Gravitational electric-magnetic duality and the (4,0) exotic theory in 6 dimensions

Marc Henneaux

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These spacetime dimensions are also the dimensions where chiral $\left(\frac{D}{2} - 1\right)$-forms can be consistently defined.

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Are there theories corresponding to the other algebras ?

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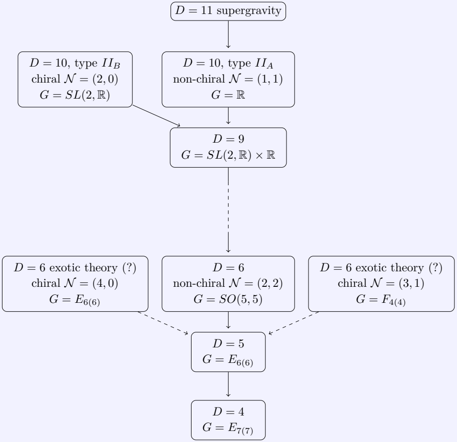

Figure 1. The various higher-dimensional origins of $D = 4$, $\mathcal{N} = 8$ supergravity, with a question mark for the so far unknown theories. We have indicated the supersymmetry in dimensions ten and six, where chirality allows for different maximal supersymmetry algebras. The (conjectured in the case of six-dimensional exotic theories) global symmetry groups G are also written.

(Taken from MH, V. Lekeu, J. Matulich and S. Prohazka (2018))

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The problem comes from the presence of chiral fields.

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  Alternative, "non-geometric" formulations of gravity where electric-magnetic duality plays a central role, are thus worth exploring.

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The last part will discuss supersymmetry algebras in six spacetime dimensions and the construction of the exotic theory of supergravity.

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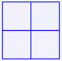

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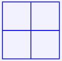

subject to self-duality conditions.

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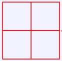
The gauge symmetries for a -field ("$(2,2)$-field") are $\delta T_{\alpha_1\alpha_2\beta_1\beta_2} = \mathbb{P}_{(2,2)}\left(\partial_{\alpha_1}\eta_{\beta_1\beta_2\alpha_2}\right)$ where $\eta_{\beta_1\beta_2\alpha_2}$ is an arbitrary $(2,1)$-tensor. Here, $\mathbb{P}_{(2,2)}$ is the projector on the $(2,2)$ symmetry.

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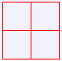
The gauge symmetries for a -field ("$(2,2)$-field") are $\delta T_{\alpha_1 \alpha_2 \beta_1 \beta_2} = \mathbb{P}_{(2,2)} \left(\partial_{\alpha_1} \eta_{\beta_1 \beta_2 \alpha_2} \right)$ where $\eta_{\beta_1 \beta_2 \alpha_2}$ is an arbitrary $(2,1)$-tensor. Here, $\mathbb{P}_{(2,2)}$ is the projector on the $(2,2)$ symmetry.

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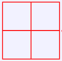
The gauge symmetries for a -field ("$(2,2)$-field") are $\delta T_{\alpha_1\alpha_2\beta_1\beta_2} = \mathbb{P}_{(2,2)}\left(\partial_{\alpha_1}\eta_{\beta_1\beta_2\alpha_2}\right)$ where $\eta_{\beta_1\beta_2\alpha_2}$ is an arbitrary $(2,1)$-tensor. Here, $\mathbb{P}_{(2,2)}$ is the projector on the $(2,2)$ symmetry.

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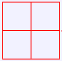
The gauge symmetries for a -field ("$(2,2)$-field") are

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Electric and magnetic fields

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Both electric and magnetic fields have the $(2,2)$ Young symmetry, are transverse and traceless ($\partial_i \mathscr{E}^{ijkl} = 0$, $\mathscr{B}^{ik} \equiv \mathscr{B}^{ijkl} \delta_{jl} = 0$ etc)

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What plays the role of the Weyl tensor is the Cotton tensor, which contains 3 derivatives of the spin-2 field.

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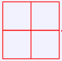
In the particular case of a  -tensor, the critical dimension turns out to be 5, i.e., 6 spacetime dimensions !

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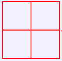
In the particular case of a  -tensor, the critical dimension turns out to be 5, i.e., 6 spacetime dimensions !

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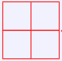
In the particular case of a  -tensor, the critical dimension turns out to be 5, i.e., 6 spacetime dimensions !

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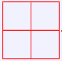
In the particular case of a -tensor, the critical dimension turns out to be 5, i.e., 6 spacetime dimensions!

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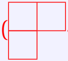
(-diffeos + -Weyl)

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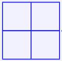
The Cotton tensor is a -tensor which is transverse and traceless,

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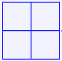
The Cotton tensor is a -tensor which is transverse and traceless,

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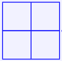
The Cotton tensor is a -tensor which is transverse and traceless,

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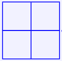
One can show that conversely, any -tensor which is transverse and traceless

can be written as the Cotton tensor of some prepotential Z_{ijmn}.

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The equations of motion obtained by varying the prepotential are just the self-duality condition on H.

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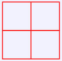
(1 chiral tensor of mixed Young symmetry  (exotic graviton), 27 chiral two-forms, 42 scalars - 128 physical degrees of freedom).

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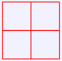
(1 chiral tensor of mixed Young symmetry  (exotic graviton), 27 chiral two-forms, 42 scalars - 128 physical degrees of freedom).

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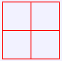
(1 chiral tensor of mixed Young symmetry  (exotic graviton), 27 chiral two-forms, 42 scalars - 128 physical degrees of freedom).

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Strong coupling limit of maximal supergravity in 5 dimensions ? (Hull)

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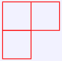
A similar construction can be achieved for the $(3,1)$ theory, which involves a  tensor with self-dual field strength. The R-symmetry is $usp(6) \oplus usp(2)$ and the global symmetry conjectured to be $F_{4(4)}$.

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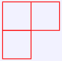
A similar construction can be achieved for the $(3, 1)$ theory, which involves a  tensor with self-dual field strength. The R-symmetry is $usp(6) \oplus usp(2)$ and the global symmetry conjectured to be $F_{4(4)}$.

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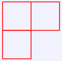
A similar construction can be achieved for the $(3, 1)$ theory, which involves a  tensor with self-dual field strength. The R-symmetry is $usp(6) \oplus usp(2)$ and the global symmetry conjectured to be $F_{4(4)}$.

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The question of interactions remain open !