Quantum scale invariance at three loops

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- "New physics" beyond SM: new symmetries?

- TeV-scale SUSY:

  - solved hierarchy problem; searched for at @ LHC; large χ^2/ndf; fine-tuned.

- Scale Invariance (SI):

  - SM with $m_h = 0$ has classical scale invariance. [Bardeen 1995]

$$x \to \rho\, x, \qquad \phi \to \rho^{-1}\,\phi, \quad [\phi] = 1. \quad \text{forbids} \qquad \int d^4x\; m^2 \phi^2 + \cdots$$

  - no dim-ful couplings; no scale $\to$ no hierarchy! all scales from vev's [M_{Planck} breaks it].
  - models with classical SI: "Higgs portal": $\lambda\phi^2\,\sigma^2$. [H Nicolai et al, P Ko, A Kobakhidze, C Csaki et al] [C Hill, B Grinstein, G. Ross, K. Allison, R Rattazzi et al]

- In this talk: 1) Scale invariance at quantum level [flat space]; 2) can protect a hierarchy of vev's [curved space: conformal/Weyl symmetry].

• Quantum level: UV regulators break SI explicitly: UV div's $\to$ subtraction scale (μ)

DR, d=4-2ϵ: $\lambda_\phi^0 = \mu^{2\epsilon}\left[\lambda_\phi + \sum_n a_n/\epsilon^n\right], \quad L = (1/2)(\partial_\mu \phi)^2 - \lambda_\phi\, \mu^{2\epsilon}\, \phi^4,$

[“Higgs portal” models: any flat direction lifted by loop effects. Pseudo-Goldstone (light)].

[Englert et al 1976, I-Z book, Shaposhnikov 2008]

• To avoid explicit S̸I: replace $\mu \to$ field σ. $\langle\sigma\rangle \neq 0$ spontaneous S̸I $\Rightarrow$ Goldstone/dilaton $\sigma = \langle\sigma\rangle\ e^{\tau}$;

$\Rightarrow$ spectrum extended by σ! different model! $x \to \rho\, x;\ \sigma \to \rho^{-1}\sigma;\ \tau \to \tau - \ln\rho$

[M_{string} moduli dep]

• WANTED! Do a scale-invariant quantum calculation & break SI spontaneously only.

- Toy model, flat space-time; S̸I, $\langle\sigma\rangle \gg \langle\phi\rangle$ stable...? loop corrections to V, m_ϕ....?

• Scale invariant regularisation (SR)

$d\!=\!4$ $$S = \int d^4x \left[(\partial_\mu \phi_j)^2 - V(\phi_j)\right] + \int d^4y\, L_h(\sigma, \partial\sigma)$$

- SI visible sector: ϕ_j; σ: hidden sector, decoupled. Poincaré symmetry $P_h \times P_v$. [Volkas, Kobakhidze, Foot 2013]
- σ: SI potential $\kappa_0\, e^{4\tau} \sim \lambda_\sigma\, \sigma^4$ but Poincaré symmetry demands $\lambda_\sigma = 0$. [Fubini 1976].

$d\!=\!4\!-\!2\epsilon,$ $\quad \mu = z\,\sigma^{2/(d-2)}, \qquad V \to \tilde V = \left[z\,\sigma^{2/(d-2)}\right]^{4-d} V(\phi_j), \qquad (z$: dim-less scale factor$)$

$$\tilde V(\phi_j,\sigma) = \mu_0^{2\epsilon}\Big[1 + 2\,\epsilon\left(\frac{\tilde\sigma}{\langle\sigma\rangle} - \frac{\tilde\sigma^2}{2\langle\sigma\rangle^2} + \ldots\right) + \epsilon^2\left(2\frac{\tilde\sigma}{\langle\sigma\rangle} + \frac{\tilde\sigma^2}{\langle\sigma\rangle^2} + \ldots.\right) + \mathcal{O}(\epsilon^3)\Big]\, V(\phi_j).$$

where: $\mu_0 = z\langle\sigma\rangle^{\frac{1}{1-\epsilon}}, \qquad \sigma = \langle\sigma\rangle + \tilde\sigma,$

$\Rightarrow$ Scale-inv reg $=$ DR $+$ dilaton with "evanescent" couplings $\propto \epsilon^n$ (∞-many) to visible sector

$\Rightarrow$ expect c-terms: $\displaystyle\sum_{m,n\geq 0;\,(m,n)\neq(0,0)} a_{mn}\,\frac{\partial^{2n}\alpha^{m+4}}{\sigma^{2n+m}}, \quad (\alpha = \phi, \sigma).$ $\qquad \sigma\sigma$ scattering: (3-loop)

[Komargodski et al]

• One-loop SI potential $(d = 4 - 2\epsilon)$:

$$L = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2}(\partial_\mu \sigma)^2 - V(\phi), \qquad V(\phi) = \frac{\lambda}{4!}\phi^4 \to \tilde{V} = z^{2\epsilon}\,\sigma^{2\epsilon/(1-\epsilon)}\,V(\phi).$$

$$V_1 = \tilde{V} - \frac{i}{2}\int \frac{d^d p}{(2\pi)^d}\ \mathrm{tr}\,\ln\left[p^2 - \tilde{V}_{\alpha\beta} + i\varepsilon\right] = \tilde{V} + \frac{1}{4\kappa}\sum_{s=\phi,\sigma}\tilde{M}_s^4\left[\frac{-1}{\epsilon} + \ln\frac{\tilde{M}_s^2}{c_0}\right].$$

$\tilde{M}_\phi^4 = M_\phi^2 + \epsilon\ldots$, $\tilde{M}_\sigma^4 \sim \epsilon^2$. Then

$$\delta L_1 = -\mu(\sigma)^{2\epsilon}\frac{1}{4!}(Z_\lambda - 1)\lambda\,\phi^4 \quad \text{with} \quad Z_\lambda - 1 = \frac{3\,\lambda}{2\kappa\epsilon}.$$

$$U = V(\phi) + \frac{1}{4\kappa}V_{\phi\phi}^2\left[\overline{\ln}\frac{V_{\phi\phi}}{(z\,\sigma)^2} - \frac{1}{2}\right]; \qquad V_{\phi\phi} = \frac{\lambda}{2}\phi^2.$$

⇒ Scale invariant result, due to dilaton σ. ($\ln\sigma$: finite quantum effect, due to symmetry, not dynamics).

$\kappa = (4\pi)^2$, $\overline{\ln}A = \ln A/(4\pi e^{1-\gamma})$

• One-loop SI potential:

No new poles: one-loop beta $\beta_\lambda^{(1)}$ unchanged from the theory without dilaton: $\lambda^B = \lambda Z_\lambda\, Z_\phi^{-2}$.

$$\frac{d\,\lambda^B}{d\ln z} = 0 \quad \Rightarrow \quad \beta_\lambda^{(1)} = \frac{d\lambda}{d\ln z} = \frac{3}{\kappa}\,\lambda^2,$$

[Shaposhnikov et al, Tamarit]

Callan-Symanzik:

$$\frac{dU}{d\ln z} = \Big(\frac{\partial}{\partial \ln z} + \beta_\lambda^{(1)}\frac{\partial}{\partial\lambda}\Big)\,U = O(\lambda^3).$$

Decouple dilaton fluctuations:

$$U \;=\; V(\phi) + \underbrace{\frac{1}{4\kappa}\,V_{\phi\phi}^2\Big[\overline{\ln}\frac{V_{\phi\phi}}{(z\langle\sigma\rangle)^2} - \frac{1}{2}\Big]}_{CW} + \underbrace{\frac{1}{4\kappa}\,V_{\phi\phi}^2\Big(-\frac{\tilde\sigma}{\langle\sigma\rangle} + \frac{1}{2}\frac{\tilde\sigma^2}{\langle\sigma\rangle^2} + \cdots\Big)}_{\to 0;\ \text{small yet maintains SI}}$$

$\sigma = \langle\sigma\rangle + \tilde\sigma$. CW term also respects CS wrt $\mu_0 = z\langle\sigma\rangle$.

$\Rightarrow$ Transmission of scale symmetry breaking to visible sector.

$\Rightarrow$ Unlike in theories without dilaton (explicit $\not{S}$I by quantum calculations), $\beta_\lambda = 0$ is not a necesary.

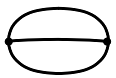
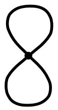
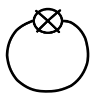
condition for SI here (spontaneous, different spectrum/symmetry) [C. Tamarit 2014]

• Two-loop SI potential: [background field method]; Jackiw 1974]

$$\tilde V(\phi,\sigma) = z^{2\epsilon}\,\sigma^{2\epsilon/(1-\epsilon)}\,V(\phi)$$

$$\tilde V(\phi+\delta_\phi,\sigma+\delta_\sigma) = \tilde V(\phi,\sigma)+\tilde V_\alpha\delta_\alpha+\frac{1}{2}\tilde V_{\alpha\beta}\,\delta_\alpha\delta_\beta+\frac{1}{3!}\tilde V_{\alpha\beta\gamma}\,\delta_\alpha\delta_\beta\delta_\gamma+\frac{1}{4!}\tilde V_{\alpha\beta\gamma\rho}\,\delta_\alpha\delta_\beta\delta_\gamma\delta_\rho+\cdots \qquad \alpha,\beta:\phi,\sigma.$$

Two-loop:

$V_2 \;=\; \frac{i}{12}$  $+\frac{i}{8}$  $+\frac{i}{2}$  $= \frac{i}{12}\,\tilde V_{\alpha\beta\gamma}\tilde V_{\alpha'\beta'\gamma'}\int\frac{d^dp}{(2\pi)^d}\frac{d^dq}{(2\pi)^d}\,(\tilde D_p)_{\alpha\alpha'}\,(\tilde D_q)_{\beta\beta'}\,(\tilde D_{p+q})_{\gamma\gamma'}+\cdots$

$$= (z\sigma)^{2\epsilon}\,\frac{\lambda^3\,\phi^4}{32\kappa^2}\Big\{-\frac{3}{\epsilon^2}+\frac{2}{\epsilon}+\mathcal{O}(\epsilon^0)\Big\}. \quad \text{(same poles if no } \sigma\text{)}. \qquad (\tilde D_p)_{\alpha\beta} = (p^2\delta_{\alpha\beta}-\tilde V_{\alpha\beta})^{-1}$$

$$\tilde V_{\alpha\beta\gamma\ldots} = V_{\alpha\beta\gamma\ldots}+\epsilon\,\times(\ldots)_{\alpha\beta\gamma\ldots}+\epsilon^2\,\times(\ldots)_{\alpha\beta\gamma\ldots}, \qquad (\tilde D_p)_{\alpha\beta} = (D_p)_{\alpha\beta}+\epsilon\;(\ldots)_{\alpha\beta}+\epsilon^2\;(\ldots)_{\alpha\beta}.$$

Then:

$$U \;=\; \frac{\lambda}{4!}\,\phi^4\Big\{1+\frac{3\lambda}{2\kappa}\Big(\overline{\ln}\frac{V_{\phi\phi}}{(z\sigma)^2}-\frac{1}{2}\Big)+\frac{3\lambda^2}{4\kappa^2}\Big(4+A_0-4\,\overline{\ln}\frac{V_{\phi\phi}}{(z\sigma)^2}+3\,\overline{\ln}^2\frac{V_{\phi\phi}}{(z\sigma)^2}\Big)+\frac{5\lambda^2}{\kappa^2}\frac{\phi^2}{\sigma^2}+\frac{7\lambda^2}{24\kappa^2}\frac{\phi^4}{\sigma^4}\Big\},$$

V $\qquad V^{(1)}$ $\qquad V^{(2)}$ $\qquad$ new $V^{(2,n)}$ finite; no z

• Two-loop: Taylor expand about $\sigma = \langle\sigma\rangle + \tilde{\sigma}$:

$$U = \frac{\lambda}{4!}\phi^4 \left\{ 1 + \frac{3\lambda}{2\kappa}\left(\overline{\ln}\frac{V_{\phi\phi}}{(z\langle\sigma\rangle)^2} - \frac{1}{2}\right) + \frac{3\lambda^2}{4\kappa^2}\left(4 + A_0 - 4\,\overline{\ln}\frac{V_{\phi\phi}}{(z\langle\sigma\rangle)^2} + 3\,\overline{\ln}^2\frac{V_{\phi\phi}}{(z\langle\sigma\rangle)^2}\right) + \mathcal{O}\Big(\frac{1}{\langle\sigma\rangle}\Big)\right.$$

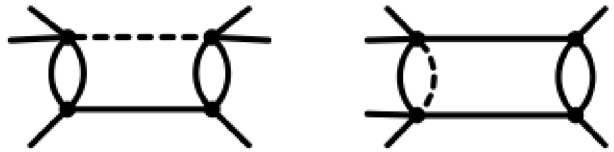
- This is the "usual" CW result with $\mu = \langle\sigma\rangle$, broken SI, no dilaton present. [Cheng, I. Jack, T. Jones, S. Martin]

- new terms comparable/larger than standard two-loop terms

$$\frac{\phi^n}{\sigma^n} \sim 1., \qquad n = 1, 2.$$

- Non-polynomial terms: - vanish for $\phi \ll \sigma$, only $\log\sigma$ is left.
  - respect symmetries of the theory.
  - finite counterterms, cannot be "seen" in a scheme that breaks this symmetry.
  - if not forbidden (by a symmetry), operators generated at quantum level.
  - non-renormalizable.

- Three-loop SI potential



Counterterm: $\delta L_3 = \frac{1}{2}\,\delta_\phi^{(3)}\,(\partial_\mu\phi)^2 - \mu^{2\epsilon}\Big(\frac{1}{4!}\,\delta_\lambda^{(3)}\,\lambda\,\phi^4 + \frac{1}{6}\,\delta_{\lambda_6}^{(3)}\,\lambda_6\,\frac{\phi^6}{\sigma^2} + \frac{1}{8}\,\delta_{\lambda_8}^{(3)}\,\lambda_8\,\frac{\phi^8}{\sigma^4}\Big)$ [Monin 2015]

$$\delta_\phi^{(3)} = -\frac{\lambda^3}{4\kappa^3}\Big(\frac{1}{6\epsilon^2} - \frac{1}{12\epsilon}\Big), \qquad \delta_{\lambda_6}^{(3)} = \frac{3}{2}\frac{\lambda^4}{\lambda_6\,\kappa^3\,\epsilon}, \qquad \delta_{\lambda_8}^{(3)} = \frac{275}{864}\frac{\lambda^4}{\lambda_8\,\kappa^3\,\epsilon.}$$

So $\gamma_\phi^{(3)} = \lambda^3/(16\kappa^3)$. With $Z_X = 1 + \delta_X$ and $\lambda_6^B = \mu^{2\epsilon}(\sigma)\lambda_6 Z_{\lambda_6} Z_\phi^{-3} Z_\sigma$ and $(d/d\ln z)\,\lambda_6^B = 0$,

$$\beta_{\lambda_6} \;=\; \frac{\lambda^2\,\lambda_6}{2\kappa^2} + \frac{\lambda^3}{\kappa^3}\Big(9\lambda - \frac{3}{8}\lambda_6\Big), \qquad \text{(similar } \beta_{\lambda_8}\text{)}.$$

Callan-Symanzik: $V^{(3)}$ ["usual" with $\mu \to \sigma$]$+V^{(3,n)}$ [new]:

$$\frac{\partial V^{(3)}}{\partial \ln z} + \beta_\lambda^{(1)}\frac{\partial V^{(2)}}{\partial\lambda} + \beta_\lambda^{(2)}\frac{\partial V^{(1)}}{\partial\lambda} + \beta_\lambda^{(3)}\frac{\partial V}{\partial\lambda} + \gamma_\phi^{(2)}\,\frac{\partial V^{(1)}}{\partial\ln\phi} + \gamma_\phi^{(3)}\,\frac{\partial V}{\partial\ln\phi} = \mathcal{O}(\lambda_j^5).$$

$$\frac{\partial V^{(3,n)}}{\partial \ln z} + \beta_{\lambda_j}^{(1)}\frac{\partial V^{(2,n)}}{\partial\lambda_j} + \beta_{\lambda_j}^{(3,n)}\frac{\partial V}{\partial\lambda_j} = \mathcal{O}(\lambda_j^5), \qquad \lambda_j = \lambda, \lambda_6, \lambda_8.$$

• Three-loop SI potential: Integrate Callan-Symanzik:

$$U = V + \Delta V + V^{(3)} + V^{(3,n)}, \quad \Delta V = \lambda_6 \frac{\phi^6}{\sigma^2} + \lambda_8 \frac{\phi^8}{\sigma^4}$$

$$V^{(3)} = \frac{\lambda^4 \phi^4}{\kappa^3} \Big\{ \mathcal{Q} + \Big(\frac{97}{128} + \frac{9}{64} A_0 + \frac{\zeta[3]}{4} \Big) \overline{\ln} \frac{V_{\phi\phi}}{(z\sigma)^2} - \frac{31}{96} \overline{\ln}^2 \frac{V_{\phi\phi}}{(z\sigma)^2} + \frac{9}{64} \overline{\ln}^3 \frac{V_{\phi\phi}}{(z\sigma)^2} \Big\}.$$

$$V^{(3,n)} = \frac{\lambda^3}{2\,\kappa^3} \phi^4 \Big\{ \Big(27\lambda - \frac{\lambda_6}{2} \Big) \frac{\phi^2}{8\,\sigma^2} + \Big(\frac{401\,\lambda}{72} - \lambda_8 \Big) \frac{\phi^4}{16\,\sigma^4} \Big\} \overline{\ln} \frac{V_{\phi\phi}}{(z\sigma)^2}. \qquad V_{\phi\phi} = \frac{\lambda}{2} \phi^2.$$

⇒ more non-polynomial operators at higher orders. Vanish for $\phi \ll \sigma$; $\ln \sigma$ left.

$\beta_\lambda^{(3)}$: unchanged (as if no dilaton; μ=const). Higher loops?

• some conclusions so far:

⇒ only terms suppressed by σ. For large σ Poincaré symmetry $P_v \times P_h$ restored.

⇒ No terms such as: $\lambda\,\phi^2\sigma^2 = \lambda\,\langle\sigma\rangle^2\,\phi^2 + \cdots$. No tunning of λ for large σ. More scalars?

⇒ since all scales generated by vev of fields, result not a DR artefact

• Dilatation current D_μ: $$D^\mu = \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_j)}(x^\nu\partial_\nu\phi_j + d_\phi) - x^\mu\,\mathcal{L}, \qquad d_\phi = (d-2)/2 \quad \text{(scalars)},$$

$$\partial_\mu D^\mu = (d_\phi+1)\,(\partial_\mu\phi_j)\,\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_j)} + d_\phi\,\phi_j\,\frac{\partial\mathcal{L}}{\partial\phi_j} - d\mathcal{L}$$

canonical k.t., SI potential $\tilde{V}$ in $d = 4-2\epsilon$, onshell:

$$\partial_\mu D^\mu = d\,\tilde{V} - \frac{d-2}{2}\phi_j\,\frac{\partial\tilde{V}}{\partial\phi_j}, \qquad \phi_j = \phi, \sigma;$$

in SI theories: $\tilde{V}$ homogeneous: $\tilde{V}(\rho\,\phi_j) = \rho^{2d/(d-2)}\,\tilde{V}(\phi_j)$. (do $\partial/\partial\rho$, $\rho\to 1$, $\partial_\mu D^\mu = 0$). $\beta_\lambda \neq 0$.

• in "traditional reg" $\tilde{V} = \mu^{2\epsilon}\,V(\phi)$, with V scale invariant in $d=4$ (not in $d = 4-2\epsilon$):

$$\partial_\mu D^\mu = 2\,\epsilon\,\mu^{2\epsilon}\,V \sim 2\,\epsilon\,\mu^{2\epsilon}\left[\lambda + \frac{\beta_\lambda}{\epsilon} + \cdots\right]\frac{\partial V}{\partial\lambda} \propto \beta_\lambda\,\frac{\partial V}{\partial\lambda} \qquad \text{(scale anomaly)}$$

[Shaposhnikov et al, Tamarit 2013]

- different field content(!); here $\beta_\lambda = 0$ for SI.

• Model building: tree level [Shaposhnikov et al 2009]

$$\mathcal{L} = \frac{1}{2}\,\partial_\mu\phi\,\partial^\mu\phi + \frac{1}{2}\,\partial_\mu\sigma\,\partial^\mu\sigma - V(\phi,\sigma), \qquad V = \frac{\lambda_\phi}{4!}\phi^4 + \frac{\lambda_m}{4}\,\phi^2\sigma^2 + \frac{\lambda_\sigma}{4!}\,\sigma^4$$

- want spontaneous $\not{S}$I: min: $\langle\phi\rangle\left[\lambda_\phi\langle\phi\rangle^2 + 3\lambda_m\langle\sigma\rangle^2\right] = 0, \quad \langle\sigma\rangle\left[3\lambda_m\langle\phi\rangle^2 + \lambda_\sigma\langle\sigma\rangle^2\right] = 0,$

$\langle\sigma\rangle \neq 0 \Rightarrow \langle\phi\rangle \neq 0$ non-trivial ground state if:

(i) $9\lambda_m^2 = \lambda_\phi\lambda_\sigma +$ loops, $\lambda_m < 0$; and **(ii)** $\dfrac{\langle\phi\rangle^2}{\langle\sigma\rangle^2} = -\dfrac{3\lambda_m}{\lambda_\phi}(1 + \text{loops}), \;\Rightarrow\; V = \dfrac{\lambda_\phi}{4}\Big(\phi^2 + \dfrac{3\lambda_m}{\lambda_\phi}\,\sigma^2\Big)^2$

flat direction: $V = 0$ on g.s.

• (i) in a given loop order: tuning $V = 0 \Rightarrow$ spontaneous $\not{S}$I $\Rightarrow$ EWSB.

$\lambda_\sigma \ll |\lambda_m| \ll \lambda_\phi$: one classical tuning so $m_{\tilde\phi} \sim \lambda_\phi\langle\phi\rangle^2 \sim -\lambda_m\langle\sigma\rangle^2 \ll \langle\sigma\rangle^2$,

• Question: quantum level: is such hierarchy protected by SI $+ P_v \times P_h$? Any dangerous $\sim \lambda_\phi^2\,\langle\sigma\rangle^2$?

$V(\phi,\sigma) = \sigma^4\,W(\phi/\sigma)$, min: $W(\phi/\sigma) = W'(\phi/\sigma) = 0, \quad \Rightarrow \; V \propto \left[\phi^2/\sigma^2 - \langle\phi\rangle^2/\langle\sigma\rangle^2\right]^2$

- Model building: one-loop SI potential

$$\mathcal{L} = \frac{1}{2}\,\partial_\mu\phi\,\partial^\mu\phi + \frac{1}{2}\,\partial_\mu\sigma\,\partial^\mu\sigma - V(\phi,\sigma), \qquad V = \frac{\lambda_\phi}{4!}\phi^4 + \frac{\lambda_m}{4}\,\phi^2\sigma^2 + \frac{\lambda_\sigma}{4!}\,\sigma^4, \qquad 9\lambda_m^2 = \lambda_\phi\,\lambda_\sigma + \text{loops}.$$

DR: $d = 4 - 2\,\epsilon$, $V \to \tilde{V} \equiv \mu^{2\epsilon}\; V(\phi,\sigma)$, $\quad \mu = z\,\sigma^{2/(d-2)}$.

$$V_1 \;=\; \tilde{V} - \frac{i}{2}\int \frac{d^dp}{(2\pi)^d}\;\text{tr}\,\ln\Big[p^2 - \tilde{V}_{\alpha\beta} + i\varepsilon\Big] = \tilde{V} + \frac{1}{4\kappa}\sum_{s=\phi,\sigma}\tilde{M}_s^4\Big[\frac{-1}{\epsilon} + \ln\frac{\tilde{M}_s^2}{c_0}\Big].$$

$$\tilde{V}_{\alpha\beta} \;=\; \mu^{2\epsilon}\Big[V_{\alpha\beta} + \epsilon\; N_{\alpha\beta}\Big] + \mathcal{O}(\epsilon^2),$$

$$N_{\alpha\beta} \;\equiv\; \frac{2}{\mu^2}\Big[\mu\,(\mu_\alpha\,V_\beta + \mu_\beta\,V_\alpha) + (\mu\,\mu_{\alpha\beta} - \mu_\alpha\,\mu_\beta)\,V\Big] \qquad \alpha,\beta = \phi,\sigma.$$

$\Rightarrow$ New, finite one-loop corrections $\epsilon \times \frac{1}{\epsilon}$ from derivative of μ wrt σ. Suppressed by σ

• Model building: one-loop SI potential

$$U = V + V^{(1)} + V^{(1,n)}$$

$$V^{(1)} = \frac{1}{64\pi^2}\Big\{\sum_{s=\phi,\sigma} M_s^4(\phi,\sigma)\Big[\ln\frac{M_s^2(\phi,\sigma)}{(z\,\sigma)^2} - \frac{3}{2}\Big]\Big\}$$

$$V^{(1,n)} = \frac{\lambda_m}{\lambda_\phi}\Big(\frac{\phi^2}{\sigma^2} + \frac{3\lambda_m}{\lambda_\phi}\Big)\big(\lambda_\phi^2\,\phi^4 - 4\,\lambda_\phi\,(4\,\lambda_\phi + \lambda_m)\,\phi^2\,\sigma^2 - 21\lambda_m^2\,\sigma^4\big) \supset \lambda_m\lambda_\phi\frac{\phi^6}{\sigma^2} + \cdots$$

- If $\lambda_m = 0$ $(\lambda_\sigma = 0)$ then $V^{(1,n)} = 0$. Also z-independent. $\beta_{\lambda_m} \propto \lambda_m \to 0$ if $\lambda_m \to 0$ $(P_v \times P_h)$
- 1 massive + 1 massles d.o.f.; Taylor expand: $\sigma = \langle\sigma\rangle + \tilde\sigma$, $\phi = \langle\phi\rangle + \tilde\phi$.

$$\frac{\phi^6}{\sigma^2} = \frac{(\tilde\phi + \langle\phi\rangle)^6}{\langle\sigma\rangle^2}\Big(1 - \frac{2\tilde\sigma}{\langle\sigma\rangle} + \frac{3\,\tilde\sigma^2}{\langle\sigma\rangle^2} + \cdots\Big)$$

$\Rightarrow V^{(1,n)}$ & non-polynomial operator already @ one-loop, finite, independent of z

• Model building: Minimise U: $\lambda_\phi \gg |\lambda_m| \gg \lambda_\sigma, \quad m_\phi^2 \sim \lambda_\phi \langle\phi\rangle^2 \sim -\lambda_m \langle\sigma\rangle^2 \ll \langle\sigma\rangle^2$ (*)

$$U = \frac{\lambda_\phi}{4}\phi^4 + \frac{\lambda_m}{2}\phi^2\sigma^2 + \frac{\lambda_\sigma}{4}\sigma^4 + \frac{1}{64\pi^2}\Big\{\sum_{s=1,2} M_s^4\Big[\ln\frac{M_s^2}{z^2\sigma^2} - \frac{3}{2}\Big] + \lambda_\phi\lambda_m\frac{\phi^6}{\sigma^2} - \big(16\,\lambda_\phi\lambda_m + 6\lambda_m^2 - 3\lambda_\phi\lambda_\sigma\big)\phi^4 - 16\,\lambda_m^2\,\phi^2\sigma^2\Big\} + \mathcal{O}(\lambda_m^3)$$

$$\text{min:} \quad \rho \equiv \frac{\langle\phi\rangle^2}{\langle\sigma\rangle^2} = -\frac{\lambda_m}{\lambda_\phi}\Big[1 - \underbrace{\frac{6\lambda_\phi}{64\pi^2}\big(4\ln 3\lambda_\phi - 17/3\big)}_{\text{one-loop}}\Big] + \mathcal{O}(\lambda_m^2)$$

- fix subtraction "parameter": $z = \langle\phi\rangle/\langle\sigma\rangle \Rightarrow \mu = \langle\phi\rangle$ (usual). $m_{\tilde\phi}^2 = \text{tr}U_{ij}\Big|_{min}$.

$$\Delta m_{\tilde\phi}^2\Big|_{\text{1-loop}} = \frac{-\lambda_m^2\langle\sigma\rangle^2}{32\pi^2} \times \text{function}(\lambda_\phi, \rho)$$

$\Rightarrow$ No tuning beyond classical one to have $\Delta m_{\tilde\phi}^2\Big|_{\text{1-loop}} \ll \langle\sigma\rangle^2$. no $\lambda_\phi^2\langle\sigma\rangle^2$; spontaneous $\to$ all orders.

• Application: scale invariant SM + dilaton: at one-loop:

$$V = \frac{\lambda_\phi}{3!}(H^\dagger H)^2 + \frac{\lambda_m}{2}(H^\dagger H)\,\sigma^2 + \frac{\lambda_\sigma}{4!}\sigma^4 + \frac{4\,\lambda_6}{3}\frac{(H^\dagger H)^3}{\sigma^2} + \cdots$$

$$= \frac{1}{4!}\lambda_\phi\,\phi^4 + \frac{1}{4}\lambda_m\,\phi^2\sigma^2 + \frac{1}{4!}\lambda_\sigma\,\sigma^4 + \frac{\lambda_6}{6}\frac{\phi^6}{\sigma^2}\cdots; \qquad (\lambda_\phi\lambda_\sigma = 9\lambda_m^2 + \text{loops}).$$

SM one-loop potential $U = V + V^{(1)} + V^{(1,n)}$

$$V^{(1)} \equiv \frac{1}{4\kappa}\sum_{j=\phi,\sigma;G,t,W,Z} n_j\, m_j^4(\phi,\sigma)\,\ln\frac{m_j^2(\phi,\sigma)}{c_j\,(z\sigma)^2}$$

$$V^{(1,n)} \equiv \frac{1}{48\kappa}\Big[(-16\lambda_m\lambda_\phi - 18\lambda_m^2 + \lambda_\phi\lambda_\sigma)\,\phi^4 - \lambda_m(48\lambda_m + 25\lambda_\sigma)\phi^2\sigma^2 - 7\lambda_\sigma^2\sigma^4$$

$$+ \; (\lambda_\phi\lambda_m + 6\lambda_6\lambda_\sigma)\frac{\phi^6}{\sigma^2} + 8\,(4\lambda_\phi - 2\lambda_m)\,\lambda_6\frac{\phi^8}{\sigma^4} + (192\lambda_6 + 2\lambda_\phi)\lambda_6\frac{\phi^{10}}{\sigma^6} + 40\lambda_6^2\frac{\phi^{12}}{\sigma^8}\Big]$$

- scale invariant. New one-loop terms. Taylor-expand about $\langle\sigma\rangle$: leading order $\mathcal{O}(1/\langle\sigma\rangle)$: usual CW.

- Implications:

$$\text{minimum} \qquad \frac{\langle\phi\rangle^2}{\langle\sigma\rangle^2} = \frac{-3\lambda_m}{\lambda_\phi}\left[1+\zeta\right], \qquad \zeta = \text{function}(\lambda_\phi)$$

- one-loop correction

$$\Delta m^2_{\tilde{\phi}} = \frac{-\lambda_m}{\lambda_\phi}\frac{\langle\sigma\rangle^2}{16\kappa}\left\{27\left[g^4\left(\ln\frac{g^2}{4}+\frac{1}{3}\right)+2\,g_2^4\left(\ln\frac{g_2^2}{4}+\frac{1}{3}\right)-16\,h_t^4\left(\ln\frac{h_t^2}{2}-\frac{1}{3}\right)\right] + 4\,\lambda_\phi^2\left[5\ln\frac{\lambda_\phi^2}{12}-8+\ln 27\right]\right\}.$$

- no extra tuning beyond classical one and that of couplings (for $V=0$).
- beta functions, similar to case with $\mu =$ constant (but same field content):

$$\beta_{\lambda_\phi} = \frac{1}{\kappa}\left[3\left(\frac{9}{4}g_2^4+\frac{3}{4}g_1^4+\frac{3}{2}g_1^2g_2^2-12h_t^4\right)-4\lambda_\phi\left(\frac{3}{4}g_1^2+\frac{9}{4}g_2^2-3h_t^2\right)+4\lambda_\phi^2+3\lambda_m^2+96\lambda_m\lambda_6\right]$$

$$\beta_{\lambda_m} = \frac{2\lambda_m}{\kappa}\left[\lambda_\phi+2\lambda_m+\frac{1}{2}\lambda_\sigma-\left(\frac{3}{4}g_1^2+\frac{9}{4}g_2^2-3h_t^2\right)\right]$$

• New couplings: beta functions:

$$\beta_{\lambda_6} = \frac{3\lambda_6}{\kappa}\Big[6\lambda_\phi - 8\lambda_m + \lambda_\sigma - 2\Big(\frac{3}{4}g_1^2 + \frac{9}{4}g_2^2 - 3h_t^2\Big)\Big]$$

$$\beta_{\lambda_8} = \frac{2}{\kappa}\Big[2\lambda_6\,(28\lambda_6 + \lambda_m) - 4\lambda_8\Big(\frac{3}{4}g_1^2 + \frac{9}{4}g_2^2 - 3h_t^2\Big)\Big]$$

$$\beta_{\lambda_{10}} = 10\Big[4\lambda_6^2 - \lambda_{10}\Big(\frac{3}{4}g_1^2 + \frac{9}{4}g_2^2 - 3h_t^2\Big)\Big]$$

$$\beta_{\lambda_{12}} = 2\Big[3\lambda_6^2 - 6\lambda_{12}\Big(\frac{3}{4}g_1^2 + \frac{9}{4}g_2^2 - 3h_t^2\Big)\Big]$$

$\Rightarrow$ if one sets $\lambda_{6,8,10,..} = 0$ at tree level, then $\beta_{\lambda_{6,8,10,12}} = 0$ at one-loop, but emerge at 2-loops.

• implications: SM vacuum stability; other: non-minimal coupling, Brans-Dicke theory.

[Ross,Hill,Ferreira; Shaposhnikov et al]

- Conclusions

$\Rightarrow$ scale-invariant UV regularization (SR) possible; additional σ beyond initial spectrum.

$\Rightarrow$ SR "=" DR + σ with "evanescent" $\propto \epsilon^n$ corrections. Spontaneous $\not{\text{SI}}$, flat direction (tuning $V = 0$....)

$\Rightarrow$ quantum level: scale invariant scalar potential. $\Rightarrow$ ϕ^6/σ^2, ϕ^8/σ^4, etc, non-polynomial c-terms

$\Rightarrow$ non-renormalizability.

SI + enhanced Poincaré symmetry:

$\Rightarrow$ Quantum stable hierarchy of vev's from tuning couplings order-by-order for $V = 0$.
All scales from fields' vevs, not an artefact of DR. More scalar fields?

$\Rightarrow$ Other applications? - SM vacuum stability? see talk by Z. Lalak at this workshop.
- curved space-time: Weyl symmetry. see talk b G.G. Ross

• Model building. One-loop: No new poles, usual counterterms:

$$\delta L_1 = -\mu(\sigma)^{2\epsilon}\Big\{\frac{1}{4!}(Z_{\lambda_\phi} - 1)\lambda_\phi\phi^4 + \frac{1}{4}(Z_{\lambda_m} - 1)\lambda_m\phi^2\sigma^2 + \frac{1}{4!}(Z_{\lambda_\sigma} - 1)\lambda_\sigma\sigma^4\Big]$$

with

$$\begin{aligned} Z_{\lambda_\phi} &= 1 + \frac{3}{2\kappa\,\epsilon}(\lambda_\phi + \lambda_m^2/\lambda_\phi), \\ Z_{\lambda_m} &= 1 + \frac{1}{2\kappa\,\epsilon}(\lambda_\phi + \lambda_\sigma + 4\lambda_m), \end{aligned}$$

$\kappa = (4\pi)^2$. From: $d\,\underbrace{(\mu(\sigma)^{2\epsilon}\lambda_\phi Z_{\lambda_\phi})}_{\lambda_\phi^B}/d\ln z = 0$ one recovers usual beta functions:

$$\begin{aligned} \beta^{(1)}_{\lambda_\phi} &\equiv \frac{d\lambda_\phi}{d\ln z} = \frac{3}{\kappa}\,(\lambda_\phi^2 + \lambda_m^2), \\ \beta^{(1)}_{\lambda_m} &\equiv \frac{d\lambda_m}{d\ln z} = \frac{1}{\kappa}\,(\lambda_\phi + 4\lambda_m + \lambda_\sigma)\lambda_m \quad [\rightarrow 0 \text{ if } \lambda_m = 0]. \end{aligned}$$

Callan Symanzik: [see C. Tamarit 2014]

$$\frac{dU}{d\ln z} = \Big(\beta^{(1)}_{\lambda_j}\frac{\partial}{\partial\lambda_j} + z\,\frac{\partial}{\partial z}\Big)\,U = \mathcal{O}(\lambda_j^3),$$

• Two-loop: No new poles. Two-loop $\beta_\lambda^{(2)}$, anom dims $\gamma_\phi^{(2)}$ unchanged. Usual counterterm:

$$\delta L_2 = \frac{1}{2}(\partial_\mu\phi)^2\,\delta_\phi^{(2)} - \mu(\sigma)^{2\epsilon}\frac{1}{4!}\,\lambda\,\phi^4\,\delta_\lambda^{(2)}, \qquad \delta_\lambda^{(2)} = \frac{\lambda^2}{\kappa^2}\Big(\frac{9}{4\,\epsilon^2} - \frac{3}{2\,\epsilon}\Big), \qquad \delta_\phi^{(2)} = \frac{-\lambda^2}{24\,\kappa^2\,\epsilon}.$$

$$\beta_\lambda^{(2)} = -\frac{17}{3\,\kappa^2}\,\lambda^3, \qquad \text{unchanged (as if no dilaton \& } \mu = \text{const).} \qquad \gamma_\sigma^{(2)} = 0.$$

Callan-Symanzik (consistency check):

$$\frac{\partial\, V^{(2)}}{\partial \ln z} + \Big[\beta_\lambda^{(2)}\frac{\partial}{\partial\lambda} + \gamma_\phi^{(2)}\phi\frac{\partial}{\partial\phi} + \gamma_\sigma^{(2)}\sigma\frac{\partial}{\partial\sigma}\Big]\,V + \beta_\lambda^{(1)}\,\frac{\partial V^{(1)}}{\partial\lambda} = O(\lambda^4), \quad \text{“usual” CS}$$

$$\frac{\partial\, V^{(2,\mathrm{n})}}{\partial \ln z} = O(\lambda^4),$$

$\beta^{(k)}$, $\gamma^{(k)}$, $V^{(k)}$: k-loop correction.

• if $V \to V + \frac{\lambda_m}{2}\,\phi^2\sigma^2$, $\quad V^{(2,n)} \supset \frac{\lambda\,\lambda_m}{96\kappa^2\,\epsilon}\Big\{7\,(2\lambda_m - \lambda)\frac{\phi^6}{\sigma^2} - \frac{3\lambda_m}{2}\frac{\phi^8}{\sigma^4}\Big\} \;\Rightarrow\; \beta_\lambda \to \beta_\lambda + \frac{\lambda_m}{\kappa^2}\Big[12\lambda_m^2 - 7\lambda_m\lambda - 40\lambda^2\Big]$

$\beta_{\lambda_m} \propto \lambda_m^2$; $\quad \lambda_m \to 0$: $P_v \times P_h$.