“Courant algebroids” for U-duality and M-theory

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Based on

- Brane Wess-Zumino terms from AKSZ and exceptional generalised geometry as an L_∞-algebroid, ASA, [arXiv:1804.07303]

What did we do?

$$
\begin{tikzcd}
\text{Courant algebroids} \arrow[r, "\text{are to}"] \arrow[d, "\text{are to}"'] & \text{strings} \arrow[r, "\text{are to}"] \arrow[d, "\text{are to}"'] & \text{Generalised geometry} \arrow[d, "\text{is to}"'] \\
\textbf{[THIS TALK]} \arrow[r, "\text{is to}"] & \begin{cases}\text{M5-branes}\\ \text{D3-branes}\end{cases} \arrow[r, "\text{are to}"] & \text{Exc. Gen. geometry.}
\end{tikzcd}
$$

- Found $E_{d(d)}$ **analogue** of the exact Courant algebroid (the one underlying $O(d,d)$-**generalised geometry**, with vector bundle $TM \oplus T^{\star}M$).
- AKSZ topological theory $\implies$ M5, D3-brane Wess-Zumino couplings
- Direct link between (exceptional) generalised geometry, brane physics, "higher algebra".

Why care?

Specifically:

- Courant algebroids $\cong$ degree 2 NPQ-manifolds [Roytenberg, Weinstein]
- **“M5-brane algebroid”** $\cong$ degree **6** NPQ-manifold over d-dimensional M-theory sections ($E_{d(d)}$ gen. geometry, $d \leq \mathbf{6}$)
- **“D3-brane algebroid”** $\cong$ degree **4** NPQ-manifold over d-dimensional IIB sections ($E_{d+1(d+1)}$ gen. geometry, $d+1 \leq \mathbf{5}$)

(NPQ-manifolds are target spaces for AKSZ, hence the brane connection)

Applications:

- novel ones I can't yet talk about. However:
- easily recover possible **twists of the gen. Lie derivative**;
- also, L_∞-algebra structure on (part of) the tensor hierarchy (overlap with [Baraglia; Cederwall-Palmkvist; Cagnacci-Codina-Marques])

NPQ manifolds

...are Nice graded supermanifolds $\mathcal{M}$ with a **(degree p)** symPlectic structure and "BRST charge" $Q = (\Theta, -)$,
$\deg Q = 1 \iff \deg \Theta = p + 1$

- PB + functions on $\mathcal{M}$ +[Getzler; Fiorenza-Manetti] $=L_\infty$-algebra structure of "tensor hierarchy" [Baraglia; Cederwall-Palmkvist; Cagnacci-Codina-Marques] see also [Hohm-Zwiebach] for $O(d,d)$
- $\mathcal{M} \xrightarrow{\text{AKSZ}}$ topological p-branes $\xrightarrow{on-shell}$ $(p-1)$-brane WZ coupling:

$(p = 6)$ M5-brane algebroid gives M5-brane WZ term, while the $(p = 4)$ D3-brane algebroid gives D3-brane WZ term*. The exact Courant algebroid $(p = 2)$ gives the string electric WZ closed 3-form coupling.

Practically, NPQ manifolds allow use of SUGRA & physicist intuition for writing things down: <u>superfields</u> and <u>hamiltonians</u>

(Synonyms for NPQ:) "dg-symplectic", "symplectic Lie-p-algebroids", "L_∞-algebroids" ...

NPQ manifold language for Courant algebroids ($p = 2$)

[Roytenberg]

Exact courant algebroid over $M \leftrightarrow \mathcal{M} = T^\star[2]T[1]M$ with obvious graded symplectic structure. Write sections of gen. tangent bundle as superfields:

$$V = v^\mu(x)\chi_\mu + \lambda_\mu(x)\psi^\mu \in (T \oplus T^\star)M$$

$$\deg x^\mu = 0\,, \quad \deg \psi^\mu = 1 \quad \deg \chi_\mu = 1 \implies \qquad \boxed{\deg V = 1}$$

Claim: antisymmetrised "Derived bracket" $\leftrightarrow$ Courant bracket:

$$(QV, V') - (V \leftrightarrow V') = \big((p_\mu\psi^\mu, V), V'\big) - (\dots) = [V, V']_{\text{Courant}}$$

using graded Poisson brackets

$$(x^\mu, p_\nu) = \delta^\mu_\nu\,, \quad (\psi^\mu, \chi_\nu) = \delta^\mu_\nu\,.$$

Hamiltonian $\Theta = p_\mu\psi^\mu$ generates the bracket; $(\Theta, \Theta) = 0 \iff$ **litany of Courant bracket identities.**

(Usual sign rule: odd degrees anticommute.) Another derived bracket yields "anchor".

NPQ manifold language for "M5-brane" algebroid ($\mathbf{p = 6}$)

"M5-brane" algebroid over $M \leftrightarrow \mathcal{M} = T^\star[6]T[1]M \times \mathbb{R}[3]$. Write sections of **exceptional** gen. tangent bundle as superfields:

$$V = v^\mu(x)\chi_\mu + \omega_{\mu\nu}(x)\psi^\mu\psi^\nu\zeta + \sigma_5(x)\psi^5 \in (T \oplus \Lambda^2 T^\star \oplus \Lambda^5 T^\star)M$$

$$\deg x^\mu = 0\,, \quad \deg \psi^\mu = 1 \quad \deg \chi_\mu = 5 \quad \deg \zeta = 3\,, \boxed{\deg V = 5}$$

Claim: antisymmetrised "Derived bracket" $\leftrightarrow$ **Exc.** Courant bracket of [Hull; Pacheco-Waldram]:

$$(QV, V') - (V \leftrightarrow V') = \big((p_\mu\psi^\mu, V), V'\big) - (\dots) = [V, V']_{\textbf{Exc.}\ \text{Courant}}$$

using graded Poisson brackets

$$(x^\mu, p_\nu) = \delta^\mu_\nu\,, \quad (\psi^\mu, \chi_\nu) = \delta^\mu_\nu\,, \quad (\zeta, \zeta) = 1\,.$$

Hamiltonian $\Theta = p_\mu\psi^\mu$ generates the bracket; $(\Theta, \Theta) = 0 \implies$ **"Leibniz algebroid" property of exceptional Dorfman bracket + other Courantesque axioms:**

Derived brackets produce operations on degrees $0, 1, 2, 3, 4, 5$: **tensor hierachy**

Branes and twists

Recall the exact Courant algebroid can be twisted by a 3-form H. This enters as

$$\Theta = p_\mu \psi^\mu + H_{\mu\nu\rho}(x)\psi^\mu\psi^\nu\psi^\rho \,, (\Theta,\Theta) = 0 \iff Q^2 \iff dH = 0$$

AKSZ yields "Courant sigma model" [Hofman-Park, Ikeda]

$$S_{\text{AKSZ}} = \int_{\Sigma_3} \text{``}p dx + \psi d\chi + \Theta\text{''} \xrightarrow{\text{some EOMs}} \int_{\Sigma_3} x^\star H$$

M5-brane algebroid: twist by F_4 and F_7

$$\Theta = p_\mu \psi^\mu + F_4(x)\psi^4 \zeta + F_7(x)\psi^7 \,, (\Theta,\Theta) = 0 \iff dF_7 + F_4 \wedge F_4 = 0$$

AKSZ yields (eliminating p_μ, ψ^μ)

$$S_{\text{AKSZ;on-shell}} = \int_{\Sigma_7} \zeta \wedge d\zeta/2 - F_4 \wedge \zeta + F_7$$

looks like M5-brane WZ coupling to SUGRA C_3, C_6-fields... (this form closely related to [Kalkinnen-Stelle; Intriligator])

M5 WZ from open 6-brane

$$S_{\text{AKSZ;on-shell}} = \int_{\Sigma_7} \zeta \wedge d\zeta/2 - F_4 \wedge \zeta + F_7$$

Let $\partial\Sigma_7 := W_6$. Consistency of variational problem dictates boundary conds. on $\boxed{\zeta = h + C_3}$. We must ensure:

$$\int_{W_6} h \wedge \delta h = 0 \iff h \text{ is in isotropic subspace}$$

for the canonical sympl. form on 3-forms on 6-fold W_6.
Natural choice is a *lagrangian* subspace. (Anti)self-dual 3-forms are lagrangian [Hitchin], so set $\boxed{\star_6 h = \pm h}$. Then ζ EOM $\iff$ $\boxed{dh = 0}$

The $\mathbb{R}[3]$ coordinate ζ on $\mathcal{M} = T^\star[6]T[1]M \times \mathbb{R}[3]$ yields M5-brane chiral gauge field!

Using ζ EOM, finally get $S = \int_{W_6} C_6 - h \wedge C_3/2$

Thank you!