Kappa symmetry, generalized supergravity equations and non-abelian T-duality

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Kappa symmetry

- Spacetime SUSY (aka Green-Schwarz) formulation of strings and branes characterized by w.s. fermionic gauge symmetry:

$$\delta X^m \sim 0\,, \qquad \delta\theta \sim \frac{1}{2}(1+\Gamma)\kappa$$

  κ – arbitrary spinor
  $\frac{1}{2}(1+\Gamma)$ – projector on half of the components of κ
- Kappa symmetry needed for correct # d.o.f. for SUSY spectrum
- In flat space (w/ other bkgr fields vanishing) kappa symmetry is automatic
- But coupling the string/brane to non-trivial background kappa symmetry restricts the form of bkgr to which the string/brane can be consistently coupled

- In 1985 Witten considered the superparticle coupled to bkgr gauge field A
- He showed that kappa symmetry $\rightarrow$ SYM e.o.m. for A
- Suggested that similarly kappa sym. of the string might imply the SUGRA e.o.m.
- Superstring action in general type II SUGRA bkgr written down shortly after [Grisaru, Howe, Mezincescu, Nilsson, Townsend]
- They showed that bkgr being SUGRA sol. was sufficient to have kappa symmetry
- Conjectured that it was also necessary, i.e. kappa sym. $\rightarrow$ SUGRA e.o.m.
- More or less assumed to be true since then ('proofs' even appeared)

η-model puzzle

Fast-forward to 2013

- Delduc, Magro and Vicedo wrote down a certain (integrable) deformation of the $AdS_5 \times S^5$ GS string σ-model – the η-model
- This model had a fermionic gauge symmetry similar to kappa symmetry
- However, when the bkgr fields were extracted from the σ-model they were found not to satisfy the SUGRA eom

  [Arutyunov,Borsato,Frolov]

- Seemed to contradict the expectation that kappa sym. $\rightarrow$ SUGRA e.o.m.

Kappa symmetry $\rightarrow$ SUGRA eom?

Given this puzzle we decided to revisit the question: Does kappa sym. $\rightarrow$ SUGRA eom?

Starting point: GS string action in general type II SUGRA bkgr

$$S = T\int d^2\xi\,\sqrt{-\det G_{ij}} + T\int_\Sigma B\,, \qquad G_{ij} = E_i{}^a E_j{}^b \eta_{ab}$$

$E^a(x,\theta)$ ($a = 0,\ldots,9$) – supervielbein one-forms

$B(x,\theta)$ – NSNS two-form potential

Requiring kappa symmetry constrains the field strengths of E^a and B:

Torsion: $T^a = dE^a + E^b \wedge \Omega_b{}^a$

Three-form field strength: $H = dB$

Conditions from kappa symmetry

Requiring $\delta_\kappa S = 0$ one finds the following conditions

$$T_{\alpha\beta}{}^{a} = -i\Gamma^{a}_{\alpha\beta}$$

$$H_{\alpha\beta\gamma} = 0\,, \qquad H_{a\beta\gamma} = -i(\Gamma_a\Gamma_{11})_{\beta\gamma}$$

in agreement with earlier analysis.

However, T^a and H satisfy Bianchi identities

$$\nabla T^A = E^B \wedge R_B{}^A\,, \qquad dH = 0$$

and further constraints arise from consistency with these

It was expected that this would lead to the type II SUGRA e.o.m.

Instead we found that it leads to a certain generalization of these – The generalized SUGRA equations

Generalized SUGRA equations

Besides the metric and B-field entering directly the GS string action there are two vector fields K, X plus RR forms $\mathcal{F}_n$

They satisfy [Tseytlin, LW]

$$\nabla_{(a}K_{b)} = 0\,, \qquad K^a X_a = 0\,, \qquad 2\nabla_{[a}X_{b]} + K^c H_{abc} = 0\,,$$

i.e. K – Killing vector and $K = 0$ implies $X = d\phi$

$$R_{ab} + 2\nabla_{(a}X_{b)} - \tfrac{1}{4}H_{acd}H_b{}^{cd} + \text{"}\mathcal{F}^2\text{"} = 0$$
$$\nabla^c H_{abc} - 2X^c H_{abc} - 4\nabla_{[a}K_{b]} + \text{"}\mathcal{F}^2\text{"} = 0$$
$$\nabla^a X_a - 2X^a X_a - 2K^a K_a + \tfrac{1}{12}H^{abc}H_{abc} + \text{"}\mathcal{F}^2\text{"} = 0$$

plus equations for the RR fields.

In fact (the bosonic part of) these eqs were written down 6 months earlier as the eqs satisfied by the bkgr fields of the η-model [Arutyunov,Frolov,Hoare,Roiban,Tseytlin]

They also argued that these should be the conditions for 1-loop scale inv. of the sigma model (as opposed to Weyl inv. which gives standard SUGRA)

- Easy to see that setting $K = 0$ leads to standard SUGRA with $X = d\phi$ and ϕ the dilaton
- In fact one can show that (formal) T-duality along isometry K gives a standard type II SUGRA sol. [AFHRT]

It is interesting to ask whether solutions with $K \neq 0$ can also directly solve the standard SUGRA eqs

Trivial solutions

To address this question we can pick a gauge s.t. the B-field respects the K isometry

$$\mathcal{L}_K B = 0$$

Then

$$dX + i_K H = 0 \qquad \rightarrow \qquad X = d\phi + i_K B$$

Crucial point: ϕ transforms under gauge transformations $B \rightarrow B + d\Lambda$

However, to interpret ϕ as the dilaton it must not transform

Gauge choice for B preserved by $\Lambda = fK$ with f an arbitrary isometric function

$$\mathcal{L}_K f = K^m \partial_m f = 0$$

Requiring ϕ not to transform we find

$$0 = i_K \delta B = i_K d(fK) = -d(fK^2)$$

Must hold for all f and therefore we must have

$$K^2 = 0$$

The remaining equations imply also $dK = i_K H$ + conditions on the RR forms [LW]

We refer to such solutions as trivial since they actually solve standard SUGRA

There are several known examples the simplest being pp-waves

Non-abelian T-duality

There is an interesting connection to NATD

- In NATD one realizes a subset of coords. as group element $g \in G$
- Then in the action one replaces $g^{-1}dg \to A$, adds $\nu F(A)$ and integrates out A
- At the quantum level the change of variables $g \to A$ gives rise to a Jacobian from the path integral measure

$\to$ Anomalous term in σ-model action if Lie(G) is not unimodular, i.e. $f_{IJ}^{I} \neq 0$ [Alvarez,Alvarez-Gaume,Lozano; Elitzur,Giveon,Rabinovici,Schwimmer,Veneziano]

- In fact one finds in such cases a bkgr that solves instead the gen. SUGRA equations [Borsato,LW]

Geometric form for anomalous terms

The anomalous non-local terms in the σ-model action can be given a geometric form in terms of gen. SUGRA fields [LW]

$$L_{\rm non-local} = \alpha' d\sigma \wedge K - \alpha' d\sigma \wedge *X - \frac{1}{2}\alpha'^2 d\sigma \wedge *d\sigma\, K^2$$

where $\sigma = \partial^{-2}\sqrt{g}R^{(2)}$ is the conformal factor which is non-local in the ws metric

Consistency check: When $K = 0$, $X = d\phi$ this reduces to the usual (local) Fradkin-Tseytlin term

$$\alpha' \phi R^{(2)}$$

In fact precisely in the case of a trivial solution, i.e. $K^2 = 0$, $dK = i_K H$, the non-local terms can be removed by a field redefinition

Conclusion

Formalism	gen. SUGRA	SUGRA
NSR	1-loop scale inv.	1-loop Weyl inv.
GS	kappa sym.	?
PS	$Q^2 = 0$ classically	$Q^2 = 0$ at 1-loop

- Many solutions of gen. SUGRA arise from so-called Yang-Baxter deformations, which can be generated through NATD [Hoare,Tseytlin; Borsato,LW]
- Interesting connections to DFT and ExFT [Sakatani,Uehara,Yoshida; Baguet,Magro,Samtleben]