The membrane sigma model for Double Field Theory

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Based on:

1802.07003 with **A. Chatzistavrakidis, F. S. Khoo** *and* **R. J. Szabo**
and work in progress

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General motivation

- String <u>geometry</u> departs from Riemannian geometry, notably in presence of fluxes
  - open strings $\rightsquigarrow$ noncommutativity - Poisson structure - $\star$-product - Kontsevich '97 DQ
    Chu, Ho '99; Seiberg, Witten '99
  - closed strings $\rightsquigarrow$ noncommutativity/nonassociativity - (twisted) Poisson - $\star$-product
    Halmagyi '09; Lüst '10; Blumenhagen, Plauschinn '10; Mylonas, Schupp, Szabo '12; & c.
- String <u>dualities</u> relate different geometries/topologies $\rightsquigarrow$ "non-geometric backgrounds"
- Manifestly duality-invariant theories - double and exceptional field theories
  Hull, Hohm, Zwiebach; Hohm, Samtleben; & c.
- Evidence that the correct language is algebroid/generalized geometry
  Courant; Liu, Weinstein, Xu, Ševera; Roytenberg; Hitchin; Gualtieri; Cavalcanti; Bouwknegt, Hannabuss, Mathai; & c.

In this talk

- Use Courant algebroid structure to unravel geometric structure of DFT.
- Construct membrane sigma model for DFT.
- Discuss DFT membrane sigma model gauge symmetry.
- DFT constraints vs. gauge invariance.

Courant Algebroids and Double Field Theory

- Courant Algebroids and Generalized Geometry double the bundle, e.g. $TM \oplus T^*M$
- DFT doubles the base, $\mathcal{M} = M \times \widetilde{M}$ — comes with constraints

- Solving the strong constraint, reduces DFT data to the data of the standard CA
- What is the geometric origin of the DFT data and the strong constraint?
  cf. Deser, Saemann '16, Freidel, Rudolph, Svoboda '17; Svoboda '18

- CAs provide membrane sigma models $\rightsquigarrow$ describe non-geometric backgrounds
  Roytenberg '06
  Mylonas, Schupp, Szabo '12; ACh, Jonke, Lechtenfeld '15; Bessho, Heller, Ikeda, Watamura '15
- Is there a "DFT algebroid" that could provide a DFT membrane sigma model?
  cf. Fech Scen talk

Membrane Sigma Model for CA

AKSZ sigma models

AKSZ sigma model - topological sigma models satisfying the classical master equation Alexandrov, Kontsevich, A. Schwarz, Zaboronsky '97.

In 2d Poisson sigma model is most general TFT Cattaneo, Felder '01. Quantization of this model lead to Kontsevich deformation quantization formula.

In 3d the AKSZ sigma model requires a dg-manifolds for source and target, symplectic form on a target, and a self-commuting hamiltonian of degree 3.

Theorem

A QP-2 manifold is in 1:1-correspondence with a Courant algebroid. (Roytenberg '02.)

Given the data of a CA, $(E, [\cdot,\cdot]_E, \langle\cdot,\cdot\rangle_E, \rho)$, one can uniquely construct a membrane sigma model Roytenberg '06.

Courant sigma model

In local coordinates

$$S[X,A,F]=\int_{\Sigma_3} F_i\wedge \mathrm{d}X^i+\tfrac{1}{2}\eta_{IJ}A^I\wedge \mathrm{d}A^J-\rho^i{}_I(X)A^I\wedge F_i+\tfrac{1}{6}T_{IJK}(X)A^I\wedge A^J\wedge A^K\ .$$

$i=1,\dots,d$ (target space index) and $I=1,\dots,2d$ (CA index).

Maps $X=(X^i):\Sigma_3\to M$, 1-forms $A\in\Omega^1(\Sigma_3,X^*E)$, and auxiliary 2-form $F\in\Omega^2(\Sigma_3,X^*T^*M)$.
Symmetric bilinear form of the CA $\rightsquigarrow$ $O(d,d)$ invariant metric

$$\eta=(\eta_{IJ})=\begin{pmatrix}0 & 1_d\\ 1_d & 0\end{pmatrix}\ ,$$

ρ and T are the anchor and twist of the CA, the latter generating a generalized Wess-Zumino term.

For a manifold with boundaries on can add both topological and non-topological terms
Cattaneo, Felder ‘01; Park ‘00

$$S_b[X,A]=\int_{\partial\Sigma_3}\tfrac{1}{2}g_{IJ}A^I\wedge *A^J+\tfrac{1}{2}\mathcal{B}_{IJ}A^I\wedge A^J\ .$$

Gauge symmetry of Courant sigma model

Gauge invariance of the Courant sigma model $\Rightarrow$ CA axioms and properties.

The gauge transformations Ikeda '02

$$\delta_\epsilon X^i = \rho^i_J \epsilon^J \ ,$$
$$\delta_\epsilon A^I = \mathrm{d}\epsilon^I + \eta^{IN} T_{NJK} A^J \epsilon^K - \eta^{IJ} \rho^i{}_J t_i \ ,$$
$$\delta_\epsilon F_m = -\epsilon^J \partial_m \rho^i{}_J F_i + \tfrac{1}{2}\epsilon^J \partial_m T_{ILJ} A^I \wedge A^L - \mathrm{d}t_m - \partial_m \rho^j{}_J A^J t_j \ ,$$

where ϵ^I and t_i are gauge parameters - world-sheet scalar and one-form, respectively, define first-stage reducible gauge symmetry of CA sigma model.

- Gauge invariance of equations of motion

$$\delta F_i(\mathrm{d}X^m - \rho^m_J A^J) := \delta F_i DX^i = 0 \ ,$$
$$\delta_\epsilon DX^i = \epsilon^J \partial_m \rho^i{}_J DX^m + \eta^{JK} \rho^i{}_J \rho^j{}_K t_j + \epsilon^J A^K (2\rho^m{}_{[K} \partial_{\underline{m}} \rho^i{}_{J]} - \rho^i{}_N \eta^{NM} T_{MKJ}) \ .$$

gives

$$\eta^{JK} \rho^i{}_J \rho^j{}_K = 0 \ , \ \ 2\rho^m{}_{[K} \partial_{\underline{m}} \rho^i{}_{J]} - \rho^i{}_N \eta^{NM} T_{MKJ} = 0.$$

- Closure of algebra of gauge transformations

$$[\delta_1, \delta_2]A^I = \delta_{12}A^I - \eta^{IJ}\partial_i T_{JKL}\epsilon_1^K \epsilon_2^L DX^i - \eta^{IJ} Z_{KLMJ}\epsilon_1^K \epsilon_2^L A^M \ ,$$
$$\epsilon_{12}^I = \eta^{IJ} T_{JKL}\epsilon_1^K \epsilon_2^L \ , \ t_{12i} = \partial_i T_{JKL}\epsilon_1^K \epsilon_2^L A^J + 2\partial_i \rho^j{}_K \epsilon_{[1}^K t_{2]j} \ ,$$

gives

$$Z_{IJKL} := 3\eta^{MN} T_{M[JK} T_{I]LN} + 3\rho^m{}_{[I}\partial_m T_{KJ]L} + \rho^m{}_L \partial_m T_{IJK} = 0 \ .$$

Thus the gauge invariance of CA sigma model reproduces all three local coordinate expressions defining CA.

The algebra of gauge transformations closes on-shell and there exists BV-BRST formulation Ikeda '02, Roytenberg '06.

From CA to DFT


C


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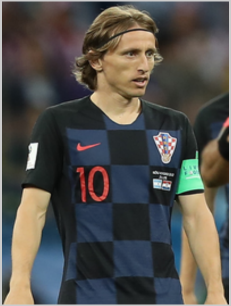
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Large CA and projection

Chatzistavrakidis, Jonke, Khoo, Szabo '18

Remember that CAs doubles the bundle, while DFT doubles the base.

Strategy:

- Double the target and construct large CA.
- Find appropriate embedding of DFT within large CA and project
  - fields A and structure functions ρ, T
  - bracket(s) and bilinear
- Define DFT algebroid structure and properties.

From CA to DFT - some details

- Doubling - take a 2nd order bundle $\mathbb{E} = (T \oplus T^*)\mathcal{M}$ with sections $\mathbb{A} \in \mathbb{E}$

$$\mathbb{A} := \mathbb{A}_V + \mathbb{A}_F = \mathbb{A}^I \partial_I + \widetilde{\mathbb{A}}_I \mathrm{d}\mathbb{X}^I \ .$$

- Embedding - $\mathbb{E} = (T \oplus T^*)\mathcal{M} = L_+ \oplus L_-$

$$\mathbb{A} = \mathbb{A}_+^I e_I^+ + \mathbb{A}_-^I e_I^- \ , \quad \text{where} \quad e_I^\pm = \partial_I \pm \eta_{IJ} \mathrm{d}\mathbb{X}^J \ ,$$

- Projecting

$$\begin{aligned} p : \mathbb{E} \quad &\to \quad L_+ \\ (\mathbb{A}_V, \mathbb{A}_F) \quad &\mapsto \quad \mathbb{A}_+ := A \ , \end{aligned}$$

Systematically project bracket, bilinear and the structure functions.

$\rightsquigarrow$ Reproduces DFT vectors, C-bracket, generalized Lie derivative with properties defining the DFT algebroid Fech Scen talk.

Membrane sigma model for DFT

The DFT Membrane Sigma Model

Using the data of DFT algebroid we propose cf. Chatzistavrakidis, Jonke, Lechtenfeld '15

$$S[\mathbb{X}, A, F] = \int_{\Sigma_3} \left(F_I \wedge \mathrm{d}\mathbb{X}^I + \eta_{IJ} A^I \wedge \mathrm{d}A^J - (\rho_+)^I{}_J A^J \wedge F_I + \tfrac{1}{3} \widehat{T}_{IJK} A^I \wedge A^J \wedge A^K \right) \ ,$$

where $\rho_+ : L_+ \to T\mathcal{M}$ is a map to the tangent bundle and $\widehat{T}$ corresponds to DFT fluxes.

Maps $\mathbb{X} = (\mathbb{X}^I) : \Sigma_3 \to \mathcal{M}$, 1-forms $A \in \Omega^1(\Sigma_3, \mathbb{X}^* L_+)$, and auxiliary 2-form $F \in \Omega^2(\Sigma_3, \mathbb{X}^* T^* \mathcal{M})$.

Symmetric bilinear form η is $O(d, d)$ invariant metric.

The DFT Membrane Sigma Model

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Use the DFT MSM to

- describe the flux backgrounds
- find the relation with flux formulation of DFT

Universal description of flux backgrounds

The standard T-duality chain relating geometric and non-geometric fluxes schematically through Shelton, Taylor, Wecht '05

$$H_{ijk} \xleftrightarrow{\mathrm{T}_k} f_{ij}{}^k \xleftrightarrow{\mathrm{T}_j} Q_i{}^{jk} \xleftrightarrow{\mathrm{T}_i} R^{ijk} \ ,$$

can be obtained using DFT membrane action.

Take a doubled torus as target of the DFT MSM and DFT structural data as $(\rho_+)^I{}_J = (\rho^i{}_j, \rho^{ij}, \rho_i{}^j, \rho_{ij})$, $T_{IJK} = (H_{ijk}, f_{ij}{}^k, Q_i{}^{jk}, R^{ijk})$ and symmetric term on boundary $g_{IJ} = (g_{ij}, g_i{}^j, g^i{}_j, g^{ij})$.

R flux background

Choose

$$(\rho_+)^I{}_J = (0, 0, \delta_i{}^j, 0) \qquad T_{IJK} = (0, 0, 0, R^{ijk}) \qquad g_{IJ} = (g_{ij}, 0, 0, 0) \ ,$$

Then the membrane action becomes

$$\begin{aligned} S &= \int_{\Sigma_3} \big(F_I \wedge \mathrm{d}\mathbb{X}^I + q^i \wedge \mathrm{d}p_i + p_i \wedge \mathrm{d}q^i - p_i \wedge F^i + \tfrac{1}{6}\, R^{ijk}\, p_i \wedge p_j \wedge p_k \big) \\ &+ \int_{\partial\Sigma_3} \tfrac{1}{2}\, g_{ij}\, q^i \wedge * q^j \ . \end{aligned}$$

After integrating out the auxiliary fields F_I the action takes form

$$\int_{\partial\Sigma_3} \big(q^i \wedge \mathrm{d}\widetilde{X}_i + \tfrac{1}{2}\, g_{ij}\, q^i \wedge * q^j \big) + \int_{\Sigma_3} \tfrac{1}{6}\, R^{ijk}\, \mathrm{d}\widetilde{X}_i \wedge \mathrm{d}\widetilde{X}_j \wedge \mathrm{d}\widetilde{X}_k \ .$$

Finally, after integrating out pq^i we obtain

$$S_R[\widetilde{X}\,] = \int_{\partial\Sigma_3} \tfrac{1}{2}\, g^{ij}\, \mathrm{d}\widetilde{X}_i \wedge * \mathrm{d}\widetilde{X}_j + \int_{\Sigma_3} \tfrac{1}{6}\, R^{ijk}\, \mathrm{d}\widetilde{X}_i \wedge \mathrm{d}\widetilde{X}_j \wedge \mathrm{d}\widetilde{X}_k \ ,$$

is the same as the sigma-model action with H-flux under the duality exchanges of all fields X^i with $\widetilde{X}_i$.

R flux and nonassociativity

On the other hand, choosing

$$(\rho_+)^I{}_J = (\delta^i{}_j, R^{ijk}\,\widetilde{X}_k, -\delta_i{}^j, 0) \qquad T_{IJK} = (0,0,0,R^{ijk}) \qquad g_{IJ} = (0,0,0,g^{ij}) \ .$$

leads to Mylonas, Schupp, Szabo '12

$$S_R[X,\widetilde{X}\,] = \int_{\partial\Sigma_3} \left(\mathrm{d}\widetilde{X}_i \wedge \mathrm{d}X^i + \tfrac{1}{2}\,R^{ijk}\,\widetilde{X}_k\,\mathrm{d}\widetilde{X}_i \wedge \mathrm{d}\widetilde{X}_j + \tfrac{1}{2}\,g^{ij}\,\mathrm{d}\widetilde{X}_i \wedge *\mathrm{d}\widetilde{X}_j\right) \ ,$$

One defines a bivector $\Theta = \frac{1}{2}\,\Theta^{IJ}\,\partial_I \wedge \partial_J$ on phase space T^*M given by

$$\Theta^{IJ} = \begin{pmatrix} R^{ijk}\,\widetilde{X}_k & \delta^i{}_j \\ -\delta_i{}^j & 0 \end{pmatrix} \ .$$

It induces a twisted Poisson bracket given by $\{\mathbb{X}^I, \mathbb{X}^J\}_\Theta = \Theta^{IJ}$, with non-vanishing Jacobiator

$$\{X^i, X^j, X^k\}_\Theta := \tfrac{1}{3}\,\{\{X^i, X^j\}_\Theta, X^k\}_\Theta + \text{cyclic} = R^{ijk} \ .$$

$\rightsquigarrow$ In terms of the doubled space of DFT, the four T-dual backgrounds with H-, f-, Q- and R-flux all correspond to the standard Courant algebroid over different submanifolds of the doubled space.

$\rightsquigarrow$ Nonassociative (nor noncommutative) background violate the strong constraint of DFT and therefore do not correspond to Courant sigma-models.

DFT fluxes from membrane sigma model

Taking a parametrization of the map ρ_+ to be

$$(\rho_+)^I{}_J = \begin{pmatrix} \delta^i{}_j & \beta^{ij} \\ B_{ij} & \delta_i{}^j + \beta^{jk} B_{ki} \end{pmatrix} ,$$

we obtain

$$\begin{aligned} &\eta^{IJ} \rho^K{}_I \rho^L{}_J = \eta^{KL} \\ &2\rho^L{}_{[I} \partial_{\underline{L}} \rho^K{}_{J]} - \rho^K{}_J \eta^{JL} \hat{T}_{LIJ} = \rho_{L[I} \partial^K \rho^L{}_{J]} \ . \end{aligned}$$

Thus we make contact with flux formulation of DFT Geissbuhler, Marques, Nunez, Penas '13:

$$\begin{aligned} &\mathcal{E}_A{}^M \mathcal{E}_B{}^N \eta_{MN} = \eta_{AB} \ , \\ &\hat{T}_{ABC} = 3\mathcal{E}^I{}_{[A} \partial_{\underline{I}} \mathcal{E}_B{}^J \mathcal{E}_{C]J} \ . \end{aligned}$$

The parametrization of the anchor map in terms of the generalized bein components has an important consequence:

- On algebroid structure - the anchor map ρ_+ has no kernel.
- On DFT - constraining bein to be an element of $O(D, D)$ is the source of the strong constraint. cf. Fech Scen talk

Gauge invariance of DFT MSM and the strong constraint

Gauge transformations obtained by projection $\{\mathbb{A},\epsilon\} \to \{A,\epsilon\}$ and gauge-fixing $t_I = \Theta_{IJK}(\mathbb{X})A^J\epsilon^K$.

$\Theta_{IJK}(\mathbb{X})$ is determined in terms of the structure functions of large CA by imposing $\delta A^I_- = 0$. .

$$
\begin{aligned}
&\delta_\epsilon \mathbb{X}^I = \rho^I{}_J\epsilon^J \ , \\
&\delta_\epsilon A^I = \mathrm{d}\epsilon^I + \eta^{IJ}(\hat{T}_{JKL} - \rho^M{}_J\Theta_{MKL})A^K\epsilon^L = \mathrm{d}\epsilon^I + \Phi^I{}_{KL}A^K\epsilon^L \ , \\
&\delta_\epsilon F_I = \Theta_{IJK}A^J\wedge \mathrm{d}\epsilon^K - \epsilon^K F_N(\partial_I\rho^N{}_K + \tfrac{1}{2}\Theta_{IJK}\eta^{JM}\rho^N{}_M) + \\
&+\epsilon^K A^N\wedge A^J(\partial_I\hat{T}_{KNJ} - \rho^L{}_N\partial_L\Theta_{IJK} - \partial_I\rho^M{}_N\Theta_{MJK} + \tfrac{1}{2}\Theta_{ILK}\eta^{LM}\hat{T}_{MNJ}) - \\
&-\partial_L\Theta_{IJK}\epsilon^K D\mathbb{X}^L\wedge A^J - \Theta_{IJK}\epsilon^K(\mathrm{d}A^J - \tfrac{1}{2}\eta^{JK}\rho^I{}_K F_I + \tfrac{1}{2}\eta^{JK}\hat{T}_{KLM}A^L\wedge A^M) \ .
\end{aligned}
$$

Note that variation of F_I now contains terms proportional to eoms, i.e., the trivial gauge transformations.

Gauge invariance continued

- Gauge invariance of equations of motion

$$\delta_\epsilon D\mathbb{X}^I = \epsilon^J \partial_K \rho^I{}_J DX^K + \epsilon^J A^K (2\rho^M{}_{[K}\partial_{\underline{M}}\rho^I{}_{J]} - \rho^I{}_N \Phi^N{}_{KJ}) \ .$$

gives

$$2\rho^M{}_{[K}\partial_{\underline{M}}\rho^I{}_{J]} - \rho^I{}_N \Phi^N{}_{KJ} = 0.$$

- Closure of algebra of gauge transformations

$$\begin{aligned}
&[\delta_1, \delta_2]A^I = \delta_{12}A^I - \partial_J \Phi^I{}_{KL}\epsilon_1^K \epsilon_2^L D\mathbb{X}^J - \eta^{IJ}\mathcal{Z}_{KLMJ}\epsilon_1^K \epsilon_2^L A^M \ , \\
&\epsilon_{12}^I = \Phi^I{}_{KL}\epsilon_1^K \epsilon_2^L \ , \\
&\mathcal{Z}_{IJKL}|_{s.c.} = \eta^{MN} T_{M[JK} T_{I]LN} + \rho^M{}_{[I}\partial_M T_{KJ]L} + \tfrac{1}{3}\rho^M{}_L \partial_M T_{IJK} \ .
\end{aligned}$$

$\rightsquigarrow$ Strong constraint needed for gauge invariance of DFT sigma model and for (on-shell) closure of gauge algebra.

Closing remarks

- We proposed membrane sigma model for DFT, gauge invariant under the strong constraint. It provides universal description of geometric and non-geometric fluxes.

- What about NC and NA examples, which do not satisfy the strong constraint? Can we find gauge invariant sigma model formulation for these cases?

  $\rightsquigarrow$ Relaxing the strong constraint by using pre-DFT (or metric Vaisman '12) algebroid structure. Can one formulate membrane sigma model for these structures?

  $\rightsquigarrow$ Changing the embedding - project to subbundle $\tilde{L}_+$ span by $\tilde{e}_I^+ = \partial_I \pm \lambda_{IJ} \mathrm{d}\mathbb{X}^J$?

  $\rightsquigarrow$ Role of the boundary terms and symplectic form? Freidel, Rudolph, Svoboda '17

- Applications - physical models where one truly needs doubled space, e.g. string gas cosmology by Brandenberger and Vafa?