Inertial Spontaneous Symmetry Breaking
and
Quantum Scale Invariance

G. Ross, CORFU Summer Institute, Sept 2018

Based on:

Inflation in a scale invariant universe
P.Ferreira, C.Hill, J.Noller, G.Ross, **Phys.Rev. D97 (2018), 123516**

Inertial Spontaneous Symmetry Breaking and Quantum Scale Invariance,
P.Ferreira, C.Hill, G.Ross , **arXiv:1801.07676;**

No fifth force in a scale invariant universe, P.Ferreira, C.Hill, G.Ross **,**
Phys.Rev. D95 (2017), 064038;

Weyl Current, Scale-invariant inflation and Planck scale generation, P.Ferreira,
C.Hill, G.Ross **, Phys.Rev. D95 (2017), 043507;**

Scale-Independent Inflation and Hierarchy Generation, P.Ferreira, C.Hill,
G.Ross **, Phys.Lett. B763 (2016) 174-178,**

See also:

M. Shaposhnikov and D. Zenhausern, Phys. Lett. B **671**, 162 (2009).
M. Shaposhnikov and D. Zenhausern, Phys. Lett. B **671**, 187 (2009)
D. Blas, M. Shaposhnikov and D. Zenhausern, Phys. Rev. D **84**, 044001 (2011)
J. Garcia-Bellido, J. Rubio, M. Shaposhnikov and D. Zenhausern, Phys. Rev. D **84**, 123504 (2011).
R. Kallosh and A. Linde, JCAP 1310 (2013), 033; J. J. M. Carrasco, R. Kallosh and A. Linde [arXiv:1506.00936[hep-th]]
K. Allison, C. T. Hill and G. G. Ross, Nucl. Phys. B **891**, 613 (2015) Phys. Lett. B **738**, 191 (2014)
R. Jackiw and S. Y. Pi, Phys. Rev. D **91**, no. 6, 067501 (2015)
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G. K. Karananas and J. Rubio, Phys. Lett. B **761**, 223 (2016)
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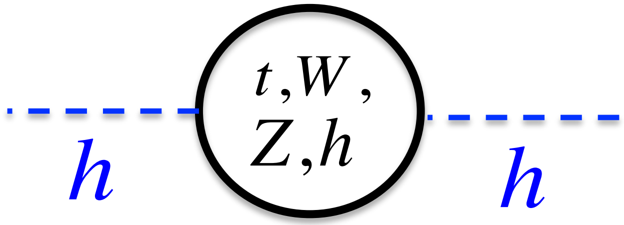
Motivation: Hierarchies

- $\Lambda \equiv \rho_{vac} \sim 10^{-122} M_P^4$
- $m_{Higgs} \sim 10^{-16} M_p$
- $m_{\text{inflaton}} \ll H_I$ (slow roll)

Scale Invariance



$$\delta m_h^2 = \frac{3G_F}{4\sqrt{2}\pi^2}\left(4m_t^2 - 2m_W^2 - m_Z^2 - m_h^2\right)\Lambda^2$$

Field theory: δm^2 not measureable

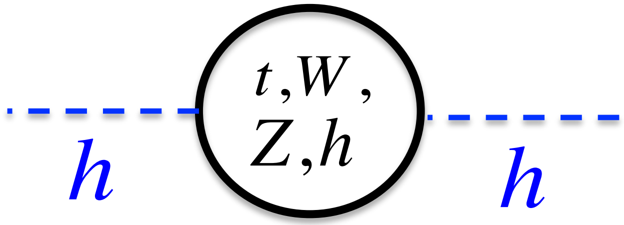
...only $m^2 = m_0^2 + \delta m^2$ "physical"

Only $m^2 = 0$ special

$$\frac{d\,m_h^2}{d\ln\mu} = \frac{3m_h^2}{8\pi^2}\left(2\lambda + y_t^2 - \frac{3g_2^2}{4} - \frac{3g_1^2}{20}\right)$$

Wheeler
Wetterich
Bardeen

Scale Invariance



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... but

What about Gravity?

How is scale symmetry broken?

Gravity and scale invariance

$$S = \int \sqrt{-g} \left(\frac{1}{2} g^{\mu \upsilon} \partial_\mu \sigma \partial_\nu \sigma - \Lambda + \frac{1}{2} M_P^2 R \right)$$

equivalent to

$$S = \int \sqrt{-g} \left(\frac{1}{2} g^{\mu \upsilon} \partial_\mu \phi \partial_\nu \phi - \frac{\lambda}{4} \phi^4 - \frac{1}{12} \alpha \phi^2 R \right)$$

$$\Lambda = \frac{\lambda}{4} \langle \phi \rangle^4, \quad M_P^2 = -\frac{\alpha}{6} \langle \phi \rangle^2$$

Brans-Dicke

Brans-Dicke gravity

$$S=\int\sqrt{-g}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-\frac{\lambda}{4}\phi^4-\frac{1}{12}\alpha\phi^2R\right)$$

Globally Weyl (Scale) invariant

$$\phi(x)\to e^{\varepsilon}\phi(x),\quad g^{\mu\nu}(x)\to e^{2\varepsilon}g^{\mu\nu}(x),\quad \sqrt{-g}\to e^{-4\varepsilon}\sqrt{-g}$$

Noether current

$$K_\mu=\frac{1}{\sqrt{\det(-g)}}\frac{\delta S}{\delta\partial_\mu\varepsilon}=(1-\alpha)\phi\partial_\mu\phi\equiv\partial_\mu K,\qquad K=\frac{1}{2}(1-\alpha)\phi^2$$

$$FRW:\quad \mathrm{D}^\mu K_\mu=\ddot{K}+3H\dot{K}=0$$

$$K(t)=c_1+c_2\int_{t_0}^{t}\left(\frac{dt'}{a(t')^3}\right)\to\text{constant}$$

Inertial symmetry breaking

$$K = \frac{1}{2}(1-\alpha)\phi^2 \rightarrow \text{constant}$$

$$M^2_{Planck} = -\frac{1}{6}\alpha\left\langle\phi\right\rangle^2$$

Scale breaking order parameter independent of potential !

$$\left(KG: \quad (1-\alpha)\left[\phi D^2\phi + \partial^\mu\phi\,\partial_\mu\phi\right] = \phi\frac{\partial V(\phi)}{\partial\phi} - 4V(\phi) = 0 \right)$$

- set by (chaotic) initial conditions

Ferreira, Hill, GGR

Hierarchy generation:

$$S=\int\sqrt{-g}\left(\frac{1}{2}g^{\mu\nu}\partial_{\mu}\phi\partial_{\nu}\phi-\frac{\lambda}{4}\phi^{4}-\frac{1}{12}\alpha\phi^{2}R\;+\frac{1}{2}g^{\mu\nu}\partial_{\mu}\chi\partial_{\nu}\chi-\frac{\xi}{4}\chi^{4}+\frac{\delta}{2}\phi^{2}\chi^{2}-\frac{1}{12}\beta\chi^{2}R\right)$$

2^{nd} singlet models Higgs sector

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Inertial symmetry breaking

$$K=\frac{1}{2}(1-\alpha)\phi^2+\frac{1}{2}(1-\alpha)\chi^2\ \to \text{constant},$$

$$M^2_{Planck}=-\frac{1}{6}\alpha\phi^2-\frac{1}{6}\beta\chi^2$$

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Hierarchy

$$\frac{\chi^2_{IR}}{\phi^2_{IR}}=\frac{4\lambda\beta-2\alpha\delta}{4\alpha\xi-2\beta\delta}$$

IR fixed point

$$\lambda\ll\delta\ll\xi$$

Shaposhnikov, Zenhausern
Garcia-Bellido, Rubio, Shaposhnikov, Zenhausern
Ferreira, Hill, GGR

Hierarchy generation:

$$S = \int \sqrt{-g}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{\lambda}{4}\phi^4 - \frac{1}{12}\alpha\phi^2 R \;+\frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - \frac{\xi}{4}\chi^4 + \frac{\delta}{2}\phi^2\chi^2 - \frac{1}{12}\beta\chi^2 R\right)$$

2nd singlet models Higgs sector

Noether current

$$K_\mu = \frac{1}{\sqrt{\det(-g)}}\frac{\delta S}{\delta\partial_\mu\varepsilon} = (1-\alpha)\phi\partial_\mu\phi + (1-\beta)\chi\partial_\mu\chi \equiv \partial_\mu K,$$

Inertial symmetry breaking

$$K = \frac{1}{2}(1-\alpha)\phi^2 + \frac{1}{2}(1-\alpha)\chi^2 \;\rightarrow \text{constant},$$

$$M^2_{Planck} = -\frac{1}{6}\alpha\phi^2 - \frac{1}{6}\beta\chi^2$$

Hierarchy

$$\frac{\chi^2_{IR}}{\phi^2_{IR}} = \frac{4\lambda\beta - 2\alpha\delta}{4\alpha\xi - 2\beta\delta}$$

IR fixed point

$$\lambda \ll \delta \ll \xi$$

Shaposhnikov, Zenhausern
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Radiative corrections?

Weyl invariant Coleman-Weinberg calculation

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{12} \sum_i^N \alpha_i \phi_i^2 R + \frac{1}{2} \sum_i^N \partial_\mu \phi_i \partial^\mu \phi_i - W(\vec{\phi}) \right]$$

Weyl invariant Coleman-Weinberg calculation

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{12}\sum_i^N \alpha_i \phi_i^2 R + \frac{1}{2}\sum_i^N \partial_\mu \phi_i \partial^\mu \phi_i - W(\vec{\phi})\right]$$

$$\phi_i = e^{-\sigma/f}\hat{\phi}_i, \;\; g_{\mu\nu} = e^{2\sigma/f}\hat{g}_{\mu\nu}$$

$$S = \int d^4x\sqrt{-\hat{g}}\left[-\frac{1}{12}\sum_i^N \alpha_i \hat{\phi}_i^2 \hat{R} + \frac{1}{2}\sum_i^N \partial_\mu \hat{\phi}_i \partial^\mu \hat{\phi}_i + \frac{1}{f^2}\bar{K}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{f}\partial_\mu\sigma\partial^\mu\bar{K} - W(\vec{\phi})\right]$$

$$\bar{K} = \frac{1}{2}\sum(1-\alpha_i)\hat{\phi}_i^2 \rightarrow \text{constant},\ f^2$$

(invariant under scale transformations)

Dilaton decouples!

$$S \approx \int d^4x \sqrt{-g}\left[\frac{1}{2}\sum_{i=1}^{2}\partial_\mu \hat{\phi}_i\,\partial^\mu \hat{\phi}_i - \frac{\lambda}{4}\hat{\phi}_1^4\right]$$

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$$\hat{\phi}_1 = \frac{f}{\sqrt{1-\alpha}}\sin\theta, \quad \hat{\phi}_2 = \frac{f}{\sqrt{1-\alpha}}\cos\theta \qquad \left(\frac{1}{2}\sum_{i-1}^{2}(1-\alpha_i)\hat{\phi}_i^2 = f^2\right)$$

For θ small:

$$S \approx \int d^4x\sqrt{-\hat{g}}\left[\frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi - \frac{\lambda}{4!}\Phi^4\right] \qquad \left(\Phi = \frac{f\hat{\theta}}{\sqrt{1-\alpha_1}}\right)$$

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} \sum_{i=1}^{2} \partial_\mu \hat{\phi}_i \partial^\mu \hat{\phi}_i - \frac{\lambda}{4} \phi_1^4 \right]$$

$$\hat{\phi}_1 = \frac{f}{\sqrt{1-\alpha}} \sin\theta, \quad \hat{\phi}_2 = \frac{f}{\sqrt{1-\alpha}} \cos\theta \qquad \left(\frac{1}{2} \sum_{i-1}^{2} (1-\alpha_i) \hat{\phi}_i^2 = f^2 \right)$$

For θ small:

$$S \approx \int d^4x \sqrt{-\hat{g}} \left[\frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi - \frac{\lambda}{4!} \Phi^4 \right] \qquad \left(\Phi = \frac{f\theta}{\sqrt{1-\alpha_1}} \right)$$

$$\Phi = \Phi_c + \hbar^{1/2} \hat{\Phi}$$ ... adding classical source term $-J\phi$

$$W(\Phi) = \frac{\lambda}{4!} \Phi_c^4 + \hbar \frac{\lambda}{4} \Phi_c^2 \widehat{\Phi}^2 + \ldots$$

$$
\begin{aligned}
W_{eff} &= \Omega + i\int \frac{d^4k}{(2\pi)^4}\sum_{n=1}^{\infty}\frac{1}{2n}\left(\frac{\frac{1}{2}\lambda\Phi_c^2}{k^2+i\varepsilon}\right)^n \\
&= \Omega + \frac{1}{2}\int \frac{d^4k}{(2\pi)^4}\ln\left(1+\frac{\lambda\Phi_c^2}{2k^2}\right) \\
&= \Omega + \frac{\lambda\Lambda^2}{128\pi^2}\Phi_c^2 - \frac{\lambda^2\Phi_c^4}{256\pi^2}\ln\left(\frac{\frac{1}{2}\lambda\Phi_c^2+\Lambda^2}{\frac{1}{2}\lambda\Phi_c^2}\right) \\
&\quad + \frac{\Lambda^4}{64\pi^2}\ln\left(\frac{\frac{1}{2}\lambda\Phi_c^2+\Lambda^2}{\Lambda^2}\right)
\end{aligned}
$$

safe

the "real" hierarchy problem

$$
W_{eff}\Big|_{\Lambda\to\infty} = \Omega + \frac{\lambda\Lambda^2}{64\pi^2}\Phi_c^2 + \frac{\lambda^2\Phi_c^4}{256\pi^2}\left(\ln\frac{\lambda\Phi_c^2}{2\Lambda^2}-\frac{1}{2}\right)
$$

where

$$
\Omega = \frac{\lambda}{4!}\Phi_c^4 - \frac{1}{2}B\Phi_c^2 - \frac{\lambda}{4!}C\Phi_c^4
$$

$$
\frac{d^2W_{eff}}{d\Phi_c^2}\Big|_{\Phi_c=0} = 0, \quad \frac{d^4W_{eff}}{d\Phi_c^4}\Big|_{\Phi_c=M} = \lambda, \quad Z|_{\Phi_c=M} = 1
$$

$$W = \frac{\lambda}{4!}\Phi_c^4 + \frac{\lambda^2\Phi_c^4}{256\pi^2}\left(\ln\frac{\Phi_c^2}{M^2} - \frac{25}{6}\right)$$

$$\Phi = \frac{f\theta}{\sqrt{1-\alpha_1}}, \qquad \theta \approx \frac{\hat{\phi}_1}{\hat{\phi}_2}$$

$$\frac{f^2}{M^2}\frac{1}{1-\alpha_2}$$

$$W \approx \frac{\lambda}{4!}\hat{\phi}_1^4 + \frac{\lambda^2\hat{\phi}_1^4}{256\pi^2}\left(\ln\left(\frac{C\hat{\phi}_{1c}^2}{\hat{\phi}_{2c}^2}\right) - \frac{25}{6}\right)$$

Inverse Weyl transformation:

Weyl invariant

$$W \approx \frac{\lambda}{4!}\phi_1^4 + \frac{\lambda^2\phi_1^4}{256\pi^2}\left(\ln\left(\frac{C\phi_1^2}{\phi_2^2}\right) - \frac{25}{6}\right)$$

$$\frac{d\lambda}{d\log C} = \frac{3\lambda^2}{32\pi^2}$$

… usual renormalisation group

Hierarchy stabilisation:

$$S=\int\sqrt{-g}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-\frac{\lambda}{4}\phi^4-\frac{1}{12}\alpha\phi^2R\ +\frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi-\frac{\xi}{4}\chi^4+\frac{\delta}{2}\phi^2\chi^2-\frac{1}{12}\beta\chi^2R\right)$$

$$\lambda\ll\delta\ll\xi \qquad \left(\frac{\chi_{IR}^2}{\phi_{IR}^2}=\frac{4\lambda\beta-2\alpha\delta}{4\alpha\xi-2\beta\delta}\right)$$

$$\phi\to\phi+\mathrm{constant} \qquad \overset{\checkmark}{\lambda}=\overset{\checkmark}{\delta}=0=\alpha$$

Hierarchy stabilisation:

$$S = \int \sqrt{-g}\left(\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{\lambda}{4}\phi^4 - \frac{1}{12}\alpha \phi^2 R \;\; + \frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - \frac{\xi}{4}\chi^4 + \frac{\delta}{2}\phi^2\chi^2 - \frac{1}{12}\beta\chi^2 R \right)$$

$$\lambda \ll \delta \ll \xi \qquad \left(\frac{\chi_{IR}^2}{\phi_{IR}^2} = \frac{4\lambda\beta - 2\alpha\delta}{4\alpha\xi - 2\beta\delta} \right)$$

$$\phi \to \phi + \text{constant} \qquad \lambda = \overset{✔}{\delta} = 0 = \overset{?}{\alpha} \qquad \frac{\delta\phi^2}{\alpha\phi^2} = \frac{\delta}{\alpha} \leq \frac{M_W^2}{M_P^2}$$

$\alpha \neq 0 \quad \Rightarrow$ gravitational corrections to λ, δ

Gravitational corrections

$\alpha \neq 0$ $\Longrightarrow$ gravitational corrections to λ, δ

Einstein gravity : $\delta \propto \alpha \left(\frac{\Lambda}{M_P} \right)^n$ ✔

Gravitational corrections

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A-gravity : $(4\pi)^2 \frac{d\delta}{d\log C} = -\alpha\beta\left(5 f_2^4 + f_0^4(6\alpha+1)\right)(6\beta+1)$ ✗

Salvio, Strumia

$$S = \int d^4x\sqrt{-g}\left[\frac{R^2}{6 f_0^2} + \frac{\frac{1}{3}R^2 - R_{\mu\nu}^2}{f_2^2} - \frac{\alpha}{12}\phi^2 R - \frac{\beta}{12}\chi^2 R \ldots\right]$$

Propagator: $\frac{1}{M_2^2 p^2 - p^4} = \frac{1}{M_2^2}\left[\frac{1}{p^2} - \frac{1}{p^2 - M_2^2}\right]$ $\quad M_2^2 = \frac{1}{2} f_2^2 M_P^2$

ghost

Hierarchy problem reintroduced through new massive (ghost) state

Gravitational corrections

$\alpha \neq 0 \quad \Rightarrow$ gravitational corrections to λ, δ

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A-gravity : $$(4\pi)^2 \frac{d\delta}{d\log C} = -\alpha\beta\left(5 f_2^4 + f_0^4 (6\alpha+1)\right)(6\beta+1)$$ ✔

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Propagator: $$\frac{1}{M_2^2 p^2 - p^4} = \frac{1}{M_2^2}\left[\frac{1}{p^2} - \frac{1}{p^2 - M_2^2} \right]$$ $$M_2^2 = \frac{1}{2} f_2^2 M_P^2$$

....unless $$\left.\frac{d^4 W_{eff}}{d\phi^4}\right|_{\phi^2 = M^2} \rightarrow \left.\frac{d^4 W_{eff}}{d\phi^4}\right|_{\phi^2 = K(\phi_i)} ,... \quad \Rightarrow C = C(\lambda, \delta, \xi ..)$$

c.f. Ghilencea, Lalak, Olszewski

Summary

- "Inertial" symmetry breaking : Weyl (scale) invariance is always spontaneously broken independently of the potential.

- Massless dilaton – decouples …no 5th force avoiding BD bounds

- Slow roll inflation with acceptable properties possible

  $r < 0.01, n_s \leq 0.967$

- Spontaneously broken scale-invariant "SM"+gravity
  - only dimensionless ratios meaningful

  Small or zero – c.c. fine tuned

$$\frac{H_I^2}{M_P^2} \propto \frac{\xi}{\beta^2}, \quad \frac{m_{"h"}^2}{M_P^2} \propto \frac{\delta}{\alpha}, \quad \frac{H_0^2}{M_P^2} \propto \frac{1}{\alpha^2}\left(\frac{\lambda}{4} + \frac{\xi\mu^4}{4} + \frac{\delta\mu^2}{2}\right) \qquad \mu^2 = \frac{\delta}{\xi}$$

  Hierarchy related to hierarchy of couplings

- Hierarchy stability :

  Non-gravitational corrections ✔

  Gravitational corrections ✔ ? Ghosts? Unitarity?

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