Exotic Branes in Extended Field Theories

Ray Otsuki

Queen Mary University of London

Dualities and Generalised Geometries
14th September 2018
Corfu, Greece

[1806.00430] David S. Berman, Edvard T. Musaev, RO
[1710.09740] Ilya Bakhmatov, David S. Berman, Axel Kleinschmidt, Edvard T. Musaev, RO

Outline

- Exotic Branes
  - In string theory
  - In Extended Field Theories (ExFT)
- ExFT = {DFT, EFT}
  - Novel non-geometric solution in $E_{7(7)} \times \mathbb{R}^+$ EFT
- Mapping out all exotic states
- Speculation + Further Work

Exotic Branes in String Theory I

- Exotic Branes: Objects with non-trivial $G(\mathbb{Z})$-monodromy [Obers, Pioline '98], [Hull '97]
- Include T-/U-folds [Hull '04]
  - **Globally** non-geometric: patched by duality transformations & fields not single-valued
- Also includes **locally** non-geometric objects [Bakhmatov, Kleinschmidt, Musaev '16]
  - Fields have dependence on coordinates not in the usual spacetime i.e. not even locally well-defined
- Obtainable from `supertube effect' (spontaneous polarisation) of regular brane configurations, puffing up into transverse space [de Boer, Shigemori '12]

Exotic Branes in String Theory II

- Low codimension $\tilde{d}$ objects destroy spacetime asymptotics
  - $\tilde{d} = 2:$ Defect Brane $H \sim c_2 + \sigma \ln|\mu/r|$
  - $\tilde{d} = 1:$ Domain Wall $H \sim c_1 - |x|$
  - $\tilde{d} = 0:$ Space-filling Brane $H \sim c_0$
- Non-standard mass/tension scaling g_s^α :
  - $\alpha = 0$: Fundamental Branes e.g. F1, P
  - $\alpha = -1:$ D-Branes e.g. D1, D4
  - $\alpha = -2:$ Solitonic Branes e.g. NS5, KK5
  - $\alpha \leq -3:$ **Exotic Branes** e.g. $7_3, 9_4$
- $\tilde{d} = 2$ states:
  - $\mathrm{M}:\ 5^3,\ 2^6,\ 0^{(1,7)},$ $\mathrm{II}:\ 5^2_2;\ p_3^{7-p},\ 1^6_4,\ 0_4^{(1,6)}$

Exotic Branes in String Theory III

- Notation for (exotic) branes:

$$b_n^{(\dots,d,c)}$$

b : Mass depends linearly on b radii
c : Mass depends quadratically on c radii
d : Mass depends cubically on d radii
n : Inverse g_s scaling; $n = -\alpha; n \geq 0$

- Coupling to mixed-symmetry potentials:
  - Each set of indices contained in all sets to the left

$$b_n^{(\dots,d,c)} \sim E_{1+b+c+d+\dots,c+d+\dots,d+\dots,\dots}$$

[Kleinschmidt '11], [Bergshoeff, Hohm, Penas, Riccioni '16], [Bergshoeff, Riccioni '17], [Lombardo, Riccioni, Risoli '17 + '18] ... etc.

$$\text{NS5} \to \text{KK5} \to \text{Q-brane} \to \text{R-brane}(\to 5_2^4)$$
$$5_2 \to 5_2^1 \to 5_2^2 \to 5_2^3(\to 5_2^4)$$

- Prototypical 5-brane chain, coupling to $\{H, f, Q, R\}$

$$\mathrm{NS5} \to \mathrm{KK5} \to \text{Q-brane} \to \text{R-brane}(\to 5_2^4)$$
$$5_2 \to 5_2^1 \to 5_2^2 \to 5_2^3(\to 5_2^4)$$

- Prototypical 5-brane chain, coupling to $\{H, f, Q, R\}$

E.g. 5_2^2 has two special isometric directions (r, θ) and is $\tilde{d} = 2$ in $d = 10$ [de Boer, Shigemori '12], [Kimura, Sasaki '13]

$$5_2^2 : \begin{cases} \mathrm{d}s^2 = \mathrm{d}x_{012345}^2 + H(dr^2 + r^2\mathrm{d}\theta^2) + HK^{-1}\mathrm{d}x_{89}^2 \\ e^{2\phi} = HK^{-1}, \qquad B_{(2)} = K^{-1}\theta\sigma\mathrm{d}x^8 \wedge \mathrm{d}x^9 \end{cases}$$

- More generally, 5_2^q are $\tilde{d} = 4 - q$; increasingly problematic to describe in string/M-theory
  - Better description in ExFT?

$$\mathrm{NS5} \to \mathrm{KK5} \to \text{Q-brane} \to \text{R-brane} \to 5_2^4$$
$$5_2 \to 5_2^1 \to 5_2^2 \to 5_2^3 \to 5_2^4$$

1) Unlike dualities in $G(\mathbb{Z})$, **solution-generating transforms** in $G(\mathbb{R})$ do not require an isometry

2) Rotate transverse coords into extended space

3) $\tilde{d} = 0$ states: entire transverse space rotated out of section

4) Such states **require** ExFT to be described

- GLSM gives interpretation:

[Tong '02], [Harvey, Jensen '05], [Kimura, Sasaki '14]

Winding-mode dependence
$$\equiv$$
String worldsheet instanton effects

Examples of Solutions in ExFT

- Solutions:
  - 5_2^q-brane chain [Berman, Rudolph '14], [Bakhmatov, Kleinschmidt, Musaev '16], [Kimura, Sasaki, Shiozawa '18]
  - Wave-String [Berkeley, Berman, Rudolph '14]
  - Self-dual (Geometric) solution [Berman, Rudolph '14]
  - KK6 - $6^{(3,1)}$ [Bakhmatov, Berman, Kleinschmidt, Musaev, RO '17]
- Actions:
  - D-Brane [Albertsson, Dai, Wao, Lin '11]
  - Doubled worldsheet action [Blair '16]
  - 5-Brane DBI [Blair, Musaev '18]

... etc.

$E_{7(7)} \times \mathbb{R}^+$ Exceptional Field Theory

- Representations: $\rho_1 = \mathbf{56}, \quad \rho_2 = \mathbf{133}$
- Fields:
$$\{g_{\mu\nu}, \mathcal{M}_{MN}, \mathcal{A}_\mu{}^M, \mathcal{B}_{\mu\nu\alpha}, \mathcal{B}_{\mu\nu}{}^M\},$$
$$\mathcal{F}_{\mu\nu}{}^M = \mathcal{F}_{\mu\nu}{}^M(\mathcal{A}_\mu{}^M, \mathcal{B}_{\mu\nu\bullet})$$
- Section condition $\quad Y^{MN}{}_{PQ}\partial_M \bullet \partial_N \bullet = 0:$
  - M : $\quad E_{7(7)} \to \mathrm{GL}(7)$
  - IIB : $\quad E_{7(7)} \to \mathrm{GL}(6) \times \mathrm{SL}(2)$
- Equations:
  - E.O.M. for $\mathcal{M}_{MN}$
  - Bianchi and Self-duality for $\mathcal{F}_{\mu\nu}{}^M$

Non-geometric Sol$^{\mathrm{n}}$ in $E_{7(7)} \times \mathbb{R}^+$ EFT

- Non-geometric counterpart to [Berman, Rudolph '14]
- Coordinates:
  - External: $x^\mu = (t, r, \theta, z)$
  - Internal: e.g. M section $Y^M = (y^m, y_{mn}, y^{mn}, y_m)$
- Ansatz:

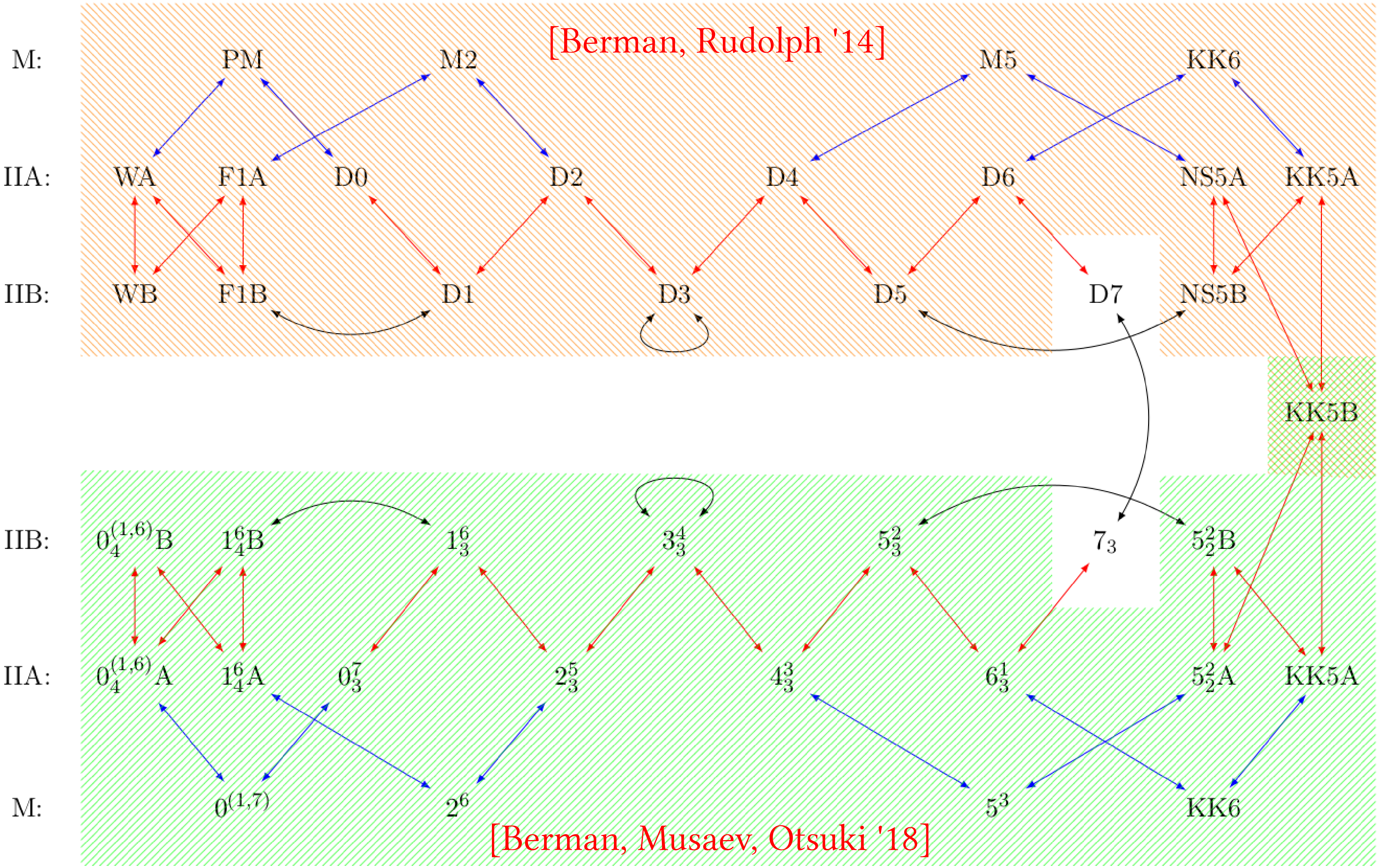
$$\mathcal{M}_{MN} \sim \mathrm{diag}\left[H^{\pm\frac{1}{2}}\delta_{(27)}, H^{\pm\frac{3}{2}}\right]$$

$$g_{\mu\nu} = \mathrm{diag}\left[-(HK^{-1})^{-\frac{1}{2}}, (HK)^{\frac{1}{2}}, r^2(HK)^{\frac{1}{2}}, (HK^{-1})^{\frac{1}{2}}\right]$$

$$H(r) = h_0 + \ln|\mu/r|, \qquad K = H^2 + \sigma^2\theta^2$$

$$\mathcal{A}_\mu{}^M = \{-H^{-1}K, -K^{-1}\theta\sigma\}, \quad \mathcal{B}_{\mu\nu,\bullet} = 0 \Rightarrow \mathcal{F}_{\mu\nu}{}^M = 2\partial_{[\mu}\mathcal{A}_{\nu]}{}^M$$

Non-geometric Sol$^{\rm n}$ in $E_{7(7)} \times \mathbb{R}^+$ EFT



Transformations

- Apply the following transformations to generate all branes:

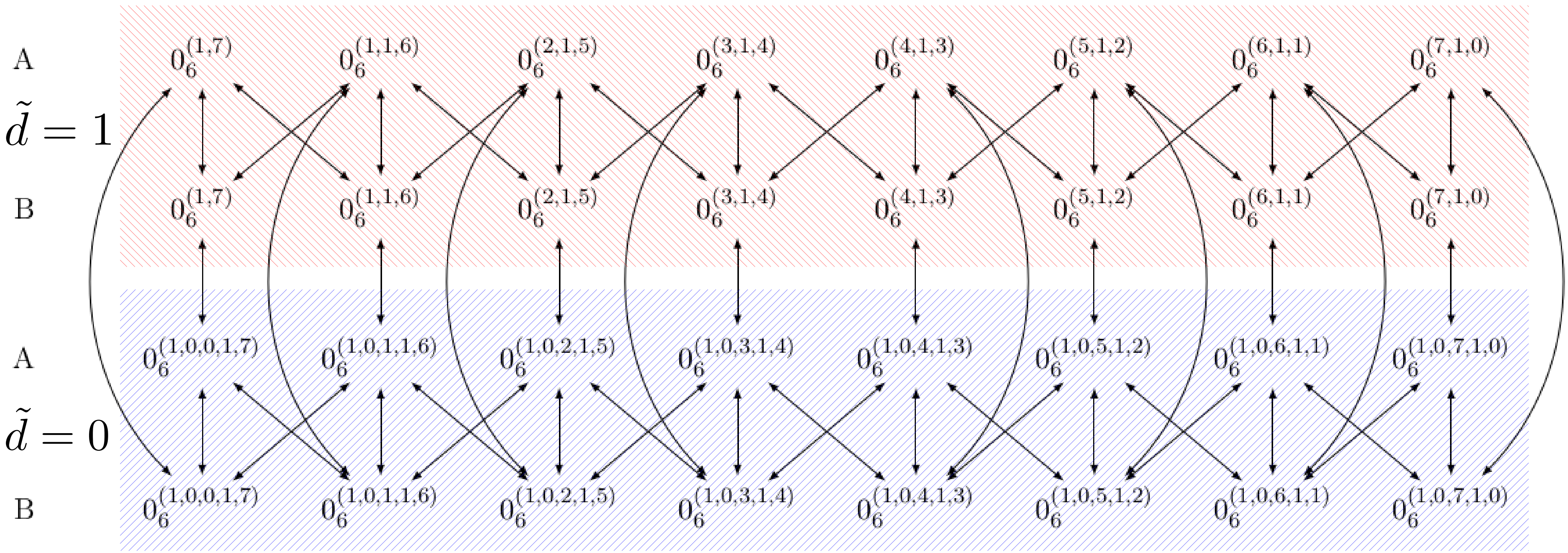
$$
\begin{aligned}
L/R: &\quad l_p = g_s^{\frac{1}{3}} l_s, \qquad R_\natural = l_s g_s \\
S: &\quad g_s \mapsto \frac{1}{g_s}, \qquad l_s \mapsto g_s^{\frac{1}{2}} l_s \\
T_y: &\quad g_s \mapsto \frac{l_s}{R_y} g_s, \quad R_y \mapsto \frac{l_s^2}{R_y}
\end{aligned}
$$

- Apply to mass formulae:

$$
\mathrm{M}: \quad M\left[b^{(\dots,d,c)}\right] \quad = \frac{\dots(R_{k_1}\dots R_{k_d})^3(R_{j_1}\dots R_{j_c})^2(R_{i_1}\dots R_{i_b})}{l_p^{1+b+2c+3d+\dots}}
$$

$$
\mathrm{II}: \quad M\left[b_n^{(\dots,d,c)}\right] \quad = \frac{\dots(R_{k_1}\dots R_{k_d})^3(R_{j_1}\dots R_{j_c})^2(R_{i_1}\dots R_{i_b})}{g_s^n l_s^{1+b+2c+3d+\dots}}
$$

T-duality Orbit e.g. @ g_s^{-6}



S-duals

$$
\begin{aligned}
&- 0_6^{(1,7)}\mathrm{B} &\leftrightarrow\ & 0_3^{(1,7)}\mathrm{B}\\
&- 0_6^{(1,1,6)}\mathrm{B} &\leftrightarrow\ & 0_4^{(1,1,6)}\mathrm{B}\\
&- 0_6(2,1,5)\mathrm{B} &\leftrightarrow\ & 0_5^{(2,1,5)}\mathrm{B}\\
&- 0_6^{(3,1,4)}\mathrm{B} &\leftrightarrow\ & 0_6^{(3,1,4)}\mathrm{B}\\
&\vdots
\end{aligned}
$$

M-theory Parent

$$
\begin{aligned}
&- 0_6^{(1,7)}\mathrm{A} &\rightarrow\ & 0^{(1,7)}\mathrm{M}\\
&- 0_6^{(1,1,6)}\mathrm{A} &\rightarrow\ & 1^{(1,1,6)}\mathrm{M}\\
&- 0_6^{(2,1,5)}\mathrm{A} &\rightarrow\ & 0^{(2,1,6)}\mathrm{M}\\
&- 0_6^{(3,1,4)}\mathrm{A} &\rightarrow\ & 0^{(3,2,4)}\mathrm{M}\\
&\vdots
\end{aligned}
$$

Potentials for $\alpha \geq -7$

α	IIA	IIB
0	B_2	
-1	C_{2n+1}	C_{2n}
-2	$D_{6+n,n}$	
-3	$E_{8+n,2m+1,n}$	$E_{8+n,2m,n}$
-4	$F_{8+n,6+m,m,n}$	
	$F_{9+n,3+m,m,n}$	
	$F_{10,2n+1,2n+1}$	$F_{10,2n,2n}$
-5	$G_{9+p,6+n,2m,n,p}$	$G_{9+p,6+n,2m+1,n,p}$
	$G_{10,4+n,2m+1,n}$	$G_{10,4+n,2m,n}$

E.g. B_2 : NS-NS 2-form
C_p : R-R forms
D_6 : NS-NS 6-form

Many orbits have, at least partially, appeared in the literature

Not included in the analysis of [Bergshoeff, Riccioni '17]
Not included in the analysis of [Fernandez-Melgarejo, Kimura, Sakatani '18]

α	IIA	IIB
-6	$H_{9+n,8+n,m+n,m+n-1,n,n}$	
	$H_{10,6+n,2+m,m,n}$	
	$H_{9+p,7+n,4+m,m,n,p}$	
-7	$I_{10,8+p,n+2,2m+1,n,p}$	$I_{10,8+p,2+n,2m,n,p}$
	$I_{9+p,8+p,5+n+p,2m+1,n+p,p,p}$	$I_{9+p,8+p,5+n+p,2m,n+p,p,p}$
	$I_{9+p,p+n+7,p+n+7,2m,n+p,n+p,p}$	$I_{9+p,p+n+7,p+n+7,2m+1,n+p,n+p,p}$
	$I_{10,7+p,4+n+p,2m,n+p,p}$	$I_{10,7+p,4+n+p,2m+1,n+p,p}$
	$I_{10,6+n,6+n,2m+1,n,n}$	$I_{10,6+n,6+n,2m,n,n}$

What Else To Expect? (New)

α	Number of Branes $N_{(\alpha)}$	Type	Breakdown $N_{(\alpha)} = A + B + 2C$									NS-NS Violation $A + B$
0	4	(NS-NS)	4	$=$	0	$+$	0	$+$	2	$\times$	2	0
-1	10	R-R	10	$=$	5	$+$	5					
-2	10	NS-NS	10	$=$	0	$+$	0	$+$	2	$\times$	5	
-3	24	R-R	24	$=$	12	$+$	12					
-4	46	NS-NS$_\delta$	46	$=$	5	$+$	5	$+$	2	$\times$	18	10
$\vdots$	$\vdots$	$\vdots$					$\vdots$					$\vdots$
-23	25320	R-R	25320	$=$	12660	$+$	12660					
-24	30374	NS-NS$_\delta$	30374	$=$	471	$+$	471	$+$	2	$\times$	14716	942
-25	38310	R-R	38310	$=$	19155	$+$	19155					
M	458124											

$A :$ # branes in IIA

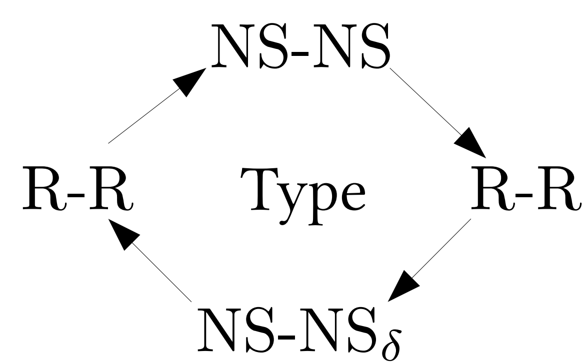
$B :$ # branes in IIB

$C :$ # branes in both IIA and IIB

$g_s^{-(4p+2)}$

$g_s^{-(2p+1)}$

g_s^{-4p}



Remarks

- Agreement with other works:
  - $E_{8(8)}$ multiplets [Fernandez-Melgarejo, Kimura, Sakatani '18]
  - E_{11} scheme [Kleinschmidt '11]
  - Exotic Potentials [Bergshoeff, Penas, Riccioni, Risoli '15] + related
- Unification of more and more states at larger $E_{n(n)}$?
  - One single orbit
  - One unique solution in E_{11}, unifying all states?
- Further work:
  - D-instanton? Tied to time-like transformations
  - Explanation of patterns amongst branes

Conclusions

- Novel solution in $E_{7(7)} \times \mathbb{R}^+$ EFT covering $\tilde{d} = 2$ exotic states
- Mapped out all duality orbits to g_s^{-7} and listed down to g_s^{-25}

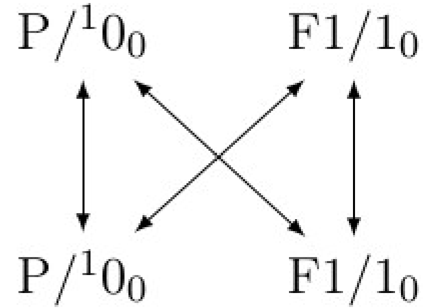
ExFT:
Solution-generating transformations can act on non-isometric directions

GLSM:
Dependence on dual coordinates $\equiv$ string worldsheet instanton corrections

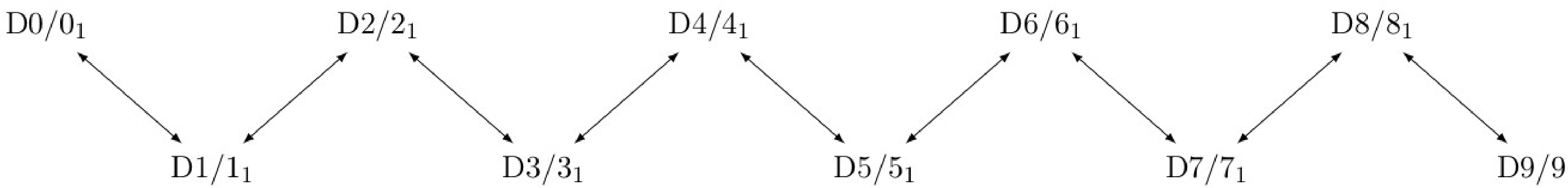
Novel $\tilde{d} = 0$ solutions, which depending only on extended coordinates, **require** an ExFT description

Spare Slides

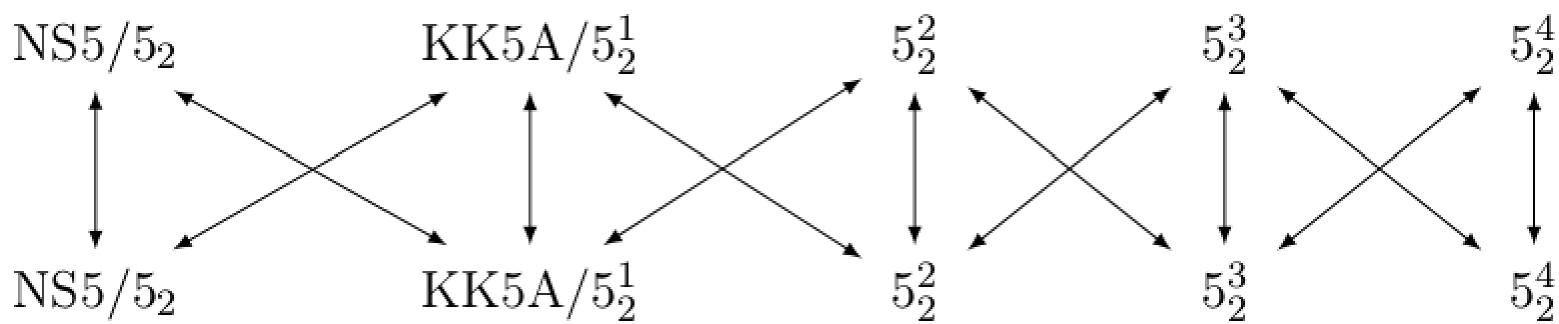
α	Number of Branes $N_{(\alpha)}$	Type	Breakdown $N_{(\alpha)} = A + B + 2C$									NS-NS Violation $A + B$
0	4	(NS-NS)	4	=	0	+	0	+	2	×	2	0
-1	10	R-R	10	=	5	+	5					
-2	10	NS-NS	10	=	0	+	0	+	2	×	5	
-3	24	R-R	24	=	12	+	12					
-4	46		46	=	5	+	5	+	2	×	18	10
-5	72	R-R	72	=	36	+	36					
-6	104	NS-NS	104	=	0	+	0	+	2	×	52	
-7	210	R-R	210	=	105	+	105					
-8	280		280	=	12	+	12	+	2	×	128	24
-9	448	R-R	448	=	224	+	224					
-10	632	NS-NS	632	=	0	+	0	+	2	×	316	
-11	942	R-R	942	=	471	+	471					
-12	1244		1244	=	36	+	36	+	2	×	586	72
-13	1926	R-R	1926	=	963	+	963					
-14	2340	NS-NS	2340	=	0	+	0	+	2	×	1170	
-15	3398	R-R	3398	=	1699	+	1699					
-16	4378		4378	=	105	+	105	+	2	×	2084	210
-17	5942	R-R	5942	=	2971	+	2971					
-18	7316	NS-NS	7316	=	0	+	0	+	2	×	3658	
-19	10050	R-R	10050	=	5025	+	5025					
-20	12252		12252	=	224	+	224	+	2	×	5902	448
-21	16134	R-R	16134	=	8067	+	8067					
-22	19388	NS-NS	19388	=	0	+	0	+	2	×	9694	
-23	25320	R-R	25320	=	12660	+	12660					
-24	30374		30374	=	471	+	471	+	2	×	14716	942
-25	38310	R-R	38310	=	19155	+	19155					
M	458124											



P/$^{1}0_0$
F1/1_0
P/$^{1}0_0$
F1/1_0



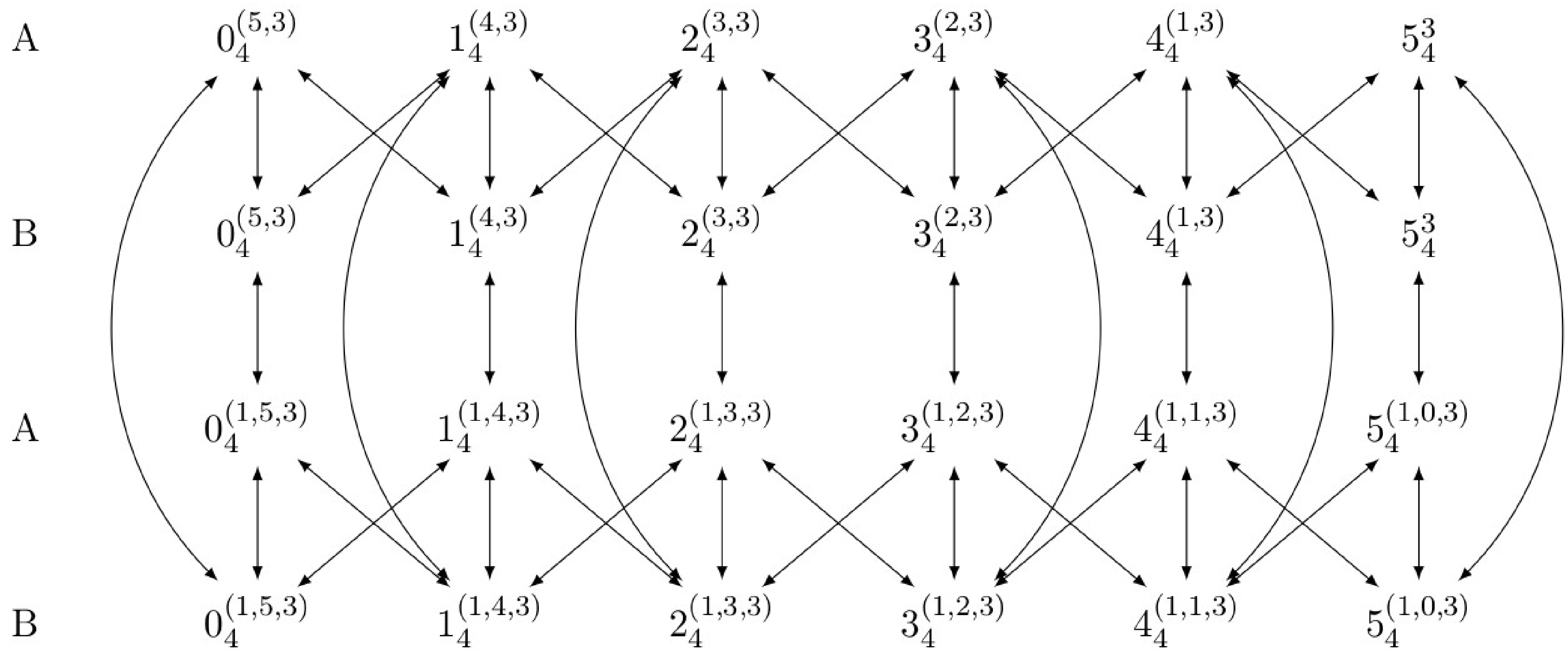
D0/0_1
D2/2_1
D4/4_1
D6/6_1
D8/8_1
D1/1_1
D3/3_1
D5/5_1
D7/7_1
D9/9_1

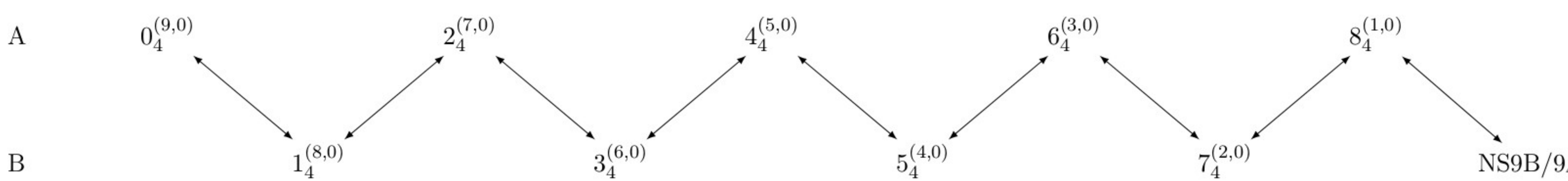


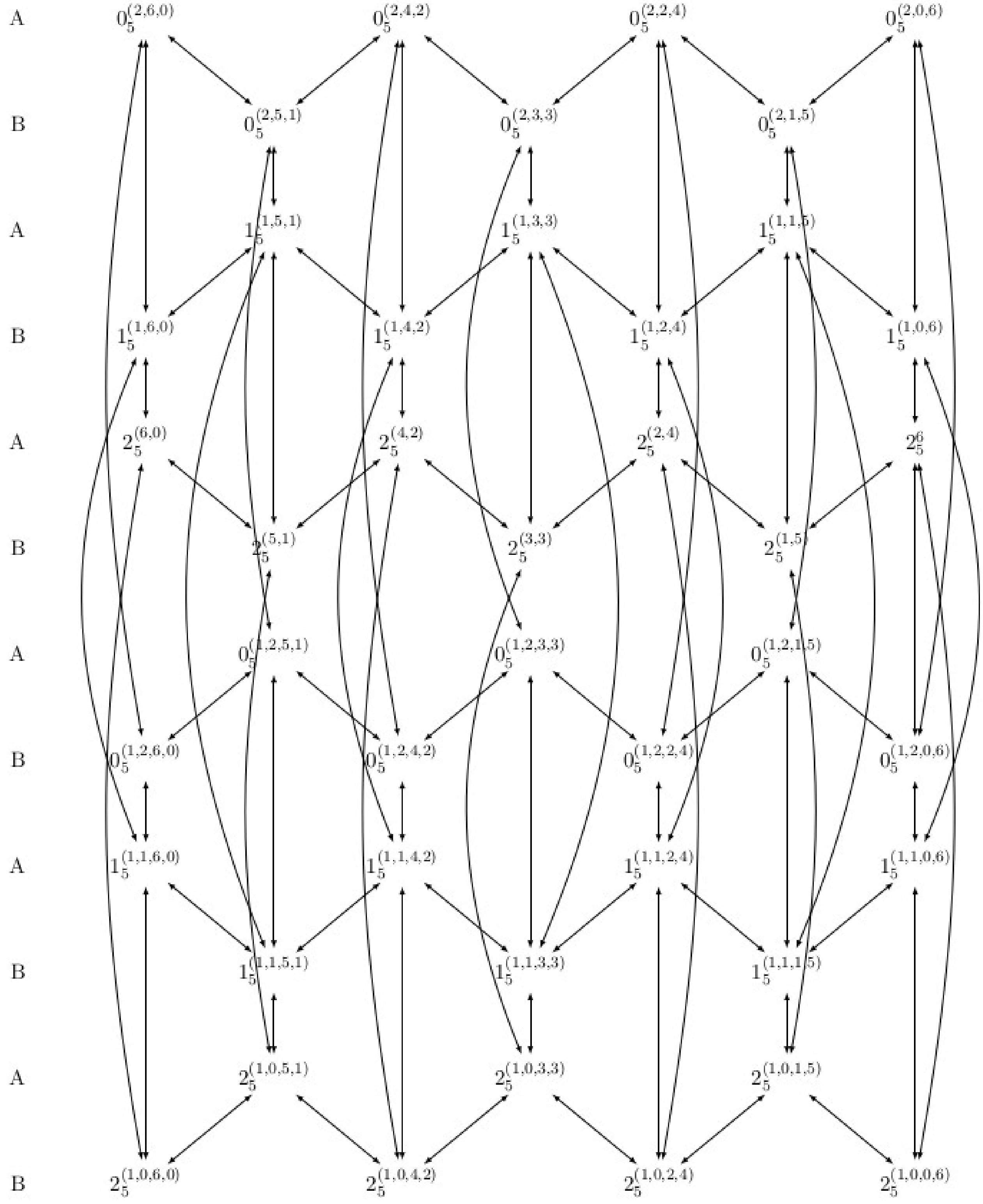
NS5/5_2
KK5A/5_2^1
5_2^2
5_2^3
5_2^4
NS5/5_2
KK5A/5_2^1
5_2^2
5_2^3
5_2^4

$$
\begin{tikzcd}
\mathrm{A} & 0_3^7 \arrow[dr, leftrightarrow] \arrow[ddd, leftrightarrow] & & 2_3^5 \arrow[dl, leftrightarrow] \arrow[dr, leftrightarrow] \arrow[ddd, leftrightarrow] & & 4_3^3 \arrow[dl, leftrightarrow] \arrow[dr, leftrightarrow] \arrow[ddd, leftrightarrow] & & 6_3^1 \arrow[dl, leftrightarrow] \arrow[dr, leftrightarrow] \arrow[ddd, leftrightarrow] & \\
\mathrm{B} & & 1_3^6 \arrow[d, leftrightarrow] & & 3_3^4 \arrow[d, leftrightarrow] & & 5_3^2 \arrow[d, leftrightarrow] & & 7_3 \arrow[d, leftrightarrow] \\
\mathrm{A} & & 1_3^{(1,6)} \arrow[dl, leftrightarrow] \arrow[dr, leftrightarrow] \arrow[ddd, leftrightarrow] & & 3_3^{(1,4)} \arrow[dl, leftrightarrow] \arrow[dr, leftrightarrow] \arrow[ddd, leftrightarrow] & & 5_3^{(1,2)} \arrow[dl, leftrightarrow] \arrow[dr, leftrightarrow] \arrow[ddd, leftrightarrow] & & 7_3^{(1,0)} \arrow[dl, leftrightarrow] \arrow[ddd, leftrightarrow] \\
\mathrm{B} & 0_3^{(1,7)} \arrow[d, leftrightarrow] & & 2_3^{(1,5)} \arrow[d, leftrightarrow] & & 4_3^{(1,3)} \arrow[d, leftrightarrow] & & 6_3^{(1,1)} \arrow[d, leftrightarrow] & \\
\mathrm{A} & 0_3^{(2,7)} \arrow[dr, leftrightarrow] & & 2_3^{(2,5)} \arrow[dl, leftrightarrow] \arrow[dr, leftrightarrow] & & 4_3^{(2,3)} \arrow[dl, leftrightarrow] \arrow[dr, leftrightarrow] & & 6_3^{(2,1)} \arrow[dl, leftrightarrow] \arrow[dr, leftrightarrow] & \\
\mathrm{B} & & 1_3^{(2,6)} & & 3_3^{(2,4)} & & 5_3^{(2,2)} & & 7_3^{(2,0)}
\end{tikzcd}
$$

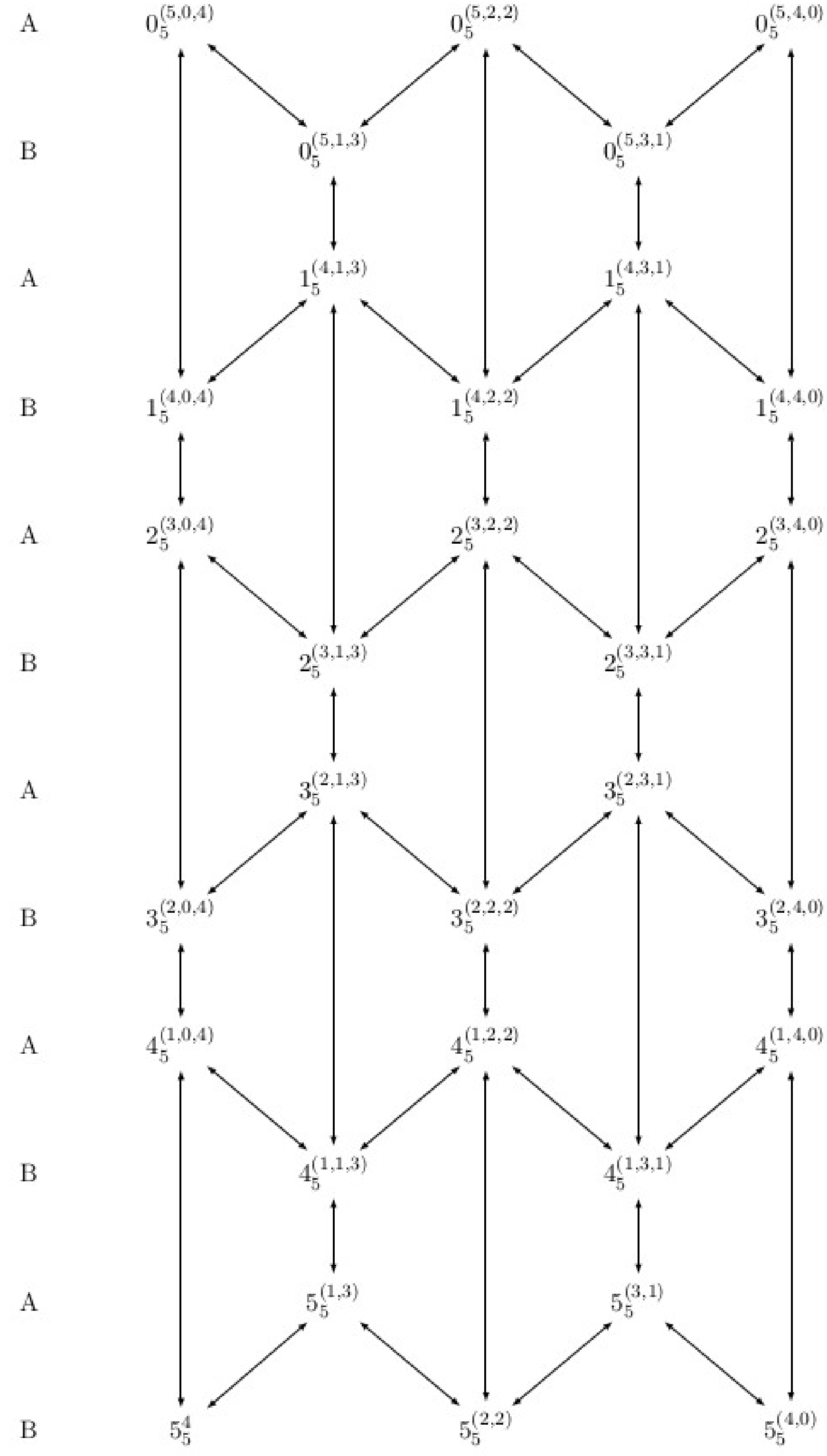
$$
\begin{tikzcd}[row sep=large]
\mathrm{A} & 0_4^{(1,6)} \arrow[d, leftrightarrow] \arrow[dr, leftrightarrow] \arrow[ddd, leftrightarrow, bend right=60] & 1_4^{6} \arrow[d, leftrightarrow] \arrow[dl, leftrightarrow] \arrow[ddd, leftrightarrow, bend left=60] \\
\mathrm{B} & 0_4^{(1,6)} \arrow[d, leftrightarrow] & 1_4^{6} \arrow[d, leftrightarrow] \\
\mathrm{A} & 0_4^{(1,1,6)} \arrow[d, leftrightarrow] \arrow[dr, leftrightarrow] \arrow[ddd, leftrightarrow, bend right=60] & 1_4^{(1,0,6)} \arrow[d, leftrightarrow] \arrow[dl, leftrightarrow] \arrow[ddd, leftrightarrow, bend left=60] \\
\mathrm{B} & 0_4^{(1,1,6)} \arrow[d, leftrightarrow] & 1_4^{(1,0,6)} \arrow[d, leftrightarrow] \\
\mathrm{A} & 0_4^{(2,1,6)} \arrow[d, leftrightarrow] \arrow[dr, leftrightarrow] & 1_4^{(2,0,6)} \arrow[d, leftrightarrow] \arrow[dl, leftrightarrow] \\
\mathrm{B} & 0_4^{(2,1,6)} & 1_4^{(2,0,6)}
\end{tikzcd}
$$


A
$0_4^{(5,3)}$
$1_4^{(4,3)}$
$2_4^{(3,3)}$
$3_4^{(2,3)}$
$4_4^{(1,3)}$
5_4^{3}
B
$0_4^{(5,3)}$
$1_4^{(4,3)}$
$2_4^{(3,3)}$
$3_4^{(2,3)}$
$4_4^{(1,3)}$
5_4^{3}
A
$0_4^{(1,5,3)}$
$1_4^{(1,4,3)}$
$2_4^{(1,3,3)}$
$3_4^{(1,2,3)}$
$4_4^{(1,1,3)}$
$5_4^{(1,0,3)}$
B
$0_4^{(1,5,3)}$
$1_4^{(1,4,3)}$
$2_4^{(1,3,3)}$
$3_4^{(1,2,3)}$
$4_4^{(1,1,3)}$
$5_4^{(1,0,3)}$

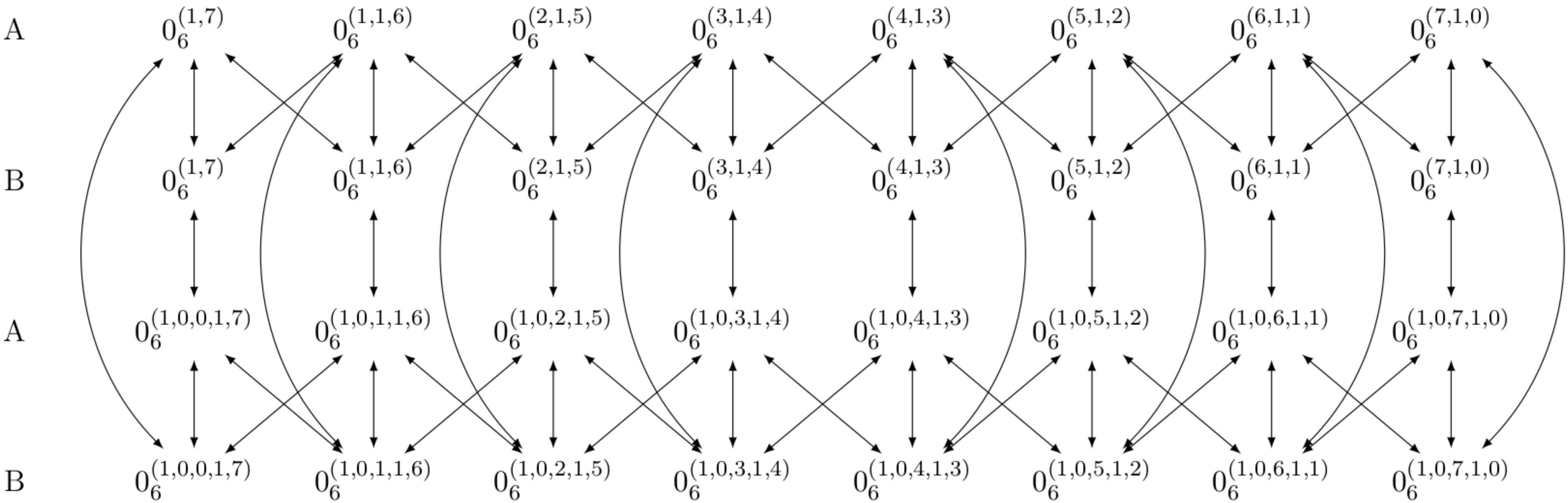

A
$0_4^{(9,0)}$
$2_4^{(7,0)}$
$4_4^{(5,0)}$
$6_4^{(3,0)}$
$8_4^{(1,0)}$
B
$1_4^{(8,0)}$
$3_4^{(6,0)}$
$5_4^{(4,0)}$
$7_4^{(2,0)}$
NS9B/9_4



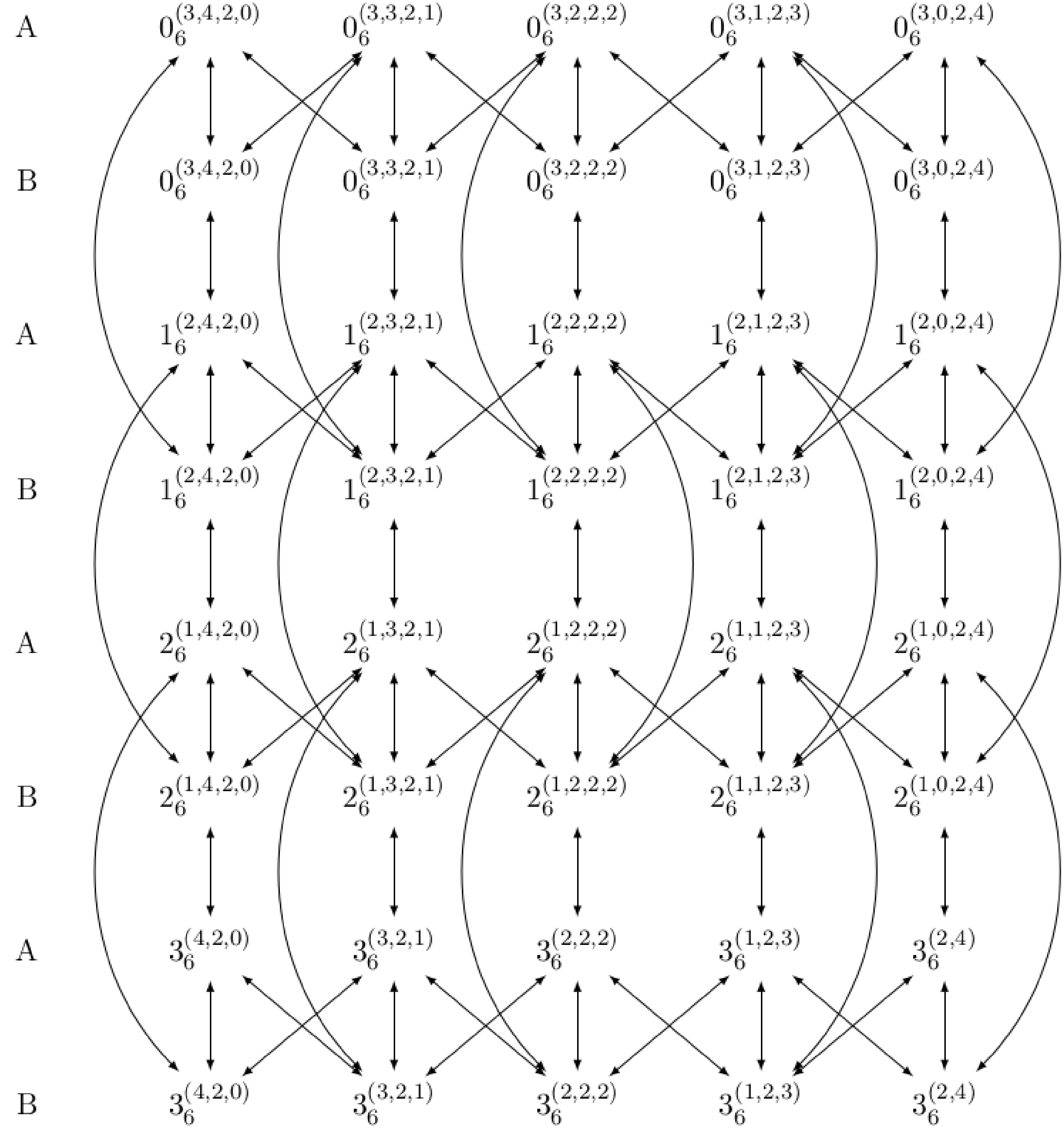
A $0_5^{(2,6,0)}$ $0_5^{(2,4,2)}$ $0_5^{(2,2,4)}$ $0_5^{(2,0,6)}$
B $0_5^{(2,5,1)}$ $0_5^{(2,3,3)}$ $0_5^{(2,1,5)}$
A $1_5^{(1,5,1)}$ $1_5^{(1,3,3)}$ $1_5^{(1,1,5)}$
B $1_5^{(1,6,0)}$ $1_5^{(1,4,2)}$ $1_5^{(1,2,4)}$ $1_5^{(1,0,6)}$
A $2_5^{(6,0)}$ $2_5^{(4,2)}$ $2_5^{(2,4)}$ 2_5^{6}
B $2_5^{(5,1)}$ $2_5^{(3,3)}$ $2_5^{(1,5)}$
A $0_5^{(1,2,5,1)}$ $0_5^{(1,2,3,3)}$ $0_5^{(1,2,1,5)}$
B $0_5^{(1,2,6,0)}$ $0_5^{(1,2,4,2)}$ $0_5^{(1,2,2,4)}$ $0_5^{(1,2,0,6)}$
A $1_5^{(1,1,6,0)}$ $1_5^{(1,1,4,2)}$ $1_5^{(1,1,2,4)}$ $1_5^{(1,1,0,6)}$
B $1_5^{(1,1,5,1)}$ $1_5^{(1,1,3,3)}$ $1_5^{(1,1,1,5)}$
A $2_5^{(1,0,5,1)}$ $2_5^{(1,0,3,3)}$ $2_5^{(1,0,1,5)}$
B $2_5^{(1,0,6,0)}$ $2_5^{(1,0,4,2)}$ $2_5^{(1,0,2,4)}$ $2_5^{(1,0,0,6)}$

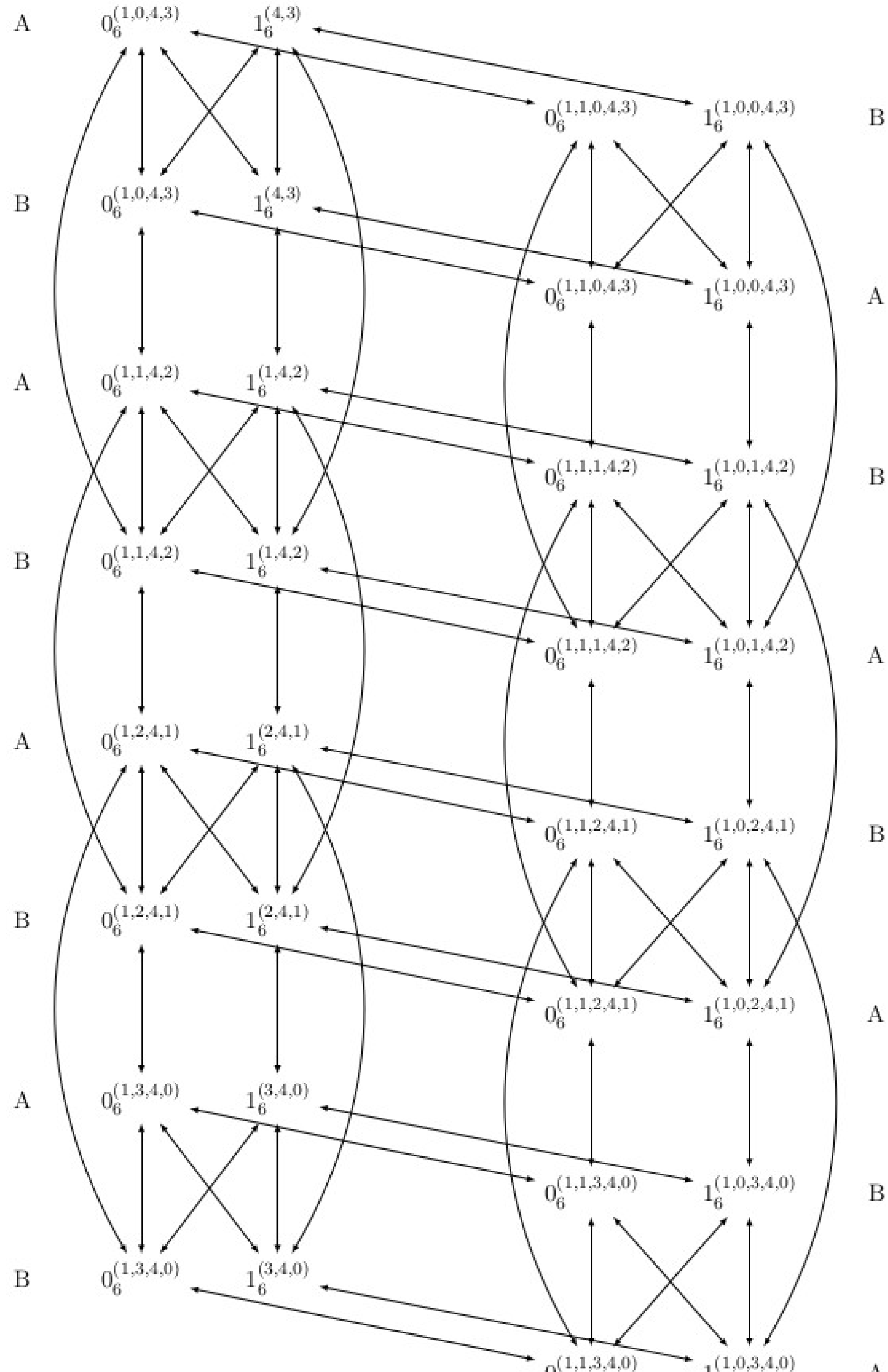


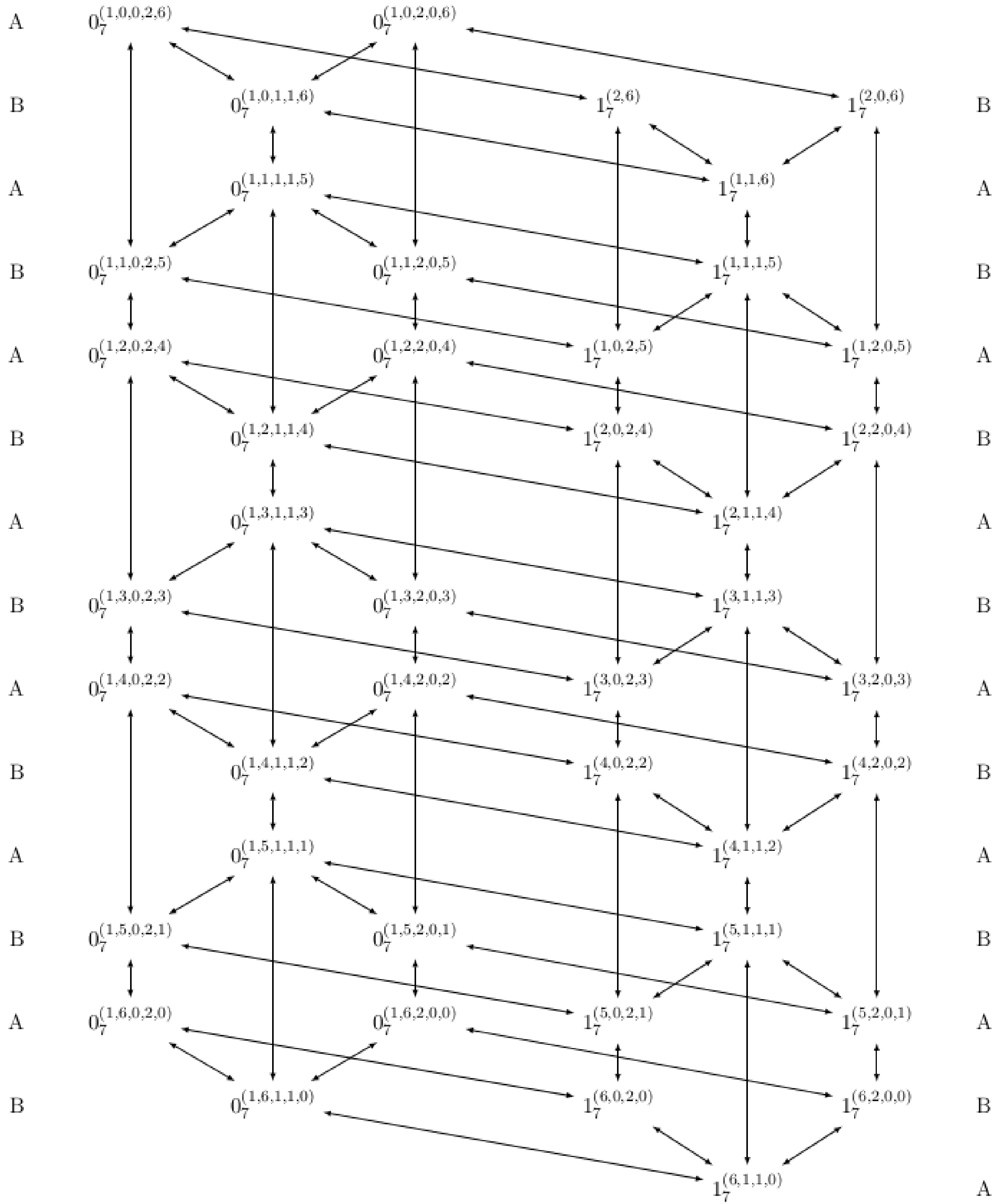
A $0_5^{(5,0,4)}$ $0_5^{(5,2,2)}$ $0_5^{(5,4,0)}$
B $0_5^{(5,1,3)}$ $0_5^{(5,3,1)}$
A $1_5^{(4,1,3)}$ $1_5^{(4,3,1)}$
B $1_5^{(4,0,4)}$ $1_5^{(4,2,2)}$ $1_5^{(4,4,0)}$
A $2_5^{(3,0,4)}$ $2_5^{(3,2,2)}$ $2_5^{(3,4,0)}$
B $2_5^{(3,1,3)}$ $2_5^{(3,3,1)}$
A $3_5^{(2,1,3)}$ $3_5^{(2,3,1)}$
B $3_5^{(2,0,4)}$ $3_5^{(2,2,2)}$ $3_5^{(2,4,0)}$
A $4_5^{(1,0,4)}$ $4_5^{(1,2,2)}$ $4_5^{(1,4,0)}$
B $4_5^{(1,1,3)}$ $4_5^{(1,3,1)}$
A $5_5^{(1,3)}$ $5_5^{(3,1)}$
B 5_5^{4} $5_5^{(2,2)}$ $5_5^{(4,0)}$

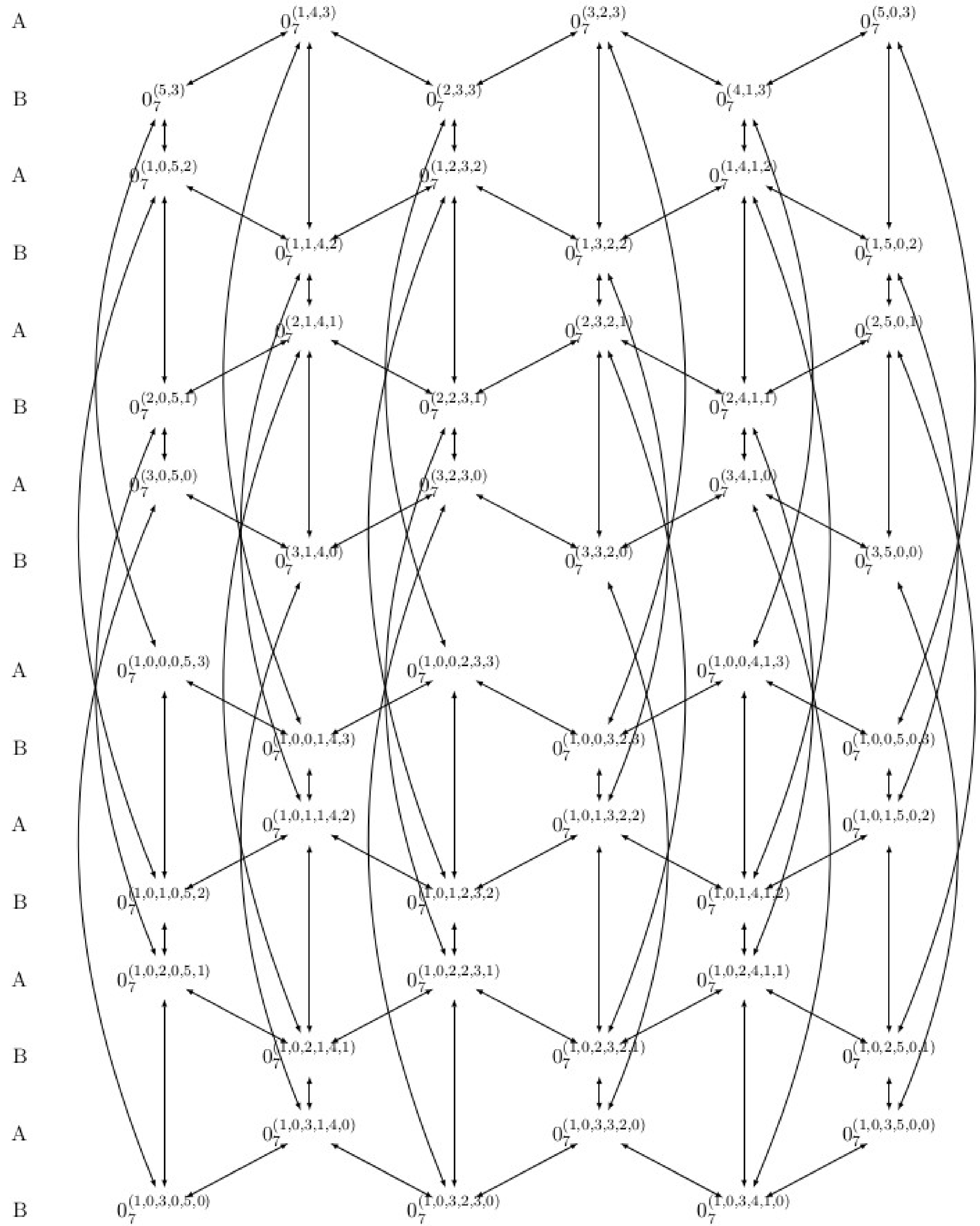


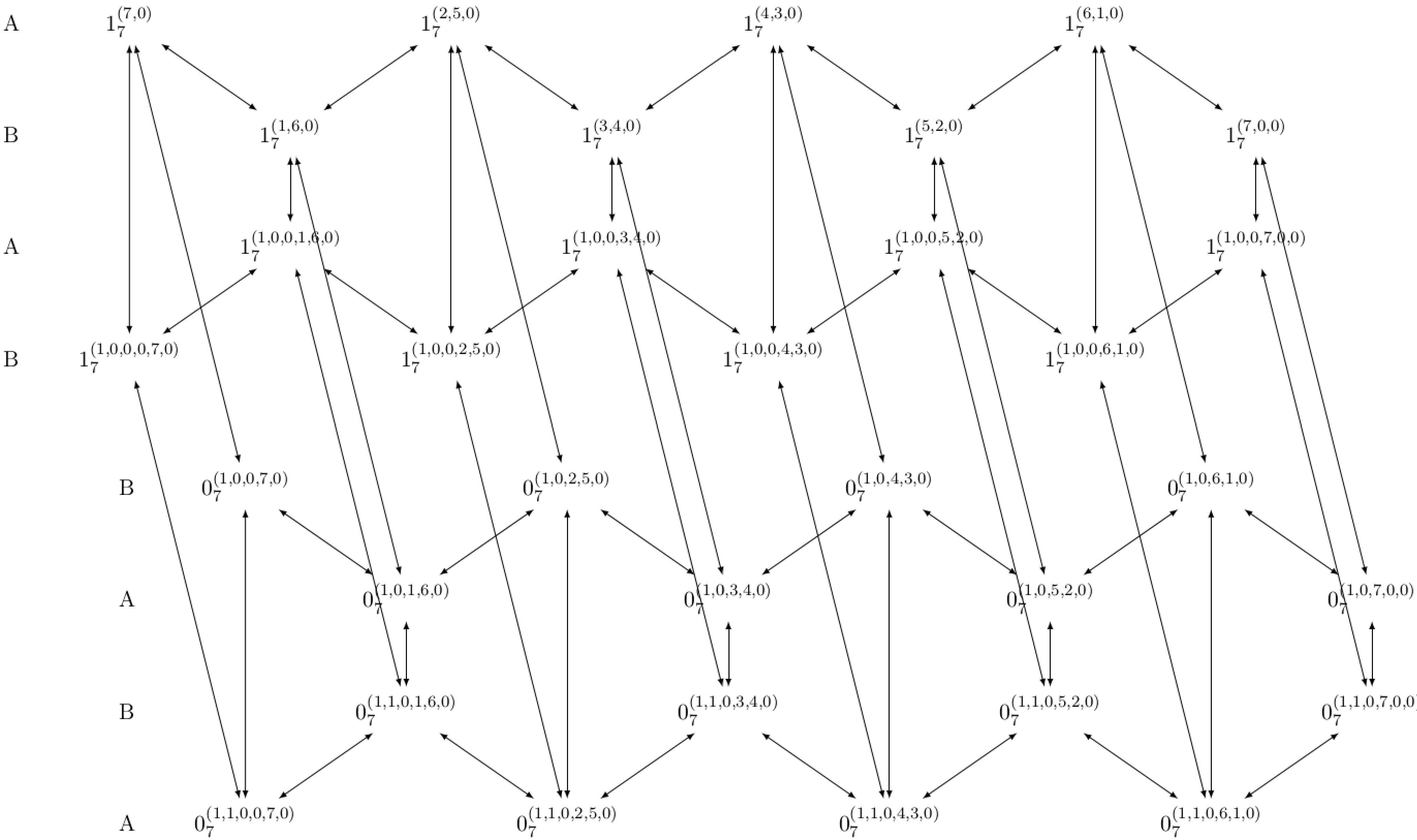
A
$0_6^{(1,7)}$ $0_6^{(1,1,6)}$ $0_6^{(2,1,5)}$ $0_6^{(3,1,4)}$ $0_6^{(4,1,3)}$ $0_6^{(5,1,2)}$ $0_6^{(6,1,1)}$ $0_6^{(7,1,0)}$
B
$0_6^{(1,7)}$ $0_6^{(1,1,6)}$ $0_6^{(2,1,5)}$ $0_6^{(3,1,4)}$ $0_6^{(4,1,3)}$ $0_6^{(5,1,2)}$ $0_6^{(6,1,1)}$ $0_6^{(7,1,0)}$
A
$0_6^{(1,0,0,1,7)}$ $0_6^{(1,0,1,1,6)}$ $0_6^{(1,0,2,1,5)}$ $0_6^{(1,0,3,1,4)}$ $0_6^{(1,0,4,1,3)}$ $0_6^{(1,0,5,1,2)}$ $0_6^{(1,0,6,1,1)}$ $0_6^{(1,0,7,1,0)}$
B
$0_6^{(1,0,0,1,7)}$ $0_6^{(1,0,1,1,6)}$ $0_6^{(1,0,2,1,5)}$ $0_6^{(1,0,3,1,4)}$ $0_6^{(1,0,4,1,3)}$ $0_6^{(1,0,5,1,2)}$ $0_6^{(1,0,6,1,1)}$ $0_6^{(1,0,7,1,0)}$

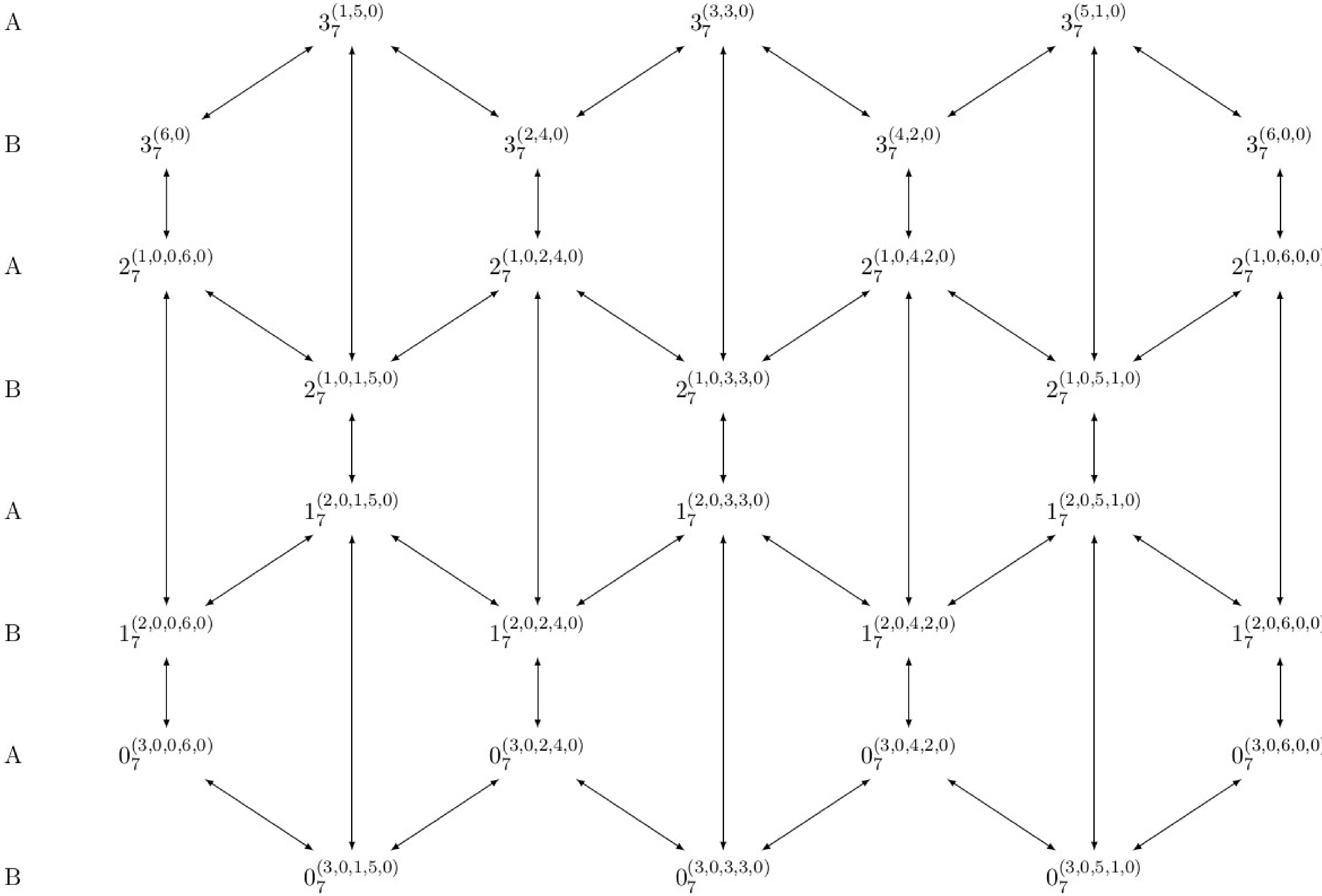

A $0_6^{(3,4,2,0)}$ $0_6^{(3,3,2,1)}$ $0_6^{(3,2,2,2)}$ $0_6^{(3,1,2,3)}$ $0_6^{(3,0,2,4)}$
B $0_6^{(3,4,2,0)}$ $0_6^{(3,3,2,1)}$ $0_6^{(3,2,2,2)}$ $0_6^{(3,1,2,3)}$ $0_6^{(3,0,2,4)}$
A $1_6^{(2,4,2,0)}$ $1_6^{(2,3,2,1)}$ $1_6^{(2,2,2,2)}$ $1_6^{(2,1,2,3)}$ $1_6^{(2,0,2,4)}$
B $1_6^{(2,4,2,0)}$ $1_6^{(2,3,2,1)}$ $1_6^{(2,2,2,2)}$ $1_6^{(2,1,2,3)}$ $1_6^{(2,0,2,4)}$
A $2_6^{(1,4,2,0)}$ $2_6^{(1,3,2,1)}$ $2_6^{(1,2,2,2)}$ $2_6^{(1,1,2,3)}$ $2_6^{(1,0,2,4)}$
B $2_6^{(1,4,2,0)}$ $2_6^{(1,3,2,1)}$ $2_6^{(1,2,2,2)}$ $2_6^{(1,1,2,3)}$ $2_6^{(1,0,2,4)}$
A $3_6^{(4,2,0)}$ $3_6^{(3,2,1)}$ $3_6^{(2,2,2)}$ $3_6^{(1,2,3)}$ $3_6^{(2,4)}$
B $3_6^{(4,2,0)}$ $3_6^{(3,2,1)}$ $3_6^{(2,2,2)}$ $3_6^{(1,2,3)}$ $3_6^{(2,4)}$

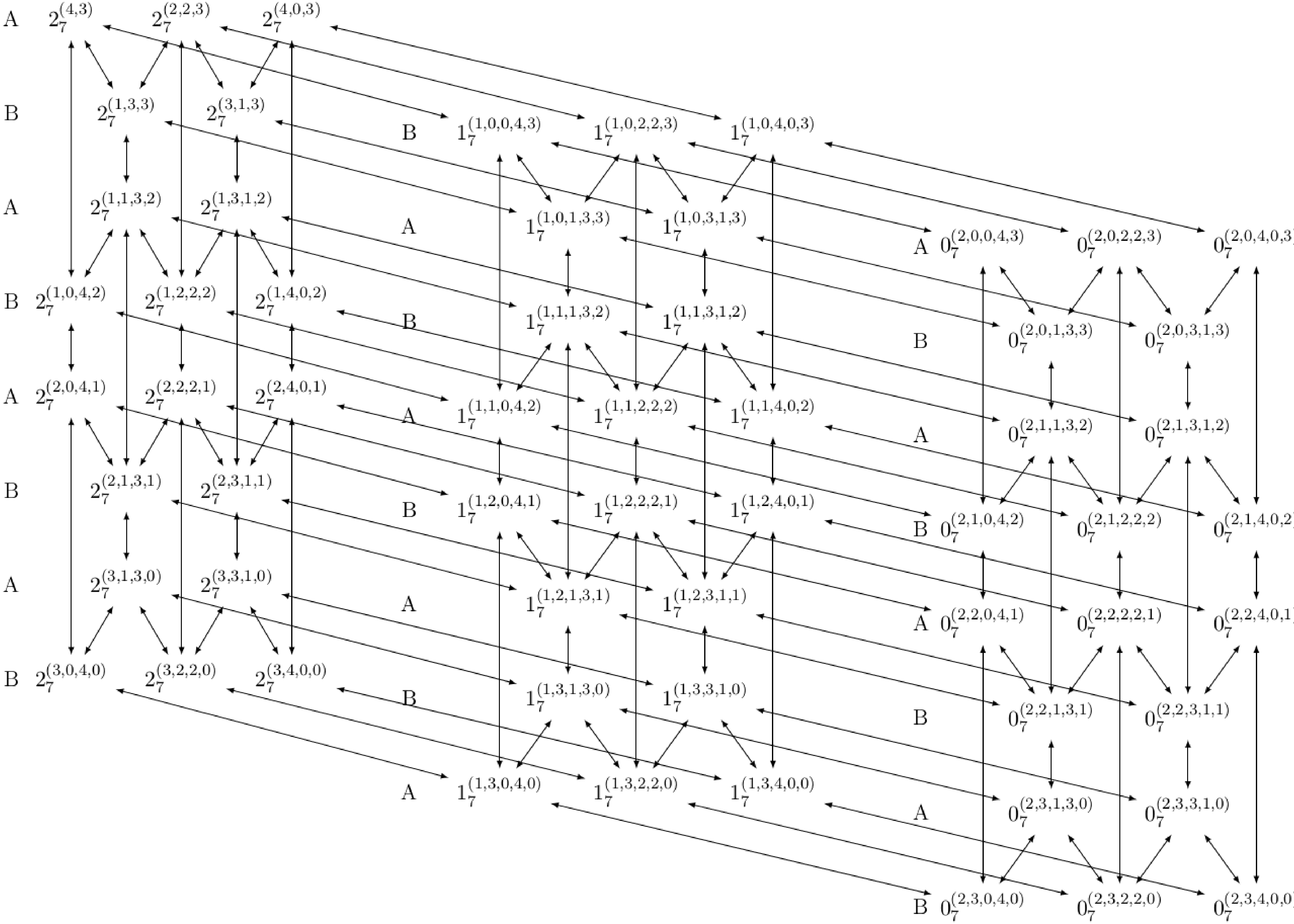

A
$0_6^{(1,0,4,3)}$
$1_6^{(4,3)}$
$0_6^{(1,1,0,4,3)}$
$1_6^{(1,0,0,4,3)}$
B
B
$0_6^{(1,0,4,3)}$
$1_6^{(4,3)}$
$0_6^{(1,1,0,4,3)}$
$1_6^{(1,0,0,4,3)}$
A
A
$0_6^{(1,1,4,2)}$
$1_6^{(1,4,2)}$
$0_6^{(1,1,1,4,2)}$
$1_6^{(1,0,1,4,2)}$
B
B
$0_6^{(1,1,4,2)}$
$1_6^{(1,4,2)}$
$0_6^{(1,1,1,4,2)}$
$1_6^{(1,0,1,4,2)}$
A
A
$0_6^{(1,2,4,1)}$
$1_6^{(2,4,1)}$
$0_6^{(1,1,2,4,1)}$
$1_6^{(1,0,2,4,1)}$
B
B
$0_6^{(1,2,4,1)}$
$1_6^{(2,4,1)}$
$0_6^{(1,1,2,4,1)}$
$1_6^{(1,0,2,4,1)}$
A
A
$0_6^{(1,3,4,0)}$
$1_6^{(3,4,0)}$
$0_6^{(1,1,3,4,0)}$
$1_6^{(1,0,3,4,0)}$
B
B
$0_6^{(1,3,4,0)}$
$1_6^{(3,4,0)}$
$0_6^{(1,1,3,4,0)}$
$1_6^{(1,0,3,4,0)}$
A


A
$0_7^{(1,0,0,2,6)}$
$0_7^{(1,0,2,0,6)}$
B
$0_7^{(1,0,1,1,6)}$
$1_7^{(2,6)}$
$1_7^{(2,0,6)}$
B
A
$0_7^{(1,1,1,1,5)}$
$1_7^{(1,1,6)}$
A
B
$0_7^{(1,1,0,2,5)}$
$0_7^{(1,1,2,0,5)}$
$1_7^{(1,1,1,5)}$
B
A
$0_7^{(1,2,0,2,4)}$
$0_7^{(1,2,2,0,4)}$
$1_7^{(1,0,2,5)}$
$1_7^{(1,2,0,5)}$
A
B
$0_7^{(1,2,1,1,4)}$
$1_7^{(2,0,2,4)}$
$1_7^{(2,2,0,4)}$
B
A
$0_7^{(1,3,1,1,3)}$
$1_7^{(2,1,1,4)}$
A
B
$0_7^{(1,3,0,2,3)}$
$0_7^{(1,3,2,0,3)}$
$1_7^{(3,1,1,3)}$
B
A
$0_7^{(1,4,0,2,2)}$
$0_7^{(1,4,2,0,2)}$
$1_7^{(3,0,2,3)}$
$1_7^{(3,2,0,3)}$
A
B
$0_7^{(1,4,1,1,2)}$
$1_7^{(4,0,2,2)}$
$1_7^{(4,2,0,2)}$
B
A
$0_7^{(1,5,1,1,1)}$
$1_7^{(4,1,1,2)}$
A
B
$0_7^{(1,5,0,2,1)}$
$0_7^{(1,5,2,0,1)}$
$1_7^{(5,1,1,1)}$
B
A
$0_7^{(1,6,0,2,0)}$
$0_7^{(1,6,2,0,0)}$
$1_7^{(5,0,2,1)}$
$1_7^{(5,2,0,1)}$
A
B
$0_7^{(1,6,1,1,0)}$
$1_7^{(6,0,2,0)}$
$1_7^{(6,2,0,0)}$
B
$1_7^{(6,1,1,0)}$
A


A $0_7^{(1,4,3)}$ $0_7^{(3,2,3)}$ $0_7^{(5,0,3)}$
B $0_7^{(5,3)}$ $0_7^{(2,3,3)}$ $0_7^{(4,1,3)}$
A $0_7^{(1,0,5,2)}$ $0_7^{(1,2,3,2)}$ $0_7^{(1,4,1,2)}$
B $0_7^{(1,1,4,2)}$ $0_7^{(1,3,2,2)}$ $0_7^{(1,5,0,2)}$
A $0_7^{(2,1,4,1)}$ $0_7^{(2,3,2,1)}$ $0_7^{(2,5,0,1)}$
B $0_7^{(2,0,5,1)}$ $0_7^{(2,2,3,1)}$ $0_7^{(2,4,1,1)}$
A $0_7^{(3,0,5,0)}$ $0_7^{(3,2,3,0)}$ $0_7^{(3,4,1,0)}$
B $0_7^{(3,1,4,0)}$ $0_7^{(3,3,2,0)}$ $0_7^{(3,5,0,0)}$
A $0_7^{(1,0,0,0,5,3)}$ $0_7^{(1,0,0,2,3,3)}$ $0_7^{(1,0,0,4,1,3)}$
B $0_7^{(1,0,0,1,4,3)}$ $0_7^{(1,0,0,3,2,3)}$ $0_7^{(1,0,0,5,0,3)}$
A $0_7^{(1,0,1,1,4,2)}$ $0_7^{(1,0,1,3,2,2)}$ $0_7^{(1,0,1,5,0,2)}$
B $0_7^{(1,0,1,0,5,2)}$ $0_7^{(1,0,1,2,3,2)}$ $0_7^{(1,0,1,4,1,2)}$
A $0_7^{(1,0,2,0,5,1)}$ $0_7^{(1,0,2,2,3,1)}$ $0_7^{(1,0,2,4,1,1)}$
B $0_7^{(1,0,2,1,4,1)}$ $0_7^{(1,0,2,3,2,1)}$ $0_7^{(1,0,2,5,0,1)}$
A $0_7^{(1,0,3,1,4,0)}$ $0_7^{(1,0,3,3,2,0)}$ $0_7^{(1,0,3,5,0,0)}$
B $0_7^{(1,0,3,0,5,0)}$ $0_7^{(1,0,3,2,3,0)}$ $0_7^{(1,0,3,4,1,0)}$


A
$1_7^{(7,0)}$
$1_7^{(2,5,0)}$
$1_7^{(4,3,0)}$
$1_7^{(6,1,0)}$
B
$1_7^{(1,6,0)}$
$1_7^{(3,4,0)}$
$1_7^{(5,2,0)}$
$1_7^{(7,0,0)}$
A
$1_7^{(1,0,0,1,6,0)}$
$1_7^{(1,0,0,3,4,0)}$
$1_7^{(1,0,0,5,2,0)}$
$1_7^{(1,0,0,7,0,0)}$
B
$1_7^{(1,0,0,0,7,0)}$
$1_7^{(1,0,0,2,5,0)}$
$1_7^{(1,0,0,4,3,0)}$
$1_7^{(1,0,0,6,1,0)}$
B
$0_7^{(1,0,0,7,0)}$
$0_7^{(1,0,2,5,0)}$
$0_7^{(1,0,4,3,0)}$
$0_7^{(1,0,6,1,0)}$
A
$0_7^{(1,0,1,6,0)}$
$0_7^{(1,0,3,4,0)}$
$0_7^{(1,0,5,2,0)}$
$0_7^{(1,0,7,0,0)}$
B
$0_7^{(1,1,0,1,6,0)}$
$0_7^{(1,1,0,3,4,0)}$
$0_7^{(1,1,0,5,2,0)}$
$0_7^{(1,1,0,7,0,0)}$
A
$0_7^{(1,1,0,0,7,0)}$
$0_7^{(1,1,0,2,5,0)}$
$0_7^{(1,1,0,4,3,0)}$
$0_7^{(1,1,0,6,1,0)}$


A
$3_7^{(1,5,0)}$
$3_7^{(3,3,0)}$
$3_7^{(5,1,0)}$
B
$3_7^{(6,0)}$
$3_7^{(2,4,0)}$
$3_7^{(4,2,0)}$
$3_7^{(6,0,0)}$
A
$2_7^{(1,0,0,6,0)}$
$2_7^{(1,0,2,4,0)}$
$2_7^{(1,0,4,2,0)}$
$2_7^{(1,0,6,0,0)}$
B
$2_7^{(1,0,1,5,0)}$
$2_7^{(1,0,3,3,0)}$
$2_7^{(1,0,5,1,0)}$
A
$1_7^{(2,0,1,5,0)}$
$1_7^{(2,0,3,3,0)}$
$1_7^{(2,0,5,1,0)}$
B
$1_7^{(2,0,0,6,0)}$
$1_7^{(2,0,2,4,0)}$
$1_7^{(2,0,4,2,0)}$
$1_7^{(2,0,6,0,0)}$
A
$0_7^{(3,0,0,6,0)}$
$0_7^{(3,0,2,4,0)}$
$0_7^{(3,0,4,2,0)}$
$0_7^{(3,0,6,0,0)}$
B
$0_7^{(3,0,1,5,0)}$
$0_7^{(3,0,3,3,0)}$
$0_7^{(3,0,5,1,0)}$


A $2_7^{(4,3)}$ $2_7^{(2,2,3)}$ $2_7^{(4,0,3)}$
B $2_7^{(1,3,3)}$ $2_7^{(3,1,3)}$
A $2_7^{(1,1,3,2)}$ $2_7^{(1,3,1,2)}$
B $2_7^{(1,0,4,2)}$ $2_7^{(1,2,2,2)}$ $2_7^{(1,4,0,2)}$
A $2_7^{(2,0,4,1)}$ $2_7^{(2,2,2,1)}$ $2_7^{(2,4,0,1)}$
B $2_7^{(2,1,3,1)}$ $2_7^{(2,3,1,1)}$
A $2_7^{(3,1,3,0)}$ $2_7^{(3,3,1,0)}$
B $2_7^{(3,0,4,0)}$ $2_7^{(3,2,2,0)}$ $2_7^{(3,4,0,0)}$
B $1_7^{(1,0,0,4,3)}$ $1_7^{(1,0,2,2,3)}$ $1_7^{(1,0,4,0,3)}$
A $1_7^{(1,0,1,3,3)}$ $1_7^{(1,0,3,1,3)}$
B $1_7^{(1,1,1,3,2)}$ $1_7^{(1,1,3,1,2)}$
A $1_7^{(1,1,0,4,2)}$ $1_7^{(1,1,2,2,2)}$ $1_7^{(1,1,4,0,2)}$
B $1_7^{(1,2,0,4,1)}$ $1_7^{(1,2,2,2,1)}$ $1_7^{(1,2,4,0,1)}$
A $1_7^{(1,2,1,3,1)}$ $1_7^{(1,2,3,1,1)}$
B $1_7^{(1,3,1,3,0)}$ $1_7^{(1,3,3,1,0)}$
A $1_7^{(1,3,0,4,0)}$ $1_7^{(1,3,2,2,0)}$ $1_7^{(1,3,4,0,0)}$
A $0_7^{(2,0,0,4,3)}$ $0_7^{(2,0,2,2,3)}$ $0_7^{(2,0,4,0,3)}$
B $0_7^{(2,0,1,3,3)}$ $0_7^{(2,0,3,1,3)}$
A $0_7^{(2,1,1,3,2)}$ $0_7^{(2,1,3,1,2)}$
B $0_7^{(2,1,0,4,2)}$ $0_7^{(2,1,2,2,2)}$ $0_7^{(2,1,4,0,2)}$
A $0_7^{(2,2,0,4,1)}$ $0_7^{(2,2,2,2,1)}$ $0_7^{(2,2,4,0,1)}$
B $0_7^{(2,2,1,3,1)}$ $0_7^{(2,2,3,1,1)}$
A $0_7^{(2,3,1,3,0)}$ $0_7^{(2,3,3,1,0)}$
B $0_7^{(2,3,0,4,0)}$ $0_7^{(2,3,2,2,0)}$ $0_7^{(2,3,4,0,0)}$