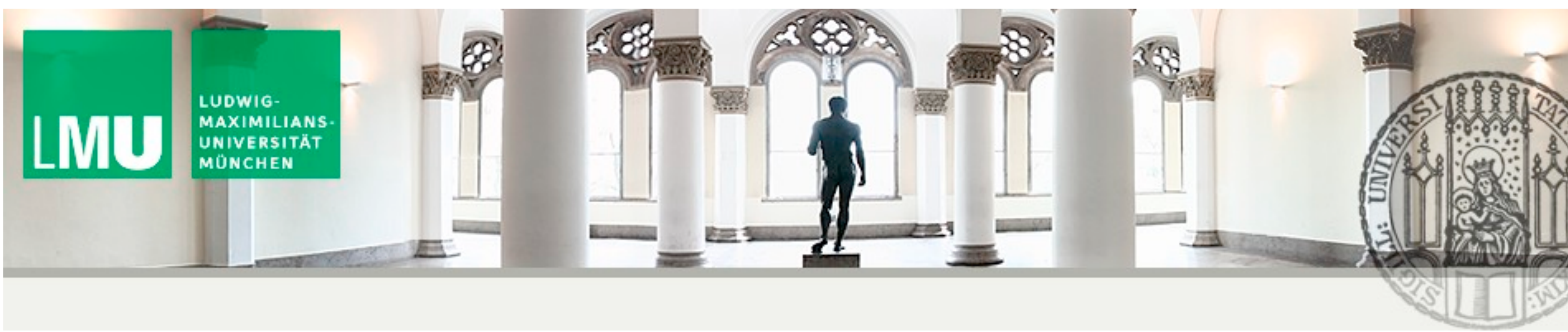
W-Supergravity

DIETER LÜST (LMU, MPI)



Corfu Workshop on Dualities and Generalised Geometries, September 2018

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Joint work with Sergio Ferrara: arXiv:1805.10022
and with Sergio Ferrara & Alex Kehagias: arXiv:1806.10016

See also earlier work with I. Florakis, I. Garcia-Etxebarria, D. Regalado:
arXiv:1712.04318

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No-go theorems in Quantum Gravity:

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Which IR consistent quantum field theories cannot be embedded into a UV complete quantum gravity theory?

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In general, all (also hidden) assumptions have to be very carefully stated.

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⇒ W - supergravity & W - superstrings

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[S. Ferrara, D.L. (2018)]

- Exist also for N=7 supersymmetry
- Topological theories: topological twist by S - duality and diffeomorphisms
- Strongly coupled, massive spin-four theories

Classification of rigid, supersymmetric 4D field theories:

exist for $\mathcal{N} = 1, 2, 4$ supersymmetry

But not for $\mathcal{N} = 3$ (automatically extends to $\mathcal{N} = 4$)

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Classification of 4D supergravity theories:

exist for $\mathcal{N} = 1, 2, 3, 4, 5, 6, 8$ supersymmetry

But not for $\mathcal{N} = 7$ (automatically extends to $\mathcal{N} = 8$)

[D. Freedman, A. Van Proeyen (2012)]

Main assumptions:

- weak coupling
- CPT invariant lagrangian description
- massless N-extended spin-two gravity multiplet

This agrees with perturbative 4D string constructions:

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4D Heterotic string: # of right-moving supercharges:

$$\mathcal{N}_H = 1\,, \quad \mathcal{N}_H = 2\,, \quad \text{and} \quad \mathcal{N}_H = 4$$

$$\mathbb{R}^{1,3}\times \quad CY_3 \quad K3 \times T^2 \quad T^6$$

Again no $\mathcal{N} = 3$ supersymmetry.

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Again no $\mathcal{N} = 3$ supersymmetry.

4D Type II string: # of left + right-moving supercharges:

$$\mathcal{N}_{II} = 1 = 1_L + 0_R\,, \quad \mathcal{N}_{II} = 2 = 2_L + 0_R\,, \quad \mathcal{N}_{II} = 2' = 1_L + 1_R\,,$$
$$\mathcal{N}_{II} = 3 = 2_L + 1_R\,, \quad \mathcal{N}_{II} = 4 = 4_L + 0_R\,, \quad \mathcal{N}_{II} = 4' = 2_L + 2_R\,,$$
$$\mathcal{N}_{II} = 5 = 4_L + 1_R\,, \quad \mathcal{N}_{II} = 6 = 4_L + 2_R\,, \quad \mathcal{N}_{II} = 8 = 4_L + 4_R\,.$$

Again no $\mathcal{N} = 7$ supersymmetry.

[H. Kawai, S. Lewellen, H. Tye (1987);
W. Lerche, D.L., A.Schellekens (1987);
A. Antoniadis, C. Bachas, C. Kounnas (1987);
S. Ferrara, D.L., S. Theisen (1989); S. Ferrara, C. Kounnas (1989)]

How to evade „no-go theorems":

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Field theory: S-fold construction

[I. Garcia-Etxebarria, D. Regalado (2015/16);
O. Aharony, M. Evtikhiev (2015);
O. Aharony, Y. Tachikawa (2016);
Y. Imamura, S. Yokoyama (2016)]

$$(\mathcal{N} = 3_{SYM}) \equiv (\mathcal{N} = 4_{SYM})/[(\mathrm{R-sym.}) \times SL(2)_S]\,.$$

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$$(\mathcal{N} = 3_{SYM}) \equiv (\mathcal{N} = 4_{SYM})/[(\mathrm{R} - \mathrm{sym.}) \times SL(2)_S] \, .$$

- Three invariant supercharges
- Strongly coupled theory $(\; S = i \;)$
- Non-lagrangian, superconformal theory

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$$\boxed{\mathcal{N}=7\ \text{W - Sugra:}\qquad \text{S-fold of } \mathcal{N}=8 \text{ Sugra}}$$

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Two factors: massless spin-one vector multiplets of $\mathcal{N} = 4$

Result: massless spin - two multiplet of $\mathcal{N} = 8$

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$$\boxed{\mathcal{N}=7 \text{ W - Sugra} \equiv (\mathcal{N}=4\ \text{ YM}) \otimes (\mathcal{N}=3\ \text{ YM})}$$

Two factors: massive spin-two multiplets of $\mathcal{N}=4$ and $\mathcal{N}=3$

Result: massive spin - four multiplet of $\mathcal{N}=7$

W - supergravity

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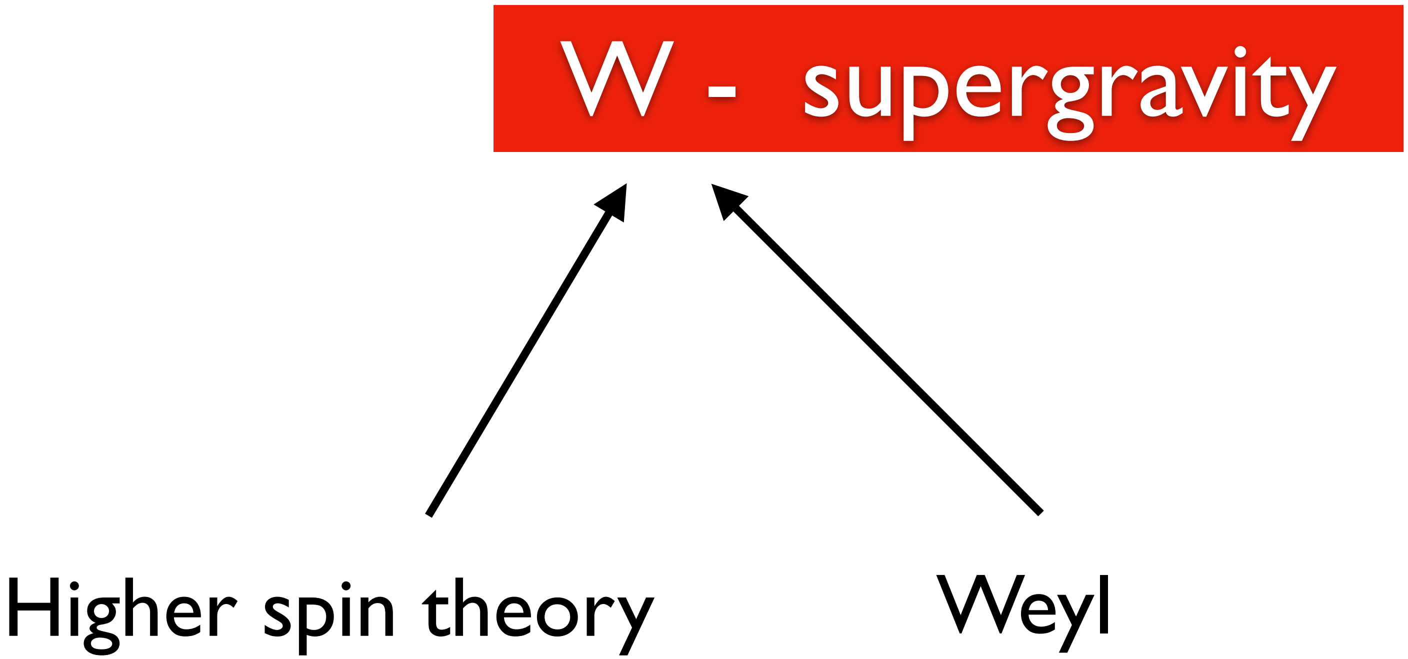
Higher spin theory


W - supergravity
Higher spin theory
Weyl

Outline:

II) Double copy construction of W-supergravity - N=7 W-supergravity

III) W - Superstring construction

IV) Conclusion and Outlook

II) Double copy construction of W - supergravity

$$\mathrm{QFT}(\mathcal{N}_L) \otimes \mathrm{QFT}(\mathcal{N}_R) \quad = \quad \mathrm{Sugra}(\mathcal{N}_L + \mathcal{N}_R)$$

[Z. Bern, J. Carrasco, H. Johansson (2008/2010)
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Operators/fields: $\Phi_{L+R} = \Phi_L \otimes \Phi_R$

$\Phi_L,\ \Phi_R$: Operators with scaling dimensions h_L, h_R

Require: $h_L = h_R = h \quad (m_L = m_R = m)$

4D Standard massless supergravity:

Tensor product of two massless vector multiplets $(h = 1)$

$$\text{Supergravity :} \quad \text{Massless Spin}(2) = V_{L,\mathcal{N}_L} \otimes V_{R,\mathcal{N}_R}$$

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(iii) $\mathcal{N}_L = 1, \mathcal{N}_R = 4 \quad \Rightarrow \mathcal{N} = 5 \text{ Sugra}$

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(vi) $\mathcal{N}_L = 1, \mathcal{N}_R = 2 \quad \Rightarrow \mathcal{N} = 3 \text{ Sugra}$

(vii) $\mathcal{N}_L = 0, \mathcal{N}_R = 2 \quad \Rightarrow \mathcal{N} = 2 \text{ Sugra}$

(viii) $\mathcal{N}_L = \mathcal{N}_R = 1 \quad \Rightarrow \mathcal{N} = 2 \text{ Sugra}$

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Non-standard W - supergravities in 4D :

We will need massive multiplets of $\mathcal{N}$ - extended supersymmetric field theories:

[S. Ferrara, C. Savoy, B. Zumino (1981)]

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Massive spin-two Weyl super-multiplets:

$$W_{\mathcal{N}=4}: \ \mathrm{Spin}(2) + \underline{8} \times \mathrm{Spin}(3/2) + \underline{27} \times \mathrm{Spin}(1) + \underline{48} \times \mathrm{Spin}(1/2) + \underline{42} \times \mathrm{Spin}(0)$$

States build $USp(8)$ representations.

$$W_{\mathcal{N}=3}: \mathrm{Spin}(2) + \underline{6} \times \mathrm{Spin}(3/2) + (\underline{14} + \underline{1}) \times \mathrm{Spin}(1) + (\underline{14'} + \underline{6}) \times \mathrm{Spin}(1/2) + \underline{14} \times \mathrm{Spin}(0)$$

States build $USp(6)$ representations.

$$W_{\mathcal{N}=2}: \ \mathrm{Spin}(2) + \underline{4} \times \mathrm{Spin}(3/2) + (\underline{5} + \underline{1}) \times \mathrm{Spin}(1) + \underline{4} \times \mathrm{Spin}(1/2) + \mathrm{Spin}(0)$$

States build $USp(4)$ representations.

$$W_{\mathcal{N}=1}: \ \mathrm{Spin}(2) + \underline{2} \times \mathrm{Spin}(3/2) + \mathrm{Spin}(1)$$

States build $USp(2)$ representations.

Massive spin-two Weyl supermultiplet in string theory

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Open string spin-two fields on D3-branes
at the first excited mass level.

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Effective action: Weyl square supergravity:

$$S = \int d^4x\sqrt{-g}\Big(\frac{a}{2}W_{\mu\nu\rho\sigma}W^{\mu\nu\rho\sigma} + \kappa^2 R\Big)$$

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Propagating degrees of freedom: Bi-gravity theory

[B. Gording, A. Schmidt-May (2018); S. Ferrara, A. Kehagias, D.L. work in progress]

Closed string: massless spin-two: metric $g_{\mu\nu}$

Open string: massive spin-two: 2nd. metric $w_{\mu\nu}$

S - fold construction of N=3 field theory:

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One performs two types of simultaneous twists/projections:

R-symmetry: $Q^{\alpha;1,2,3} \rightarrow e^{-i\pi/k} Q^{\alpha;1,2,3}, \; Q^{\alpha;4} \rightarrow e^{3\pi/k} Q^{\alpha;4}$

S-duality: $Q^{\alpha;1,2,3,4} \rightarrow e^{i\pi/k} Q^{\alpha;1,2,3,4}$

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Massless spin-one vector field $V_i(x,\theta)$ is not invariant under the S - fold projection.

Invariant operators: $\Phi^{2p} = \mathrm{Tr}(V_{i\,1} \bar{V}_{i\,2} \ldots V_{i\,2p-1} \bar{V}_{i\,2p})$

$(h = 2p)$

Lowest invariant operator: $(h = 2)$

massive spin - two super-Weyl multiplet:

$$W_{\mathcal{N}=3}: \quad \mathrm{Spin}(2) + \underline{6} \times \mathrm{Spin}(3/2) + (\underline{14} + \underline{1}) \times \mathrm{Spin}(1) + (\underline{14'} + \underline{6}) \times \mathrm{Spin}(1/2) + \underline{14} \times \mathrm{Spin}(0)$$

$W_{\mathcal{N}=3}$ is a true $\mathcal{N} = 3$ supermultiplet.

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Tensor product of two massive, spin-two Weyl multiplets:

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$$\Phi^4_{L+R,\mathcal{N}_L+3} = W_{L,\mathcal{N}_L} \otimes W_{R,\mathcal{N}_R=3}$$

spin=4 spin=2 spin=2 (+matter)

All possible cases:

(i) $\mathcal{N}_L = 4,\ \mathcal{N}_R = 3 \quad \Longrightarrow \quad \mathcal{N} = 7\ \mathrm{W-Sugra}$

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Most prominent example: $\mathcal{N}=7$ W - supergravity:

$$\Big(\mathrm{Spin}(2) + 8\times\mathrm{Spin}(\frac{3}{2}) + 27\times\mathrm{Spin}(1) + 48\times\mathrm{Spin}(\frac{1}{2}) + 42\times\mathrm{Spin}(0)\Big)_{\mathcal{N}=4}$$

$$\otimes\Big(\mathrm{Spin}(2) + 6\times\mathrm{Spin}(3/2) + 15\times\mathrm{Spin}(1) + 20\times\mathrm{Spin}(1/2) + 14\times\mathrm{Spin}(0)\Big)_{\mathcal{N}=3}$$

$$= \Big(\mathrm{Spin}(4) + \underline{14}_1\times\mathrm{Spin}(\frac{7}{2}) + (\underline{90}_2 + \underline{1})\times\mathrm{Spin}(3) + (\underline{350}_3 + \underline{14}_1)\times\mathrm{Spin}(\frac{5}{2})$$

$$+ (\underline{90}_2 + \underline{910}_4)\times\mathrm{Spin}(2) + (\underline{350}_3 + \underline{1638}_5)\times\mathrm{Spin}(\frac{3}{2}) + (\underline{2002}_6 + \underline{910}_4)\times\mathrm{Spin}(1)$$

$$+ (\underline{1430}_5 + \underline{1638}_5)\times\mathrm{Spin}(\frac{1}{2}) + \underline{2002}_6\times\mathrm{Spin}(0)\Big)_{\mathcal{N}=7}$$

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Number of states $n_B+n_F=2^{15}=32768$

All massive states form representations of $USp(14)$

Strongly coupled theory without Langrangian

Can also be obtained directly by S-fold of $\mathcal{N}=8$

Precisely appears at first mass level in corresponding string construction.

Outline:

II) Double copy construction of W-supergravity - N=7 W-supergravity

III) W - Superstring construction

IV) Conclusion and Outlook

III) W - Superstring construction

Basic requirements for $\mathcal{N}=7$ (or $\mathcal{N}=3$ heterotic) in 4D:

Left-moving sector is untouched:

Type II: Four supercharges

Heterotic: Zero supercharges

Right-moving sector gets projected/twisted as in field theory:

Three invariant supercharges

No massless excitations

4D heterotic (type II) string in light cone gauge on

$$\mathbb{R}^{1,1} \times \mathbb{R}^2 \times T^6$$

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1 complex uncompactified, transversal coordinate:

$$X(z, \bar{z}) = X_L(\bar{z}) + X_R(z)$$

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$$X(z, \bar{z}) = X_L(\bar{z}) + X_R(z)$$

3 complex compactified coordinates:

$$Z^i(z, \bar{z}) = Z^i_L(\bar{z}) + Z^i_R(z) \quad (i = 1, 2, 3)$$

S-fold projection in string theory:

1.) R-symmetry projection in the internal sector:

Projection by a discrete element of T-duality group SO(6,6) in the internal sector

Can be represented by discrete $\mathbb{Z}_4$ rotation on the internal coordinates:

$$Z^i_L \rightarrow Z^i_L$$

$$Z^i_R \rightarrow e^{-\frac{i\pi}{2}} Z^i_R$$

Moduli have to be frozen at their fixed point values:

$$\tau = \rho = i$$

The internal space is a completely left-right asymmetric $\mathbb{Z}_4$ orbifold .

2.) S - duality projection in the space-time sector:

It is given by a discrete element of the (string) Geroch group.

[R. Geroch (1972);
I. Bakas (1994)]

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The Geroch group contains:

(i) Axion-dilaton transformations:

$$S = a + ie^{-2\phi}$$

$$SL(2)_S : \quad S \to \frac{\tilde{a}' S + \tilde{b}'}{\tilde{c}' S + \tilde{d}'}, \quad \tilde{a}'\tilde{d}' - \tilde{b}'\tilde{c}' = 1$$

[A. Font, L. Ibanez, D.L., F. Quevedo (1990);
J.H. Schwarz, A. Sen (1993)]

(ii) Ehlers transformations - G-duality:

Four-dimensional, transversal, complex graviton field:

$$G = \frac{g_{12}}{g_{11}} + i\ \frac{\sqrt{\det g}}{g_{11}}$$

Large 4D diffeomorphisms (Ehlers transformation):

$$SL(2)_G: \quad G \to \frac{\tilde{a}\, G + \tilde{b}}{\tilde{c}\, G + \tilde{d}}, \quad \tilde{a}\tilde{d} - \tilde{b}\tilde{c} = 1$$

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Combined action of $SL(2)_S$ and $SL(2)_G$:

$O(2,2)_{S,G} \simeq SL(2)_S \times SL(2)_G$ (Geroch group)

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$$G = \frac{g_{12}}{g_{11}} + i \, \frac{\sqrt{\det g}}{g_{11}}$$

Large 4D diffeomorphisms (Ehlers transformation):

$$SL(2)_G : \quad G \to \frac{\tilde{a}\, G + \tilde{b}}{\tilde{c}\, G + \tilde{d}} , \quad \tilde{a}\tilde{d} - \tilde{b}\tilde{c} = 1$$

Combined action of $SL(2)_S$ and $SL(2)_G$:

$$O(2,2)_{S,G} \simeq SL(2)_S \times SL(2)_G \quad \text{(Geroch group)}$$

$SL(2)_S$ and $SL(2)_G$ also act on the transversal string coordinates X_L, X_R as certain in general left-right asymmetric rotations.

Particular discrete $\mathbb{Z}_4$ element:

$$X_L(\bar{z}) \to X_L(\bar{z})\,, \quad X_R(z) \to e^{-\frac{i\pi}{2}} X_R(z)$$

Dilaton and metric must be frozen: $S = G = i$.

This is a completely asymmetric $\mathbb{Z}_4$ rotation of the transversal, uncompactified coordinates.

Geroch transformation becomes T-duality when further compactifying to two dimensions

[I. Florakis, I. Garcia-Etxebarria, D. Lüst, D. Regalado (2018)]

Invariant right-moving (open string) spectrum:

64 + 64 invariant states at the first mass level:

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$$b^i_{-1/2} b^j_{-1/2} b^I_{-1/2}|0\rangle\,, \qquad b^i_{-1/2} b^I_{-1/2} b^J_{-1/2}|0\rangle\,, \qquad b^I_{-1/2} b^J_{-1/2} b^K_{-1/2}|0\rangle\,,$$

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$$(8)_c + (56)_c :\quad b^A_{-1}|a\rangle\,,\qquad (8)_s + (56)_s :\quad \alpha^A_{-1}|\dot{a}\rangle$$

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This precisely agrees with the massive $\mathcal{N} = 3$ Weyl multiplet.

Now we can consider the following type II S - fold:

$$(\mathcal{N} = 7_{II}) \equiv (\mathcal{N} = 8_{II})/(\mathrm{T-duality} \times \mathrm{Geroch})$$

3 invariant right-moving supercharges .

4 invariant left-moving supercharges .

- 7 invariant supercharges
- $S = i \quad \Rightarrow$ strongly coupled
- $G = i \quad \Rightarrow$ no massless graviton

Spectrum of type II $\mathcal{N}=7$ W - superstring theory:

- No massless states, no massless graviton & gravitini.

- Invariant states at first mass level:

Tensor product: (Fermionic left) x (fermionic right):

$$\begin{aligned}
\mathrm{B}: \quad & [\mathrm{Spin}(2)+15\times\mathrm{Spin}(1)+14\times\mathrm{Spin}(0)]_R\times[\mathrm{Spin}(2)+27\times\mathrm{Spin}(1)+42\times\mathrm{Spin}(0)]_L\,,\\
& [6\times\mathrm{Spin}(3/2)+20\times\mathrm{Spin}(1/2)]_R\times[8\times\mathrm{Spin}(3/2)+48\times\mathrm{Spin}(1/2)]_L\\
\mathrm{F}: \quad & [6\times\mathrm{Spin}(3/2)+20\times\mathrm{Spin}(1/2)]_R\times[\mathrm{Spin}(2)+27\times\mathrm{Spin}(1)+42\times\mathrm{Spin}(0)]_L\,,\\
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\end{aligned}$$

$$\begin{aligned}
\mathrm{B}: \quad & [\mathrm{Spin}(4)+91\times\mathrm{Spin}(3)+1000\times\mathrm{Spin}(2)+2912\times\mathrm{Spin}(1)+2002\times\mathrm{Spin}(0)]\,,\\
\mathrm{F}: \quad & [14\times\mathrm{Spin}(\tfrac{7}{2})+364\times\mathrm{Spin}(\tfrac{5}{2})+1988\times\mathrm{Spin}(\tfrac{3}{2})+3068\times\mathrm{Spin}(\tfrac{1}{2})
\end{aligned}$$

Massive spin - four multiplet of $\mathcal{N}=7$.

Agrees with double copy construction.

IV) Conclusion and Outlook

We have provided evidence for the existence of new W - supergravity and W - superstring theories.

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Strongly coupled theories with frozen graviton !

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Strongly coupled theories with frozen graviton !
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Field theory W - supergravity:

- Double copy involving $\mathcal{N}=3$ field theory.
- Non-perturbative, massive $\mathcal{N}=7$ W - supergravity with massive spin-four supermultiplet.

W - superstring theories:

- The internal space is non-geometric
  - modding out by T-duality.
- In 4D also modding out by S-duality
  - strongly coupled theory.
- In 4D also modding by large diffeomorphism - massive, topological theory.
- 4D space is „non-geometric“ - relation to non-associativity?
- No proof (yet) that these theories are fully consistent and full-fledged string theories.

But there exist fully consistent 2D non-geometric string construction with N=(20,8) supercharges (and also with N=(24,24) and N=(48,0)).

[I. Garcia-Etxebarria, I. Florakis, D.L., D. Regalado (2017)]

Final conjecture about holography:

Following the same kind of arguments as in standard holography, it is tempting to conjecture that $\mathcal{N} = 4$ spin-two superconformal Weyl supergravity on a 4D boundary is holographically dual to $\mathcal{N} = 8$ spin-four W - supergravity in the 5 D bulk theory.

[S. Ferrara, A. Kehagias, D.L., work in progress]

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Thank you!