Attempts to invert non-abelian T-duality: a gauging approach

Mark Bugden

Mathematical Sciences Institute
Australian National University

Dualities and Generalized Geometries
Corfu
9-16 September 2018

Joint work with P. Bouwknegt and K. Wright

Outline

1. Review of T-duality
2. (ω, ϕ)-modified gauging

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Setting up notation

Consider a non-linear sigma model $X : \Sigma \to M$ described by the following action:

$$S = \int_\Sigma g_{ij}\, dX^i \wedge \star dX^j + \int_\Sigma B_{ij}\, dX^i \wedge dX^j$$

In this talk we will ignore the dilaton, and assume that both g and B are globally defined fields on M.

Gauging isometries

Suppose now that there are vector fields generating the following global symmetry:

$$\delta_\epsilon X^i = v_a^i \epsilon^a$$

for ϵ^a constant. The sigma model action is invariant under this transformation if

$$\mathcal{L}_{v_a} g = 0 \qquad \mathcal{L}_{v_a} B = 0$$

If this is the case, we can gauge the model by promoting the global symmetry to a local one.

The gauged action

Introducing gauge fields A^a and Lagrange multipliers η_a, the gauged action is

$$S_G = \int_\Sigma g_{ij}\, DX^i \wedge \star DX^j + \int_\Sigma B_{ij}\, DX^i \wedge DX^j + \int_\Sigma \eta_a F^a$$

where

- $F = dA + A \wedge A$ is the standard Yang-Mills field strength
- $DX^i = dX^i - v_a^i A^a$ are the gauge covariant derivatives.

Gauge invariance

The gauged action is invariant with respect to the following (local) gauge transformations:

$$\begin{aligned}
\delta_\epsilon X^i &= v_a^i \, \epsilon^a \\
\delta_\epsilon A^a &= d\epsilon^a + C_{bc}^a A^b \epsilon^c \\
\delta_\epsilon \eta_a &= -C_{ab}^c \epsilon^b \eta_c
\end{aligned}$$

T-duality

$$
\begin{tikzcd}
 & S_G[X, A, \eta] \\
S[X] \arrow[ur, bend right, "\text{gauge isometries}"'] &
\end{tikzcd}
$$

T-duality

$$
\begin{tikzcd}[row sep=large, column sep=large]
 & S_G[X, A, \eta] \arrow[dl, bend right, "\text{integrate } \eta \text{ and fix gauge}"'] \\
S[X] \arrow[ur, bend right, "\text{gauge isometries}"'] &
\end{tikzcd}
$$

Varying the Lagrange multipliers forces the field strength F to vanish. If we then fix the gauge $A = 0$ we recover the original model.

T-duality

$$\begin{tikzcd}
 & S_G[X, A, \eta] \arrow[dl, bend right, "\text{integrate } \eta \text{ and fix gauge}"'] \arrow[dr, bend left, "\text{integrate } A \text{ and fix gauge}"] & \\
S[X] \arrow[ur, bend right, "\text{gauge isometries}"'] & & \widetilde{S}[\eta]
\end{tikzcd}$$

Varying the Lagrange multipliers forces the field strength F to vanish. If we then fix the gauge $A = 0$ we recover the original model.

On the other hand, we can eliminate the non-dynamical gauge fields A, obtaining the dual sigma model.

Can we do it without isometries?

The existence of global symmetries is a very stringent requirement. A generic metric will not have any Killing vectors.

Question

Is it possible to follow the same procedure when the vector fields are not Killing vectors?

Keep in mind: Since we generically lose isometries when we perform a non-abelian T-duality, there is no known way to invert non-abelian T-duality from a gauging perspective.

Outline

1. Review of T-duality

2. (ω, ϕ)-modified gauging

Gauging without isometry

A modified form of gauging was introduced by Chatzistavrakidis, Deser, Jonke, and Strobl, based on earlier work of Kotov and Strobl.

They introduce matrix-valued one-forms ω_a^b and ϕ_a^b satisfying

$$\mathcal{L}_{v_a} g = \omega_a^b \vee \iota_{v_b} g - \phi_a^b \vee \iota_{v_b} B$$
$$\mathcal{L}_{v_a} B = \omega_a^b \wedge \iota_{v_b} B - \phi_a^b \wedge \iota_{v_b} g$$

These conditions are the **modified Killing equations.**

The gauged action and modified gauge invariance

The gauged action is the same:

$$S_G^{(\omega,\phi)} = \int_\Sigma g_{ij}\, DX^i \wedge \star DX^j + \int_\Sigma B_{ij}\, DX^i \wedge DX^j$$

The gauged action is invariant under the following (local) gauge transformations:

$$\begin{aligned}\delta_\epsilon X^i &= v_a^i\, \epsilon^a \\ \delta_\epsilon A^a &= d\epsilon^a + C^a_{bc} A^b \epsilon^c + \omega^a_{bi} \epsilon^b DX^i + \phi^a_{bi} \epsilon^b \star DX^i .\end{aligned}$$

The field strength

In order to have any chance of being covariant, the field strength must have the form:

$$F^a_{(\omega,\phi)} = dA^a + \frac{1}{2} C^a_{bc}(X) A^b \wedge A^c - \omega^a_{bi} A^b \wedge DX^i - \phi^a_{bi} A^b \wedge \star DX^i$$

Constraints

There are two conditions that place constraints on the allowed (ω, ϕ):

- The gauge algebra should close on the space of fields.
- F^a should transform covariantly.
  - This is to ensure that the $\int_\Sigma \eta_a F^a$ term is gauge invariant

Satisfying these constraints is non-trivial. For $\phi = 0$, they imply that the sigma model has isometries, and the entire procedure is equivalent to non-abelian T-duality.[1]

[1] [1705.09254] - P. Bouwknegt, M.B., C. Klimcik, K. Wright.

Gauging Poisson-Lie T-duality

If a sigma model possesses the Poisson-Lie symmetry:

$$\mathcal{L}_{v_a}(g+B)_{ij} = \widetilde{C}_a^{bc} v_b^m v_c^n (g+B)_{mj}(g+B)_{in}.$$

Then we can choose[2]

$$\omega_{ai}^b = \frac{1}{2}\widetilde{C}_a^{bc} v_c^n E_{in}$$

$$\phi_{ai}^b = -\omega_{ai}^b$$

and the modified Killing equations become the Poisson-Lie symmetry conditions.

[2] This choice is not unique!

So close, yet so far...

Does this choice satisfy the constraints?

- Closure of gauge algebra
- Covariant F

So close, yet so far...

Does this choice satisfy the constraints?

- Closure of gauge algebra ✓
- Covariant F

So close, yet so far...

Does this choice satisfy the constraints?

- Closure of gauge algebra ✓
- Covariant F ✗

So close, yet so far...

Does this choice satisfy the constraints?

- Closure of gauge algebra ✓
- Covariant F ✗

So we can gauge the Poisson-Lie symmetry with *this choice* of (ω, ϕ), but we can't use it to perform T-duality.

Question

Is there a choice of (ω, ϕ) reproducing the PL symmetry conditions, which satisfies both constraints?

Answer: I don't know, but I'm working on it!

If there is, then we can provide a gauging derivation of Poisson-Lie T-duality. In particular, it could be used to invert non-abelian T-duality via the Buscher procedure.

Thanks for listening!