Aspects of Gauge-Strings Duality

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Outline

1. I will discuss some work in progress on AdS/CFT. The focus will be on the general ideas and outcomes.
2. The knowledge of field theory results at strong coupling allows us to say things about a geometry. The viewpoint adopted in this talk will be mostly field theoretical.
3. The particular example I will discuss today will be in the context of an $N = 2$ SCFT in four dimensions.
4. This talk mostly based on work in preparation and material developed with: S. Zacarías, G. Itsios, J. Montero, D. Roychowdhury, J. van Gorsel, J. M. Penin, S. Speziale, K. Sfetsos, D. Thompson, Y. Lozano, H. Nastase.

SCFTs in diverse dimensions (16 SUSY). An *incomplete* picture.

- d=6:Hanany-Zaffaroni, Brunner-Karch —D6-D8-NS5— Gaiotto-Tomasielo, Apruzzi, Fazzi, Rosa, Passias, Cremonesi.
- d=5:Aharony-Hanany-Kol —D5-D7-NS5– D'Hoker, Gutperle, Uhlemann, Trivella, Karch.
- d=4:Gaiotto —D4-D6-NS5— Gaiotto, Maldacena; Aharony, Berkooz, Berdichevsky; Stefanski, Reid-Edwards.
- d=3: Gaiotto-Witten —D3-D5-NS5— D'Hoker, Estes, Gutperle; Assel, Bachas, Gomis.
- d=2:(0,4) SCFT —D2-D4-NS5.
- d=1 (not a SCFT): Lin, Lunin, Maldacena —D0-D2-NS5— Lin, Maldacena.

In all these examples, there is (at least) an $SU(2)$ R-symmetry. The dual backgrounds have the form

$$ds^2 \sim f_1 AdS_{d+1} + f_2 d\Omega_2 + f_3 d\Omega_{5-d} + f_4 d\eta^2 + f_5 d\sigma^2; \quad f_i(\sigma, \eta).$$

There are also NS B_2, Φ and RR fields respecting the isometries above.

In particular, for four dimensional $\mathcal{N}=2$ CFTs, Lin, Lunin and Maldacena wrote in 2005 the Type IIA backgrounds ($\alpha' = g_s = 1$)

$$ds_{10}^2 = 4f_1 ds_{AdS_5}^2 + f_2(d\sigma^2 + d\eta^2) + f_3 d\Omega_2(\chi,\xi) + f_4 d\beta^2.$$
$$B_2 = f_5 d\Omega_2(\chi,\xi), \quad C_1 = f_6 d\beta, \; A_3 = f_7 d\beta \wedge d\Omega_2, \; e^{2\phi} = f_8.$$

The functions $f_i(\sigma,\eta)$ can be all written in terms of a function $V(\sigma,\eta)$ and its derivatives, $f_i \sim f_i(V, \partial_\sigma V, \partial_\eta V)$

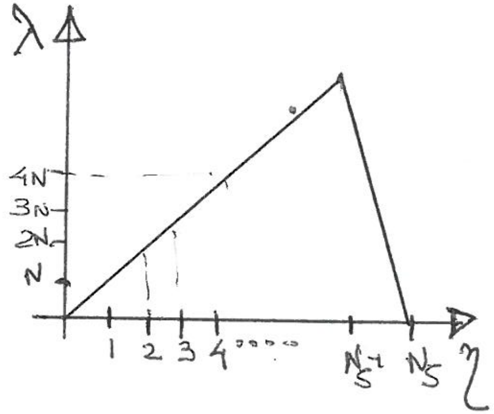
The function $V(\sigma,\eta)$ satisfies a Laplace-like equation with certain given boundary conditions

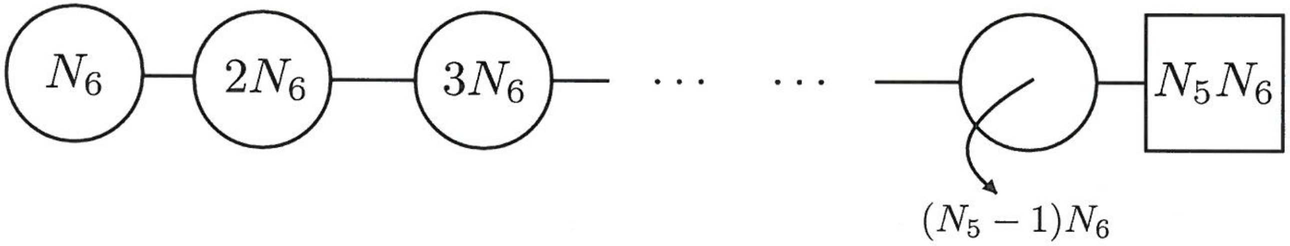
$$\sigma\partial_\sigma\left[\sigma\partial_\sigma V\right] + \sigma^2\partial_\eta^2 V = 0,$$
$$V(\sigma\to\infty,\eta)\to 0, \;\; \sigma\partial_\sigma V(\sigma,\eta)|_{\sigma=0} = \lambda(\eta) \to \lambda(0) = \lambda(N_5) = 0.$$

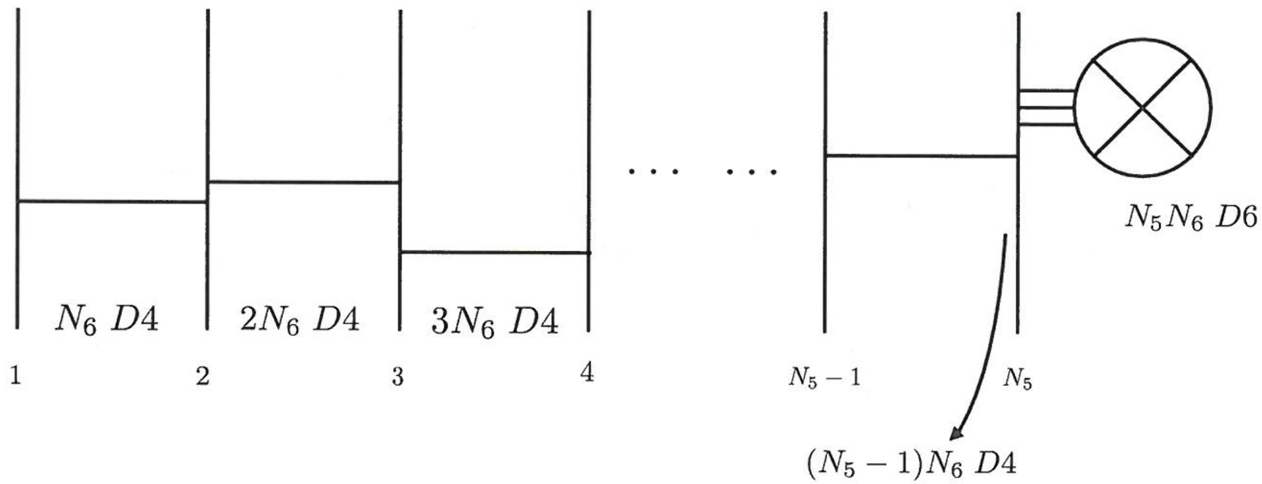
Various aspects of the Physics of the CFT are encoded in the function $\lambda(\eta)$.

A very simple example of an acceptable $\lambda(\eta)$ is,

$$\lambda(\eta) = N_6 \begin{cases} \eta & 0 \leq \eta \leq (N_5 - 1) \\ (N_5 - 1)(N_5 - \eta) & (N_5 - 1) \leq \eta \leq N_5. \end{cases}$$







Given $\lambda(\eta)$, one can write the solution for $V(\sigma,\eta)$ as a Fourier series

$$V(\sigma,\eta) = -\sum_{n=1}^{\infty} \frac{c_n}{w_n} K_0(w_n\sigma)\sin(w_n\eta), \qquad w_n = \frac{n\pi}{N_5}.$$

$$c_n = \frac{n\pi}{N_5^2}\int_{-N_5}^{N_5} \lambda(\eta)\sin(w_n\eta)d\eta, \qquad w_n = \frac{n\pi}{N_5}.$$

Using this, one can calculate the Page charges and make a correspondence with the Hanany-Witten brane set-up.

$$\hat{F} = Fe^{-B_2}, Q_{D_p} = \frac{1}{2\kappa_{10}^2 T_{Dp}}\int_\Sigma \hat{F}_{8-p}, \;\; 2\kappa_{10}^2 T_{D_p} = g_s(4\pi^2\alpha')^{\frac{7-p}{2}}.$$

$$Q_{NS5} = N_5, \qquad Q_{D6} = \lambda'(0) - \lambda'(N_5), \quad Q_{D4} = \int_0^{N_5} \lambda(\eta)d\eta.$$

One can check that these expressions work for any quiver CFT/Hanany-Witten set-up.

Other quantities can be calculated in terms of $\lambda(\eta)$, for example the linking numbers of the different branes

$$K_i = N_{D4}^{right} - N_{D4}^{left} - N_{D6}^{right}, \quad L_j = N_{D4}^{right} - N_{D4}^{left} + N_{NS}^{left}.$$

$$\sum_{i=1}^{N_5} K_i + \sum_{j=1}^{N_6} L_j = 0.$$

One can find expressions that compute the linking numbers, purely in terms of $\lambda(\eta)$,

$$\sum_{i=1}^{N_5} K_i = \lambda'(N_5)N_5 = \frac{1}{2\kappa_{10}^2 T_{D4}} \int_{\Sigma_4} F_4 - B_2 \wedge F_2.$$

$$\sum_{j=1}^{N_6} L_j = -\sum_{j=1}^{N_6} \lambda'(\eta_j)\eta_j = -\lambda'(N_5)N_5 = \frac{1}{2\kappa_{10}^2 T_{D4}} \int_{\tilde{\Sigma}_4} F_4 + C_1 \wedge H_3.$$

Similarly, the central charge of the CFT can be calculated using the gravity solution. After various manipulations one obtains,

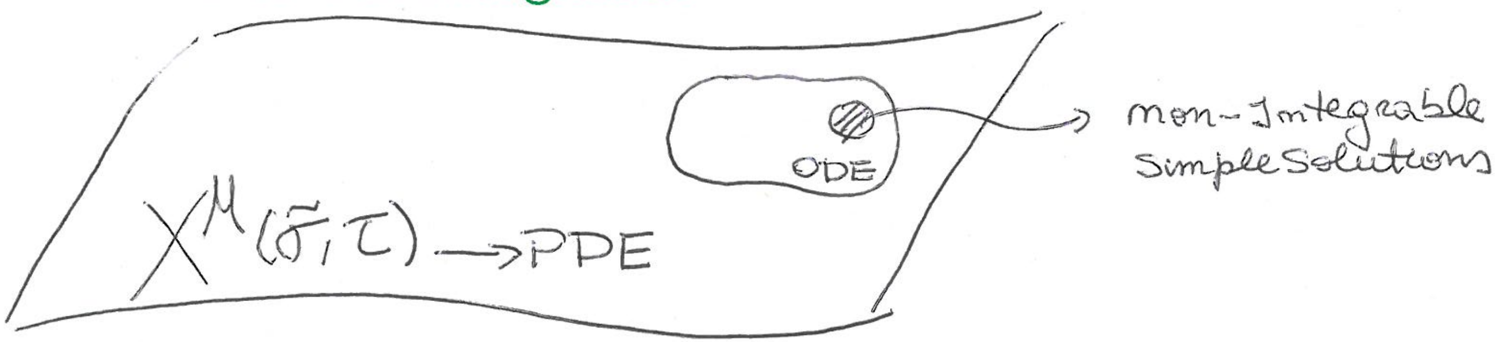
$$c = \frac{\pi^3}{8} \int_0^{N_5} \lambda^2(\eta) d\eta = \frac{\pi N_5^3}{2^6} \sum_{m=1}^{\infty} \frac{c_m^2}{m^2}.$$

One can show, that this result coincides with the field theoretical calculation (in the limit of large N_5),

$$c = \frac{2n_v + n_h}{12\pi}, \qquad a = \frac{5n_v + n_h}{24\pi}.$$

Where we count certain combinations of the number of vector multiplets and hypermultiplets of the $\mathcal{N} = 2$ SCFT. In some sense, the central charge is the 'power' of the function $\lambda(\eta)$.
For any generic quiver CFT or $\lambda(\eta)$, we have shown that the expression above is correct. There are similar expressions for the Entanglement Entropy. Some other observables should also be calculable in terms of $\lambda(\eta)$. Others will depend on $\dot{V}$, $\dot{V}'$ and $\ddot{V}$.

Let me focus on a particular aspect of these systems: Integrability.
One can show that the string sigma models in a given background is classically integrable, if the equations of motion can be written in terms of a Lax pair. In general, it is very difficult to find such Lax pair.
It is much easier to 'disprove Integrability'. By proposing a semiclassical string soliton $X^{\mu}(\tilde{\sigma}, \tau)$ and studying the coupled non-linear partial differential equations of motion. This is still quite complicated to do in practice!
A more modest approach is to consider a simple string soliton, whose equations of motion admit a one-dimensional truncation and reduce to ordinary differential equations.
If this truncation is Liouville non-Integrable, the whole sigma model is also non-Integrable.



Consider the NS sector of the $\mathcal{N}=2$ Gaiotto-Maldacena solutions

$$ds_{10}^2 = 4f_1 ds_{AdS_5}^2 + f_2(d\sigma^2 + d\eta^2) + f_3 d\Omega_2(\chi,\xi) + f_4 d\beta^2.$$
$$B_2 = f_5 d\Omega_2(\chi,\xi).$$

Propose a string solution of the form,

$$t = t(\tau), \quad \sigma = \sigma(\tau), \quad \eta = \eta(\tau), \quad \chi = \chi(\tau); \quad \xi = k\tilde{\sigma}, \quad \beta = \lambda\tilde{\sigma}.$$

Carefully studying the equations of motion and Virasoro constraint, one finds a set of non-linear and couple ordinary differential equations for $\ddot{\sigma}, \ddot{\eta}, \ddot{\chi}$, in terms of first derivatives and the potential function $V(\sigma,\tau)$.

One solution is $\sigma = 0$, $\eta = E\tau$, $\chi = 0$, $\dot{t} = E/f_1$.

One then follows developments by mathematicians.
Consider the previous simple solution and a variation of it

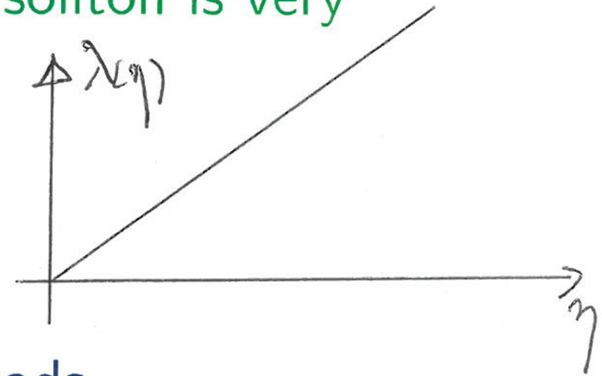
$$\eta(\tau) = \eta_s = E\tau, \quad \chi = 0 + z(\tau).$$

This leads us to a linear second order differential equation

$$\ddot{z}(\tau) + \mathcal{B}\dot{z}(\tau) + \mathcal{A}z(\tau) = 0,$$
$$\mathcal{A} = (k^2 - k\dot{\eta}\frac{\partial_\eta f_5}{f_3})|_{\eta=\eta_s}, \quad \mathcal{B} = (\dot{\eta}\partial_\eta \log f_3)|_{\eta=\eta_s}.$$

There are criteria due to Kovacic to decide the Liouvillian integrability (or not) of this differential equation.
When applied to the potentials written in terms of a Fourier-Bessel series, we find that—unless the wrapping $k = 0$— all of them are non-integrable.
Except for one $V(\sigma, \eta)$!

The only potential, for which one finds an integrable soliton is very simple,



$$V_{ST} = \eta \log \sigma - \eta \frac{\sigma^2}{2} + \frac{\eta^3}{3}.$$

The background that is derived from this potential reads,

$$ds^2 = AdS_5 + \frac{d\sigma^2 + d\eta^2}{1-\sigma^2} + \frac{\eta^2(1-\sigma^2)}{4\eta^2 + (1-\sigma^2)^2} d\Omega_2 + \sigma^2 d\beta^2,$$

$$e^{-2\phi} = (1-\sigma^2)[4\eta^2 + (1-\sigma^2)^2], \quad B_2 = \frac{2\eta^3}{4\eta^2 + (1-\sigma^2)^2} d\Omega_2,$$

$$A_1 = 2(1-\sigma^2)^2 d\beta, \quad F_4 = B_2 \wedge F_2.$$

This background was written by Sfetsos and Thompson in 2011. They obtained it by applying non-Abelian T-duality on $AdS_5 \times S^5$. Further, for this background a Lax pair was written by Borsato and Wulff (2017). The system is classically integrable.

Let me draw some general lessons from this.

We started by considering $\mathcal{N} = 2$ linear quiver SCFTs. We worked with their dual string description.

All the dynamical information is encoded in a function $V(\sigma, \eta)$ solving a Laplace equation with boundary condition in terms of $\lambda(\eta)$.

Inside all of these different potentials V and functions λ, there is a particular one. It is the potential leading to the Sfetsos-Thompson background.

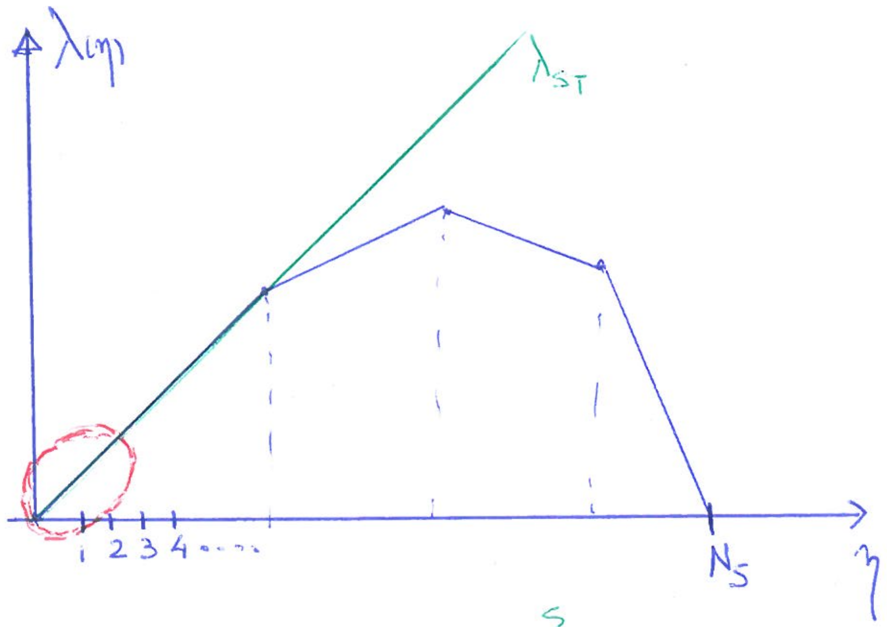
The 4d field theoretical interpretation of such isolated background is not solid. Aside from this, the Sfetsos-Thompson background has the special property of having an integrable sigma model.

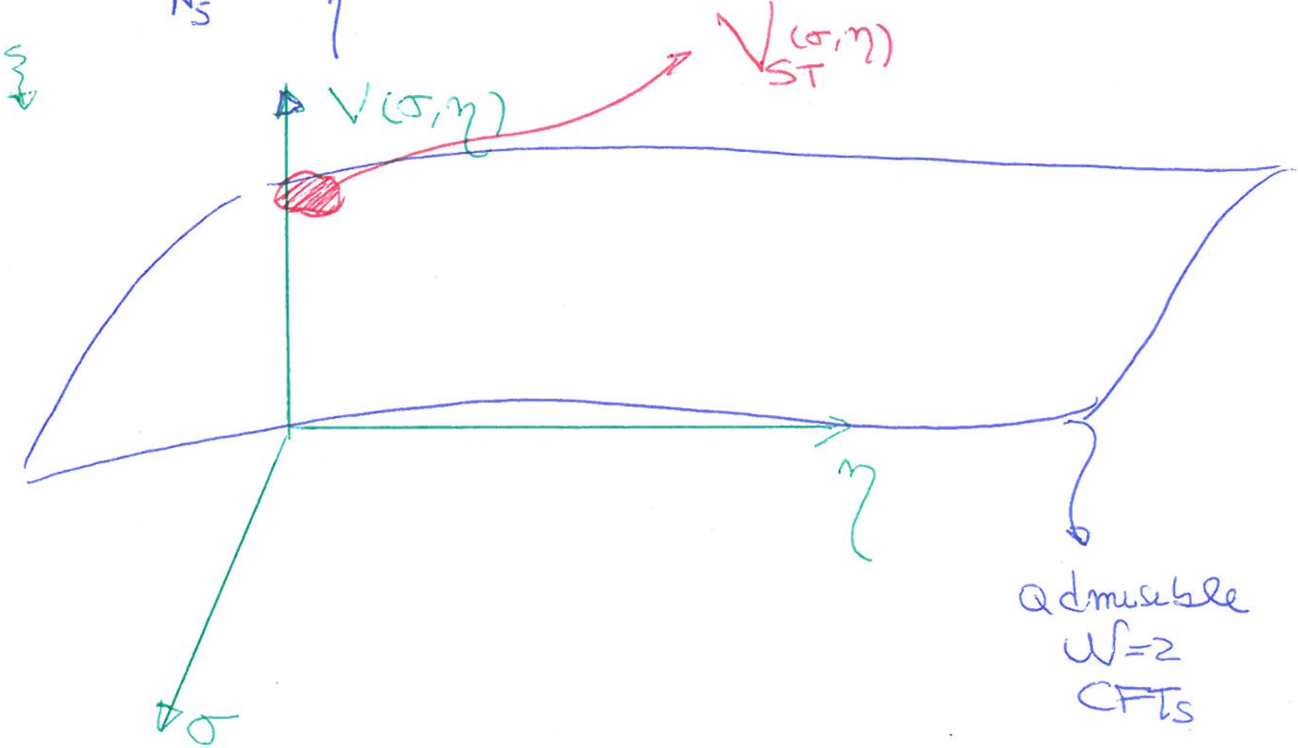
Probably, the Sfetsos-Thompson solution should be understood as 'needing a completion' that the Gaiotto-Maldacena backgrounds provide.

A graphic may clarify this!

$V_{ST} = \eta \log \sigma + \eta \frac{\sigma^2}{2} + \frac{\eta^3}{3}$; $\lambda_{ST}(\eta) = \eta$ $\rightsquigarrow$ (N) — (2N) — (3N) — (4N) - - - -

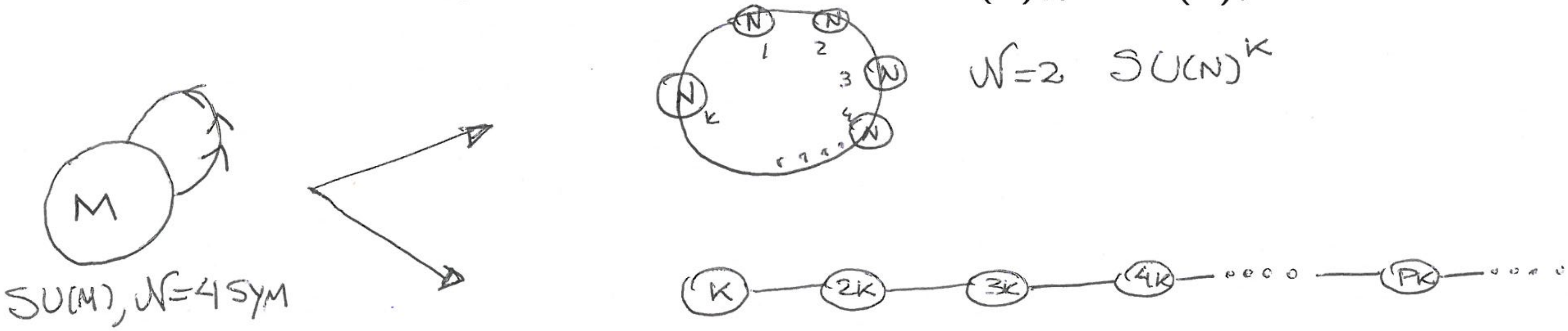
(NOT a good CFT in 4d!)





The effect of NATD on the CFT

Consider $\mathcal{N} = 4$ SYM. It contains $A^a_\mu, 4 \times \lambda^a, 6 \times \phi^a$. The global symmetries are $SO(2,4) \times SU(4)$. We consider an operation that preserves conformality, $\mathcal{N} = 2$ SUSY and $SU(2)_R \times U(1)_r$.



This last quiver should be closed by a flavour group. This is not possible, as a new global symmetry in the CFT (background isometry) should be produced by the duality. The Sfetsos-Thompson background reflects this impossibility by creating a singularity.

What about integrability? in the case of the Z_k-theory, this was proven by Roiban and Beisert in 2005.

The CFT view on the integrability of the string on the Sfetsos-Thompson background is more interesting!

Indeed, various authors: Gadde, Liendo, Rastelli, Pomoni, Sieg, Yan in papers written between 2011-2017, have studied linear quivers (mostly $\mathcal{N}=2$ SQCD) and shown that the presence of dimer-operators $\mathcal{M}^{ab}=\sum_{i=1}^{N_f} Q^{a,i}\tilde{Q}^{b,i}$ is enough to show that the theory is not-integrable. The argument is (3-loop) perturbative.

The CFT corresponding to the Sfetsos-Thompson background avoids this, simply by not having dimer-operators, as no flavour groups are present. Completing the Sfetsos-Thompson solution by a flavour group induces non-integrability.

All these CFTs have a sub-sector, made out of fields in the vector multiplet only, that is integrable. The holographic calculation knows this: it is choosing the wrapping $k=0$ in the trial solutions discussed above. In this case, these are all integrable solitons.

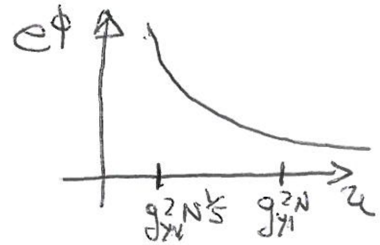
Let us end the presentation with an analogy that may serve for a better view of the Sfetsos-Thompson solution.

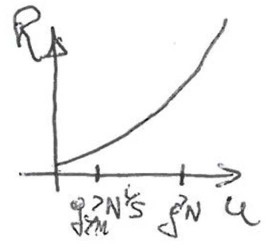
A loose analogy: D2 branes in ten and eleven dimensions

Consider the near horizon solution for D2 branes

$$\frac{ds^2}{\alpha'} = \frac{u^{5/2}}{\sqrt{g_{YM}^2 N}} dx_{1,2}^2 + \frac{\sqrt{g_{YM}^2 N}}{u^{5/2}} du^2 + \frac{\sqrt{g_{YM}^2 N}}{u^{1/2}} d\Omega_6,$$

$$e^{4\phi} = \frac{g_{YM}^{10} N}{u^5}, \qquad C_3 = \frac{u^5}{g_{YM}^2 N}.$$

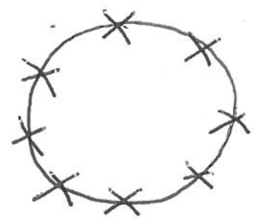




When going to low energies (small values of u), the dilaton grows and we should lift to eleven dimensions. This does *not* produce $AdS_4 \times S^7$! Instead, to get the correct Physics, we should use

$$ds_{11}^2 = H^{-2/3} dx_{1,2}^2 + H^{1/3}(dr^2 + r^2 d\Omega_6 + dx_{11}^2),$$

$$H = \sum_{m=-\infty}^{\infty} \frac{1}{(r^2 + (x_{11} + x_{11,0} + mR_{11})^2)^2}.$$



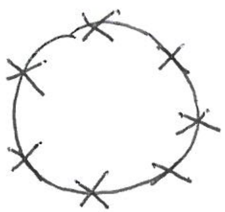
What about non-Abelian T-duality and the Sfetsos-Thompson solution?

Consider a generic Gaiotto-Maldacena potential $\dot{V}(\sigma,\eta)$

$$\dot{V}_{GM}(\sigma,\eta)=\sum_{k=1}^{\infty}c_k\sigma K_1(\frac{k\pi}{N_5}\sigma)\sin(\frac{k\pi}{N_5}\eta)\cong\int d\sigma\sigma\sum_{m=-\infty}^{\infty}\frac{1}{\sqrt{\sigma^2+(\eta-\eta_0-m)^2}}$$

If we go very close to $\sigma=0,\eta=0$ and keep only one term in the sum, we get $\dot{V}_{GM}=\dot{V}_{ST}$

Somewhat, the Sfetsos-Thompson solution is a zoom-in on the 'physically correct' solution.



Some conclusions

The features discussed here for 4d $\mathcal{N} = 2$ SCFTs: dual description in terms of a single function V and its derivatives. Writing of observable quantities in terms of V. Linearity of the PDE to determine V, etc.

Should repeat in the cases of SUSY CFTs in 2d, 3d, 5d, 6d. There should be 'core' solution, obtained by the application of non-Abelian T-duality on a given background. This core solution is integrable in some cases.

The systems can be thought in terms of D_p-D_{p+2}-NS_5 branes. There is always an AdS and an S^2, realising the $SO(2, p)$ and $SU(2)$ global symmetries.

It may be useful to study the taxonomy of these backgrounds and their deformations. There may be relations with integrable deformations.