Generalized Geometry, A/B-models and topological M-theory

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based on 1802.04581 and 1805.11485
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Outline

We reformulate AKSZ constructions of topological sigma-models (in topological string theory) on doubled space and relate them to generalized complex structures.

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We describe the known membrane theories related to topological M-theory with an AKSZ 3-brane, which has the higher Courant bracket on $T \oplus \bigwedge^2 T^*$ as its underlying derived bracket.

[1805.11485]

Topological string theory

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- Where does it appear in physical string theory?

a) IIA or IIB compactifications $\rightsquigarrow$ superpotential

b) IIA compactification $\rightsquigarrow$ entropy of BPS black hole

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- The space of fields is the mapping space

$$\boldsymbol{\mathcal{M}} = \mathrm{Map}\big(T[1]\Sigma_d\,,\,\mathcal{M}\big)$$

$\Longrightarrow$ An arbitrary coordinate $\phi \in \mathcal{M}$ of degree $|\phi|$ corresponds to a field $\boldsymbol{\phi} \in \boldsymbol{\mathcal{M}}$ of ghost number $|\phi|$

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- Gauge fixing:

  1) Assign 'fields' and 'antifields' (paired in the BV symplectic srtucture $\boldsymbol{\omega}$)
  2) Gauge fix the antifields $\longleftrightarrow$ Choose a Lagrangian submanifold $\boldsymbol{\mathcal{L}} \in \boldsymbol{\mathcal{M}}$ $(\boldsymbol{\omega}|_{\boldsymbol{\mathcal{L}}} = 0)$.

A- and B-models

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[Ikeda, Tokunaga]

Doubled Poisson sigma-model for A/B-models

- Poisson sigma-model on doubled targetspace $\mathcal{M} = T^*[1]T^*M$

$$\omega = \mathrm{d}\underset{1}{\chi_I} \wedge \mathrm{d}\underset{0}{X^I} \qquad \text{and} \qquad \gamma = \frac{1}{2}\,\Omega^{IJ}(X)\,\chi_I\,\chi_J$$

$$\text{with} \qquad X^I = \begin{pmatrix} X^i \\ \widetilde{X}_i \end{pmatrix} \qquad \text{and} \qquad \chi_I = \begin{pmatrix} \chi_i \\ \widetilde{\chi}^i \end{pmatrix}$$

$$\mathcal{S}^{(2)}_{\Omega} = \int_{T[1]\Sigma_2} \left(\boldsymbol{\chi}_I\,\boldsymbol{D}\boldsymbol{X}^I + \frac{1}{2}\,\boldsymbol{\Omega}^{IJ}(\boldsymbol{X})\,\boldsymbol{\chi}_I\,\boldsymbol{\chi}_J \right) .$$

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- Observation: it gives the AKSZ formulation of A- or B-models

$$\Omega^{IJ} = \begin{pmatrix} \pi^{ij} & 0 \\ 0 & 0 \end{pmatrix} \qquad \Longrightarrow \qquad \text{A-model}$$

$$\Omega^{IJ} = \begin{pmatrix} 0 & J^i{}_j \\ -J^j{}_i & -2\partial_{[i}J^k{}_{j]}\widetilde{X}_k \end{pmatrix} \qquad \Longrightarrow \qquad \text{B-model}$$

AKSZ membrane formulation for A/B-models

- We lift up the doubled Poisson sigma-model to an AKSZ membrane (contravariant Courant sigma-model [Bessho,Heller,Ikeda,Watamura] on $\mathcal{M} = T^*[2]T[1]T^*M$)

$$\omega = \mathrm{d}\underset{2}{F_I} \wedge \mathrm{d}\underset{0}{X^I} + \mathrm{d}\underset{1}{\chi_I} \wedge \mathrm{d}\underset{1}{\psi^I}$$

$$\boldsymbol{\mathcal{S}}^{(3)}_{\Omega,\mathcal{R}} = \int_{T[1]\Sigma_3} \Big(\boldsymbol{F}_I\, \boldsymbol{D}\boldsymbol{X}^I - \boldsymbol{\chi}_I\, \boldsymbol{D}\boldsymbol{\psi}^I + \boldsymbol{\Omega}^{IJ}\, \boldsymbol{F}_I\, \boldsymbol{\chi}_J - \frac{1}{2}\, \boldsymbol{\partial}_I \boldsymbol{\Omega}^{JK}\, \boldsymbol{\psi}^I\, \boldsymbol{\chi}_J\, \boldsymbol{\chi}_K + \frac{1}{3!}\, \boldsymbol{\mathcal{R}}^{IJK}\, \boldsymbol{\chi}_I\, \boldsymbol{\chi}_J\, \boldsymbol{\chi}_K \Big)$$

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- It allows the definition of fluxes

$$\mathcal{R}^{IJK} \quad \longrightarrow \quad H_{ijk},\ F^i{}_{jk},\ Q^{ij}{}_k,\ R^{ijk}$$

and the master equation gives them Bianchi identities $[\Omega, \mathcal{R}]_{\mathrm{S}} = 0$.

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it gives back the doubled Poisson sigma-model.

Relation to generalized complex structure and topological S-duality

Halving degree 1 fields with DFT projection [Chatzistavrakidis,Jonke,Khoo,Szabo] and imposing the master equation (it has the role of a section condition) we get the Courant sigma-model

$$\mathcal{S}^{(3)}_{\pi,J} = \int_{T[1]\Sigma_3} \Big(F_i\, DX^i - \chi_i\, D\psi^i + \pi^{ij}\, F_i\, \chi_j + J^i{}_j\, F_i\, \psi^j - \frac{1}{2}\, \partial_i \pi^{jk}\, \psi^i\, \chi_j\, \chi_k + \partial_i J^k{}_j\, \psi^i\, \psi^j\, \chi_k \Big)$$

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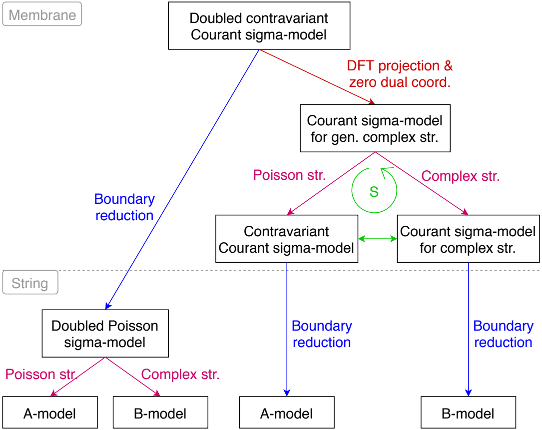
- Topological S-duality between A- and B-models arises ($g_{\mathrm{A}} \longleftrightarrow 1/g_{\mathrm{B}}$) from $\mathbb{J}$, as a canonical transformation:

$$\mathcal{S}^{(2)}_{\mathrm{B}} \longleftarrow \mathcal{S}^{(3)}_{\pi,J} \longrightarrow \mathcal{S}^{(2)}_{\mathrm{A}}$$

Different reductions and connections between the AKSZ string and membrane sigma-models related to the topological A- and B-models



Topological membranes on G_2-manifold

- Topological M-theory: Hitchin-type form theory of G_2-manifolds

  It is a unification of the form theory of A- and B-models.

Topological membranes on G_2-manifold

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- They can be unified within one single AKSZ 3-brane ($\mathcal{M} = T^*[3]T[1]M$)

$$\omega = \mathrm{d}\underset{0}{X^i} \wedge \mathrm{d}\underset{3}{F_i} + \mathrm{d}\underset{2}{\chi_i} \wedge \mathrm{d}\underset{1}{\psi^i} \qquad \text{and} \qquad \gamma = F_i \psi^i$$

$$\boldsymbol{\mathcal{S}}^{(4)}_{G_2} = \int_{T[1]\Sigma_4} \left(\boldsymbol{F}_i \boldsymbol{D}\boldsymbol{X}^i + \boldsymbol{\psi}^i \boldsymbol{D}\boldsymbol{\chi}_i + \boldsymbol{F}_i \boldsymbol{\psi}^i \right)$$

Worldvolume dimension reductions of $\boldsymbol{\mathcal{S}}^{(4)}_{G_2}$ gives both topological membranes in different gauges.

Relation to higher Courant bracket

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$$A^i(X)\chi_i + \frac{1}{2}\,\alpha_{ij}(X)\,\psi^i\psi^j \qquad \longleftrightarrow \qquad A^i(X)\frac{\partial}{\partial X^i} + \frac{1}{2}\,\alpha_{ij}(X)\,\mathrm{d}X^i \wedge \mathrm{d}X^j\ ,$$

and the derived bracket

$$\begin{aligned}[][\,A+\alpha\,,\,B+\beta\,]_C &= \{\{\,\gamma\,,\,A+\alpha\,\}\,,\,B+\beta\,\} \\ &= [A,B]\,+\,\mathcal{L}_A\beta\,-\,\mathcal{L}_B\alpha\,+\,\frac{1}{2}\,\mathrm{d}(\iota_B\,\alpha-\iota_A\,\beta)\end{aligned}$$

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- This construction allows the definition of geometric fluxes.

Summary

- We reformulated the AKSZ constructions of A- and B-models on doubled space and introduced a Courant sigma-model for generalized complex structure, which reduces to the A- and B-models on the boundary.
- As an application, we derived topological S-duality from generalized complex structure.
- Our approach led to new classes of Courant algebroids associated to (generalized) complex geometry.
- We constructed an AKSZ 3-brane, which unifies the known topological membranes on G_2-manifold.
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Outlook

- Explore the relation of our Courant sigma-model for generalized complex structure to topological string theory.
- Investigate the AKSZ 3-brane theory in the context of non-geometric flux backgrounds.

Thank you for your attention!