

Attempts to invert non-abelian T-duality: a gauging approach

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Outline

- 1 Review of T-duality
- 2 (ω, ϕ) -modified gauging

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Setting up notation

Consider a non-linear sigma model $X : \Sigma \rightarrow M$ described by the following action:

$$S = \int_{\Sigma} g_{ij} dX^i \wedge \star dX^j + \int_{\Sigma} B_{ij} dX^i \wedge dX^j$$

In this talk we will ignore the dilaton, and assume that both g and B are globally defined fields on M .

Gauging isometries

Suppose now that there are vector fields generating the following global symmetry:

$$\delta_\epsilon X^i = v_a^i \epsilon^a$$

for ϵ^a constant. The sigma model action is invariant under this transformation if

$$\mathcal{L}_{v_a} g = 0 \quad \mathcal{L}_{v_a} B = 0$$

If this is the case, we can gauge the model by promoting the global symmetry to a local one.

The gauged action

Introducing gauge fields A^a and Lagrange multipliers η_a , the gauged action is

$$S_G = \int_{\Sigma} g_{ij} DX^i \wedge \star DX^j + \int_{\Sigma} B_{ij} DX^i \wedge DX^j + \int_{\Sigma} \eta_a F^a$$

where

- $F = dA + A \wedge A$ is the standard Yang-Mills field strength
- $DX^i = dX^i - v_a^i A^a$ are the gauge covariant derivatives.

Gauge invariance

The gauged action is invariant with respect to the following (local) gauge transformations:

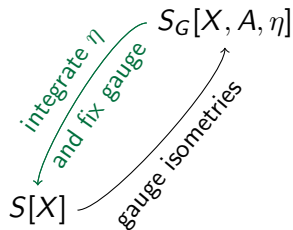
$$\delta_\epsilon X^i = v_a^i \epsilon^a$$

$$\delta_\epsilon A^a = d\epsilon^a + C_{bc}^a A^b \epsilon^c$$

$$\delta_\epsilon \eta_a = -C_{ab}^c \epsilon^b \eta_c$$

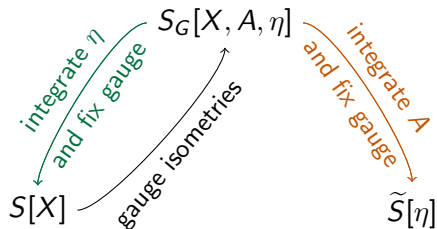
$$S[X] \xrightarrow{\text{gauge isometries}} S_G[X, A, \eta]$$

T-duality



Varying the Lagrange multipliers forces the field strength F to vanish. If we then fix the gauge $A = 0$ we recover the original model.

T-duality



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On the other hand, we can eliminate the non-dynamical gauge fields A , obtaining the dual sigma model.

Can we do it without isometries?

The existence of global symmetries is a very stringent requirement. A generic metric will not have any Killing vectors.

Question

Is it possible to follow the same procedure when the vector fields are not Killing vectors?

Keep in mind: Since we generically lose isometries when we perform a non-abelian T-duality, there is no known way to invert non-abelian T-duality from a gauging perspective.

Outline

- 1 Review of T-duality
- 2 (ω, ϕ) -modified gauging**

Gauging without isometry

A modified form of gauging was introduced by Chatzistavrakidis, Deser, Jonke, and Strobl, based on earlier work of Kotov and Strobl.

They introduce matrix-valued one-forms ω_a^b and ϕ_a^b satisfying

$$\begin{aligned}\mathcal{L}_{v_a} g &= \omega_a^b \vee \iota_{v_b} g - \phi_a^b \vee \iota_{v_b} B \\ \mathcal{L}_{v_a} B &= \omega_a^b \wedge \iota_{v_b} B - \phi_a^b \wedge \iota_{v_b} g\end{aligned}$$

These conditions are the **modified Killing equations**.

The gauged action and modified gauge invariance

The gauged action is the same:

$$S_G^{(\omega, \phi)} = \int_{\Sigma} g_{ij} DX^i \wedge \star DX^j + \int_{\Sigma} B_{ij} DX^i \wedge DX^j$$

The gauged action is invariant under the following (local) gauge transformations:

$$\delta_{\epsilon} X^i = v_a^i \epsilon^a$$

$$\delta_{\epsilon} A^a = d\epsilon^a + C_{bc}^a A^b \epsilon^c + \omega_{bi}^a \epsilon^b DX^i + \phi_{bi}^a \epsilon^b \star DX^i.$$

The field strength

In order to have any chance of being covariant, the field strength must have the form:

$$F_{(\omega, \phi)}^a = dA^a + \frac{1}{2} C_{bc}^a(X) A^b \wedge A^c - \omega_{bi}^a A^b \wedge DX^i - \phi_{bi}^a A^b \wedge \star DX^i$$

Constraints

There are two conditions that place constraints on the allowed (ω, ϕ) :

- The gauge algebra should close on the space of fields.
- F^a should transform covariantly.
 - This is to ensure that the $\int_{\Sigma} \eta_a F^a$ term is gauge invariant

Satisfying these constraints is non-trivial. For $\phi = 0$, they imply that the sigma model has isometries, and the entire procedure is equivalent to non-abelian T-duality.¹

¹[1705.09254] - P. Bouwknegt, M.B., C. Klimcik, K. Wright.

Gauging Poisson-Lie T-duality

If a sigma model possesses the Poisson-Lie symmetry:

$$\mathcal{L}_{v_a}(g + B)_{ij} = \tilde{C}_a^{bc} v_b^m v_c^n (g + B)_{mj} (g + B)_{in}.$$

Then we can choose²

$$\omega_{ai}^b = \frac{1}{2} \tilde{C}_a^{bc} v_c^n E_{in}$$

$$\phi_{ai}^b = -\omega_{ai}^b$$

and the modified Killing equations become the Poisson-Lie symmetry conditions.

²This choice is not unique!

So close, yet so far...

Does this choice satisfy the constraints?

- Closure of gauge algebra
- Covariant F

So close, yet so far...



Does this choice satisfy the constraints?

- Closure of gauge algebra
- Covariant F



So close, yet so far...

Does this choice satisfy the constraints?

- Closure of gauge algebra 
- Covariant F 

So close, yet so far...

Does this choice satisfy the constraints?

- Closure of gauge algebra ✓
- Covariant F ✗

So we can gauge the Poisson-Lie symmetry with *this choice* of (ω, ϕ) , but we can't use it to perform T-duality.

Question

Is there a choice of (ω, ϕ) reproducing the PL symmetry conditions, which satisfies both constraints?

Answer: I don't know, but I'm working on it!

If there is, then we can provide a gauging derivation of Poisson-Lie T-duality. In particular, it could be used to invert non-abelian T-duality via the Buscher procedure.

Thanks for listening!