O-folds: Orientifolds and Orbifolds in Exceptional Field Theory

Chris Blair

Dualities and Generalized Geometries, Corfu Summer Institute
14th September 2018

Based on: 1805.04524 *with E. Malek and D. Thompson*

VRIJE UNIVERSITEIT BRUSSEL

Fonds Wetenschappelijk Onderzoek Vlaanderen
Opening new horizons

Exceptional field theory (ExFT)

- Reformulates **maximal SUGRA** in 10/11 dims
- Extended coordinates: X^{μ} (external $(11-d)$-dimensional)
  $Y^{M} \in R_1$ rep of $E_{d(d)}$
- Solution of the section condition (SSC): $(X^{\mu}, Y^{M}) \to (X^{\mu}, Y^{i})$
- 11-dimensional SSC (**M-theory**) $\to$ 10 dimensional SSC (**IIA**)
- 10-dimensional SSC (**IIB**)

Exceptional field theory (ExFT)

- Reformulates **maximal SUGRA** in 10/11 dims
- Extended coordinates: X^{μ} (external $(11-d)$-dimensional)
  $Y^{M} \in R_1$ rep of $E_{d(d)}$
- Solution of the section condition (SSC): $(X^{\mu}, Y^{M}) \to (X^{\mu}, Y^{i})$
- 11-dimensional SSC (**M-theory**) $\to$ 10 dimensional SSC (**IIA**)
- 10-dimensional SSC (**IIB**)
- What about: 10-dim **half-max theories:** type I, heterotic $\mathrm{SO}(32)$, $\mathrm{E}_8 \times \mathrm{E}_8$?

Half-maximal duality web: I

- Type I = Type IIB / Ω, with worldsheet parity $\Omega = \sigma \to -\sigma$
- $\Rightarrow$ **orientifold**
- Orientifold plane: O9. Negative tension $\Rightarrow$ add 16 D9 branes $\Rightarrow$ $\mathrm{SO}(32)$ gauge sector (cancels anomaly)
- Action on SUGRA fields:
  $\Omega : (g, B_2, \phi, C_0, C_2, C_4) \to (g, -B_2, \phi, -C_0, C_2, -C_4)$.

  $\Rightarrow$ type I SUGRA: (g, C_2, ϕ, A^α)
- T-duality on X^i: Op + Dp planes at fixed points of transverse $T^{9-p}/\mathbb{Z}_2$ with $\mathcal{I} : \tilde{X}^i \to -\tilde{X}^i$

  $\Omega : X_L \leftrightarrow X_R$. T-duality: $X = X_L + X_R \to \tilde{X} = X_L - X_R \Rightarrow \Omega : \tilde{X} \to -\tilde{X}$

Half-maximal duality web: heterotic $\mathrm{E}_8 \times \mathrm{E}_8$

- **Hořava-Witten:** M-theory on interval
- View interval as $I = S^1_R/\mathbb{Z}_2$, $\mathcal{I} : z \to -z$. Fixed points: $z = 0, \pi R$.
- $(g, C_3) \to (g, -C_3)$
- Anomaly cancellation: E_8 SYM at fixed points = 10-dimensional "end-of-the-world" or M9 branes
- For small R, gives **heterotic $\mathbf{E}_8 \times \mathbf{E}_8$**
- At fixed points: (g, B_2, ϕ) from even components

$$g^{\text{het}}_{\mu\nu} \equiv g_{\mu\nu} \quad B_{\mu\nu} \equiv C_{\mu\nu z}\,, \quad e^{\Phi} \equiv g_{zz}$$

odd components projected out, for $(x^\mu, z) \to (x^\mu, -z)$,

$$g_{\mu z}(x, z) \sim -g_{\mu z}(x, -z) \quad C_{\mu\nu\rho}(x, z) \sim -C_{\mu\nu\rho}(x, -z)$$

Half-maximal duality web: heterotic $\mathrm{SO}(32)$

- Strong coupling limit of type I = heterotic $\mathrm{SO}(32)$
- T-duality between heterotic theories (with Wilson lines)

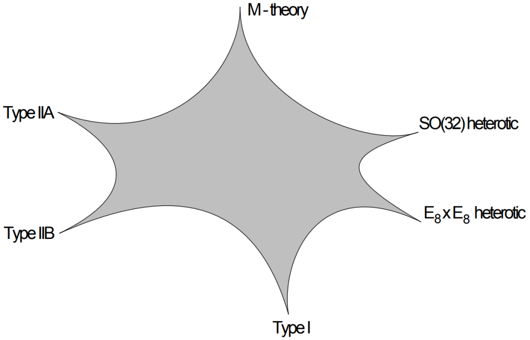

Figure 1: A portrait of the theory as a young M

Generic orbifolds and orientifolds

- **Orbifold** = manifold / G, G = discrete group.
- **Orientifold** = additional quotient by worldsheet parity
- At **fixed points:** additional degrees of freedom (twisted sector) $\rightarrow$ determined by charge/anomaly cancellation
- N.b. Hořava-Witten not a geometric orbifold as $C_3 \rightarrow -C_3$. Will become "geometric" in ExFT!
- Will break some SUSY. We focus on: half-maximal quotients.

Half-maximal generalised orbifolds in ExFT

- Assume $\exists$ $E_{d(d)}$ ExFT **half-maximal structure** [Malek]

$$J_u^M \in R_1 \,, \quad \hat{K} \in R_{D-4}$$

  well-defined, nowhere vanishing, $u = 1, \ldots d-1$, $D = 11 - d$, obey compatibility conditions

- Structure group reduction to: $\mathrm{Spin}(d-1) \subset \mathrm{E}_{d(d)}$ preserving half-maximal structure

- $\Rightarrow$ Quotient by discrete subgroups of $\mathrm{Spin}(d-1) \subset \mathrm{E}_{d(d)} \rightarrow$ half-maximal **generalised orbifolds** or **O-folds**

O-folds in practice

- Given $Z^M{}_N \in G_{\textbf{discrete}} \subset E_{d(d)}$ (preserving half-max structure), quotient via identifications on extended coords:

$$(X^\mu, Y^M) \sim (X^\mu, Y'^M) = (X^\mu, Z^M{}_N Y^N)$$

and generalised fields e.g.

$$\begin{aligned} g_{\mu\nu}(X,Y) &\sim g_{\mu\nu}(X,Y')\,, \\ \mathcal{M}_{MN}(X,Y) &\sim (Z^{-1})^P{}_M (Z^{-1})^Q{}_N \mathcal{M}_{PQ}(X,Y')\,, \\ \mathcal{A}_\mu{}^M(X,Y) &\sim Z^M{}_N \mathcal{A}_\mu{}^N(X,Y') \end{aligned}$$

O-folds in practice

- Coordinate identification $Y^M \sim Z^M{}_N Y^N$.
- Depending on SSC, may be **geometric** $Y^i \sim Y^j$ (conventional orbifolds/orientifolds) or **non-geometric** $Y^i \sim \tilde{Y}_{ij}$ (asymmetric orbifolds and "non-perturbative" generalisations)
- Fixed points: $Y^M = Z^M{}_N Y^N$.
- Depending on SSC, fixed points may be **physical** or **only in dual directions**.
- Can introduce extra dofs at fixed points
  discussing $\mathbb{Z}_2$ example later
- General picture consistent with T/U-folds [Dabholkar, Hull]: O-folds appear at fixed points in moduli space

$\mathrm{SL}(5)$ O-folds

- $a = 1, \ldots, 5$, coordinates $Y^{ab} = -Y^{ba} \in \mathbf{10}$
- Half-max structure ${J_u}^{ab}$, $\hat{K}^a$
- Stabilised by $\mathrm{SU}(2) \subset \mathrm{SL}(5)$. Discrete subgroups: ADE classification. Can construct generators explicitly (see paper).
- e.g. $\mathbb{Z}_2$ generated by

$$Z^a{}_b = \mathrm{diag}(-1, -1, -1, -1, +1)$$

Coordinates $Y^{ab} \Rightarrow$ 6 even, 4 odd

$\mathrm{SL}(5)$ $\mathbb{Z}_2$ O-fold

- Impose $\mathbb{Z}_2$ identification on coordinates and ExFT fields $g_{\mu\nu}, m_{ab}, \mathcal{A}_\mu{}^{ab}, \mathcal{B}_{\mu\nu a}, \mathcal{C}_{\mu\nu\rho}{}^a, \ldots$ in different SSCs

  M-theory SSCs: $Y^{ab} = (Y^{i5}, Y^{ij})$, $i, j = 1, \ldots, 4$.

- **Hořava-Witten:** $a = (i, 5)$ parity $(+----)$

$$\begin{array}{rll} \text{physical:} & Y^{i5} & -+++ \\ \text{dual (M2 } w\text{):} & Y^{ij} & ---+++ \end{array}$$

  Fields: e.g. $\mathcal{C}_{\mu\nu\rho}{}^5$ odd $\Rightarrow C_3 \to -C_3$

- **Geometric orbifold:** $a = (i, 5)$ parity $(----+)$

$$\begin{array}{rll} \text{physical:} & Y^{i5} & ---- \\ \text{dual (M2 } w\text{):} & Y^{ij} & ++++++ \end{array}$$

  No action on fields $\to$ geometric orbifold

$\mathrm{SL}(5)$ $\mathbb{Z}_2$ O-fold

- **IIA SSCs:** $Y^{ab} = (Y^{i5}, Y^{45}, Y^{ij}, Y^{i4})$ with $i = 1, 2, 3$.

- **Heterotic "$\mathrm{E}_8 \times \mathrm{E}_8$":** $a = (i, 4, 5)$ with parity $(- - - + -)$

physical:	Y^{i5}	$+++$
M-theory:	Y^{45}	$-$
dual (F1 w):	Y^{ij}	$+++$
dual (D2 w):	Y^{i4}	$---$

  10-d spacetime, C_1, C_3 projected out.

- **Type I′ (O8 planes):** $a = (i, 4, 5)$ with parity $(+ - - - -)$

physical:	Y^{i5}	$-++$
M-theory:	Y^{45}	$+$
dual (F1 w):	Y^{ij}	$--+$
dual (D2 w):	Y^{i4}	$-++$

  Fields: $(g, B_2, \Phi, C_1, C_3) \to (g, -B_2, \Phi, C_1, -C_3)$

$\mathrm{SL}(5)$ $\mathbb{Z}_2$ O-fold

- **IIA with O6 planes:** $a = (i, 4, 5)$ with parity $(- - - - +)$

$$\begin{array}{rcl} \text{physical:} & Y^{i5} & - - - \\ \text{M-theory:} & Y^{45} & - \\ \text{dual (F1 } w\text{):} & Y^{ij} & + + + \\ \text{dual (D2 } w\text{):} & Y^{i4} & + + + \end{array}$$

Reflection $Y^{i5} \to -Y^{i5}$. Fields:
$(g, B_2, \Phi, C_1, C_3) \to (g, -B_2, \Phi, -C_1, C_3)$

SL(5) $\mathbb{Z}_2$ O-fold

- **IIB SSCs:** $Y^{ab} = (Y^{ij}, Y^{i\alpha}, Y^{\alpha\beta})$, $i = 1,2,3$, $\alpha = 1,2$ SL(2)

- **Heteretic "SO(32)":** $a = (i, \alpha)$ with parity $(- - - + -)$

$$
\begin{array}{rll}
\text{physical:} & Y^{ij} & + + + \\
\text{dual (D1/F1 } w\text{):} & Y^{i\alpha} & \left\{ \begin{array}{l} - - - \\ + + + \end{array} \right. \\
\text{dual (D3 } w\text{):} & Y^{\alpha\beta} & -
\end{array}
$$

C_0, C_2, C_4 projected out

- **Type I:** $a = (i, \alpha)$ with parity $(- - - - +)$

$$
\begin{array}{rll}
\text{physical:} & Y^{ij} & + + + \\
\text{dual (D1/F1 } w\text{):} & Y^{i\alpha} & \left\{ \begin{array}{l} + + + \\ - - - \end{array} \right. \\
\text{dual (D3 } w\text{):} & Y^{\alpha\beta} & -
\end{array}
$$

C_0, B_2, C_4 projected out

SL(5) $\mathbb{Z}_2$ O-fold

- **IIB with O7 planes:** $a = (i, \alpha)$ with parity $({\color{red}+} - - - -)$

$$\begin{array}{rcl} \text{physical:} & Y^{ij} & - - + \\ \text{dual (D1/F1 } w\text{):} & Y^{i\alpha} & \left\{ \begin{array}{l} + + - \\ + + - \end{array} \right. \\ \text{dual (D3 } w\text{):} & Y^{\alpha\beta} & + \end{array}$$

Fields: B_2, C_2 odd.

Localised vector multiplets (in this $\mathbb{Z}_2$ O-fold)

- Expand half-max structure:

$$J_u{}^M(X,Y) = J_u{}^A(X,Y)\omega_A{}^M(Y) + \dots\,,$$
$$\hat{K}(X,Y) = e^{-2d(X,Y)}\hat{n}(Y) + \dots$$

  and fields:

$$\mathcal{A}_\mu{}^M(X,Y) = \mathcal{A}_\mu{}^A(X,Y)\omega_A{}^M(Y) + \dots$$
$$\mathcal{B}_{\mu\nu}(X,Y) = B_{\mu\nu}(X,Y)n(Y) + \dots$$

- Using **basis for even generalised tensors**:

$$\omega_A{}^M = (\omega_{\underline{k}}{}^M, \omega^{\underline{k}M}, \omega_\alpha{}^M)\,, \quad n \in R_2\,, \quad \hat{n} \in R_{D-4}\,,$$

  $\underline{k} = 1, \dots, d-1$
  ω_α are **localised** at fixed point(s)
  $\alpha = 1, \dots, \dim G$ for G Lie group

Localised vector multiplets

- Consistency conditions include (using $\wedge : R_p \otimes R_q \to R_{p+q}$)

$$\omega_A \wedge \omega_B = \eta_{AB}\, n\,, \quad \mathcal{L}_{\omega_A}\omega_B = -f_{AB}{}^C\omega_C\,,$$

- Let $\mathbf{y}$ denote all (physical) **odd** coordinates. Take:

$$\eta_{AB} = \begin{pmatrix} 0 & \delta_{\underline{i}}{}^{\underline{j}} & 0 \\ \delta^{\underline{i}}{}_{\underline{j}} & 0 & 0 \\ 0 & 0 & \delta(\mathbf{y})\kappa_{\alpha\beta} \end{pmatrix}$$

- $\kappa_{\alpha\beta}$ Killing form for Lie group G
- $f_{AB}{}^C \neq 0$ only for $f_{\alpha\beta}{}^\gamma$ structure constants of G

- $\Rightarrow$ (think generalised Scherk-Schwarz) modified generalised Lie derivative

$$\mathcal{L}_\Lambda V^A = L_\Lambda V^A + \eta^{AB}\eta_{CD}\partial_B\Lambda^C V^D + f_{BC}{}^A\Lambda^B V^C$$

Here $\partial_A = \omega_A{}^M\partial_M$, we always take $\partial_\alpha = 0$

Localised vector multiplets

- Look at $\mathcal{A}_\mu{}^A = (\underbrace{\mathcal{A}_\mu{}^{\underline{k}}}_{\text{KK vector}}, \underbrace{\mathcal{A}_{\mu\underline{k}}}_{\text{2-form}}, \underbrace{\mathcal{A}_\mu{}^\alpha}_{\text{localised gauge field}})$

- Find **localised** modified gauge transformations e.g.

$$\delta \mathcal{A}_{\mu\underline{k}} \propto \delta(\mathbf{y}) \kappa_{\alpha\beta} \Lambda^\alpha (D_\mu A_{\underline{k}}{}^\beta - \partial_k A_\mu{}^\beta)$$

Localised $A_{\underline{k}}{}^\alpha$ from expansion of $J_u{}^A$

- Depending on SSC: all, some or none of $\mathbf{y}$ are physical directions
  - e.g. Hořava-Witten $\mathbf{y} \equiv (Y^{12}, Y^{13}, Y^{14}, Y^{15})$, physical coordinates Y^{i5}, $i = 1, \ldots, 4 \Rightarrow \delta(\mathbf{y}) \to \delta(Y^{15})$
  - e.g. heterotic SSC $\mathbf{y}$ all dual coordinates $\Rightarrow \delta(y) \to 1$

- Reproduce field content, extra gauge fields, modified (Green-Schwarz) gauge transformations, and actions

Summary

- **Discrete quotients of ExFT = O-folds**
- Compatibility with half-maximal structure $\rightarrow$ half-maximal O-folds
- Generic quotient is **non-geometric** (in some SSC)
- For $\mathbb{Z}_2$ quotient: unified description of type I/orientifolds, heterotic, Hořava-Witten including **localised vector multiplets** within ExFT
- General idea works for general $E_{d(d)}$ groups, but can be new features e.g. $d = 5$ $\mathrm{SO}(5,5)$ chiral and non-chiral half-max structures see paper, appendix A!

Open problems

- For $\mathbb{Z}_2$: how to **determine** the gauge group?
  Anomaly cancellation in ExFT?

- **Moduli space?** = Location of D-branes/value of Wilson lines. ExFT "on" K3?
- **Gauge enhancement?** For heterotic DFT see [Aldazabal et al, Fraiman et al] - embeddable in ExFT?

- **Beyond $\mathbb{Z}_2$:** classification/control. Generically have $Y \sim \tilde{Y}$. Can this be studied in ExFT?

- Full extension to $E_{d(d)}$ $d = 5, 6, 7, 8, 9, \ldots$

Open problems

- For $\mathbb{Z}_2$: how to **determine** the gauge group?
  Anomaly cancellation in ExFT?
- **Moduli space?** = Location of D-branes/value of Wilson lines.
  ExFT "on" K3?
- **Gauge enhancement?** For heterotic DFT see [Aldazabal et al, Fraiman et al] - embeddable in ExFT?
- **Beyond $\mathbb{Z}_2$:** classification/control. Generically have $Y \sim \tilde{Y}$. Can this be studied in ExFT?
- Full descriptions for $E_{d(d)}$ $d = 5, 6, 7, 8, 9, \ldots$

Thanks for listening!