Space-filling branes & gaugings

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Dualities and generalized geometries
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Outline

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Introduction

We are all familiar with the idea that fluxes in $\mathcal{N} = 1$ models generate a superpotential

Example: CY O3-orientifold of IIB with NS and RR 3-form fluxes turned on compatibly with susy results in $\mathcal{N} = 1$ supergravity with GVW superpotential

$$W = \int (\mathcal{F}_3 - iS\mathcal{H}_3) \wedge \Omega$$

We also know that fluxes induce RR tadpoles

The IIB theory has a Chern-Simons term

$$\int C_4 \wedge \mathcal{H}_3 \wedge \mathcal{F}_3$$

implying that $\mathcal{H}_3$ and $\mathcal{F}_3$ act as sources for the RR 4-form C_4

This source can be compensated by changing the net amount of D3-branes in the orientifold theory

Introduction

Consider a specific model: Type-IIB on a $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold

Antoniadis, Dudas, Sagnotti (1999)

Antoniadis, D'Appollonio, Dudas, Sagnotti (2000)

We take $T^6 = \bigotimes_{i=1}^{3} T^2_{(i)}$, coordinates (x^i, y^i)

The two $\mathbb{Z}_2$'s act as $(-,-,+)$ and $(+,-,-)$ on the coordinates of the 2-tori

There are seven complex moduli: axion-dilaton S, three complex Kähler moduli T^i (made of C_4's and the volumes of the tori) and three complex structure moduli U^i (which are the τ's of the tori)

Moduli space $(\mathrm{SL}(2,\mathbb{R})/\mathrm{SO}(2))^7$

Non-geometric fluxes

Turn on fluxes $\mathcal{H}_3$ and $\mathcal{F}_3$, indices on the three different tori

Superpotential is cubic in U moduli

Fluxes induce tadpoles for D3 branes

This can be generalised to contain the non-geometric $\mathcal{Q}$ and $\mathcal{P}$ fluxes

$W_B = \int [(\mathcal{F}_3 - iS\mathcal{H}_3) + (\mathcal{Q} - iS\mathcal{P})J_c] \wedge \Omega$

Shelton, Taylor, Wecht (2005)

Aldazabal, Cámara, Font, Ibáñez (2006)

Fluxes belong to the $(\mathbf{2},\mathbf{2},\mathbf{2},\mathbf{2},\mathbf{2},\mathbf{2},\mathbf{2})$ of $\mathrm{SL}(2,\mathbb{R})^7$

Including all possible fluxes one ends up with a superpotential which is cubic in U and T moduli

Aldazabal, Cámara, Rosabal (2009)

Aldazabal, Andres, Cámara, Graña (2010)

What about tadpoles?

Non-geometric fluxes

The $\mathcal{H}_3$ and $\mathcal{F}_3$ fluxes can be turned on also in the $\mathcal{N}=2$ theory, that is before the orientifold projection

In this case you can't include D3-branes, which means you must impose the Bianchi identity

$\mathcal{H}_3 \wedge \mathcal{F}_3 = 0$

Considering all the $\mathcal{N}=2$ fluxes and performing the orientifold projection, the Bianchi identities are projected on the representations

$16 \times (\mathbf{3},\mathbf{3},\mathbf{3},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$

(all representations made of three triplets and four singlets with either zero or two triplets of $\mathrm{SL}(2,\mathbb{R})_{U^i}$)

The Bianchi identities of the orbifold $\mathcal{N}=2$ theory are not required by $\mathcal{N}=1$ supersymmetry, so we would like to have space-filling branes compatible with the orientifold projection to cancel the tadpoles. What are these branes?

Exotic branes

Consider an NS5-brane smeared along the directions x and y

The NS field B_{xy} has monodromy

$$B_{xy} \to B_{xy} + 1$$

If one performs two T-duality transformations in the directions x and y, these act on the complex scalar

$$\rho = B_{xy} + i\sqrt{\det G}$$

exactly like S-duality acts on τ, that is $\rho \to -\frac{1}{\rho}$

This means that one ends up with a solution with monodromy

$$\mathrm{Re}\left(-\frac{1}{\rho}\right) \to \mathrm{Re}\left(-\frac{1}{\rho}\right) + 1$$

where

$$\mathrm{Re}\left(-\frac{1}{\rho}\right) = -\frac{B_{xy}}{B_{xy}^2 + \det G}$$

de Boer, Shigemori (2010)

Exotic branes

In general, using S and T duality one finds all the solutions of branes of this type

Lozano-Tellechea, Ortín (2001)

Using the dualities, one derives the tension of all such branes as functions of the string coupling and the compactification radii

Obers, Pioline, Rabinovici (1998)

de Boer, Shigemori (2013)

The S-dual of the D7 has tension $g_s^{-3} \to 7_3$

The double-T-dual of the NS5 has tension $g_s^{-2} R_x R_y \to 5_2^2$

In general one can get higher power of the radii. For instance by performing two T-dualities on the 7_3 one gets a brane of tension $g_s^{-3} R_x^2 R_y^2 \to 7_3^{(0,2)}$

Exotic branes

Natural correspondence between exotic branes and mixed-symmetry potentials

$p_n^{(a,b,c,\dots)}$ brane $\rightarrow$ $A_{p+1+a+b+c+\dots,a+b+c+\dots,b+c+\dots,c+\dots,\dots}$

In particular:

5_2^2 brane $\rightarrow$ $A_{8,2}$

$7_3^{(0,2)}$ brane $\rightarrow$ $A_{10,2,2}$

Universal T-duality rule: Lombardo, FR, Risoli (2016)

$$g_s^{-n} \; : \qquad \underbrace{a, a, \dots, a}_{p} \overset{T_a}{\longleftrightarrow} \underbrace{a, a, \dots, a}_{n-p}$$

This implies, under T-dualities in the directions x and y,

$$n = 2 : \qquad A_6 \rightarrow A_{6\ xy,xy}$$

$$n = 3 : \qquad A_8 \rightarrow A_{8\ xy,xy,xy}$$

Space-filling branes

Consider D7 and 7_3 space-filling branes in $D = 8$

These two branes preserve the same supersymmetry

One can construct bound states which are still 1/2-BPS (familiar in F-theory)

Performing two T-dualities in the internal directions, these two branes are mapped to the D9 and the $7_3^{(0,2)}$

Therefore we find that the $7_3^{(0,2)}$ brane preserves the same susy of the D9

Space-filling branes

We now move to $D = 7$. In seven dimensions there are three different $7_3^{(0,2)}$ branes because there are three possible ways of choosing two isometry directions

Obviously all these branes preserve the same susy because each of them preserves the same susy of the D9

If we now perform two T-dualities, say along x and y, the D9 goes back to the D7 wrapping the direction z and the $7_3^{(0,2)}$ with isometries along x and y goes to the 7_3

On the other hand, the other two branes go to two $7_3^{(1,1)}$ branes with tensions

$$g_s^{-3} R_y R_z^2 \qquad\qquad g_s^{-3} R_x R_z^2$$

corresponding to the components $A_{7\ yz,yz,z}$ and $A_{7\ xz,xz,z}$ of the mixed-symmetry potential $A_{9,2,1}$

Space-filling branes

Performing the same analysis, we find that the 3-branes that preserve the same supersymmetry as the O7/O3 orientifold of IIB on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ are in the representations of $\mathrm{SL}(2,\mathbb{R})^7$ made of three triplets and four singlets

More precisely, we get 16 representations of this kind, which are those with either zero or two triplets of $\mathrm{SL}(2,\mathbb{R})_{U^i}$

Remarkably, these are exactly the representations of the tadpole conditions, that is the representations of the Bianchi identities of the $\mathcal{N}=2$ theory that survive the orientifold truncation but are not required by $\mathcal{N}=1$ susy

Lombardo, FR, Risoli (2017)

Gaugings & embedding tensor

Form the supergravity point of view, fluxes induce a gauging, which is described in terms of the embedding tensor θ

de Wit, Samtleben, Trigiante (2002)

If the ungauged theory has a global symmetry G, the embedding tensor belongs to a specific representation of G

In our $\mathcal{N} = 1$ model the global symmetry of the ungauged supergravity theory is $\mathrm{SL}(2, \mathbb{R})^7$, and as we have mentioned the embedding tensor belongs to the $(\mathbf{2}, \mathbf{2}, \mathbf{2}, \mathbf{2}, \mathbf{2}, \mathbf{2}, \mathbf{2})$

The components of this embedding tensor are all the geometric and non-geometric fluxes that can be turned on compatibly with the orientifold

Gaugings & embedding tensor

In general gauge invariance and supersymmetry imply that the embedding tensor has to satisfy specific quadratic constraints

$\theta^2|_R = 0$

The Bianchi identities of the $\mathcal{N} = 2$ theory are an example of a quadratic constraint

As we have seen, when the embedding tensor is projected on the $\mathcal{N} = 1$ theory, the quadratic constraint is projected on the representation containing all the space-filling branes that are compatible with the projection

Therefore the constraint can be lifted, compatibly with the fact that no quadratic constraint is needed in $\mathcal{N} = 1$

Gaugings & embedding tensor

We find that this result is completely general

We consider in each dimension the $\mathbb{Z}_2$ truncation of the maximal theory to the half-maximal theory

We project the embedding tensor to the one of the half-maximal theory

We find that the quadratic constraint is projected as

$$\theta^2|_R \rightarrow \theta^2|_{R_1} \oplus \theta^2|_{R_2}$$

where R_1 is the representation of the quadratic constraint of the half-maximal theory, while R_2 is the representation containing the space-filling branes that preserve the same supersymmetry of the truncation

We find exactly the same result when we consider further truncations to theories with 8 supersymmetries and then to theories with 4 supersymmetries

Gaugings & embedding tensor

Consider as an example the $\mathcal{N}=8$ theory in four dimensions

The theory can be truncated to the $\mathcal{N}=4$ theory with global symmetry $\mathrm{SL}(2,\mathbb{R})\times\mathrm{SO}(6,6)$

We find that the space-filling branes preserving the same susy of the truncation are in the $(\mathbf{1},\mathbf{462})$

The embedding tensor of the maximal theory is in the $\mathbf{912}$ of $\mathrm{E}_{7(7)}$

The quadratic constraint is in the $\mathbf{8645}\oplus\mathbf{133}$

Gaugings & embedding tensor

Decomposing the embedding tensor one finds

$$\mathbf{912} = (\mathbf{3}, \mathbf{32}) \oplus (\mathbf{2}, \mathbf{220}) \oplus (\mathbf{1}, \mathbf{352}) \oplus ....$$

Only the second representation survives the projection, that is the embedding tensor of the half-maximal theory is in the $(\mathbf{2}, \mathbf{220})$

The quadratic constraint is projected on

$$(\mathbf{3}, \mathbf{495}) \oplus (\mathbf{1}, \mathbf{2079}) \oplus (\mathbf{1}, \mathbf{462})$$

But only the first two are required by $\mathcal{N} = 4$ supersymmetry!

Dibitetto, Guarino, Roest (2011)

The extra representation is the one containing all the space-filling branes compatible with susy

Conclusions

- We have found a relation between the quadratic constrains of the embedding tensor and the space-filling branes that all preserve the same supersymmetry
- In particular, this explains how the Bianchi identities of the $\mathcal{N} = 2$ theory can be uplifted generating tadpoles for these branes
- There is an elegant relation between these results and the structure of the supersymmetry algebra. The number of truncations to the theory with half the supersymmetry is always given by the number of vector central charges
- We have a universal T-duality rule for all the branes in string theory
- It would be extremely interesting to have some understanding of the physics of exotic branes