

Finite heterotic deformations and holomorphic Courant algebroids

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General motivation

- ▶ \exists “good” understanding of sugra/strings on “nice” geometries (Special holonomy manifolds e.g. CY3 [Candelas, Horowitz, Strominger & Witten '85])
- ▶ **Want:** to understand general SUSY backgrounds (with fluxes)
 - for phenomenology (moduli stabilisation, SUSY breaking)
 - for AdS/CFT (CFT data \leftrightarrow geometry)
 - for theoretical understanding
- ▶ Much work on this! [Too many people to list!]
- ▶ At first: Mathematical picture much worse than for e.g. CY3
- ▶ **Recently:** New mathematical picture... **Generalised geometry!**

[Hitchin '02] [Gualtieri '04]

Generalised geometry and supergravity

- ▶ Geometry of extended tangent bundle $E \simeq TM \oplus$ ("forms")
- ▶ Combines : $\begin{cases} \text{diffeo} + \text{gauge symmetry} \rightarrow \text{Generalised diffeos} \\ \text{metric} + \text{gauge fields} \rightarrow \text{Generalised metric} \end{cases}$
- ▶ Enhanced sym $\begin{cases} GL(d, \mathbb{R}) \rightarrow O(d, d) \times \mathbb{R}^+, E_{d(d)} \times \mathbb{R}^+, \text{etc.} \\ SO(d) \rightarrow SO(d) \times SO(d), H_d, \text{etc.} \end{cases}$
- ▶ Also combines : various G -structures on TM for \mathcal{N} SUSYs
→ Unique generalised structure group $\mathcal{G}_{\mathcal{N}}$ for each \mathcal{N}
- ▶ Notion of integrability [Coimbra, CS-C & Waldram '14; Coimbra & CS-C '16]

\mathcal{N} -SUSY Minkowski background \Leftrightarrow Integrable $\mathcal{G}_{\mathcal{N}}$ structure

$O(d, d)$ Generalised geometry [Hitchin '02; Gualtieri '04]

E.g. $E \simeq TM \oplus T^*M$ NS-NS fields (g, B_2) (& ϕ via \mathbb{R}^+)

▶ Natural $O(d, d)$ metric $\langle v + \lambda, v + \lambda \rangle = i_v \lambda$

▶ Dorfman derivative $L_V = (\mathcal{L}_v - d\lambda \cdot)$

▶ Generalised metric $G = \begin{pmatrix} g - Bg^{-1}B & Bg^{-1} \\ -g^{-1}B & g^{-1} \end{pmatrix} \in \frac{O(6,6)}{O(6) \times O(6)}$

▶ Generalised Levi-Civita D (**torsion-free!**)

$$\rightarrow \text{e.g. SUSY} \begin{cases} \delta\psi_a^- = D_a \epsilon^- = (\nabla_a - \frac{1}{8} H_{abc} \gamma^{bc}) \epsilon^- \\ \delta\rho^- = \gamma^{\bar{a}} D_{\bar{a}} \epsilon^- = (\nabla - \frac{1}{4} \not{H} - \not{\phi}) \epsilon^- \end{cases}$$

▶ For e.g. $\mathcal{N} = 2$, $\epsilon = (\epsilon_1, \epsilon_2)$ defines $SU(3) \times SU(3)$ structure

[Grana, Minasian, Petrini & Tomasiello '04]

▶ SUSY \Leftrightarrow Integrability $\Leftrightarrow \exists$ **torsion-free** D s.t. $D\epsilon = 0$

Generalised geometry for heterotic?

Heterotic supergravity : [Garcia-Fernandez '13] [Coimbra, Minasian, Triendl & Waldram '14]

$$E \simeq T \oplus \text{End}(V) \oplus T^*$$

Find much structure the same:

\exists generalised connection \rightarrow SUSY and eqns of motion etc.

But : Connection is **not** torsion-free (Torsion $\sim F$)

SUSY \neq Integrability

Holomorphic structures and $\mathcal{N} = 1$ heterotic moduli

- ▶ Consider $\mathcal{N} = 1$ Minkowski backgrounds of heterotic sugra
(always \mathbb{C} -manifolds!)
- ▶ Infinitesimal moduli given by $H_{\bar{D}}^{0,1}(Q)$ where:

$$Q \simeq T^{1,0} \oplus \text{End}(V) \oplus T^{*(1,0)}$$

has holomorphic structure [de la Ossa & Svanes '14][Anderson, Gray & Sharpe '14]

$$\begin{aligned} \bar{D} : \Omega^{0,p}(Q) &\longrightarrow \Omega^{0,p+1}(Q) \quad \text{s.t.} \quad \bar{D}^2 = 0 \\ \bar{D} &\sim \bar{\partial} + F + H \quad (\Leftrightarrow \text{Bianchi}) \end{aligned}$$

- ▶ How to extend this to (small but) **finite deformations**?
- ▶ Does some **generalised geometry** based on Q play a role?

Outline of talk

- ▶ Review: Heterotic supergravity and Hull-Strominger system
 - Derive F-terms from superpotential
- ▶ Space of off-shell $\mathcal{N} = 1$ geometries
 - Expand superpotential in \mathbb{C} -coords
- ▶ Holomorphic Courant algebroids
 - Superpotential as holomorphic Chern-Simons
- ▶ F-term conditions from holomorphic L_3 algebra
- ▶ Comments

Heterotic supergravity

Field content : $(g_{MN}, B_{MN}, \phi, A^\alpha; \psi_M, \lambda, \chi^\alpha)$

$$H_3 := dB + \frac{\alpha'}{4}(\omega_{\text{CS}}(A) - \omega_{\text{CS}}(\Theta))$$

$$\Rightarrow \text{ Bianchi } dH_3 = \frac{\alpha'}{4}(\text{tr } F \wedge F - \text{tr } R \wedge R)$$

Trick :

Think of Θ as extra gauge connection!

$$H_3 := dB + \frac{\alpha'}{4}\omega_{\text{CS}}(A)$$

$$\Rightarrow dH_3 = \frac{\alpha'}{4} \text{tr } F \wedge F$$

[Bergshoeff & de Roo '89] [de la Ossa & Svanes '14] [Coimbra, Minasian, Triendl & Waldram '14]

Minkowski backgrounds : $M^{3,1} \times_w X_6$

(Off-shell) $\mathcal{N} = 1$ SUSY :

$\Rightarrow \exists$ spinor field $\eta \neq 0$ on $X_6 \rightarrow SU(3)$ structure

$\Rightarrow \exists$ forms : $\omega_{mn} = -i\eta^\dagger \gamma_{mn} \eta$, $\Omega_{mnp} = e^{-2\phi} \eta^T \gamma_{mnp} \eta$,

$\mathcal{N} = 1$ SUSY :

$$\left. \begin{aligned} d\Omega &= 0 \\ i(\partial - \bar{\partial})\omega &= H \\ \Omega \wedge F &= 0 \end{aligned} \right\} \text{F-terms}$$

$$\left. \begin{aligned} \omega \lrcorner F &= 0 \\ d(e^{-2\phi}\omega \wedge \omega) &= 0 \end{aligned} \right\} \text{D-terms}$$

$$\left[\text{And Bianchi! : } dH_3 = -2i\partial\bar{\partial}\omega = \frac{\alpha'}{4} \text{tr } F \wedge F \right]$$

The Superpotential

Superpotential :

[Gurrieri, Lukas & Micu '04]

$$W = \int (H + i d\omega) \wedge \Omega$$

Stationary zero of W

$$\delta W = 0$$

\Leftrightarrow geometry satisfies F-term equations

[see: de la Ossa & Svanes '15]

$$\begin{aligned} d\Omega &= 0 \quad \Rightarrow \quad \text{Complex manifold} \\ i(\partial - \bar{\partial})\omega &= H \\ \Omega \wedge F &= 0 \quad \Rightarrow \quad \text{Holomorphic bundle} \end{aligned}$$

But: What is the space of geometries?

Space of $\mathcal{N} = 1$ geometries

Degrees of freedom

- ▶ For now, **ignore gauge field** A^α !
- ▶ $\mathcal{N} = 1 \Rightarrow \exists SU(3)$ structure on TX $(\omega, \Omega) \sim \frac{GL(6, \mathbb{R})}{SU(3)}$
- ▶ Total d.o.f: $SU(3)$ structure (ω, Ω) , 2-form B and ϕ

Geometry

- ▶ $\mathcal{N} = 1 \Rightarrow$ **complex structure** on space of fields? (Kähler)

How can we understand this?

- ▶ Useful tool: “holomorphic variation” operator

$$\Delta = \sum_{n=1}^{\infty} \frac{1}{n!} \epsilon^n \frac{\partial^n}{\partial t^n} \quad \begin{array}{l} t \rightarrow t + \epsilon \\ \bar{t} \rightarrow \bar{t} \end{array}$$

Review: Space of almost complex structures

Almost \mathbb{C} -structure

$$(GL(3, \mathbb{C}) \subset GL(6, \mathbb{R}))$$

- ▶ $J \in \text{End}(TX)$ s.t. $J^2 = -1$
- ▶ $TX_{\mathbb{C}} \rightarrow T^{1,0} \oplus T^{0,1}$ J specified by choice of $T^{0,1}$

Deforming $J \rightarrow J'$

- ▶ New $T^{0,1}$ graph of map $\mu : T^{0,1} \rightarrow T^{1,0}$
- ▶ Think $\mu \in \Omega^{0,1}(T^{1,0})$ local \mathbb{C} -coord
- ▶ $J' = \infty$ -series in μ and $\bar{\mu}$
- ▶ But $\Delta J = -2i\mu$ (Only $\mu \neq 0$ by definition)

Review: \mathbb{C} -structure and Integrability

\mathbb{C} -structure

- ▶ Integrable $\Leftrightarrow \bar{\partial}^2 = 0$
- ▶ Deformed $(\bar{\partial} + \mu^a \partial_a)^2 = 0 \rightarrow$ Maurer-Cartan equation

$$\bar{\partial}\mu + \frac{1}{2}[\mu, \mu] = 0$$

Kodaira-Spencer DLGA

- ▶ On $\Omega^{0,\bullet}(T^{1,0})$ have $\begin{cases} \bar{\partial} : \Omega^{0,p}(T^{1,0}) \rightarrow \Omega^{0,p+1}(T^{1,0}) \\ [\cdot, \cdot] : \Omega^{0,p}(T^{1,0}) \times \Omega^{0,q}(T^{1,0}) \rightarrow \Omega^{0,p+q}(T^{1,0}) \end{cases}$

$$\bar{\partial}_{[\bar{a}_1 \mu_{\bar{a}_2 \dots \bar{a}_{p+1}}]^c} \quad v_{[\bar{a}_1 \dots \bar{a}_p}{}^c \partial_c w_{\bar{b}_1 \dots \bar{b}_q]}^a \pm w_{[\bar{b}_1 \dots \bar{b}_q}{}^c \partial_c v_{\bar{a}_1 \dots \bar{a}_p]}^a$$

- ▶ $\bar{\partial}[v, w] = [\bar{\partial}v, w] \pm [v, \bar{\partial}w]$ and (graded) Jacobi

Key point: Elegant expressions due to nice parameterisation μ

How to pick nice variable μ ?

- ▶ Use group theory
- ▶ $\mathfrak{gl}(6, \mathbb{R}) \rightarrow \mathfrak{gl}(3, \mathbb{C}) \oplus \{\mu\} \oplus \{\bar{\mu}\}$
- ▶ Complex coordinate on homogenous space $\frac{GL(6, \mathbb{R})}{GL(3, \mathbb{C})}$

Space of $\mathcal{N} = 1$ geometries

Think **generalised geometry!**

Generalised tangent bundle $E \simeq T \oplus T^*$

$$\rightarrow \text{Generalised metric} \sim \frac{SO(6,6) \times \mathbb{R}^+}{SO(6) \times SO(6)}$$

$\mathcal{N} = 1$ SUSY \rightarrow generalised **$SU(3) \times SO(6)$ structure**

$$\tilde{\chi} \sim \frac{SO(6,6) \times \mathbb{R}^+}{SU(3) \times SO(6)}$$

Lie algebra decomposition provides analogue of μ

\rightarrow corresponds to $(\mu \propto \Delta J, x := \Delta(B + i\omega)^{1,1}, b := \Delta(B + i\omega)^{0,2})$

\rightarrow **\mathbb{C} -coords** on space of $\mathcal{N} = 1$ structures

The superpotential

$\mathcal{N} = 1 \Rightarrow$ Superpotential **holomorphic**: $W' = W + \Delta W$

Expand **superpotential** around SUSY vacuum ($W = \delta W = 0$)

$$\begin{aligned}\Delta W &= \int_X \left(H + i d\omega + d(\Delta B + i\Delta\omega) \right) \wedge \left(\Omega + \Delta\Omega \right) \\ &= 2 \int_X \left(\mu^d \wedge \bar{\partial} x_d + \frac{1}{2} \mu^d \wedge \mu^e \wedge H_{de\bar{c}} e^{\bar{c}} \right. \\ &\quad \left. + \mu^d \wedge \mu^e \wedge \partial_d x_e - \frac{1}{2} \mu^d \wedge \partial_d \tilde{b} \right) \wedge \Omega\end{aligned}$$

Recall:

$$\mu \in \Omega^{0,1}(T^{1,0}) \quad x \in \Omega_{\mathbb{C}}^{1,1} \sim \Omega^{0,1}(T^{*(1,0)}) \quad b \in \Omega^{0,2}$$

Exact sequence of bundles

$$0 \longrightarrow T^{*(1,0)} \longrightarrow Q \longrightarrow T^{(1,0)} \longrightarrow 0$$

Other formulae as for Courant algebroids but with $\frac{\partial}{\partial x^m} \rightarrow \frac{\partial}{\partial z^a}$

Pairing: $\langle v + \lambda, v + \lambda \rangle = i_v \lambda$

Courant bracket:

$$[v + \lambda, v' + \lambda'] = [v, v'] + i_v \partial \lambda' - i_{v'} \partial \lambda + \frac{1}{2} \partial (i_v \lambda' - i_{v'} \lambda) - i_v i_{v'} H^{3,0}$$

Holomorphic structure

$$\bar{D} : \Omega^{0,p}(Q) \longrightarrow \Omega^{0,p+1}(Q)$$
$$\bar{D}(v + \lambda) = \bar{\partial}v + (\bar{\partial}\lambda - i_v \wedge H^{2,1})$$

$$\bar{D}^2 = 0 \quad \Leftrightarrow \quad \bar{\partial}H^{2,1} = 0$$

More on $\Omega^{0,\bullet}(Q)$

Courant bracket extends to bracket on $\Omega^{0,\bullet}(Q)$

$$\bar{D}[y, y'] = [\bar{D}y, y'] \pm [y, \bar{D}y']$$

$\therefore [,]$ well-defined on \bar{D} -holomorphic sections of Q (\rightarrow sheaf \mathcal{Q})

The superpotential revisited

Package variables: $\mu + x \in \Omega^{0,1}(Q)$ $b \in \Omega^{0,2}$

Expression becomes

$$\Delta W = \int_X \langle y, \bar{D}y - \frac{1}{3}[y, y] - \partial b \rangle \wedge \Omega$$

(c.f. **Holomorphic Chern-Simons** ...)

⇒ F-term conditions

$$\begin{aligned} \bar{D}y - \frac{1}{2}[y, y] - \frac{1}{2}\partial b = 0 &\quad \rightarrow \quad \text{Maurer-Cartan? DGLA?} \\ \bar{\partial}b - \frac{1}{2}\langle y, \partial b \rangle + \frac{1}{3!}\langle y, [y, y] \rangle = 0 &\quad \rightarrow \quad ??? \\ \partial \iota_\mu \Omega = 0 &\quad \rightarrow \quad \text{volume preservation} \end{aligned}$$

Graded vector space :

$$\mathcal{Y} = \bigoplus_n \mathcal{Y}_n, \quad n \in \mathbb{Z}$$

Multilinear brackets : $\ell_1(\cdot)$ $\ell_2(\cdot, \cdot)$ $\ell_3(\cdot, \cdot, \cdot)$... degree $2 - k$

Field equation / Maurer-Cartan equation : $Y \in \mathcal{Y}_1$

$$\mathcal{F}(Y) = \ell_1(Y) - \frac{1}{2}\ell_2(Y, Y) - \frac{1}{3!}\ell_3(Y, Y, Y) + \dots$$

Gauge transformation / Symmetry : $\Lambda \in \mathcal{Y}_0$

$$\delta_\Lambda Y = \ell_1(\Lambda) + \ell_2(\Lambda, Y) - \frac{1}{2}\ell_3(\Lambda, Y, Y) - \frac{1}{3!}\ell_4(\Lambda, Y, Y, Y) + \dots$$

Graded vector space :

$$\mathcal{Y}_0 = \mathcal{Q} \quad \mathcal{Y}_{-1} = \mathcal{O}_X$$

Multilinear brackets : $V, V' \in \mathcal{Q} \quad \varphi \in \mathcal{O}_X$

$$\begin{aligned} \ell_1(V) &= 0 & \ell_2(V, V') &= [V, V'] & \ell_3(V, V', V'') &= \langle V, [V', V''] \rangle \\ \ell_1(\varphi) &= 0 + \partial\varphi & \ell_2(V, \varphi) &= \langle V, \partial\varphi \rangle & &+ \text{cycles} \end{aligned}$$

Crucial point : Jacobiator of Courant bracket is ∂ -exact

$$\text{Jac}(V, V', V'') = -\frac{1}{3}\partial\langle V, [V', V''] \rangle + \text{cycles}$$

Another L_3 algebra

Graded vector space :

$$\mathcal{Y} = \bigoplus_{n \geq -1} \mathcal{Y}_n = \bigoplus_{n \geq -1} \Omega^{0,n}(Q) \oplus \Omega^{0,n+1}$$

Multilinear brackets : $Y = (y, b)$

$$\ell_1(Y) = (\bar{D}y + \frac{1}{2}(-1)^Y \partial b, \bar{\partial}b),$$

$$\ell_2(Y, Y') = ([y, y'], \frac{1}{2}(\langle y, \partial b' \rangle + (-1)^{1+Y Y'} \langle y', \partial b \rangle)),$$

$$\begin{aligned} \ell_3(Y, Y', Y'') &= \frac{1}{3}(-1)^{Y+Y'+Y''} (0, \langle y, [y', y''] \rangle \\ &\quad + (-1)^{Y(Y'+Y'')} \langle y', [y'', y] \rangle, \\ &\quad + (-1)^{Y''(Y+Y')} \langle y'', [y, y'] \rangle), \end{aligned}$$

$$\ell_{k \geq 4} = 0$$

Package variables: $Y = (y, b) \in \Omega^{0,1}(Q) \oplus \Omega^{0,2} = \mathcal{Y}_1$

Result: F-term conditions

$$\left. \begin{aligned} \bar{D}y - \frac{1}{2}[y, y] - \frac{1}{2}\partial b &= 0 \\ \bar{\partial}b - \frac{1}{2}\langle y, \partial b \rangle + \frac{1}{3!}\langle y, [y, y] \rangle &= 0 \end{aligned} \right\} \leftrightarrow \mathcal{F}(Y) = 0$$
$$\partial_{\nu\mu}\Omega = 0 \} \rightarrow \text{volume preservation}$$

Including the gauge fields

New \mathbb{C} -coordinate: $\alpha \in \Omega^{0,1}(\text{End}(V))$ joins (μ, x, b) from before

New Courant algebroid [c.f. Garcia-Fernandez '13] [Coimbra, Minasian, Triendl & Waldram '14]

$$y = (\mu, \alpha, x) \in \Omega^{0,1}(Q) \quad \text{with} \quad Q \simeq T^{1,0} \oplus \text{End}(V) \oplus T^{*(1,0)}$$

\exists natural modifications of $[\ , \]$, \bar{D} , etc. including this part

Then...

Superpotential, L_3 algebra and Maurer-Cartan

...take exactly same expressions with new Q objects

Conclusions

Holomorphic Courant algebroid $\rightarrow L_3$ algebra:

$$Q \simeq T^{1,0} \oplus \text{End}(V) \oplus T^{*(1,0)} \quad \mathcal{Y} = \bigoplus_{n \geq -1} \Omega^{0,n}(Q) \oplus \Omega^{0,n+1}$$

Superpotential

$$\Delta W = \int_X \langle y, \bar{D}y - \frac{1}{3}[y, y] - \partial b \rangle \wedge \Omega$$

(extension of **Holomorphic Chern-Simons...**)

F-term conditions

$$\left. \begin{aligned} \bar{D}y - \frac{1}{2}[y, y] - \frac{1}{2}\partial b &= 0 \\ \bar{\partial}b - \frac{1}{2}\langle y, \partial b \rangle + \frac{1}{3!}\langle y, [y, y] \rangle &= 0 \end{aligned} \right\} \leftrightarrow \mathcal{F}(Y) = 0$$
$$\partial_{\nu\mu}\Omega = 0 \} \rightarrow \text{volume preservation}$$

Matching infinitesimal story

At first order:

$$\bar{D}y = \frac{1}{2}\partial b \quad \bar{\partial}b = 0 \quad \partial\iota_\mu\Omega = 0$$

$\partial\bar{\partial}$ -lemma \Rightarrow ∂b & $\partial\iota_\mu\Omega$ are $\bar{\partial}$ -exact

$$\Rightarrow \begin{cases} \text{can shift } y \text{ to get } \bar{D}y = 0 \\ \text{up to diffeo } \partial\iota_\mu\Omega = 0 \end{cases}$$

So matches earlier result:

$$\text{Infinitesimal moduli space} = H_{\bar{D}}^{0,1}(Q)$$

[de la Ossa & Svanes '14] [Anderson, Gray & Sharpe '14]

Relation to holomorphic L_3 algebra

Dolbeault resolutions: of \mathcal{O}_X and \mathcal{Q} (holomorphic sections of Q)

$$\begin{array}{ccccccccc}
 0 & \longrightarrow & \mathcal{O}_X & \xrightarrow{\iota} & \mathcal{C}^\infty(\mathbb{C}) & \xrightarrow{\bar{\partial}} & \Omega^{(0,1)} & \xrightarrow{\bar{\partial}} & \Omega^{(0,2)} & \xrightarrow{\bar{\partial}} & \longrightarrow \\
 & & \partial \downarrow & & \partial \downarrow & & \partial \downarrow & & \partial \downarrow & & \\
 0 & \longrightarrow & \mathcal{Q} & \xrightarrow{\iota} & \mathcal{Q} & \xrightarrow{\bar{D}} & \Omega^{(0,1)}(Q) & \xrightarrow{\bar{D}} & \Omega^{(0,2)}(Q) & \xrightarrow{\bar{D}} & \longrightarrow
 \end{array}$$

Total complex \cong original complex

$$\begin{array}{ccccccccc}
 0 & \longrightarrow & \mathcal{O}_X & \xrightarrow{\partial} & \mathcal{Q} & \longrightarrow & 0 & \longrightarrow & \longrightarrow \\
 & & \downarrow \iota & & \downarrow \iota & & \downarrow & & \\
 0 & \longrightarrow & \mathcal{C}^\infty(\mathbb{C}) & \xrightarrow{\ell_1} & \Gamma(Q) \oplus \Omega^{(0,1)} & \xrightarrow{\ell_1} & \Omega^{(0,1)}(Q) \oplus \Omega^{(0,2)} & \xrightarrow{\ell_1} & \longrightarrow
 \end{array}$$

Relation to holomorphic L_3 algebra

L_∞ quasi-isomorphism :

$$(\mathcal{O}_X \xrightarrow{\partial} \mathcal{Q}) \stackrel{\text{quasi}}{\cong} \left(\mathcal{Y} = \bigoplus_{n \geq -1} \Omega^{0,n}(Q) \oplus \Omega^{0,n+1} \right)$$

Our L_3 algebra is rewriting of LHS using \mathcal{C}^∞ objects

Further points (see paper!)

- ▶ Sol^n of F-terms $\Rightarrow \exists$ Sol^n of D-terms
 - \rightarrow D-terms are “gauge-fixing conditions”
 - \rightarrow Our Maurer-Cartan set is **full moduli space**
- ▶ Effective field theory

Decompose into massless and massive parts: $y = y_0 + y_h$

$$y = y_0 + y_h \quad \bar{D}y_0 = 0 \quad \rightarrow \quad \text{modes } [y] \in H_{\bar{D}}^{0,1}(Q)$$

Integrate out $b^{0,2}$ and argue that low energy **Yukawa couplings** are

$$\Delta W_{\text{Yuk}} = \int \langle y_0, [y_0, y_0] \rangle \wedge \Omega \quad \text{c.f. [Witten '86]}$$

- ▶ **Obstructions** : analyse by formal power series expansion ?
[c.f. Tian-Todorov construction?]
- ▶ **Superpotential** \sim **holomorphic Chern-Simons** functional ?
 - Anti-holomorphic coords \sim spacetime
 - Holomorphic generalised diffeos \sim gauge group
 - Couples \mathbb{C} -structure and “Kähler” moduli to hCS
 - Quantisation? TQFT? Donaldson-Thomas invariants?
 - SFT for some topological string?
(c.f. Witten '92, BCOV '93)

- ▶ Worksheet picture ?
 - How to see L_3 algebra? (Extending Melnikov & Sharpe '11)
 - Holomorphic β - γ system? [see Nekrasov '05]
- ▶ G_2 or $Spin(7)$ manifolds? [de la Ossa, Larfors & Svanes]
- ▶ How to impose Θ is connection $\nabla + \frac{1}{2}H$ on TM ?
- ▶ Kähler potential/metric and more on effective theory?
(See Candelas, de la Ossa & McOrist '16; McOrist '16)

The End

Thanks for your attention!