partial^\mu Y + \frac{\Omega^2}{4v^2} \partial_\mu Y \partial^\mu Y \, |H|^2 + \mathcal{O}(\partial^4)$$

* No other terms exist to the leading order of ∂^2
* Only derivative coupling exists (low energy theorem)

$\longrightarrow$ strong suppresion $\frac{\Omega^2}{v^2} \sim \mathcal{O}(\Omega^{-2})$

③ $H \to \gamma\gamma$ @ tree level

Our Lagrangian has $\mathcal{L}_{5D} \supset -\dfrac{|\mathcal{H}|^2}{4\mu^2} B_{MN} B^{MN}$

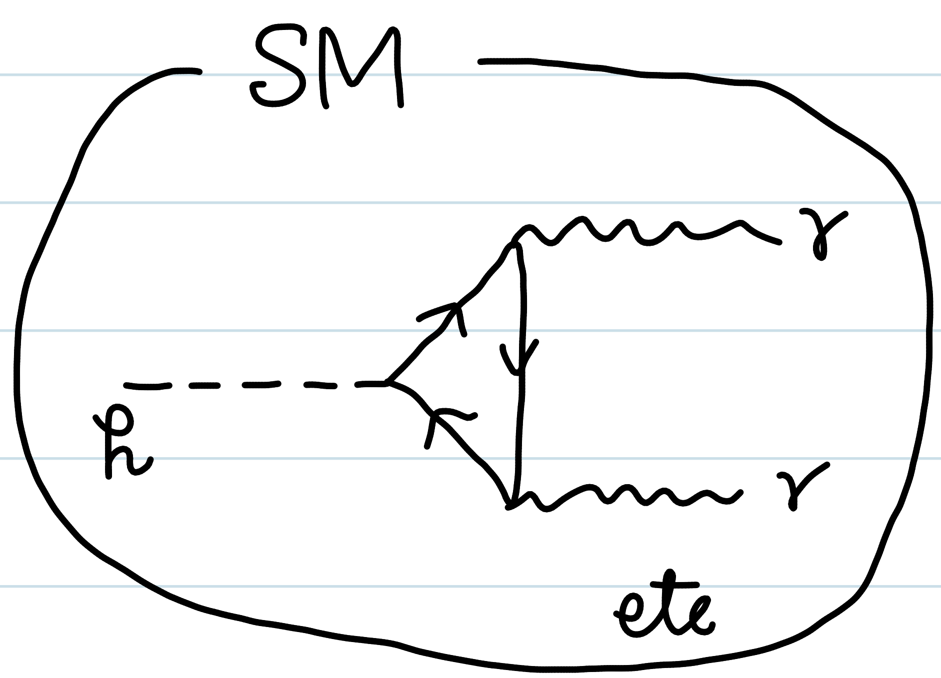
$$\int_{-\infty}^{\infty} \mathcal{L}_{5D}\, dy \supset -\frac{1}{4} \frac{2}{v_H^2} |H(x)|^2 \left(B_{\mu\nu}^{(0)}\right)^2$$

$$H = \begin{pmatrix} 0 \\ \frac{v_H}{\sqrt{2}} + h \end{pmatrix}$$

$$= -\frac{1}{4}\left(1 + 2\sqrt{2}\frac{h}{v_H} + 2\frac{h^2}{v_H^2}\right)\left(B_{\mu\nu}^{(0)}\right)^2$$

Triple gauge-Higgs coupling : $\dfrac{1}{\sqrt{2}} \dfrac{h}{v_H} \left(B_{\mu\nu}^{(0)}\right)^2$

Too big !



To be consistent with LHC data,

$$\bar{c}_\gamma \frac{h}{v_H} B_{\mu\nu}^2 \quad \text{w/} \quad \bar{c}_\gamma < 10^{-3}$$

(Ellis-Sanz-You, JHEP'15)

Need to modify $\mathcal{L}_{5D}$

$$-\underbrace{\frac{|\mathcal{H}|^2}{4\mu^2}}_{\text{monomial}} B_{MN} B^{MN} \Longrightarrow \underbrace{f(\mathcal{H})}_{\text{polynomial}} B_{MN} B^{MN}$$

e.g.) $$f(\mathcal{H}) = \frac{1}{2\mu^2}\left(|\mathcal{H}|^2 - b\frac{|\mathcal{H}|^4}{\tilde{v}^2}\right)$$

tune $b = \frac{3}{4}$

$$\int_{-\infty}^{\infty} dy\, f(\mathcal{H}) B_{MN} B^{MN} \supset \left\{-\frac{1}{4} + 0\,\frac{h}{v_h} + 2\left(\frac{h}{v_h}\right)^2 + \cdots\right\}\left(B_{\mu\nu}^{(0)}\right)^2$$

If b slightly deviates from $b = \frac{3}{4}$, small but non-zero triple Higgs-gauge coupling appears.

$\longrightarrow$ It would be an evidence of DW.

④ Finite electroweak monopole

Cho - Maison EW monopole (Cho-Maison PLB '97)

"

A spherical numerical solution with magnetic charge $Q = \frac{4\pi}{e}$

$$
\begin{cases}
H = \frac{1}{\sqrt{2}} \rho(r) \, i \begin{pmatrix} \sin\frac{\theta}{2} \, e^{-i\varphi} \\ -\cos\frac{\theta}{2} \end{pmatrix} \\
W_i = \frac{1}{g} \left(f(r) - 1 \right) \underbrace{\hat{r} \times \partial_i \hat{r}}_{\text{Wu-Yang monopole}} \\
B_i = -\frac{1}{g'} \underbrace{(1 - \cos\theta) \, \partial_i \varphi}_{\text{Dirac monopole}}
\end{cases}
$$

As a Dirac monopole, its energy diverges due to singularity.

To detect the monopole in an experiment, it is better to know the mass, at least order of magnitude.

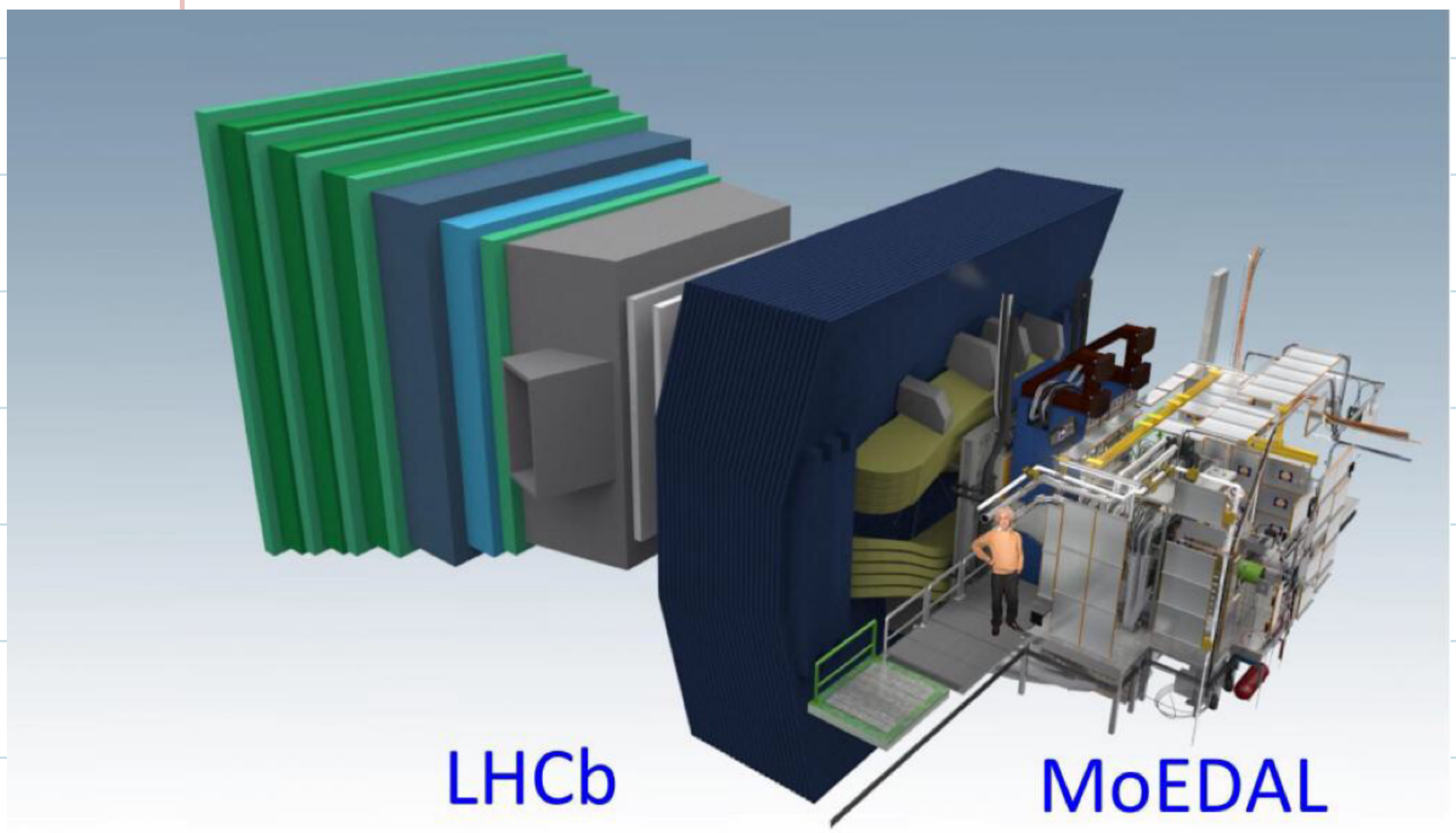
Improvement by Cho-Kim-Yoon (Euro. Phys. J 2015)

$$\mathcal{L}'_{EW} = |D_\mu H|^2 - V(H) - \frac{1}{4}\,\epsilon\left(\frac{H}{v_H}\right)(B_{\mu\nu})^2 - \frac{1}{4}(W^a_{\mu\nu})^2$$

CKY found an EW monopole becomes regular if

$$\epsilon = \left(\frac{H}{v_H}\right)^n \text{ and } n > 4+2\sqrt{3} \sim 6.4$$

$$M_{\text{monopole}} \sim 4\ \text{TeV}$$



EW monopoles could be created by LHC.

A target of MoEDAL.

However, a monomial $\epsilon(H)$ is excluded.

(Ellis-Sanz-You JHEP'15)

Ellis-Mavromatos-You (JHEP'16) investigatec generic $\epsilon(H)$

e.g.) $$\epsilon = 5\left(\frac{H}{v_H}\right)^8 - 4\left(\frac{H}{v_H}\right)^{10}$$

$$\epsilon = 6\left(\frac{H}{v_H}\right)^{10} - 5\left(\frac{H}{v_H}\right)^{12}$$

$$\vdots$$

These are consistent because triple gauge-Higgs coupling is exactly zero.

Investigating various ϵ, EMY concluded

$$M_{\text{monopole}} \lesssim 5.5\ \text{TeV}$$

Our model can provide such term from necessity of localizing the SM gauge fields on the domain wall.

If LHC finds an EW monopole, it would be an evidence of non-compact extra dimension and domain wall.

Summary

We consider a model in non-compact 5 dimensions with the SM field + a real singlet scalar field T.

$\mathbb{Z}_2$ SSB

↓

dynamical compactification by domain wall

↳ { chiral fermions
  Higgs condensation → ① EW symmetry SSB
  → ② fermion mass
  → ③ localization of gauge fields

The 3rd fundamental role of Higgs field !!

Four footprints of DW are

① typical KK spectrum ② translatinal NG

③ $h \to \gamma\gamma$ ④ finite EW monopole

$\boxed{\Omega}$ is a typical mass scale of domain wall.

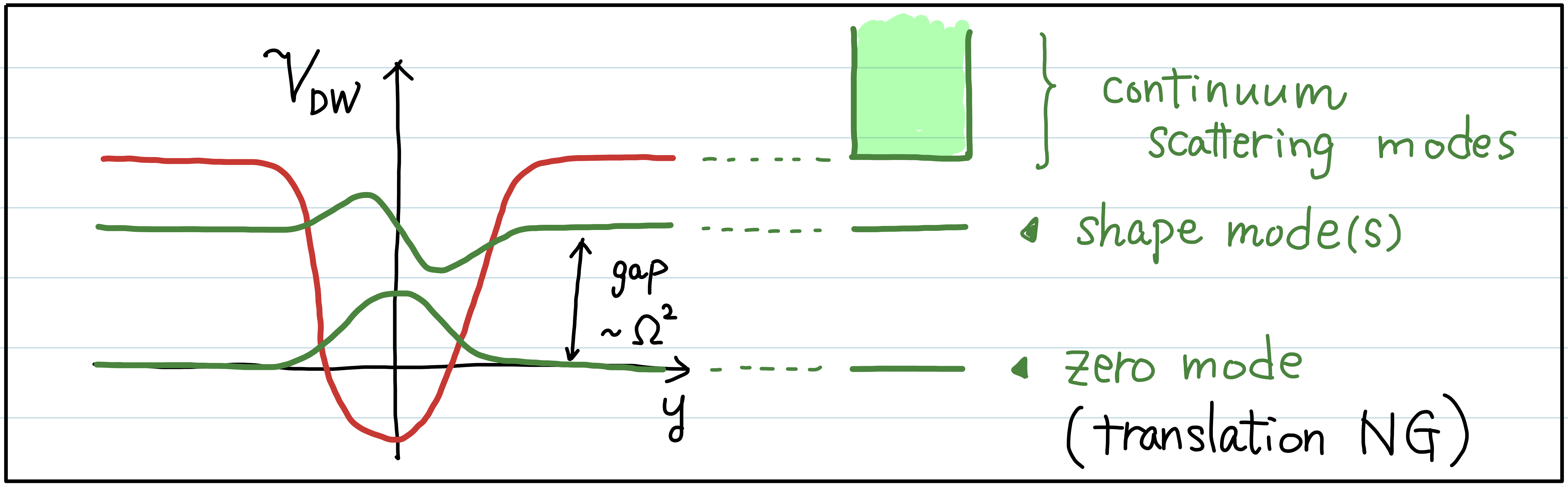
We will set $\Omega \gg M_{EW} \sim 10^2\ \text{GeV}$

Small fluctuations about domain wall background:

$$T(x^\mu, y) = T_{DW}(y) + \delta T(x^\mu, y)$$

KK spectrum is determined by

$$\left(-\frac{d^2}{dy^2} + \mathcal{V}_{DW}\right) f_n(y) = m_n^2 f_n(y)$$

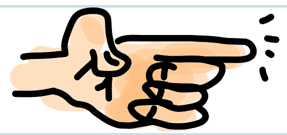


Short summary so far:

$$\mathbb{Z}_2 \text{ SSB in } \mathbb{R}^{4,1} \longrightarrow \begin{cases} \text{dynamical compactification} \\ \text{4D chiral fermions} \end{cases}$$

These are independent of details of models, since they are topological.

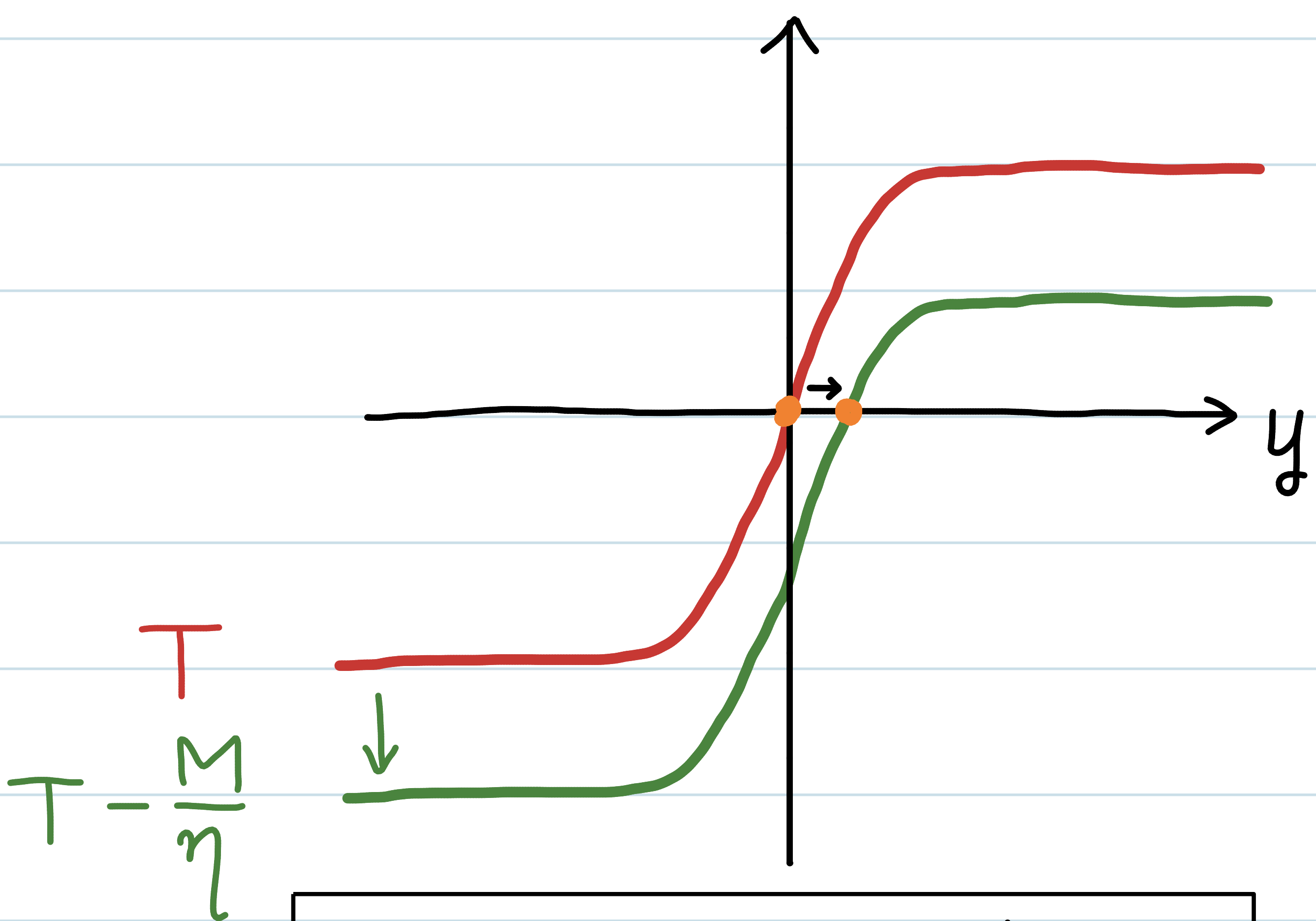
Another advantage thanks to geometry of extra dimensions.

 mass hierarchy of fermions

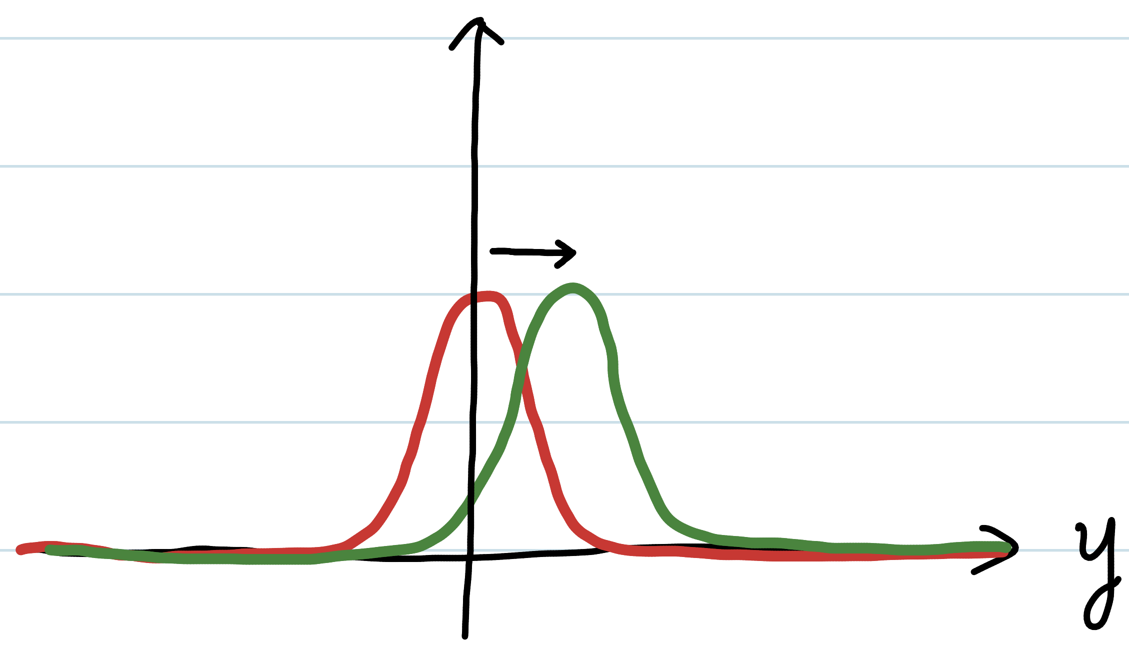
[Arkani-Hamed = Schmaltz PRD'00]

Introduce small bulk masses

$$\eta T \bar{\Psi} \Psi \longrightarrow (\eta T - M) \bar{\Psi} \Psi$$



zero point shift

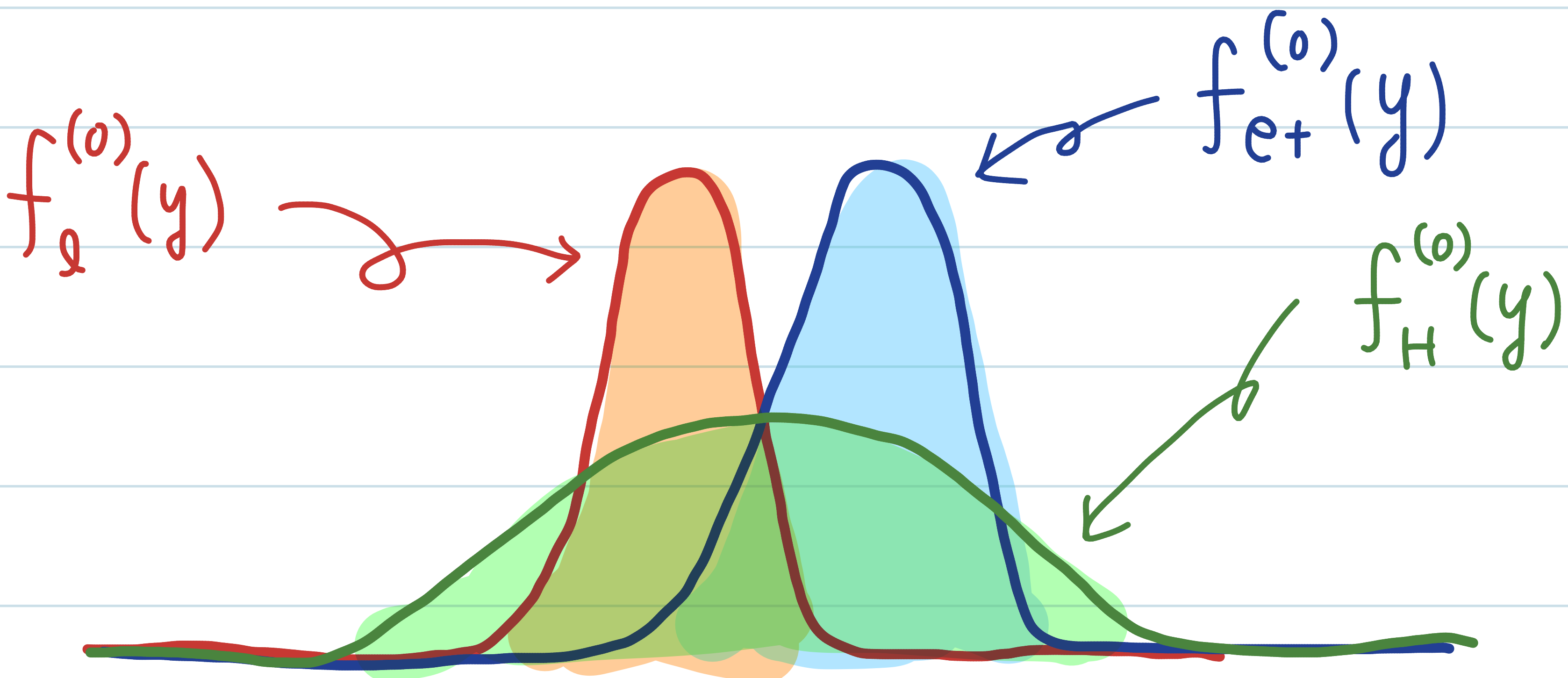


zero mode wave function shift

$$l = l^{(0)}(x^\mu)\, f_l^{(0)}(y) + \cdots \qquad \text{left-hand lepton}$$

$$e^+ = e^{+(0)}(x^\mu)\, f_{e^+}^{(0)}(y) + \cdots \qquad \text{right-hand lepton}$$

$$H = H^{(0)}(x^\mu)\, f_H^{(0)}(y) + \cdots \qquad \text{Higgs}$$



4D effective Yukawa coupling

$$Y_{4D} = Y_{5D} \times \underbrace{\int dy\, f_l^{(0)} f_{e^+}^{(0)} f_H^{(0)}}_{\text{exponentially small}}$$