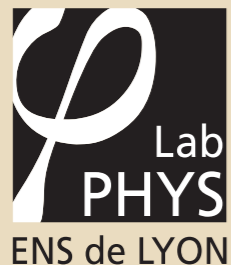

Introduction to exceptional field theory II

— towards affine algebras —

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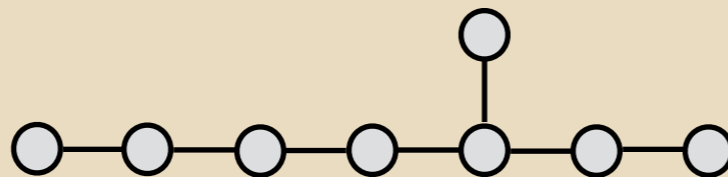


motivation

structure of exceptional field theory (ExFT)

- ▶ $E_{d(d)}$ generalized diffeomorphisms
- ▶ unique invariant two-derivative actions
- ▶ reproduce the bosonic sector of maximal supergravity
- ▶ powerful tool for construction of vacua and consistent truncations
- ▶ unlike DFT: based on a split external / internal coordinates $\{x^\mu, Y^M\}$
 $\mu = 1, \dots, D$
- ▶ constructed separately for every exceptional group
- ▶ so far: until $E_{8(8)}$ (symmetry of 3D sugra) – $D=3$ external dimensions

[Hohm, HS] [Abzalov, Bakhmatov, Musaev, Hohm, Wang, Berman, Blair, Malek, Rudolph]

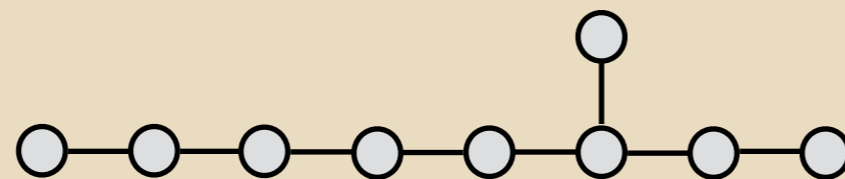


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→ now : $E_{9(9)}$ (symmetry of 2D sugra) – $D=2$ external dimensions



infinite-dimensional algebra and representations
(integrable structure of 2D sugra)

ExFT tensor hierarchy and the dual graviton

- ▶ tensor hierarchy
- ▶ dual graviton

affine symmetries in 2D supergravity

- ▶ 2D supergravity
- ▶ affine symmetries

exceptional field theory for the affine algebra E_9

- ▶ generalized diffeomorphisms & section constraints
- ▶ invariant actions

based on work with Olaf Hohm, Guillaume Bossard, Martin Cederwall, Axel Kleinschmidt, Jakob Palmkvist, Franz Ciceri, Gianluca Inverso

ExFT tensor hierarchy and the dual graviton

- ▶ generalized diffeomorphisms (general)
- ▶ tensor hierarchy
- ▶ dual graviton

ExFT generalized diffeomorphisms

■ generalized diffeomorphisms parameter ξ^M vector $V^M \in \mathcal{R}_1$ irrep of \mathfrak{g}

$$\mathcal{L}_\xi V^M = \xi^N \partial_N V^M + \kappa (\mathbb{P}_{\text{adj}})^N{}_{P^M}{}_Q (\partial_N \xi^P) V^Q + \beta \partial_N \xi^N V^M \quad \beta = \frac{1}{D-2}$$

compatible with $E_{d(d)}$ structure (respects invariant tensors)

[Coimbra, Strickland-Constable, Waldram]

[Berman, Cederwall, Kleinschmidt, Thompson]

$$= \xi^N \partial_N V^M - V^N \partial_N \xi^M + Y^{MN}{}_{PQ} \partial_N \xi^P V^Q$$

closure of the algebra up to section condition $Y^{MN}{}_{PQ} \partial_M \otimes \partial_N = 0$

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closure of the algebra up to section condition $Y^{MN}{}_{PQ} \partial_M \otimes \partial_N = 0$

- ▶ modified YM field strengths

$$\mathcal{F}_{\mu\nu}{}^M = 2 \partial_{[\mu} A_{\nu]}{}^M - [A_\mu, A_\nu]_{\mathbb{E}}{}^M - Y^{MN}{}_{PQ} \partial_N B_{\mu\nu}{}^{PQ}$$

Stückelberg type coupling to 2-forms

- tensor hierarchy (schematically)

$$\mathcal{F}^{[p+1]} = DC^{[p]} + \dots + DC^{[p+1]} \quad \mathcal{D} : \mathcal{R}_{p+1} \longrightarrow \mathcal{R}_p$$

similar to gauged supergravity, where \mathcal{D} is the *embedding tensor*
(embedding of gauged supergravity by suitable Scherk-Schwarz ansatz)
precisely compatible with the supergravity field content

ExFT tensor hierarchy

■ tensor hierarchy (schematically)

$$\mathcal{F}^{[p+1]} = DC^{[p]} + \dots + \mathcal{D}C^{[p+1]} \quad \mathcal{D} : \mathcal{R}_{p+1} \longrightarrow \mathcal{R}_p$$

precisely compatible with the supergravity field content

► until (D−3) forms $\mathcal{C}_M \in \overline{\mathcal{R}}_1$ (dual to the vector fields)

whose field strength requires further correction

$$\mathcal{F}_M = DC_M + \dots + (t^\alpha)_M{}^N \partial_N \mathcal{C}_\alpha + \mathcal{B}_M$$

by a *covariantly constrained* (D−2) form \mathcal{B}_M

related to symmetries from higher-dim dual graviton

$$Y^{MK}{}_{NL} \mathcal{B}_M \otimes \partial_K = 0$$

$$Y^{MK}{}_{NL} \mathcal{B}_M \mathcal{B}_K = 0$$

► D=4, $E_{7(7)}$ FT : modified YM field strength (vectors dual to vectors)

$$\mathcal{F}_{\mu\nu}{}^M \equiv 2\partial_{[\mu} A_{\nu]}{}^M - 2[A_\mu, A_\nu]_{\mathbb{E}}{}^M - 12(t^\alpha)^{MN} \partial_N B_{\mu\nu\alpha} - \frac{1}{2} \Omega^{MN} \mathcal{B}_{\mu\nu N}$$

► D=3, $E_{8(8)}$ FT : modified scalar currents (vectors dual to scalars)

$$D_\mu \mathcal{V} \mathcal{V}^{-1} \equiv \partial_\mu \mathcal{V} \mathcal{V}^{-1} - \mathcal{L}_{A_\mu} \mathcal{V} \mathcal{V}^{-1} + \mathcal{B}_{\mu M} T^M$$

in particular: standard algebra of generalized diffeomorphisms no longer closes

ExFT tensor hierarchy

- tensor hierarchy (schematically)

$$\mathcal{F}^{[p+1]} = DC^{[p]} + \dots + DC^{[p+1]} \quad \mathcal{D} : \mathcal{R}_{p+1} \longrightarrow \mathcal{R}_p$$

- ▶ D=4, E₇₍₇₎FT : modified YM field strength

$$\mathcal{F}_{\mu\nu}{}^M \equiv 2\partial_{[\mu}A_{\nu]}{}^M - 2[A_{\mu}, A_{\nu}]_E^M - 12(t^\alpha)^{MN} \partial_N B_{\mu\nu\alpha} - \frac{1}{2}\Omega^{MN} \mathcal{B}_{\mu\nu N}$$

- ▶ D=3, E₈₍₈₎FT : modified scalar currents

$$\tilde{D}_\mu \mathcal{V} \mathcal{V}^{-1} \equiv \partial_\mu \mathcal{V} \mathcal{V}^{-1} - \mathcal{L}_{A_\mu} \mathcal{V} \mathcal{V}^{-1} + \mathcal{B}_{\mu M} T^M$$

in particular: standard algebra of generalized diffeomorphisms no longer closes

- ▶ D=2, E₉₍₉₎FT : modified scalar field content! $\{\hat{\mathcal{V}}, \chi_M\}$

together with additional vector fields $\{A_\mu{}^M, \mathcal{B}_{\mu M}{}^N\}$

again: standard algebra of generalized diffeomorphisms no longer closes

affine symmetries in D=2 supergravity

affine symmetries in D=2 supergravity

2D Lagrangian (dimensional reduction of D=11 supergravity)

$$\mathcal{L} = -\frac{1}{4}\sqrt{-g}\rho\left(-R + \text{tr}[P^\mu P_\mu]\right) + \mathcal{L}_{\text{ferm}}(\psi^I, \psi_2^I, \chi^A)$$

coset space sigma model coupled to dilaton gravity $\mathcal{V}^{-1}\partial_\mu\mathcal{V} = Q_\mu + P_\mu \in \mathfrak{e}_{8(8)}$
 off-shell symmetry (target space isometries): $E_{8(8)}$ \cap $\mathfrak{so}(16)$

symmetries

has a remarkable structure :

(infinite tower of) dual scalar potentials

$$\partial_\mu\tilde{\rho} = \varepsilon_{\mu\nu}\partial^\nu\rho$$

dual dilaton

$$\partial_\mu Y_1 = \varepsilon_{\mu\nu}J_{\text{Noeth}}^\nu$$

dual scalars

→ classical integrability, affine Lie-Poisson symmetry E_9

realised as a coset action $SL(2) \times E_{9(9)} / SO(1,1) \times K(E_9)$

$$\hat{\mathcal{V}} = \dots e^{Y_3 w^{-3}} e^{Y_2 w^{-2}} e^{Y_1 w^{-1}} \mathcal{V} e^{\tilde{\rho} L_{-1}} \rho^{L_0}$$

Virasoro $L_{-1}\tilde{\rho} = 1$

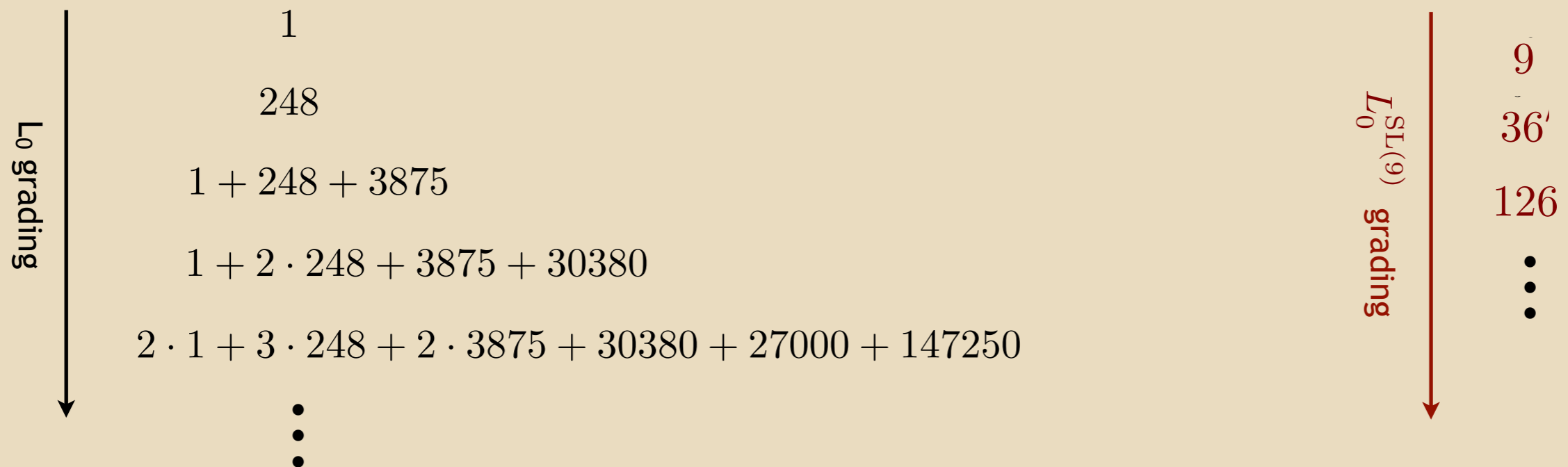
central extension $k\sigma = 1$ [Julia]

affine symmetries in 2D supergravity

vector fields $A_\mu{}^M \in \mathcal{R}_1$

(non-propagating in D=2)

restore by embedding known examples: level 1 (basic) representation of E_9



$$\chi_{\omega_0} = 1 + 248q + 4124q^2 + 34752q^3 + 213126q^4 + 1057504q^5 + 4530744q^6 + \dots$$

ExFT coordinates $Y^M \in \mathcal{R}_1 \quad Y^M \longrightarrow \{x^i, y_{ij}, y_{ijklm}, \dots\}$

construct the associated generalised diffeomorphisms

exceptional field theory for the affine algebra E_9

- ▶ section constraints
- ▶ generalized diffeomorphisms

exceptional field theory for the affine algebra E_9

■ section constraints $Y^{MN}{}_{PQ} \partial_M \otimes \partial_N = 0$??

lives in the tensor product $\mathcal{R}_1 \otimes \mathcal{R}_1$: linear combination of level 2 representations

$$\mathcal{R}(2\Lambda_0), \mathcal{R}(\Lambda_7), \mathcal{R}(\Lambda_1)$$

$$\begin{aligned} \mathcal{R}_1 \otimes \mathcal{R}_1 = & (1 + q^2 + q^3 + \dots) \mathcal{R}(2\Lambda_0) \oplus (q^2 + q^3 + q^4 + \dots) \mathcal{R}(\Lambda_7) \\ & \oplus (q + q^2 + q^3 + \dots) \mathcal{R}(\Lambda_1) \end{aligned}$$

where the multiplicities carry representations of the $c = \frac{1}{2}$ coset CFT $\frac{(\mathbb{E}_9)_1 \oplus (\mathbb{E}_9)_1}{(\mathbb{E}_9)_2}$

specifically

[Goddard,Olive]

$$\mathcal{R}_1 \otimes \mathcal{R}_1 = \chi_{(1,1)} \otimes \mathcal{R}(2\Lambda_0) \oplus \chi_{(2,1)} \otimes \mathcal{R}(\Lambda_7) \oplus \chi_{(2,2)} \otimes \mathcal{R}(\Lambda_1)$$

rescaled coset Virasoro generators $C_n \equiv 32 L_n^{\text{coset}} = 32 \left(\mathbb{I} \otimes L_n^{(1)} + L_n^{(1)} \otimes \mathbb{I} - L_n^{(2)} \right)$

exceptional field theory for the affine algebra E_9

$$\mathcal{R}_1 \otimes \mathcal{R}_1 = \chi_{(1,1)} \otimes \mathcal{R}(2\Lambda_0) \oplus \chi_{(2,1)} \otimes \mathcal{R}(\Lambda_7) \oplus \chi_{(2,2)} \otimes \mathcal{R}(\Lambda_1)$$

rescaled coset Virasoro generators $C_n \equiv 32 L_n^{\text{coset}} = 32 \left(\mathbb{I} \otimes L_n^{(1)} + L_n^{(1)} \otimes \mathbb{I} - L_n^{(2)} \right)$

■ section constraints

$$(\langle \partial_1 | \otimes \langle \partial_2 | + \langle \partial_2 | \otimes \langle \partial_1 |) C_1 = 0$$

$$(\langle \partial_1 | \otimes \langle \partial_2 |) (C_0 - \mathbb{I} + \sigma) = 0$$

$$\forall n > 0 : (\langle \partial_1 | \otimes \langle \partial_2 |) C_{-n} = 0$$

$$\iff \partial_{(M} \otimes \partial_{N)} C_1^{MN}{}_{PQ} = 0$$

etc.

- ▶ reproduce correct higher-dimensional constraints & solutions
- ▶ sufficient in order to define consistent generalised diffeomorphisms

exceptional field theory for the affine algebra E_9

■ generalised diffeomorphisms full affine algebra $\{ T^{\mathcal{A}} \} = \{ T_{\alpha,n}, L_0, K \}$

$$\begin{aligned} \mathcal{L}_\xi V^M &= \xi^N \partial_N V^M + \kappa (\mathbb{P}_{\text{adj}})^N{}_{P^M}{}_{Q} (\partial_N \xi^P) V^Q + \beta \partial_N \xi^N V^M \\ &= \xi^N \partial_N V^M - \underbrace{\eta_{AB} (T^{\mathcal{A}})_P{}^N (T^{\mathcal{B}})_Q{}^M (\partial_N \xi^P) V^Q}_{(C_0)^{NM}{}_{PQ}} - \partial_N \xi^N V^M \\ &\qquad\qquad\qquad (C_0)^{NM}{}_{PQ} = 32 \left(\mathbb{I} \otimes L_0^{(1)} + L_0^{(1)} \otimes \mathbb{I} - L_0^{(2)} \right)^{NM}{}_{PQ} \end{aligned}$$

index-free notation

$$\mathcal{L}_\xi |V\rangle^1 = \langle \partial_V | \xi \rangle^2 \otimes |V\rangle^1 + \langle \partial_\xi | (C_0 - \mathbb{I}) | \xi \rangle^2 \otimes |V\rangle^1$$

▶ as expected: do not close into an algebra (but come close...)

▶ additional symmetry

$$\mathcal{L}_\Sigma V^M = C_{-1}{}^{NM}{}_{PQ} \Sigma_N{}^P V^Q$$

weight compensating internal derivative

with covariantly constrained parameter $\Sigma_N{}^P$

exceptional field theory for the affine algebra E_9

■ generalised diffeomorphisms full affine algebra $\{ T^{\mathcal{A}} \} = \{ T_{\alpha,n}, L_0, K \}$

$$\begin{aligned} \tilde{\mathcal{L}}_{\xi} V^M &= \xi^N \partial_N V^M + \kappa (\mathbb{P}_{\text{adj}})^N{}_P{}^M{}_Q (\partial_N \xi^P) V^Q + \beta \partial_N \xi^N V^M \\ &= \xi^N \partial_N V^M - \underbrace{\eta_{AB} (T^{\mathcal{A}})_P{}^N (T^{\mathcal{B}})_Q{}^M (\partial_N \xi^P) V^Q}_{(C_0)^{NM}{}_{PQ}} - \partial_N \xi^N V^M \\ &\qquad\qquad\qquad (C_0)^{NM}{}_{PQ} = 32 \left(\mathbb{I} \otimes L_0^{(1)} + L_0^{(1)} \otimes \mathbb{I} - L_0^{(2)} \right)^{NM}{}_{PQ} \end{aligned}$$

index-free notation

$$\mathcal{L}_{\xi} |V\rangle^1 = \langle \partial_V | \xi \rangle^2 \otimes |V\rangle^1 + \langle \partial_{\xi} | (C_0 - \mathbb{I}) | \xi \rangle^2 \otimes |V\rangle^1 + \langle \pi_{\Sigma} | C_{-1}^{12} | \Sigma \rangle^2 \otimes |V\rangle^1$$

► additional symmetry

$$\tilde{\mathcal{L}}_{\Sigma} V^M = C_{-1}{}^{NM}{}_{PQ} \Sigma_N{}^P V^Q$$

with covariantly constrained parameter $\Sigma_N{}^P \sim |\Sigma\rangle \langle \pi_{\Sigma}|$

► close into an algebra (modulo section constraints)

exceptional field theory for the affine algebra E_9

■ generalised diffeomorphisms

$$\mathcal{L}_\xi |V^1\rangle = \langle \partial_V | \xi^2 \rangle \otimes |V^1\rangle + \langle \partial_\xi | (C_0^{12} - \mathbb{I}) | \xi^2 \rangle \otimes |V^1\rangle + \langle \pi_\Sigma^2 | C_{-1}^{12} | \Sigma^2 \rangle \otimes |V^1\rangle$$

▶ algebra closes $[\mathcal{L}_{\xi_1, \Sigma_1}, \mathcal{L}_{\xi_2, \Sigma_2}] = \mathcal{L}_{\xi_{12}, \Sigma_{12}}$

$$\xi_{12} \equiv [[\xi_1, \xi_2]] \equiv \frac{1}{2} (\mathcal{L}_{\xi_1} \xi_2 - \mathcal{L}_{\xi_2} \xi_1)$$

$$\begin{aligned} |\Sigma_{12}\rangle \langle \pi_{\Sigma_{12}}| &\equiv \mathcal{L}_{\xi_1} (|\Sigma_2\rangle \langle \pi_{\Sigma_2}|) + \frac{1}{2} \langle \pi_{\Sigma_1} | C_{-1} | \Sigma_2 \rangle \otimes |\Sigma_1\rangle \langle \pi_{\Sigma_2}| \\ &\quad + \frac{1}{4} \langle \partial_{\xi_2} | C_1 | (|\xi_2\rangle \otimes |\xi_1\rangle - |\xi_1\rangle \otimes |\xi_2\rangle) \rangle \langle \partial_{\xi_2} | - (1 \leftrightarrow 2) \end{aligned}$$

modulo section constraints, non-associative, Jacobiator

▶ the computation is remarkably simple !

only relies on commutation relations of the coset Virasoro generators

$$\left[C_m^{13}, C_n^{23} \right] = \frac{m-n}{2} \left(C_{m+n}^{13} + C_{m+n}^{23} - C_{m+n}^{12} \right) + \frac{2}{3} m(m^2 - 1) \delta_{m+n,0} + C_{m+n}^{123}$$

not on any details on the $E_{8(8)}$ structure constants...

exceptional field theory for the affine algebra E_9

▶ invariant action

exceptional field theory for the affine algebra E_9

■ recall: invariant actions

e.g. $E_{6(6)}$ ExFT with $D=5$

$$\mathcal{L} = \widehat{R} + \frac{1}{24} \mathcal{D}_\mu \mathcal{M}_{MN} \mathcal{D}^\mu \mathcal{M}^{MN} - \frac{1}{4} \mathcal{M}_{MN} \mathcal{F}_{\mu\nu}{}^M \mathcal{F}^{\mu\nu N} + \mathcal{L}_{\text{top}} - V_{\text{pot}}(\mathcal{M}_{MN}, g_{\mu\nu})$$

with ‘potential’ (invariant under generalised diffeomorphisms) for $\mathcal{M} = \mathcal{V}\mathcal{V}^T$

$$V_{\text{pot}} = \frac{1}{24} \mathcal{M}^{MN} \partial_M \mathcal{M}^{KL} (12 \partial_L \mathcal{M}_{NK} - \partial_N \mathcal{M}_{KL}) - \frac{1}{2} g^{-1} \partial_M g \partial_N \mathcal{M}^{MN} - \frac{1}{4} \mathcal{M}^{MN} g^{-1} \partial_M g g^{-1} \partial_N g - \frac{1}{4} \mathcal{M}^{MN} \partial_M g^{\mu\nu} \partial_N g_{\mu\nu}$$

■ main challenge: construct the invariant ‘potential’ for $E_{9(9)}$

building blocks: $(J_M)^K{}_L = \mathcal{M}_{LP} \partial_M \mathcal{M}^{PK}$

in a Borel gauge, such that at given level, \mathcal{M}^{MN} has only finitely many entries

exceptional field theory for the affine algebra E_9

- main challenge: construct the invariant ‘potential’ for $E_{9(9)}$

building blocks: $(J_M)^K{}_L = \mathcal{M}_{LP} \partial_M \mathcal{M}^{PK}$

expand on $E_{9(9)}$, (for simplicity truncate to $\tilde{\rho} = 0$)

$$J_M \Big|_{\tilde{\rho}=0} = J_M^{\alpha m} T_{\alpha m} + J_M^0 L_0 + J_M^{(K)} K = J_M^A T_A$$

- generic contribution $V_1 = \rho^{-1} \mathcal{M}^{MN} J_M^A J_M^B \eta_{AB}$
- generic contribution $V_2 = \rho^{-1} \mathcal{M}^{MN} (J_M)_K{}^L (J_L)_N{}^K$
- specific contribution $V_3 = \rho \mathcal{M}^{MN} (\mathbb{J}_{-1,K})_M{}^L (\mathbb{J}_{-1,L})_N{}^K$

in terms of the shifted current

$$(\mathbb{J}_{-1})_M \Big|_{\tilde{\rho}=0} = J_M^{\alpha m} T_{\alpha, m-1} + J_M^0 L_{-1} + \chi_M K$$

bringing in the extra scalar fields χ_M

exceptional field theory for the affine algebra E_9

- final step: restore $\tilde{\rho}$ dependence

$$J_M \Big|_{\tilde{\rho}=0} = J_M^{\alpha m} T_{\alpha m} + J_M^0 L_0 + J_M^{(K)} K + J_M^+ L_{+1} + J_M^- L_{-1}$$

- ▶ generic contribution

$$V_1 = \rho^{-1} \mathcal{M}^{MN} J_M^A J_M^B \eta_{AB}$$

there is no longer a bilinear invariant on $SL(2) \times E_{9(9)}$

exceptional field theory for the affine algebra E_9

- final step: restore $\tilde{\rho}$ dependence

$$J_M \Big|_{\tilde{\rho}=0} = J_M^{\alpha m} T_{\alpha m} + J_M^0 L_0 + J_M^{(K)} K + J_M^+ L_{+1} + J_M^- L_{-1}$$

- ▶ complete contribution

$$V_1 = \rho^{-1} \mathcal{M}^{MN} J_M^A J_M^B \eta_{AB} + \mathcal{M}^{MN} (J_M^+ \chi_{+,N} + J_M^- \chi_{-,N})$$

with the $\tilde{\chi}_{\pm,M}$ related to the above χ_M as

$$\chi_{+,M} = \chi_M$$

$$\chi_{-,M} = \rho^{-2} \chi_M - \omega_-(\mathcal{M})_A J_M^A$$

← cocycle on E_9

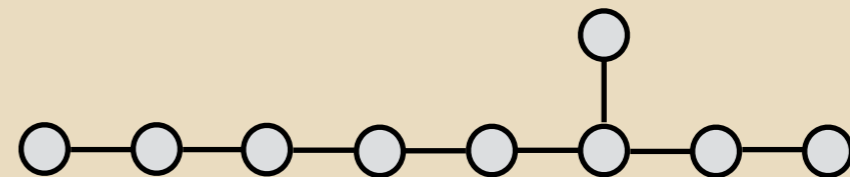
- final result $V(\mathcal{M}^{MN}, \chi_M) = \hat{V}_1 + \hat{V}_2 + \hat{V}_3 + \dots$

(upon completing into $\tilde{\rho}$ power series and adding standard terms ...)

invariant under generalized diffeomorphisms (up to total derivatives)

exceptional field theory for affine algebras

- ▶ infinite-dimensional algebra and representations
- ▶ algebra of generalized diffeomorphisms with additional gauge symmetry
- ▶ dynamics: invariant action – potential



■ complete the E_9 construction:

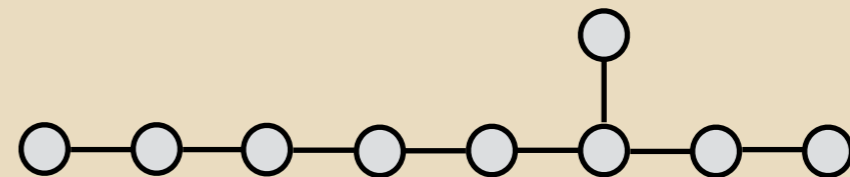
- ▶ external part of the action
- ▶ duality equations for scalar fields \longrightarrow linear system
- ▶ remnants of the integrable structure

■ supersymmetry

- ▶ representation theory of $K(E_9)$
- ▶ potential as an internal curvature scalar
- ▶ predictions for D=2 supergravity

exceptional field theory for affine algebras

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- ▶ algebra of generalized diffeomorphisms with additional gauge symmetry
- ▶ dynamics: invariant action – potential



- extend to the higher-rank algebras:
 - ▶ (E_{10} , E_{11} , Borcherds, tensor hierarchy, ...)
 - ▶ relate to [Damour, Henneaux, Nicolai][West]
- tool for analyzing existing theories
 - or hints towards a more fundamental structure ..?