Exceptional field theory and supersymmetric $\mathrm{AdS}_{6,7}$ vacua

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Based on EM: arXiv:1612.01692, arXiv:1612.01990, arXiv:1707.00714,
EM, Samtleben, Vall Camell: arXiv:1808.05597 + *ongoing work*

Motivation

- "Good" understanding of flux-less supersymmetric vacua
  - $\delta_\epsilon \psi \sim \nabla \epsilon = 0 \Rightarrow$ "Special holonomy"
  - Type II SUGRA $\longrightarrow D = 4\ \mathcal{N} = 2 \Rightarrow$ Calabi-Yau geometry
- Fluxes? AdS?
  - $\delta_\epsilon \psi \sim \nabla \epsilon + \not{F} \epsilon = 0$

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  - Exceptional field theory / generalised geometry
  - "Generalised special holonomy"

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  - $\delta_\epsilon \psi \sim \nabla\epsilon + \not{F}\epsilon = \nabla_{ExFT}\epsilon = 0$
  - Exceptional field theory / generalised geometry
  - "Generalised special holonomy"
- Powerful tool"
  - ExFT $\rightarrow$ "good variables" for global solutions.
  - Universal consistent truncations
  - Consistent truncations with matter (c.f. Valentí's talk)
  - Moduli e.g. [Ashmore, Gabella, Graña, Petrini, Waldram 2016]

Outline

- Half-maximal SUSY structures
- AdS vacua and consistent truncations
- Applications:
  - SUSY AdS_7 vacua of mIIA & "minimal" consistent truncations.
  - SUSY AdS_6 vacua of IIB & "minimal" consistent truncations.
- Valentí's talk: Consistent truncations with vector multiplets.

Exceptional field theory & half-maximal SUSY

- Local symmetries of 11-d/IIB SUGRA $\longrightarrow$ generalised diffeomorphisms (local $E_{d(d)}$)
- Should also capture half-maximal SUSY.
- Consistent truncations to half-maximal gSUGRA.

Reminder of G-structures: Calabi-Yau

- CY_3 (no flux, preserves $\mathcal{N} = 2$ SUSY of type II).
- **Algebraic condition**: One well-defined internal spinor ψ.
  Structure group $\mathrm{SU}(3) \subset \mathrm{SU}(4)$

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$$\omega_{(2)} \wedge \Omega_{(3)} = 0\,, \qquad \Omega_{(3)} \wedge \overline{\Omega}_{(3)} = \omega_{(2)}^3 = vol_6 \neq 0\,.$$

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- **Differential condition**: SUSY Minkowski vacuum $\nabla\psi = 0 \Longrightarrow \mathrm{SU}(3)$ holonomy.

$$d\omega_{(2)} = d\Omega_{(3)} = 0 \,.$$

Algebraic condition

- Half-maximal set of H_d spinors Ψ.
- Generalised structure group $\mathrm{G}_{\mathrm{half}} = \mathrm{Spin}(d-1) \subset H_d \subset E_{d(d)}$.
- $\tilde{H}_d \longrightarrow \mathrm{Spin}(d-1)_S \times \mathrm{Spin}(d-1)_R$

D	$E_{d(d)}$	$\tilde{H}_d$	$\mathrm{G}_{\mathrm{half}}$	G_R
7	$\mathrm{SL}(5)$	$\mathrm{USp}(4)$	$\mathrm{SU}(2)$	$\mathrm{SU}(2)$
6a	$\mathrm{Spin}(5,5)$	$\mathrm{USp}(4) \times \mathrm{USp}(4)$	$\mathrm{SU}(2) \times \mathrm{SU}(2)$	$\mathrm{SU}(2) \times \mathrm{SU}(2)$
6b	$\mathrm{Spin}(5,5)$	$\mathrm{USp}(4) \times \mathrm{USp}(4)$	$\mathrm{USp}(4)$	$\mathrm{USp}(4)$
5	$E_{6(6)}$	$\mathrm{USp}(8)$	$\mathrm{USp}(4)$	$\mathrm{USp}(4)$
4	$E_{7(7)}$	$\mathrm{SU}(8)$	$\mathrm{SU}(4)$	$\mathrm{SU}(4) \times \mathrm{U}(1)$

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- $D = 5$, $6a$, 7 similar pattern.
- $D = 4$, $6b$ slightly different.

$\mathrm{G}_{\mathrm{half}}$-structures and half-maximal SUSY

- Describe bosonically using "generalised differential forms".
- Sections of $\mathcal{R}_1$, $\mathcal{R}_2$, $\mathcal{R}_3$, $\ldots$, $\mathcal{R}_{D-3}$

D	$E_{d(d)}$	R_1	R_2	R_3	R_4	R_c
7	$\mathrm{SL}(5)$	**10**	$\overline{\mathbf{5}}$	**5**	$\overline{\mathbf{10}}$	$\emptyset$
6a	$\mathrm{Spin}(5,5)$	**16**	**10**	$\overline{\mathbf{16}}$	**48**	**1**
5	$E_{6(6)}$	**27**	$\overline{\mathbf{27}}$	**78**	n/a	**27**
4	$E_{7(7)}$	**56**	**133**	n/a	n/a	**1539**

$$\mathcal{R}_1 \xrightarrow{\wedge} \mathcal{R}_2 \xrightarrow{\wedge} \ldots \xrightarrow{\wedge} \mathcal{R}_{D-3}$$

$$\mathcal{R}_{D-3} \xrightarrow{d} \ldots \xrightarrow{d} \mathcal{R}_2 \xrightarrow{d} \mathcal{R}_1$$

$\mathrm{G}_{\mathrm{half}}$-structures and half-maximal SUSY

- Generalised tensors stabilised by $\mathrm{Spin}(d-1) \subset E_{d(d)}$:

$$J_u{}^M \in \Gamma\left(\mathcal{R}_1\right) , \qquad \hat{K}^I \in \Gamma\left(\mathcal{R}_{D-4}\right) ,$$

$u = 1, \ldots, d-1$ of $\mathrm{Spin}(d-1)_R$, satisfying

$$\left(\delta_u^w \delta_v^x - \frac{1}{d-1}\delta_{uv}\delta^{wx}\right) J_w \wedge J_x = 0 \,,$$
$$\hat{K} \times_{R_c} \hat{K} = 0 \,, \qquad \delta^{uv} J_u \wedge J_v \wedge \hat{K} > 0 \,.$$

- Generalisations of $\omega_{(2)}$, $\Omega_{(2)}$ on K3.

Ex: $\mathrm{Spin}(4) \subset \mathrm{Spin}(5,5)$ structure

- Generalised tensors stabilised by $\mathrm{Spin}(4) \subset \mathrm{Spin}(5,5)$:

$$J_u{}^M \in \Gamma\left(\mathcal{R}_1\right) , \qquad \hat{K}^I \in \Gamma\left(\mathcal{R}_2\right) ,$$

$u = 1, \ldots, 4$ of $\mathrm{Spin}(4)_R$,
$M = 1, \ldots, 16$,
$I = 1, \ldots, 10$, satisfying

$$\left(\delta_u^w \delta_v^x - \frac{1}{d-1}\delta_{uv}\delta^{wx}\right) J_w{}^M J_x{}^N \left(\gamma^I\right)_{MN} = 0 ,$$

$$\hat{K}_I\, \hat{K}_J\, \eta^{IJ} = 0 , \qquad \delta^{uv} J_u{}^M J_v{}^N \hat{K}_I \left(\gamma^I\right)_{MN} > 0 .$$

SUGRA background from G_{half}-structures

- For CY, $g_{ij} = I_i{}^k \omega_{kj}$.
- J_u and $\hat{K}$ define SUGRA background:

$$\{\text{SUGRA fields}\} = \mathcal{M}_{MN} \sim \hat{J}_{u,M} \hat{J}^u{}_N - d_{MNI} \hat{K}^I + \epsilon^{u_1 \ldots u_{d-1}} \left(J_{u_1} J_{u_2} \ldots J_{u_{d-1}} \right)_{MN} ,$$

d_{MNI} is an $E_{d(d)}$ invariant and $\hat{J}_u = J_u \wedge \hat{K}$.

SUGRA background from G_{half}-structures

- For CY, $g_{ij} = I_i{}^k \omega_{kj}$.
- Ex $\mathrm{Spin}(5,5)$ ExFT:

$$
\begin{aligned}
\sqrt{2}\mathcal{M}_{MN} &= 4\, \hat{J}_{u,M} \hat{J}^u - \left(\gamma^I\right)_{MN} \hat{K}_I \\
&\quad - \frac{1}{4!} \epsilon^{uvwx} \left(\gamma_I\right)_{MP} \left(\gamma_J\right)_{NQ} \left(\gamma^{IJ}\right)^S{}_R J_u{}^P J_v{}^Q J_w{}^R \hat{J}_{x\,S}\,, \\
\hat{J}_{u\,M} &= \left(\gamma^I\right)_{MN} J_u{}^N \hat{K}_I\,.
\end{aligned}
$$

Differential conditions: 1/2-maximal AdS vacua

- BPS equations = "Special holonomy" ("intrinsic torsion")
  1/2-max AdS vacua:

$$\mathcal{L}_{J_u} J_v = \underbrace{R_{uvw}}_{\text{c.c. } \Lambda} J^w \,, \qquad \mathcal{L}_{J_u} \hat{K} = 0 \,,$$

  and
  - $D = 7$: $d\hat{K} = \epsilon^{uvw} R_{uvw} J^x \wedge J_x$,
  - $D = 6$: $d\hat{K} = \epsilon^{uvwx} R_{uvw} J_x$,

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D	d	$\mathrm{Spin}(d-1)_R$	AdS R-symmetry
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- J_u generalised Killing vector fields, generate R-symmetry
- $\hat{K}$ invariant under R-symmetry

Consistent truncations

- Consistent truncation $\rightarrow$ study AdS vacua via lower-dim gSUGRA.

Conjecture (Gauntlett & Varela)

For any SUSY solution of $D = 10$ or $D = 11$ SUGRA that consists of a warped product $AdS_D \times_w M$, there is a consistent truncation on M to a D-dimensional gauged SUGRA keeping only the gravitational supermultiplet.

- Can we get a free lunch?

AdS consistent truncation: SUGRA multiplet

- Given $J_u(Y)$, $\hat{K}(Y)$ of 1/2-max AdS, the Ansatz

$$\mathcal{J}_u(x,Y) = X^{-1}(x)\, J_u(Y)\,, \qquad \hat{\mathcal{K}}(x,Y) = X^2(x)\, \hat{K}(Y)\,,$$
$$\mathcal{A}_\mu(x,Y) = A_\mu{}^u(x)\, J_u(Y)\,, \qquad \ldots\,,$$

  gives a *consistent* truncation.
- Consistency follows from special holonomy.
- $X(x)$ scalar, $A_\mu{}^u(x)$ vector fields of grav. supermultiplet.
- "Minimal" truncation $\Rightarrow$ no vector multiplets.
- Spacetime fields from $\mathcal{M}_{MN} \longrightarrow$ non-linear truncation Ansatz.

Constructing AdS solutions from ExFT

- Geometric Ansatz $\rightarrow J_u$, $\hat{K}$
  - Algebraic conditions (existence of spinors)
  - Differential conditions (SUSY vacuum)
- Construct generalised metric

$$\mathcal{M}_{MN} = \hat{J}_{u,M}\hat{J}^u{}_N - d_{MNI}\hat{K}^I + \left(J^{d-1}\right)_{MN}.$$

- ExFT dictionary: $\mathcal{M}_{MN} = \{\mathrm{SUGRA\ fields}\}$.
- Minimal consistent truncation for free.

Example 1: AdS_7 from ExFT

- Consider $\mathrm{AdS}_7 \times S^3$ of mIIA in $\mathrm{SL}(5)$ ExFT.
- Ansatz: S^2 fibred over I.
- "Constrained coords" y_u on $S^2 \subset \mathbb{R}^3$ with $y^u y_u = 1$.
- z coord on I.

$$\underbrace{\mathcal{L}_{J_u}}_{m} J_v = R_{uvw} J^w \,, \qquad \mathcal{L}_{J_u} \hat{K} = 0\,.$$

- $J_u \to \mathrm{SU}(2)_R$ triplets, $\hat{K} \to \mathrm{SU}(2)_R$ singlets:
  - y_u, $f(z)$,
  - Killing vecs v_u, ∂_z,
  - dy_u, $vol_{S^2} = \frac{1}{2}\epsilon_{uvw} y^u dy^v \wedge dy^w$,
  - dz, $dy_u \wedge dz$, $vol_{S^2} \wedge dz$.

$\mathrm{G}_{\mathrm{half}}$ structure on S^3

IIA: $\partial_{[ab]} = (\partial_{i5}, \not{\partial}_{45}, \not{\partial}_{i4})$, $\quad i = 1, \ldots, 3,\ a = (i, 4, 5)$
$J_u \in \Gamma\left(\mathcal{R}_1\right)$, $\quad \mathcal{R}_1 = TM \oplus T^*M \oplus \Lambda^0 T^*M \oplus \Lambda^2 T^*M$
$\hat{K} \in \Gamma\left(\mathcal{R}_3\right)$, $\quad \mathcal{R}_3 = \Lambda^0 T^*M \oplus \Lambda^2 T^*M \oplus \Lambda^3 T^*M$
Algebraic conditions uniquely give:

$$\begin{aligned} J_u &= \sqrt{2} v_u + \frac{1}{2}\left(-\frac{h(z)}{q(z)} y_u\, dz + g(z) dy_u\right) - q(z)\, y_u \\ &\quad + \frac{1}{2\sqrt{2}}\left(y_u\, q(z)\, g(z)\, z\, vol_{S^2} + h(z) \epsilon_{uvw} y^v\, dy^w \wedge dz\right), \\ \hat{K} &= s(z) + \frac{1}{2\sqrt{2}}\left(g(z)\, s(z) - t(z)\right) vol_{S^2}, \end{aligned}$$

Natural gauge choice: $h(z) = q(z)$.

$\mathrm{G}_{\mathrm{half}}$ structure on S^3

IIA: $\partial_{[ab]} = (\partial_{i5}, \cancel{\partial_{45}}, \cancel{\partial_{i4}})$, $\quad i = 1, \ldots, 3, \; a = (i, 4, 5)$
$J_u \in \Gamma(\mathcal{R}_1), \quad \mathcal{R}_1 = TM \oplus T^*M \oplus \Lambda^0 T^*M \oplus \Lambda^2 T^*M$
$\hat{K} \in \Gamma(\mathcal{R}_3), \quad \mathcal{R}_3 = \Lambda^0 T^*M \oplus \Lambda^2 T^*M \oplus \Lambda^3 T^*M$
Algebraic and differential conditions uniquely gives:

$$J_u = \sqrt{2} v_u - \frac{1}{2} d\left(y_u\, z\right) + \ddot{t}(z)\, y_u + \frac{1}{2\sqrt{2}} \ddot{t}(z) \left(y_u\, z\, vol_{S^2} - \epsilon_{uvw} y^v\, dy^w \wedge dz\right) ,$$

$$\hat{K} = -\dot{t}(z) + \frac{1}{2\sqrt{2}} \left(z\, \dot{t}(z) - t(z)\right) vol_{S^2} ,$$

with $\dddot{t} = -\frac{m}{2}$, $t \geq 0$.
Infinitely many solutions determined by $t(z)$.

$\mathrm{AdS}_7 \times S^3$ solution

Generalised metric $\mathcal{M}(J_u, \hat{K})$ and EFT $\leftrightarrow$ IIA SUGRA dictionary:

$$
\begin{aligned}
ds_{10}^2 &= 4\sqrt{-\frac{t}{\ddot{t}}}\, ds^2_{AdS_7} + \frac{1}{2}\sqrt{-\frac{\ddot{t}}{t}}\left(\frac{t^2}{\dot{t}^2 - 2\,\ddot{t}\,t} ds^2_{S^2} + dz^2\right)\,, \\
e^{\psi} &= \left(-\frac{t}{\ddot{t}}\right)^{3/4} \frac{1}{\sqrt{\dot{t}^2 - 2\,\ddot{t}\,t}}\,, \\
B_2 &= \frac{1}{2\sqrt{2}}\left(z - \frac{\dot{t}\,t}{\dot{t}^2 - 2\,\ddot{t}\,t}\right) vol_2\,, \\
F_2 &= \frac{1}{2\sqrt{2}}\left(2\ddot{t} + \frac{m\,\dot{t}\,t}{\dot{t}^2 - 2\,\ddot{t}\,t}\right) vol_2\,,
\end{aligned}
$$

with $\dddot{t} = -m/2$.

These are the general AdS_7 solutions of mIIA [Apruzzi, Fazzi, Rosa, Tomasiello] in coords of [Cremonesi, Tomasiello] .

AdS_7 minimal consistent truncation

Consistent truncation for $\mathrm{AdS}_7 \times S^3$.
$\mathcal{J}_u(x, Y) = X^{-1}(x) J_u(Y)$, $\quad \hat{\mathcal{K}}(x, Y) = X^2(x) \hat{K}(Y)$:

$$ds_{10}^2 = 4\sqrt{-\frac{t}{\ddot{t}}} X^{1/2} ds_7^2 + \frac{1}{2}\sqrt{-\frac{\ddot{t}}{t}} \left[X^{-5/2} dz^2 + X^{5/2} \frac{t^2}{\dot{t}^2 X^5 - 2\,\ddot{t}\,t} ds_{S^2}^2 \right] ,$$

$$e^{\psi} = X^{5/4} \left(-\frac{t}{\ddot{t}}\right)^{3/4} \frac{1}{\sqrt{X^5 \dot{t}^2 - 2\,\ddot{t}\,t}} ,$$

$$B_2 = \frac{1}{2\sqrt{2}} \left(z - \frac{\dot{t}\, t X^5}{\dot{t}^2 X^5 - 2\,\ddot{t}\,t} \right) vol_{S^2} ,$$

$$F_2 = \frac{1}{2\sqrt{2}} \left(2\ddot{t} + X^5 \frac{m \dot{t}\, t}{\dot{t}^2 X^5 - 2\,\ddot{t}\,t} \right) vol_{S^2} ,$$

$\dddot{t} = -m/2$. Reproduces [Passias, Rota, Tomasiello].

Example 2: AdS$_6$ from ExFT

- Consider AdS$_6$ of IIB in $\mathrm{SO}(5,5)$ ExFT.

$$\mathcal{L}_{J_u} J_v = R_{uvw} J^w \,, \qquad d\hat{K} = \epsilon^{uvwx} R_{uvw} J_x \,, \qquad \mathcal{L}_{J_u} \hat{K} = 0 \,,$$

with $u = 1, \ldots, 4$, $R_{uvw} = \epsilon_{uvwx} \Lambda^x$. Wlog $\Lambda^u = (0,0,0,1)$.

$$\mathcal{L}_{J_I} J_J = \epsilon_{IJK} J^K \,, \qquad \mathcal{L}_{J_I} J_4 = 0 \,, \qquad \mathcal{L}_{J_I} \hat{K} = 0 \,, \qquad d\hat{K} = J_4 \,.$$

- Ansatz: IIB on S^2 fibred over Riemann surface Σ_2.
- "Constrained coords" y_I on $S^2 \subset \mathbb{R}^3$ with $y^I y_I = 1$.
- χ_α coord on Σ_2, $\alpha = 1, 2$.
- $J_I \to \mathrm{SU}(2)_R$ triplets, J_4, $\hat{K} \to \mathrm{SU}(2)_R$ singlets

IIB: $\partial_M = \left(\partial_i, \cancel{\partial}^{i\alpha}, \cancel{\partial}^{ijk}\right), \quad i = 1, \ldots, 4, \ \alpha = (1,2)$

$J_u \in \Gamma\left(\mathcal{R}_1\right), \quad \mathcal{R}_1 = TM \oplus 2 \cdot T^*M \oplus \Lambda^3 T^*M$

$\hat{K} \in \Gamma\left(\mathcal{R}_2\right), \quad \mathcal{R}_2 = 2 \cdot \Lambda^0 T^*M \oplus \Lambda^2 T^*M \oplus 2 \cdot \Lambda^4 T^*M$

Most general structure after algebraic and differential conditions:

$$
\begin{aligned}
J_I &= v_I + d(k^\alpha y_I) + \frac{1}{2} d\left(k^\alpha \epsilon_{IJK} y^J dy^K \wedge dk_\alpha\right) , \\
J_4 &= dp^\alpha - k_\alpha\, dp^\alpha \wedge vol_{S^2} , \\
\hat{K} &= p_\alpha - (r + p_\alpha k^\alpha) vol_{S^2} .
\end{aligned}
$$

Depends on four real functions, k^α, p^α, on Σ_2 satisfying

$$
|dk|^2 \equiv \star\left(dk^\alpha \wedge dk_\alpha\right) = \star\left(dp^\alpha \wedge dp_\alpha\right) \geq 0 , \qquad dk^\alpha \wedge dp_\alpha = 0 ,
$$
$$
dr = -p_\alpha\, dk^\alpha .
$$

AdS$_6$ solutions in IIB

- From generalised metric and ExFT $\leftrightarrow$ IIB dictionary we find SUGRA fields

$$ds^2 = \frac{\sqrt{2}\, r^{5/4}\, \Delta^{1/4}}{3^{3/4}\, |dk|^{1/4}} \left(\frac{12}{r} ds^2_{AdS_6} + \frac{|dk|}{\Delta} ds^2_{S^2} + \frac{4}{r^2} dk^\alpha \otimes dp_\alpha \right) , \qquad C_{(4)} = 0 \, ,$$

$$C_{(2)}{}^\alpha = -\frac{4}{3} vol_{S^2} \left(k^\alpha + \frac{r\, p_\gamma\, \partial_\beta k^\gamma\, \partial^\beta p^\alpha}{2\Delta} |dk|^{1/2} \right) , \qquad H_{\alpha\beta} = \frac{|dk|^{1/2}\, p_\alpha p_\beta + 6\, r\, \partial_\gamma k_\alpha \partial^\gamma p_\beta}{2\sqrt{3\,\Delta\, r}} \, ,$$

$$\Delta = \frac{3}{4}\, r\, |dk| + \frac{1}{2} |dk|^{1/2}\, p_\alpha p_\beta \partial_\gamma k^\alpha \partial^\gamma p^\beta \, , \qquad H_{\alpha\beta} = \frac{1}{\mathrm{Im}\,\tau} \begin{pmatrix} |\tau|^2 & \mathrm{Re}\,\tau \\ \mathrm{Re}\,\tau & 1 \end{pmatrix} .$$

- Can use diffeos on Σ_2 to make $g|_{\Sigma_2}$ conformally flat

$$dk^\alpha = I \cdot dp^\alpha \, .$$

  I complex structure on Σ_2.

- Solutions defined by two holomorphic functions $f^\alpha = k^\alpha + i\, p^\alpha$ with $df^\alpha \wedge d\bar{f}_\alpha \geq 0$.
- Upon field redefinition matches [D'Hoker, Gutperle, Karch, Ulhemann 2016]

AdS$_6$ minimal consistent truncations

- Minimal consistent truncation for free:

$$\mathcal{J}_u(x, Y) = X^{-1}(x)\, J_u(Y)\,, \qquad \hat{\mathcal{K}}(x, Y) = X^2(x)\, \hat{K}(Y)\,.$$

- From generalised metric and ExFT/IIB dictionary:

$$ds^2 = \frac{\sqrt{2}\, r^{5/4}\, \Delta^{1/4}}{3^{3/4}\, |dk|^{1/4}} \left(\frac{12}{r} ds_6^2 + \frac{X^2\, |dk|^{1/2}}{\Delta}\, ds_{S^2}^2 + \frac{4}{X^2\, r^2} dk^\alpha \otimes dp_\alpha \right)\,, \qquad C_{(4)} = 0\,,$$

$$C_{(2)}{}^\alpha = -\frac{4}{3} vol_{S^2} \left(k^\alpha + \frac{X^4\, r\, p_\gamma\, \partial_\beta k^\gamma\, \partial^\beta p^\alpha}{2\Delta} |dk|^{1/2} \right)\,, \quad H^{\alpha\beta} = \frac{X^4\, |dk|^{1/2}\, p^\alpha p^\beta + 6\, r\, \partial_\gamma k^\alpha \partial^\gamma p^\beta}{2\sqrt{3}\, \Delta\, r}\,,$$

$$\Delta = \frac{3}{4}\, r\, |dk| + \frac{1}{2} X^4 |dk|^{1/2}\, p_\alpha p_\beta \partial_\gamma k^\alpha \partial^\gamma p^\beta\,.$$

- Can study AdS$_6$ solution using minimal 6d gSUGRA.

Conclusions

- Exceptional geometry underlying 1/2-max AdS vacua.
- ExFT useful new tool to construct AdS vacua.
- Reproduce all mIIA $\mathrm{AdS}_7 \times S^3$, IIB $\mathrm{AdS}_6 \times S^2$ solutions.
- Consistent truncation (grav. multiplet) for free.
- Useful for studying & finding new solutions (e.g. domain-wall solutions for holographic RG flow)

Outlook

- Consistent truncations with matter (c.f. Valentí's talk)
- Uplift of moduli, e.g. $\mathcal{N} = 4$ AdS$_5$ vacua.
- Classification of AdS vacua?
- Other amounts of SUSY, e.g. $\mathcal{N} = 2$ [Ashmore, Gabella, Graña, Petrini, Waldram].
- Also, $\mathcal{N} = 6$ AdS_5 vacua dual to 4d $\mathcal{N} = 3$ SCFT?