# Relating duality covariant approaches to higher derivatives

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Dualities and generalized geometry
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| | | |
|---|---|---|
| Based on | Bedoya, DM and Nuñez | 1407.0365 |
| | DM and Nuñez | 1507.00652 |

# Situation

There are two approaches to T-duality covariant first order corrections in heterotic supergravity:

- Extending the duality structure

  Bedoya, DM and Nunez 2014

  Coimbra, Minasian, Triendl and Waldram 2014

- Deforming the gauge symmetries

  Hohm and Zwiebach 2014

  DM and Nunez 2015

# Puzzles

- How are they related?

- How are they extended to higher orders?

# Case 1: extended duality group

Bedoya, DM and Nunez 2014

Coimbra, Minasian, Triendl and Waldram 2014

Lee 2015

# Extended duality group approach

It is based on two facts related to heterotic supergravity

$$\mathcal{L} = R + 4(\partial\phi)^2 - \frac{1}{12}\widehat{H}^2 - \frac{1}{4}F^2 + \text{fermions}$$

where

$$\widehat{H} = dB + CS(A) + \text{fermions}$$

# Extended duality group approach

The first observation is due to Bergshoeff and de Roo 1988

$$\begin{array}{rcccl} \text{gauge fields} & A & \leftrightarrow & \omega_- = \omega - \dfrac{1}{2}\widehat{H} & \text{spin con. w/torsion} \\ \text{gauginos} & \chi & \leftrightarrow & D\psi & \text{gravitino curvature} \end{array}$$

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The Bergshoeff-de Roo identification is based on the transformation behavior

$$\begin{array}{lcl} \delta A = \bar{\epsilon}\gamma\chi & \leftrightarrow & \delta\omega_- = \bar{\epsilon}\gamma D\psi \\ \delta\chi = F_{\mu\nu}\gamma^{\mu\nu}\epsilon & \leftrightarrow & \delta D\psi = R_{-\mu\nu}\gamma^{\mu\nu}\epsilon \end{array}$$

The pair $(\omega_-, D\psi)$ *effectively* behaves as a gauge multiplet.

# Extended duality group approach

First order corrections are obtained by including extra Lorentz multiplets and *identifying* them with $(\omega^-, D\psi)$

$$\mathcal{L} = R + 4(\partial\phi)^2 - \frac{1}{12}\widehat{H}^2 - \frac{1}{4}F^2 + \frac{1}{4}R_-^2 + \text{fermions}$$

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- $CS(\omega_-)$ deforms the transformation of $\omega^-$ itself, rendering the identification ill-defined to second order.
- Noether procedure for higher orders, Bergshoeff and de Roo 1989.

# Extended duality group approach

The second observation is due to Hohm and Kwak 2011

Gauge multiplets are incorporated into DFT through extensions of the duality group and local symmetries

$$\mathcal{G} = O(D, D + k) \ , \qquad \mathcal{H} = \underline{O(D)} \times \overline{O(D + k)}$$

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$$\mathcal{G} = O(D, D + k) \ , \qquad \mathcal{H} = \underline{O(D)} \times \overline{O(D + k)}$$

Under a $GL(D)$ and $O(D)$ decomposition the generalized fields include the gauge multiplet components

$$\mathcal{E}[e, B, A] \in \mathcal{G} \ , \qquad \Psi = (\psi, \ \chi) \ \ \text{vector} \ \overline{O(D + k)}$$

Generalized diffeomorphisms $=$ ($GL(D)$ diffs, B-shifts, $\mathcal{K}$).

# Extended duality group approach

Based on these observations one makes a further extension of the duality group

$$\mathcal{E}[e,\ B,\ A,\ A'] \ , \qquad \Psi = (\psi,\ \chi,\ \chi')$$

and performs a Bergshoeff-de Roo identification

$$\mathcal{K} \quad \leftrightarrow \quad O(D) \in \overline{O(D+k)}$$

such that

$$A' = \omega_- \ , \qquad \chi' = D\psi$$

to lowest order.

# Extended duality group approach

Pro

- It is guaranteed to work to first order.

Cons

- It is guaranteed not to work to higher orders.
- The identification is done *after* the $GL(D)$ and $O(D)$ decomposition, so the procedure is not duality covariant.

# Case 2: deformed gauge symmetries

Hohm and Zwiebach 2014

DM and Nunez 2014

Baron, F. Melgarejo, DM and Nunez 2017

# Deformed gauge transformations approach

It is based on the transformation of the Kalb-Ramond field due to Green and Schwarz 1984

$$\delta B = \frac{1}{2} tr \left( d\Lambda \wedge \omega_- \right)$$

which requires and fixes higher derivative terms

$$\widehat{H} = dB + CS(A) - CS(\omega_-) + \text{fermions}$$

# Deformed gauge transformations approach

The generalized Green-Schwarz transformation is the T-duality covariant extension

$$\begin{aligned}
\delta E_M{}^{\underline{a}} &= -E_M{}^{\overline{d}}\, D_{\overline{d}} \Lambda^{\overline{bc}}\, F^{\underline{a}}{}_{\overline{bc}} \\
\delta E_M{}^{\overline{a}} &= E_M{}^{\underline{d}}\, D^{\overline{a}} \Lambda^{\overline{bc}}\, F_{\underline{d}\overline{bc}}
\end{aligned}$$

It requires and fixes higher derivative terms in DFT, which translate into

$$\mathcal{L} = R + 4(\partial\phi)^2 - \frac{1}{12}\widehat{H}^2 - \frac{1}{4}F^2 + \frac{1}{4}R_-^2$$

T-duality forces the inclusion of quadratic Riemann interactions.

# Deformed gauge transformations approach

Pro

- It is duality covariant: helps in clarifying the role of T-duality as an organizing principle for higher interactions.

Cons

- Only known to first order.
- Not supersymmetric yet.

# How are they related?

...soon in the arXiv...

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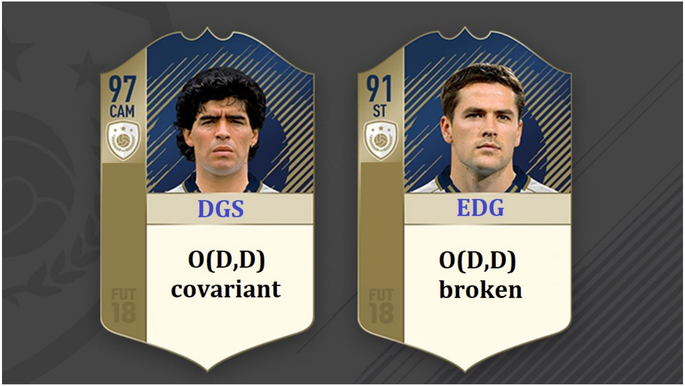
Comparing them looks like a crazy thing to do...



## How are they related?

- Instead of decomposing $O(D, D+k)$ w.r.t. $GL(D)$, it must be decomposed w.r.t. $O(D, D)$ as in Hohm, Sen and Zwiebach 2014

$$\mathcal{E}[E, \mathcal{C}] \in O(D, D+k) \ , \quad E \in O(D, D) \ , \quad E^{M\bar{a}} \mathcal{C}_M{}^{\alpha} = 0$$

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- Only then one should try a first order generalized Bergshoeff-de Roo identification

$$A \ \leftrightarrow \ \omega_- \quad \rightarrow \quad \mathcal{C} \ \leftrightarrow \ ?$$

# How are they related?

The double frame transforms as follows WRT $\mathcal{K}$

$$\delta E_M{}^{\overline{a}} = \mathcal{C}_{M\,\alpha} D^{\overline{a}} \xi^{\alpha}$$

while in the generalized Green-Schwarz transformation one has

$$\delta E_M{}^{\underline{d}} = E_M{}^{\underline{d}} F_{\underline{d}\overline{bc}} D^{\overline{a}} \Lambda^{\overline{bc}}$$

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The identification follows naturally (similar story for susy)

$$\begin{array}{rcl} \mathcal{K} & \leftrightarrow & O(D) \in \overline{O(D+k)} \\ \alpha & \leftrightarrow & \overline{bc} \\ \xi^{\alpha} & \leftrightarrow & \Lambda^{\overline{bc}} \\ \mathcal{C}_{M\alpha} & \leftrightarrow & E_M{}^{\underline{d}} F_{\underline{d}\overline{bc}} \end{array}$$

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The extension to higher orders can only work if the $O(D,D)$ covariant identification is improved.

Proposal...

$$\mathcal{K} \quad \leftrightarrow \quad \overline{O(D+k)}$$

is an *exact* identification.

## How are they extended to higher orders?

The extension to higher orders can only work if the $O(D, D)$ covariant identification is improved.
Proposal...

$$\mathcal{K} \quad \leftrightarrow \quad \overline{O(D+k)}$$

is an *exact* identification.
A priori this seems impossible because for finite $k$

$$k = \dim(\mathcal{K}) \neq \dim(\overline{O(D+k)}) = \frac{(D+k)(D+k-1)}{2}$$

An infinite extension of the tangent space is required.

# How are they extended to higher orders?

We call this the generalized Bergshoeff-de Roo identification

$$
\begin{array}{rcl}
\mathcal{K} & \leftrightarrow & \overline{O(D+k)} \\
\alpha & \leftrightarrow & \overline{\mathcal{BC}} \\
\xi^{\alpha} & \leftrightarrow & \Lambda^{\overline{\mathcal{BC}}} \\
f(\mathcal{C})_{M\alpha} & \leftrightarrow & E_M{}^{\underline{d}}\mathcal{F}_{\underline{d}\overline{\mathcal{BC}}}(\mathcal{E})
\end{array}
$$

# How are they extended to higher orders?

Some interesting features

- It is exact and supersymmetric.
- It can be worked out perturbatively to find higher derivative corrections.
- It is profoundly generalized in nature: the $\overline{O(D+k)}$ is broken to $O(D)$ in supergravity.

## Summary

generalized Bergshoeff-de Roo identification

$=$

generalized Green-Schwarz transformation

# Outlook

- Imposed *by hand*
- Interactions?
- Bi-parametric deformations?
- Maximal supergravity?
- Non-perturbative (exact) treatment?