Poisson-Lie duals of η-deformed superstrings

Ben Hoare

Based on `arXiv:1709.01448` and `arXiv:1807.04608` with FK Seibold

ETH Zurich

Motivation

Motivation

- **Integrability** in the AdS/CFT correspondence
- Type IIB superstring theory on $\mathrm{AdS}_5 \times \mathrm{S}^5$ is dual to $\mathcal{N} = 4$ super Yang-Mills
- In the planar limit the two theories are integrable
  - Spectrum of energies / Anomalous dimensions
  - Minimal surfaces / Scattering amplitudes and Wilson loops
  - Correlation functions and form factors

From 2002 onwards, ...

- Are there are other superstring theories for which integrability can be of use?

Motivation

Lower dimensional AdS backgrounds

$$\mathrm{AdS}_4 \times \mathbb{C}\mathbf{P}^3$$

$$\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{S}^3 \times \mathrm{S}^1$$

$$\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{T}^4$$

$$\mathrm{AdS}_2 \times \mathrm{S}^2 \times \mathrm{S}^2 \times \mathrm{T}^4$$

$$\mathrm{AdS}_2 \times \mathrm{S}^2 \times \mathrm{T}^6$$

- Number of supersymmetries decreases
- New features appear, for example,
  - massless modes,
  - extra parameters,
  - B field,
  - cubic terms,
  - long representations, ...

and many more...

Motivation

- **Deformed AdS backgrounds**
  - Deformations are a powerful tool in integrability
  - Rich algebraic structure, including twists and *q*-deformations
- Well-studied example: the TsT transformation
  - Combination of T dualities and coordinate shifts
  - Can be explicitly implemented in the string worldsheet action
  - Maps solutions of supergravity to solutions of supergravity
  - Corresponds to introducing non-commutative structures in the dual theory

Motivation

- **Deformed AdS backgrounds**
  - Deformations are a powerful tool in integrability
  - Rich algebraic structure, including twists and *q*-deformations
- More general deformations?
  - Natural to construct from a 2-d worldsheet perspective
  - How do deformations play with string theory?
  - Are the models scale and Weyl invariant?
  - What, if they exist, are the dual theories?

Yang-Baxter deformations

Yang-Baxter deformations

- Integrable deformations of the ...

  - *i)* principal chiral model (plus WZ term)
  - *ii)* symmetric space sigma model
  - *iii)* semi-symmetric space sigma model

These are all (classically) integrable sigma models of interest in the context of the AdS/CFT correspondence

  - and more...

Yang-Baxter deformations

Integrable deformations of the ...

- *i)* principal chiral model (plus WZ term)
- *ii)* symmetric space sigma model

Sigma models on AdS_n, S^n, $\mathbb{C}\mathbf{P}^n$, ...

Polyakov; Pohlmeyer;
Eichenherr, Forger; ...

- *iii)* semi-symmetric space sigma model

Green-Schwarz superstring on $\mathrm{AdS}_5 \times \mathrm{S}^5$, ...

Metsaev, Tseytlin; Henneaux, Mezincescu; Metsaev;
Berkovits, Bershadsky, Hauer, Zhukov, Zwiebach; ...
Bena, Roiban, Polchinski; ...

- and more...

Semi-symmetric space sigma model

- The semi-symmetric space sigma model for $\widehat{\mathrm{F}}/\mathrm{G}$
  - $\widehat{\mathrm{F}}$ is a supergroup, whose Lie superalgebra $\widehat{\mathfrak{f}}$ is basic and admits a $\mathbb{Z}_4$ grading

$$\widehat{\mathfrak{f}} = \mathfrak{f}_0 \oplus \mathfrak{f}_1 \oplus \mathfrak{f}_2 \oplus \mathfrak{f}_3 \qquad\qquad [\mathfrak{f}_i, \mathfrak{f}_j\} \subset \mathfrak{f}_{i+j} \quad \text{mod } 4$$

  - This should be such that $\mathfrak{f}_0$ and $\mathfrak{f}_2$ are even, while $\mathfrak{f}_1$ and $\mathfrak{f}_3$ are odd
  - $\mathfrak{f}_0$ forms a subalgebra that is identified with the Lie algebra of the group G
  - We introduce projectors P_i onto the spaces $\mathfrak{f}_i$
  - We also introduce the ad-invariant non-degenerate bilinear form on $\widehat{\mathfrak{f}}$, denoted STr

Semi-symmetric space sigma model

➡ Examples include

$$\frac{\mathrm{PSU}(2,2|4)}{\mathrm{Sp}(1,1)\times\mathrm{Sp}(2)} \longrightarrow \mathrm{AdS}_5\times S^5$$

$$\frac{\mathrm{OSp}(6|4)}{\mathrm{U}(3)\times\mathrm{SL}(2;\mathbb{C})} \longrightarrow \mathrm{AdS}_4\times\mathbb{C}\mathbf{P}^3$$

$$\left(\frac{\mathrm{PSU}(1,1|2)\times\mathrm{PSU}(1,1|2)}{\mathrm{SU}(1,1)\times\mathrm{SU}(2)}\right) \longrightarrow \mathrm{AdS}_3\times S^3(\times T^4)$$

$$\frac{\mathrm{PSU}(1,1|2)}{\mathrm{SO}(1,1)\times\mathrm{SO}(2)} \longrightarrow \mathrm{AdS}_2\times S^2(\times T^6)$$

Zarembo; Wulff; ...

Semi-symmetric space sigma model

- To write the action we take a supergroup-valued field, $f \in \widehat{\mathsf{F}}$, and construct the left-invariant Maurer-Cartan form, $J = f^{-1} df \in \widehat{\mathfrak{f}}$

- Introducing light-cone coordinates on the worldsheet, $\partial_\pm = \partial_0 \pm \partial_1$, the action in conformal gauge is

$$\mathcal{S} = T \int d^2x \, \mathrm{STr} \left[J_+ P J_- \right] \qquad\qquad P = P_2 + \tfrac{1}{2}(P_1 - P_3)$$

Metsaev, Tseytlin; Henneaux, Mezincescu; Metsaev;
Berkovits, Bershadsky, Hauer, Zhukov, Zwiebach; ...

Semi-symmetric space sigma model

$$\mathcal{S} = T \int d^2x \, \mathrm{STr}\left[J_+ P J_-\right] \qquad\qquad P = P_2 + \tfrac{1}{2}(P_1 - P_3)$$

- Global $\widehat{\mathrm{F}}$ symmetry, $f \to f_0 f$
- Local symmetries: G gauge symmetry and, on a curved worldsheet, diffeomorphisms, Weyl symmetry and fermionic κ-symmetry
- Classical integrability via existence of Lax connection

Bena, Roiban, Polchinski; ...

The Yang-Baxter deformed model

$$\mathcal{S} = T \int d^2x \, \mathrm{STr}\left[J_+ P J_-\right] \qquad P = P_2 + \tfrac{1}{2}(P_1 - P_3)$$

The Yang-Baxter deformation depends on a linear operator

$$R : \widehat{\mathfrak{f}} \to \widehat{\mathfrak{f}}$$

The Yang-Baxter deformed model

$$\mathcal{S} = T \int d^2x \, \mathrm{STr}\left[J_+ P J_-\right] \qquad P = P_2 + \tfrac{1}{2}(P_1 - P_3)$$

$$\mathcal{S} = T \int d^2x \, \mathrm{STr}\left[J_+ P_\eta \frac{1}{1 - \frac{2}{1-\sigma\eta^2} R_f P_\eta} J_-\right] \qquad \begin{aligned} P_\eta &= P_2 + \tfrac{1-\sigma\eta^2}{2}(P_1 - P_3) \\ R_f &= \mathrm{Ad}_f^{-1} \, R \, \mathrm{Ad}_f \end{aligned}$$

Klimčík;
Delduc, Magro, Vicedo;
Matsumoto, Yoshida;

The Yang-Baxter deformed model

$$\mathcal{S} = T\int d^2x\,\mathrm{STr}\left[J_+ P_\eta \frac{1}{1-\frac{2}{1-\sigma\eta^2}R_f P_\eta} J_-\right] \qquad \begin{aligned} P_\eta &= P_2 + \tfrac{1-\sigma\eta^2}{2}(P_1 - P_3) \\ R_f &= \mathrm{Ad}_f^{-1}\, R\, \mathrm{Ad}_f \end{aligned}$$

- Global $\widehat{\mathrm{F}}$ symmetry is broken: the residual symmetry depends on R
- Local symmetries: G gauge symmetry and, on a curved worldsheet, diffeomorphisms, Weyl symmetry and fermionic κ-symmetry
- What about classical integrability?

The Yang-Baxter deformed model

- The deformation preserves classical integrability when R is an antisymmetric solution to the (modified) classical Yang-Baxter equation

$$\mathrm{STr}[X(RY)] + \mathrm{STr}[(RX)Y] = 0 \qquad X, Y \in \mathfrak{f}$$
$$[RX, RY] - R([RX, Y] + [X, RY]) = \sigma\eta^2[X, Y]$$

- Restricting to real deformations we take $\eta \in \mathbb{R}$ and $\sigma \in \{-1, 0, 1\}$

- We take R to scale as η, $R \sim \eta$

- Taking $\eta \to 0$ we recover the undeformed model

The Yang-Baxter deformed model

- Three classes of YB deformations, characterised by $\sigma \in \{-1, 0, 1\}$ with deformation parameter η

- $\sigma \in \{-1, 1\}$ correspond to q-deformations of the symmetry algebra

  - *Inhomogeneous Yang-Baxter (iYB) deformations*

Delduc, Magro, Vicedo;
Vicedo;

- $\sigma = 0$ corresponds to twists of the symmetry algebra

  - *Homogeneous Yang-Baxter (hYB) deformations*

Matsumoto, Yoshida;
van Tongeren;
Vicedo;

The Yang-Baxter deformed model

- iYB deformation of $S^2 = \dfrac{SO(3)}{SO(2)}$

$$\boxed{ds^2 = \frac{1}{1+\eta^2 r^2}\Big[\frac{dr^2}{1-r^2} + (1-r^2)d\varphi^2\Big]}$$

- Deformation of the O(3) sigma model, known as the 2-d sausage model

Fateev, Onofri, Zamolodchikov;

The Yang-Baxter deformed model

- iYB deformation of $S^3 = \dfrac{SO(4)}{SO(3)}$

$$ds^2 = \frac{1}{1+\eta^2 r^2}\Big[\frac{dr^2}{1-r^2} + (1-r^2)d\varphi^2\Big] + d\phi^2$$

- Contained within Fateev's 2-parameter deformation of the O(4) sigma model

Fateev;

The Yang-Baxter deformed model

- The YB deformations define deformations of the Green-Schwarz string on $AdS_5 \times S^5,\ AdS_4 \times \mathbb{C}\mathbf{P}^3,\ AdS_3 \times S^3(\times T^4),\ AdS_2 \times S^2(\times T^6),\ \ldots$

- The local symmetries of the model are preserved, and hence also the counting of degrees of freedom

Are these superstring theories themselves?
Is Weyl invariance preserved?

- The trace of the structure constants of the *R*-bracket should vanish, i.e. the corresponding algebra is unimodular

$$\boxed{[X, Y]_R = [RX, Y] + [X, RY] \qquad \tilde{f}_{ab}{}^{b} = 0}$$

Borsato, Wulff;

Lessons from hYB deformations

- Can we say something more?

hYB deformation based on a classical r-matrix

$\Updownarrow$

Dualising wrt a certain centrally-extended subalgebra $\mathfrak{h}_{\mathrm{ext}}$

$\downarrow$ *integrate out the central extension*

Adding a total derivative and dualising wrt to the subalgebra $\mathfrak{h}$

BH, Tseytlin;
Borsato, Wulff;
BH, Thompson;

- Recent developments in the context of generalised geometry, generalised fluxes, T folds, etc.

Sakatani, Uehara, Yoshida;
Baguet, Magro, Samtleben;
Sakamoto, Sakatani, Yoshida;
Fernández-Melgarejo, Sakamoto, Sakatani, Yoshida;
Lüst, Osten; Sakamoto, Sakatani;

Lessons from hYB deformations

- The structure constants of the subalgebra $\mathfrak{h}$ are also given by the R-bracket

- For the duality transformation to preserve Weyl invariance the trace of these structure constants should vanish, i.e. the subalgebra $\mathfrak{h}$ is unimodular

$$[X, Y]_R = [RX, Y] + [X, RY] \qquad \tilde{f}_{ab}{}^{b} = 0$$

Álvarez, Álvarez-Gaumé, Lozano;
Elitzur, Giveon, Rabinovici, Schwimmer, Veneziano;

- This is the same condition as before, providing a path integral explanation for the Weyl anomaly of the hYB deformation when $\tilde{f}_{ab}{}^{b} \neq 0$

BH, Tseytlin;

iYB deformation of $\mathrm{AdS_2 \times S^2 \times T^6}$ and its Poisson-Lie duals

iYB deformation of $\mathrm{AdS}_2 \times \mathrm{S}^2 \times \mathrm{T}^6$

- Consider the iYB deformation of the $\mathrm{AdS}_2 \times \mathrm{S}^2 \times \mathrm{T}^6$ superstring with $\sigma = 1$

$$\mathcal{S} = T \int d^2x \, \mathrm{STr}\left[J_+ P_\eta \frac{1}{1 - \frac{2}{1-\eta^2} R_f P_\eta} J_-\right] \qquad P_\eta = P_2 + \frac{1-\eta^2}{2}(P_1 - P_3), \quad R_f = \mathrm{Ad}_f^{-1} R \, \mathrm{Ad}_f$$

$$\frac{\widehat{\mathrm{F}}}{\mathrm{G}} = \frac{\mathrm{PSU}(1,1|2)}{\mathrm{SO}(1,1) \times \mathrm{SO}(2)} \sim \mathrm{AdS}_2 \times \mathrm{S}^2 + 8 \text{ fermions}$$

- The torus and remaining fermions go along for the ride

iYB deformation of $AdS_2 \times S^2 \times T^6$

$$\mathcal{S} = T \int d^2x \, \mathrm{STr}\left[J_+ P_\eta \frac{1}{1 - \frac{2}{1-\eta^2} R_f P_\eta} J_-\right] \qquad \begin{aligned} P_\eta &= P_2 + \tfrac{1-\eta^2}{2}(P_1 - P_3) \\ R_f &= \mathrm{Ad}_f^{-1} \, R \, \mathrm{Ad}_f \end{aligned}$$

- Take R to be the standard Drinfel'd-Jimbo solution of the modified classical Yang-Baxter equation
  - The residual global symmetry algebra is the Cartan subalgebra of $\widehat{\mathfrak{f}}$
  - The Poisson brackets of the conserved charges now satisfy the relations of the q-deformed Poisson-Hopf algebra $\mathcal{U}_q(\widehat{\mathfrak{f}})$, associated to a Poisson-Lie symmetry

Delduc, Magro, Lacroix, Vicedo;
Klimčík;

iYB deformation of $AdS_2 \times S^2 \times T^6$

- The resulting deformation of the $AdS_2 \times S^2 \times T^6$ superstring has a Weyl anomaly

  Arutyunov, Borsato, Frolov; BH, Tseytlin;
  Arutyunov, Frolov, BH, Roiban, Tseytlin;
  Borsato, Wulff;

  - The background fields do not satisfy the type II supergravity equations

- There are (at least) three ways to reach a supergravity background by dualities:

  - via T dualities to a theory with a dilaton linear in the dual coordinates (shown indirectly)

    BH, Tseytlin;

  - via Poisson-Lie duality wrt the bosonic subalgebra (shown directly)

    Sfetsos; Hollowood, Miramontes, Schmidtt;
    Sfetsos, Thompson; Demulder, Sfetsos, Thompson;
    Borsato, Tseytlin, Wulff;

  - via Poisson-Lie duality wrt the full superalgebra (conjectured)

    Vicedo; BH, Tseytlin;
    Sfetsos, Siampos, Thompson;
    Klimčík; BH, Seibold;

iYB deformation of $\mathrm{AdS}_2 \times \mathrm{S}^2 \times \mathrm{T}^6$

Undeformed model	→	**Deformed model**
T dualities wrt the Cartan subalgebra of $\widehat{\mathfrak{f}}$		T dualities wrt the Cartan subalgebra of $\mathcal{U}_q(\widehat{\mathfrak{f}})$
Non-abelian duality wrt the bosonic subalgebra of $\widehat{\mathfrak{f}}$		Poisson-Lie duality wrt the bosonic “subalgebra” of $\mathcal{U}_q(\widehat{\mathfrak{f}})$
Non-abelian duality wrt the full superalgebra $\widehat{\mathfrak{f}}$		Poisson-Lie duality wrt the full “superalgebra” $\mathcal{U}_q(\widehat{\mathfrak{f}})$
T duality and non-abelian duality wrt other subalgebras of $\widehat{\mathfrak{f}}$		???

- Can these different dualisations be unified in a single framework?
- What are the possible dualities that give Weyl-invariant theories?

iYB deformations and Poisson-Lie duals

- Our starting point is the first-order action of Klimčík and Ševera on the Drinfel'd double

Klimčík, Ševera; Sfetsos; Squellari;
Tseytlin; Giveon, Roček,
Hull; Hull, Reid-Edwards;

- For the iYB deformation with $\sigma = 1$ the relevant "doubled" space is the complexification of the Lie algebra

$$\boxed{\widehat{\mathfrak{f}}^{\mathbb{C}} = \mathfrak{psl}(2|2;\mathbb{C})}$$

Vicedo; Klimčík;
Klimčík, Ševera; Sfetsos;

iYB deformations and Poisson-Lie duals

- Our starting point is the first-order action of Klimčík and Ševera on the Drinfel'd double

Klimčík, Ševera; Sfetsos; Squellari;

$$\mathcal{S} = \frac{1}{2}\int d\tau d\sigma \Big[\langle l^{-1}\partial_\sigma l, l^{-1}\partial_\tau l\rangle - \mathcal{H}(l^{-1}\partial_\sigma l)\Big] + \mathrm{WZ}(l) \qquad l \in \widehat{\mathrm{F}}^{\mathbb{C}}$$

- The non-degenerate ad-invariant bilinear form on $\widehat{\mathfrak{f}}^{\mathbb{C}}$ is $\langle X, Y\rangle = -\operatorname{Im}\operatorname{STr}[XY]$
- $\mathrm{WZ}(l)$ is the standard Wess-Zumino term
- $\mathcal{H}$ is a bilinear form that characterises the model: we consider a specific choice that depends on the operator R and ensures G gauge symmetry

iYB deformations and Poisson-Lie duals

- Given a maximally isotropic subalgebra $\tilde{\mathfrak{k}} \subset \widehat{\mathfrak{f}}^{\mathbb{C}}$, $\langle \tilde{\mathfrak{k}}, \tilde{\mathfrak{k}} \rangle = 0$, $\dim \tilde{\mathfrak{k}} = \dim \widehat{\mathfrak{f}}$, we parametrise

$$\boxed{l = \tilde{k} k \qquad \tilde{k} \in \tilde{\mathrm{K}}, \; k \in \tilde{\mathrm{K}} \backslash \widehat{\mathrm{F}}^{\mathbb{C}} / \mathrm{G}}$$

- The first-order action then only depends on $\tilde{k}$ through $p = \tilde{k}^{-1} \partial_\sigma \tilde{k}$

- Integrating out p gives a second-order Lorentz-invariant model on

$$\boxed{\tilde{\mathrm{K}} \backslash \widehat{\mathrm{F}}^{\mathbb{C}} / \mathrm{G}}$$

Klimčík;
Klimčík, Ševera;

iYB deformations and Poisson-Lie duals

- We denote by $\widehat{\mathfrak{b}}$ the particular Borel subalgebra picked out by the operator R, whose structure constants are given by the R-bracket

- We have the following vector space decomposition

$$\widehat{\mathfrak{f}}^{\mathbb{C}} = \widehat{\mathfrak{f}} \oplus \widehat{\mathfrak{b}} \qquad \begin{aligned} \widehat{\mathfrak{f}} &= \{ih_i, i(f_m + \# e_m), (f_m - \# e_m)\} \\ \widehat{\mathfrak{b}} &= \{h_i, e_m, ie_m\} \end{aligned}$$

- The two real subalgebras are isotropic wrt the ad-invariant bilinear form

$$\langle \widehat{\mathfrak{f}}, \widehat{\mathfrak{f}} \rangle = \langle \widehat{\mathfrak{b}}, \widehat{\mathfrak{b}} \rangle = 0$$

Vicedo; Klimčík;
Klimčík, Ševera; Sfetsos;

iYB deformations and Poisson-Lie duals

- Take a subalgebra $\mathfrak{h}$ of $\widehat{\mathfrak{f}}$ associated to a sub-Dynkin diagram, possibly plus additional Cartan generators
  - In this case $\mathcal{U}_q(\mathfrak{h})$ is a sub-Hopf algebra of $\mathcal{U}_q(\widehat{\mathfrak{f}})$
- There exists $\tilde{\mathfrak{m}} \subset \widehat{\mathfrak{b}}$ such that $\tilde{\mathfrak{k}} = \mathfrak{h} \oplus \tilde{\mathfrak{m}}$ is a maximally isotropic subalgebra
- We can Poisson-Lie dualise wrt $\mathcal{U}_q(\mathfrak{h})$
  - Adding in additional Cartan generators corresponds to additional T dualities

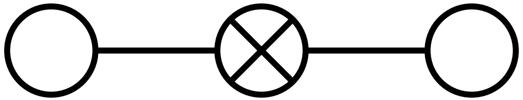
BH, Seibold;

- These are certain explicit elements of the general Poisson-Lie duality group

Osten, Lüst;

Poisson-Lie duals of iYB deformed $\mathrm{AdS}_2 \times \mathrm{S}^2 \times \mathrm{T}^6$

- We use the following Dynkin diagram of $\mathrm{PSU}(1,1|2)$



- The deformation parameter $\varkappa$ is related to η by

$$\boxed{\varkappa = \frac{2\eta}{1-\eta^2}}$$

Arutyunov, Borsato, Frolov;

Poisson-Lie duals of iYB deformed $AdS_2 \times S^2 \times T^6$

$\mathfrak{h} = \emptyset$

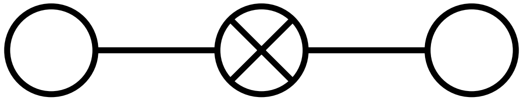
Case 1: The iYB deformation

Klimčík; Vicedo;

- The algebra $\tilde{\mathfrak{k}} = \widehat{\mathfrak{b}}$ is not unimodular
- The deformed background has the 3-form and 5-form R-R fluxes turned on, but does not solve the type II supergravity equations

BH, Tseytlin;
Arutyunov, Frolov, BH, Roiban; Tseytlin;
Araujo, Ó Colgáin, Yavartanoo;

Poisson-Lie duals of iYB deformed $AdS_2 \times S^2 \times T^6$



$\mathfrak{h} = \widehat{\mathfrak{f}}$

Case 2: Poisson-Lie dual wrt the full “superalgebra”

- The algebra $\tilde{\mathfrak{k}} = \widehat{\mathfrak{f}}$ is unimodular
- Conjectured to give an analytic continuation of the λ-deformation of $AdS_2 \times S^2 \times T^6$

Sfetsos; Hollowood, Miramontes, Schmidtt;

Vicedo; BH, Tseytlin;
Sfetsos, Siampos, Thompson;
Klimčík; BH, Seibold;

- Assuming this conjecture is correct, the deformed background solves the type II / II* supergravity equations

Borsato, Tseytlin, Wulff;

Poisson-Lie duals of iYB deformed $\mathrm{AdS}_2 \times \mathrm{S}^2 \times \mathrm{T}^6$

$\mathfrak{h} = \mathfrak{f}_0 \oplus \mathfrak{f}_2$

Case 3: Poisson-Lie dual wrt the bosonic “subalgebra”

- The algebra $\tilde{\mathfrak{k}}$ is unimodular
- The deformed background solves the type II / II* supergravity equations

BH, Seibold;
Sfetsos, Thompson;

Poisson-Lie duals of iYB deformed $AdS_2 \times S^2 \times T^6$

Case 3: Poisson-Lie dual wrt the bosonic "subalgebra"

$$ds^2 = \eta_{ab} e^a e^b + dx_i dx_i \qquad \Phi = \text{const} - \tfrac{1}{2}\log(y^2+z^2-1)(p^2+q^2-1)$$

$$e^\Phi F_5 = i\sqrt{1+\varkappa^2}(1+\star)e^2 \wedge e^1 \wedge \mathrm{Re}\,\Omega$$

$$e^0 = \frac{\varkappa^{-1}dy}{\sqrt{y^2+z^2-1}} \qquad e^1 = \frac{dz}{\sqrt{y^2+z^2-1}} \qquad e^2 = \frac{\varkappa^{-1}dp}{\sqrt{p^2-q^2-1}} \qquad e^3 = \frac{dq}{\sqrt{p^2-q^2-1}}$$

BH, Seibold;

- Ω is the holomorphic 3-form on T^6
- The imaginary R-R flux is a consequence of T duality in a timelike direction
- Solution of type II* supergravity equations

Hull; ...

Poisson-Lie duals of iYB deformed $AdS_2 \times S^2 \times T^6$

Case 3: Poisson-Lie dual wrt the bosonic “subalgebra”

$$ds^2 = \eta_{ab} e^a e^b + dx_i dx_i \qquad \Phi = \text{const} - \tfrac{1}{2}\log(y^2+z^2-1)(p^2+q^2-1)$$

$$e^{\Phi} F_5 = i\sqrt{1+\varkappa^2}(1+\star)e^2 \wedge e^1 \wedge \mathrm{Re}\,\Omega$$

$$e^0 = \frac{\varkappa^{-1} dy}{\sqrt{y^2+z^2-1}} \qquad e^1 = \frac{dz}{\sqrt{y^2+z^2-1}} \qquad e^2 = \frac{\varkappa^{-1} dp}{\sqrt{p^2-q^2-1}} \qquad e^3 = \frac{dq}{\sqrt{p^2-q^2-1}}$$

BH, Seibold;

- Related by analytic continuation to alternative known and simpler embedding of the bosonic λ-deformation of $AdS_2 \times S^2 \times T^6$ into type II / II* supergravity

Sfetsos, Thompson;

- Proves the integrability of the superstring on this background

Ševera;

- Mechanism giving different R-R fluxes and dilaton supporting the same metric and B field

Poisson-Lie duals of iYB deformed $\mathrm{AdS}_2 \times \mathrm{S}^2 \times \mathrm{T}^6$

Case 4: T dual wrt the Cartan subalgebra $\mathfrak{h} = \{ih_i\}$

BH, Seibold;

- The algebra $\tilde{\mathfrak{k}}$ is unimodular
- The deformed background solves the type II / II* supergravity equations
- Can be recovered as a scaling limit of cases 2 and 3:

$$\{y, z, p, q\} \to \gamma\{y, z, p, q\} \qquad \gamma \to \infty$$

$$y = e^{\varkappa \hat{t}} \qquad z = \rho e^{\varkappa \hat{t}} \qquad p = e^{\varkappa \hat{\varphi}} \qquad q = r e^{\varkappa \hat{\varphi}}$$

BH, Tseytlin;
Borsato, Wulff, Tseytlin;

Poisson-Lie duals of iYB deformed $AdS_2 \times S^2 \times T^6$

Case 4: T dual wrt the Cartan subalgebra

$$ds^2 = \eta_{ab} e^a e^b + dx_i dx_i \qquad \Phi = \text{const} - \varkappa(\hat{t} + \hat{\varphi}) - \frac{1}{2}\log(1+\rho^2)(1-r^2)$$

$$e^\Phi F_5 = i\sqrt{1+\varkappa^2}(1+\star) e^2 \wedge e^1 \wedge \text{Re}\,\Omega$$

$$e^0 = \frac{d\hat{t}}{\sqrt{1+\rho^2}} \quad e^1 = \frac{d\rho + \varkappa\rho d\hat{t}}{\sqrt{1+\rho^2}} \quad e^2 = \frac{d\hat{\varphi}}{\sqrt{1-r^2}} \quad e^3 = \frac{dr + \varkappa r d\hat{\varphi}}{\sqrt{1-r^2}}$$

BH, Tseytlin;

- The imaginary R-R flux is a consequence of T duality in a timelike direction

Hull; ...

- Solution of type II* supergravity equations
- In the $\varkappa \to 0$ limit we recover the 2-fold T dual of the $AdS_2 \times S^2 \times T^6$ supergravity background

Bertotti, Robinson;

Concluding remarks

Concluding remarks

- For these four cases the vanishing of the trace of the structure constants corresponds to the background being a solution of type II / II* supergravity

- Weyl anomaly associated to integrating out the degrees of freedom associated to a non-unimodular algebra

Tyurin, von Unge;
Bossard, Mohammedi;
Von Unge; Hlavatý, Šnobl;
Valent, Klimčík, Squellari;
Hassler; Jurčo, Vysoký; ...

Concluding remarks

- Remains to explain the Weyl anomaly of the iYB deformation in the path integral (as for hYB deformations based on non-unimodular *r*-matrices)

  - How do we implement PL duality starting from the iYB deformation, or its duals, in a systematic way?

  - How do we justify starting from the first-order action on the Drinfel'd double and demonstrate its Weyl invariance?

Thank you!

The Yang-Baxter deformed model

- Start from classical antisymmetric r-matrix that solves the (modified) classical Yang-Baxter equation

$$r = r^{AB} T_A \wedge T_B \in \widehat{\mathfrak{f}} \otimes \widehat{\mathfrak{f}}$$

$$[r_{12}, r_{13}] + [r_{12}, r_{23}] + [r_{13}, r_{23}] = \sigma \eta^2 \Omega$$

  - Ω is the canonical invariant element of $\widehat{\mathfrak{f}} \wedge \widehat{\mathfrak{f}} \wedge \widehat{\mathfrak{f}}$

- Construct the associated solution of the operator form of the (modified) classical Yang-Baxter equation

$$RX \equiv \mathrm{STr}_2[r(1 \otimes X)]$$

Examples

- iYB deformation of $S^2 = \dfrac{SO(3)}{SO(2)}$

$$\boxed{r = i\eta\, e \wedge f}$$

$$\boxed{ds^2 = \frac{1}{1+\eta^2 r^2}\Big[\frac{dr^2}{1-r^2} + (1-r^2)d\varphi^2\Big]}$$

- Deformation of the O(3) sigma model, known as the 2-d sausage model

Fateev, Onofri, Zamolodchikov;

Examples

- iYB deformation of $S^3 = \frac{SO(4)}{SO(3)}$

$$r = i\eta\,(e_1 \wedge f_1 + e_2 \wedge f_2)$$

$$ds^2 = \frac{1}{1+\eta^2 r^2}\Big[\frac{dr^2}{1-r^2} + (1-r^2)d\varphi^2\Big] + d\phi^2$$

- Contained within Fateev's 2-parameter deformation of the O(4) sigma model

Fateev;

Examples

- A hYB deformation of $S^3 = \dfrac{SO(4)}{SO(3)}$

$$\boxed{r = \tfrac{1}{2}\eta\, h_1 \wedge h_2}$$

$$ds^2 = \frac{(1-r^2)d\varphi^2 + r^2 d\phi^2}{1+\eta^2 r^2(1-r^2)} + \frac{dr^2}{1-r^2} \qquad B = \frac{\eta r^2(1-r^2)}{1+\eta^2 r^2(1-r^2)} d\varphi \wedge d\phi$$

- TsT transformation of the original model

Giveon, Roček; Klimčík, Tseytlin;
Kiritsis, Kounnas; Tseytlin;
Russo, Tseytlin; ...

Matsumoto, Yoshida, et al.;
Osten, van Tongeren;

Lessons from hYB deformations

- Models that come from dualising wrt non-unimodular $\mathfrak{h}$ are related by T duality to Weyl-invariant theories with a dilaton linear in the dual coordinate

BH, Tseytlin;

- This implies that these hYB deformed backgrounds solve a generalisation of the type II supergravity equations

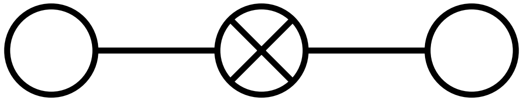
Arutyunov, Frolov, BH, Roiban, Tseytlin;
Wulff, Tseytlin;

  - Caveat: there are exceptions to these general statements when $n_a = \tilde{f}_{ab}{}^b$ is associated to a null direction

Wulff;

Poisson-Lie duals of iYB deformed $AdS_2 \times S^2 \times T^6$

- We use the following Dynkin diagram of $PSU(1,1|2)$



- $AdS_2 \times S^2 \times T^6$ supergravity background in global coordinates

$$ds^2 = -(1+\rho^2)dt^2 + \frac{d\rho^2}{1+\rho^2} + (1-r^2)d\varphi^2 + \frac{dr^2}{1-r^2} + dx_i dx_i$$

$$e^{\Phi} F_5 = (1+\star)(dt \wedge d\rho \wedge \mathrm{Re}\,\Omega) \qquad \Phi = \text{const}$$

  - Ω is the holomorphic 3-form on T^6

Bertotti, Robinson;

- The deformation parameter $\varkappa$ is related to η by $\varkappa = \dfrac{2\eta}{1-\eta^2}$

Arutyunov, Borsato, Frolov;

Poisson-Lie duals of iYB deformed $AdS_2 \times S^2 \times T^6$

- Poisson-Lie dual wrt the bosonic "subalgebra"

$$ds^2 = \eta_{ab} e^a e^b + dx_i dx_i \qquad \Phi = \text{const} - \tfrac{1}{2}\log(y^2+z^2-1)(p^2+q^2-1)$$

$$e^\Phi F_5 = i\sqrt{1+\varkappa^2}(1+\star)e^2 \wedge e^1 \wedge \mathrm{Re}\,\Omega$$

$$e^0 = \frac{\varkappa^{-1}dy}{\sqrt{y^2+z^2-1}} \quad e^1 = \frac{dz}{\sqrt{y^2+z^2-1}} \quad e^2 = \frac{\varkappa^{-1}dp}{\sqrt{p^2-q^2-1}} \quad e^3 = \frac{dq}{\sqrt{p^2-q^2-1}}$$

BH, Seibold;

- In the $\varkappa \to 0$ limit we recover the non-abelian dual of the $AdS_2 \times S^2 \times T^6$ supergravity background wrt the bosonic "subalgebra"

$$y \to 1 + \varkappa^2 y \quad z \to \varkappa z$$
$$p \to 1 + \varkappa^2 p \quad q \to \varkappa q$$

Sfetsos; Sfetsos, Thompson;