New results on the determination of the strong coupling

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based on

arXiv:1603.08927, 1606.03453, 1708.04093, 1804.09146, 1807.11472

and unpublished results of ongoing work

Workshop on the Standard Model and Beyond

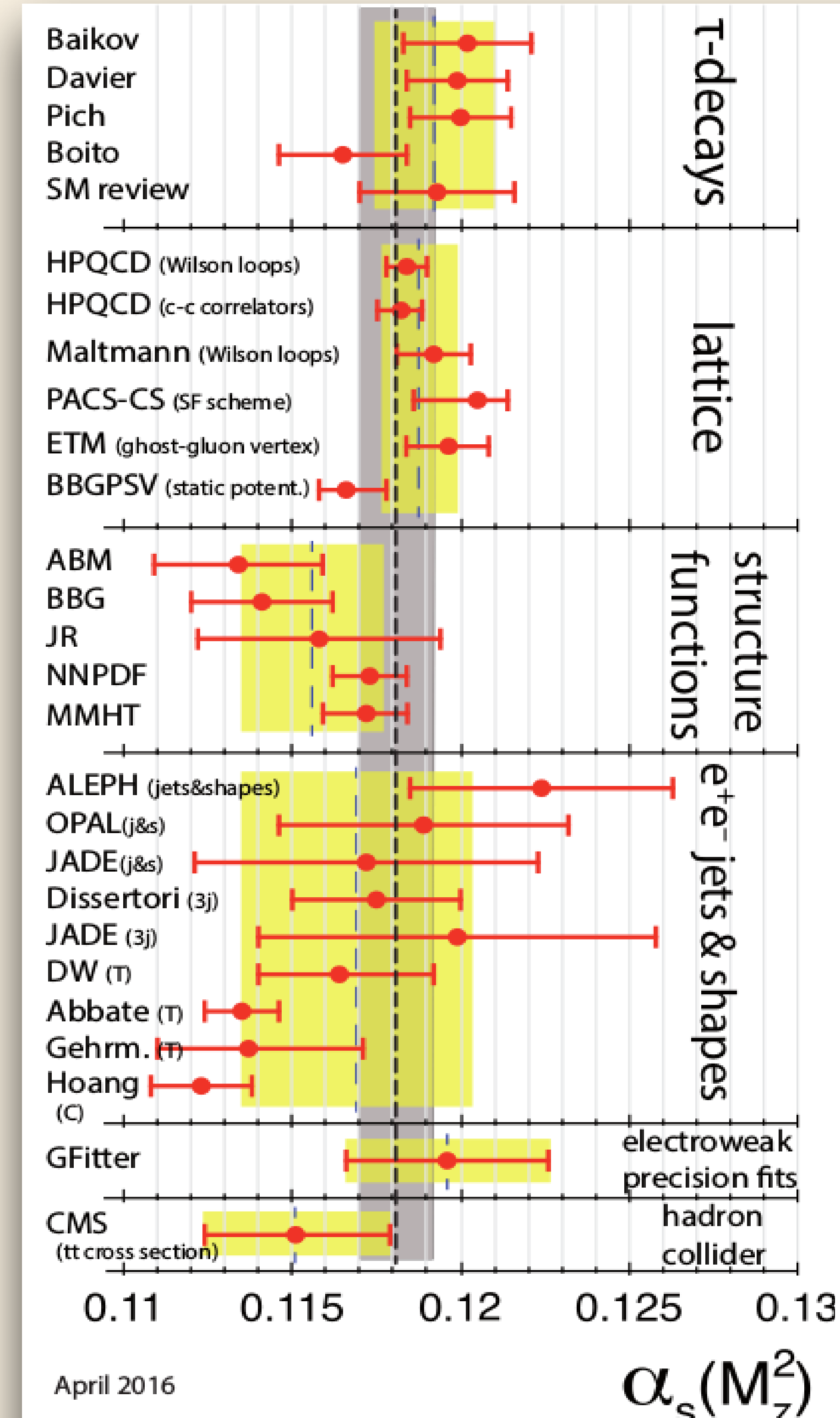
September 1, 2018

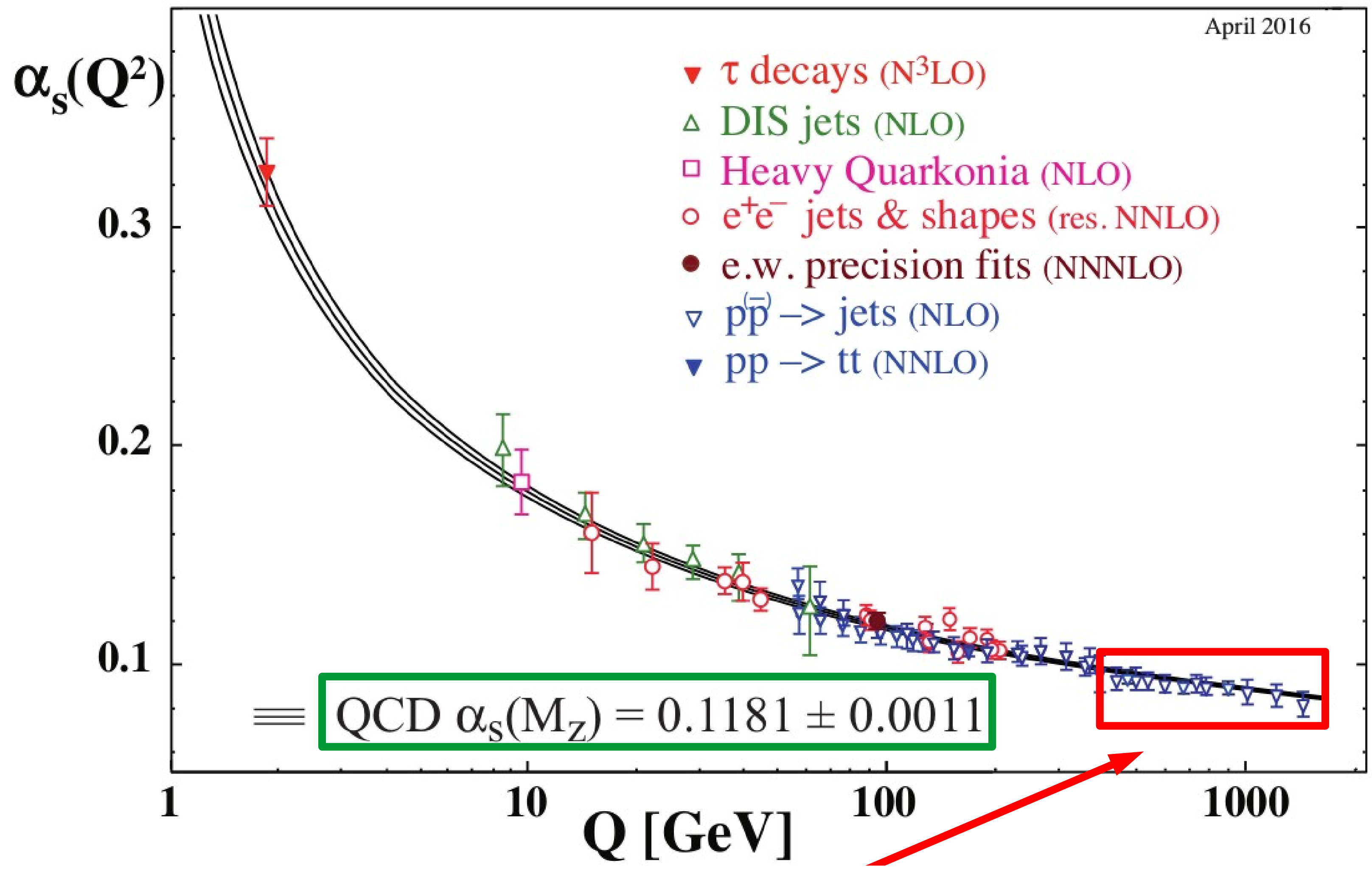
Outline

- Status of the strong coupling
- New measurements of α_s
- New prospects: soft drop event shapes
- Conclusions

Status of the strong coupling

PDG 2016 on α_s





not included in average:
LHC data, but only NLO theory

dominated by lattice

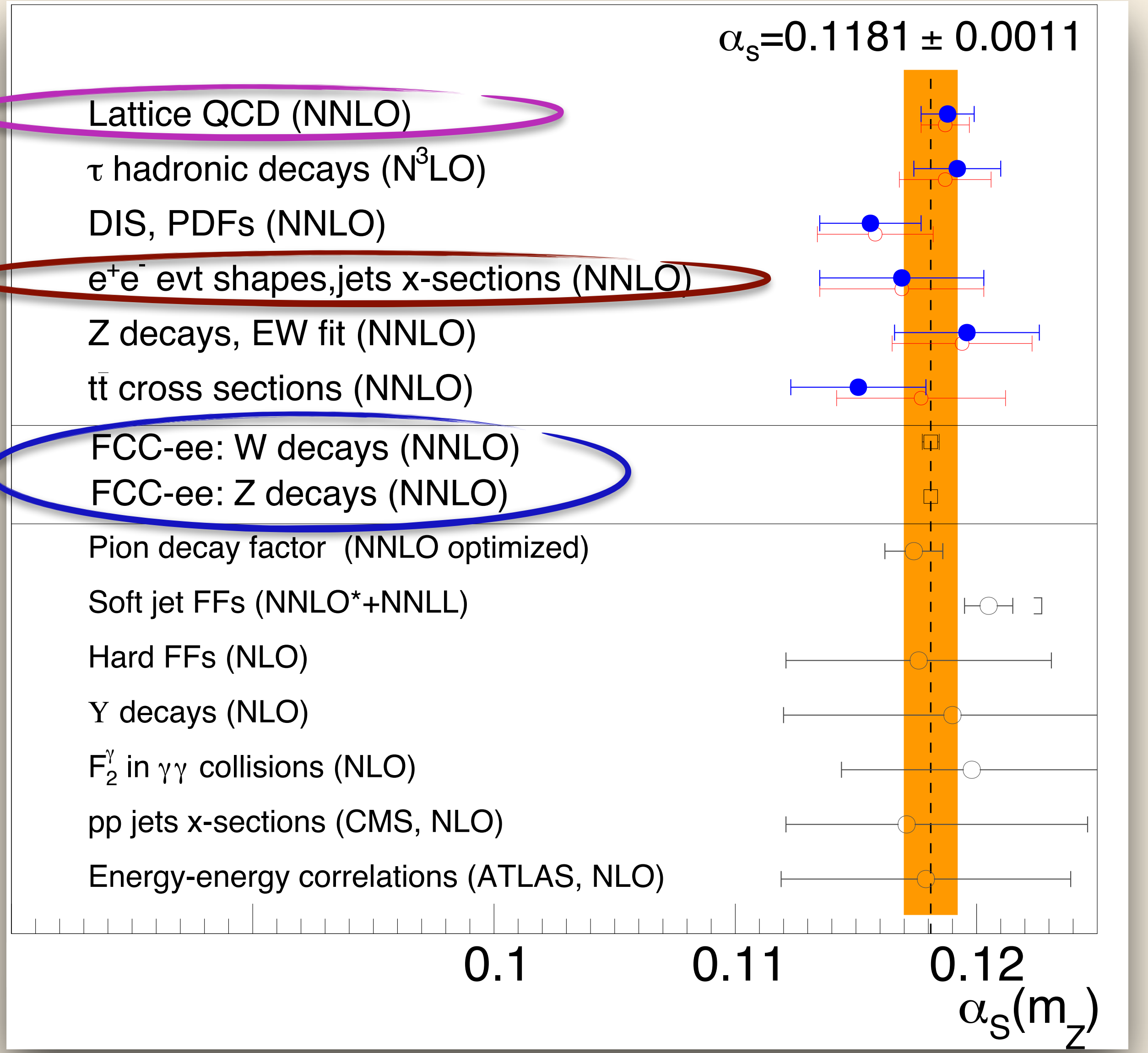
$$\frac{\Delta\alpha_S(M_Z)}{\alpha_S(M_Z)} = 0.9\%$$

PDG, Chin. Phys. C40 (2016) 100001

PDG 1992: 2.4%

Lattice unbeatable?

- recent prevailing view: lattice is unbeatable
- yet determination of α_s from experiments remains desirable (or at least a fancy)



D. d'Enterria, arXiv: 1806.06156

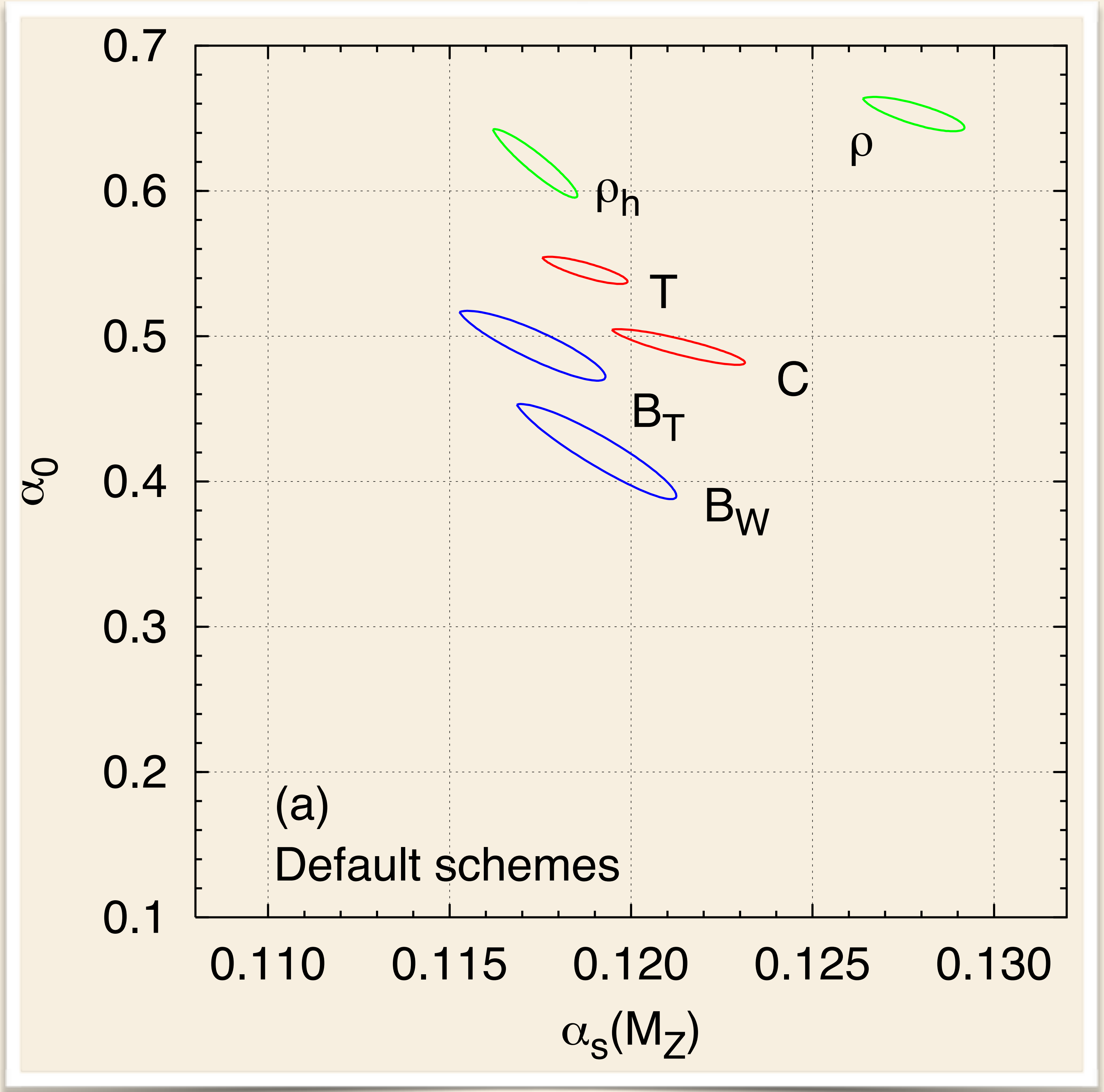
- e^+e^- event shapes, jets

✓ are sensitive to α_s

✓ are measured extensively

✓ can almost be computed from first principles (assuming local parton-hadron duality)

Shapes at NLO+NLL+power corr.+had. mass at LEP

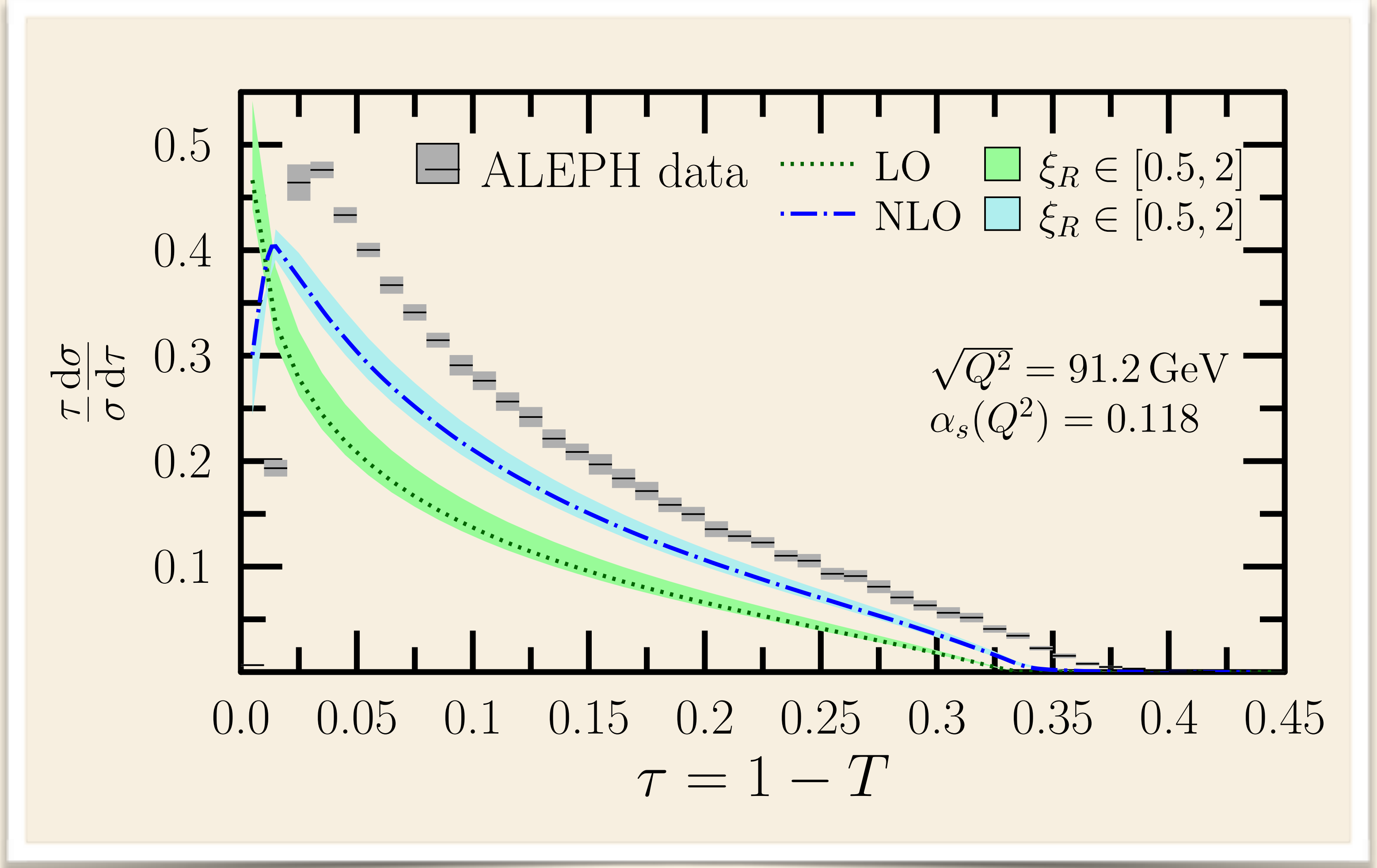


 D. Wicke, G. Salam hep-ph/0102343

Three-jet event shapes at LEP

- LO vs. NLO vs. data:

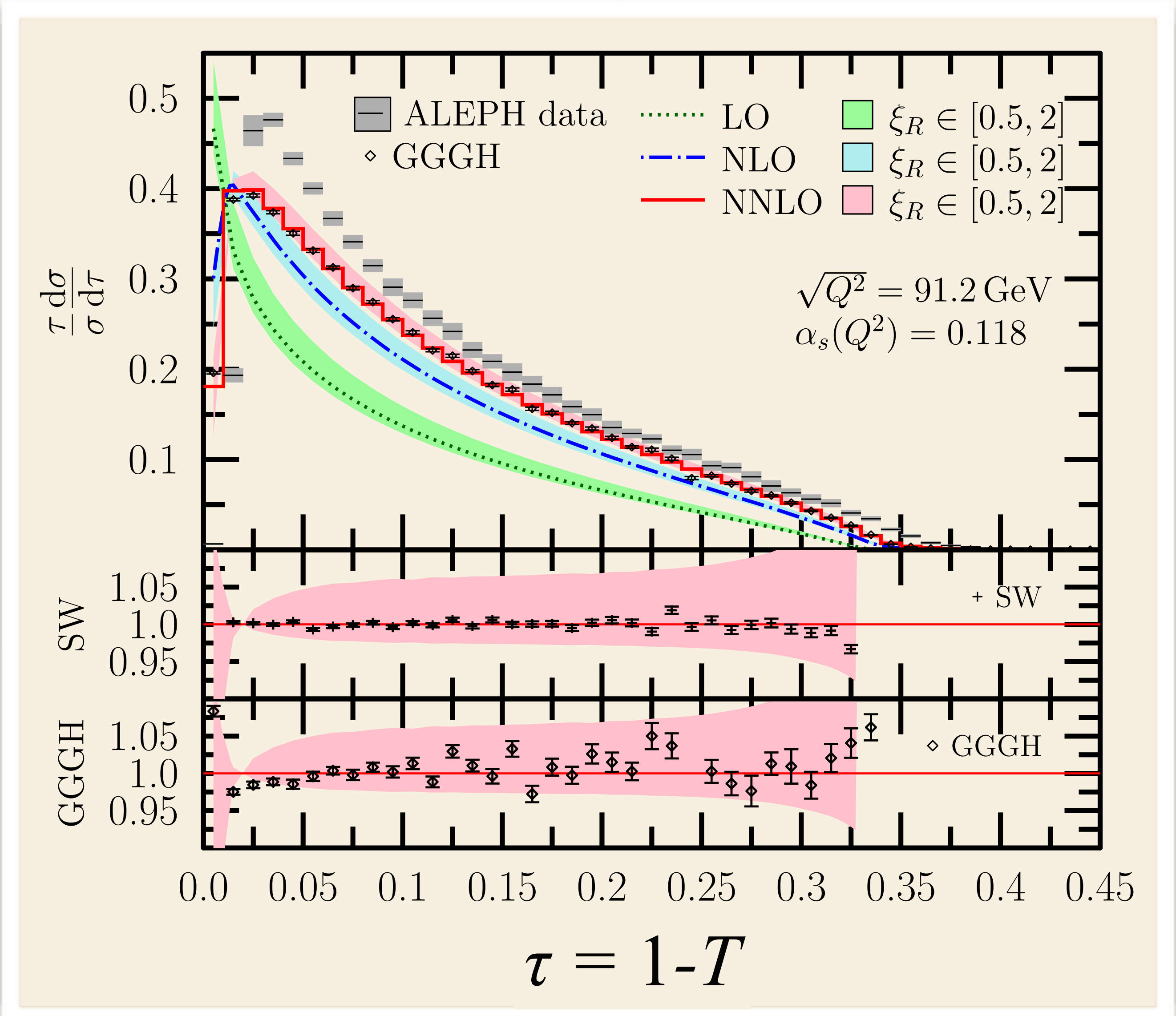
  suffer large perturbative & hadronization corrections



- new since LEP:
  - ✓ NNLO corrections
  - ✓ N^2LL or N^3LL resummation

New measurements of α_s

NNLO is not enough



$$T = \max_{\vec{n}} \left(\frac{\sum_i |\vec{n} \cdot \vec{p}_i|}{\sum_i |\vec{p}_i|} \right)$$

$$\frac{\tau}{\sigma}\frac{d\sigma}{d\tau} = \left(\frac{\alpha_s}{2\pi}\right) A(\tau) + \left(\frac{\alpha_s}{2\pi}\right)^2 B(\tau) + \left(\frac{\alpha_s}{2\pi}\right)^3 C(\tau)$$

A, B and C computed with MCCSM (=Monte Carlo for CoLoRFulNNLO Subtraction Method)

V. Del Duca et al, arXiv:1603.08927

Analytic structure of perturbative expansion

$$\frac{\tau}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = \left(\frac{\alpha_s}{2\pi}\right) A(\tau) + \left(\frac{\alpha_s}{2\pi}\right)^2 B(\tau) + \left(\frac{\alpha_s}{2\pi}\right)^3 C(\tau)$$

$$L = -\ln\tau$$

$$\begin{aligned}
A(\tau) &= \boxed{A_1 L} + A_0\,,\\
B(\tau) &= \boxed{B_3 L^3} + \boxed{B_2 L^2} + \boxed{B_1 L} + B_0\,,\\
C(\tau) &= \boxed{C_5 L^5} + \boxed{C_4 L^4} + \boxed{C_3 L^3} + \boxed{C_2 L^2} + C_1 L + C_0\\
&\quad\ \vdots \qquad\quad \vdots \qquad\quad \vdots \qquad\quad \vdots
\end{aligned}$$

LL NLL N^2LL N^3LL …

needs resummation of all orders

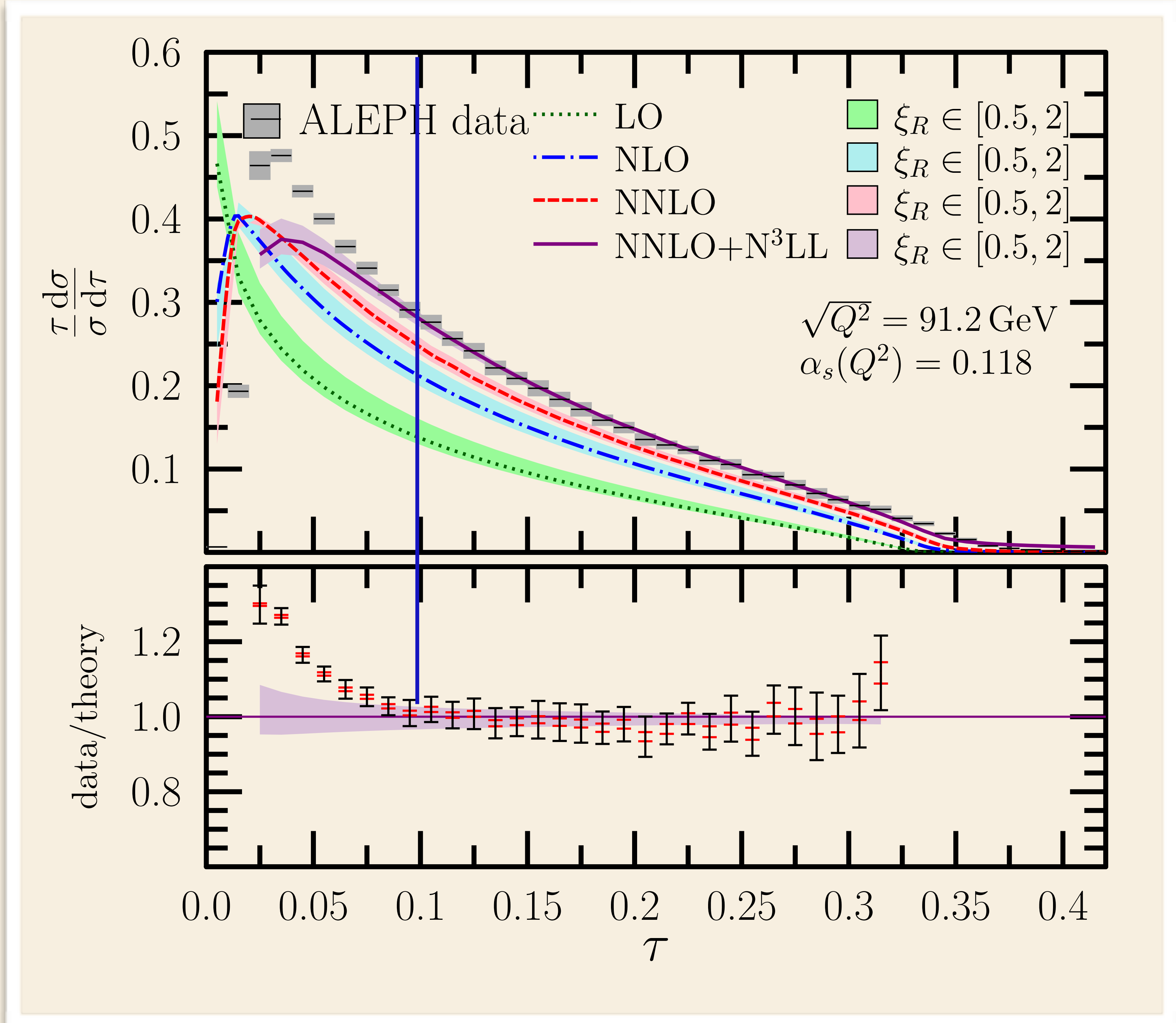
How to improve?

✓ Match to approximate predictions that resum large logarithms of the event shapes

precise predictions are available, e.g.:

- N^3LL for thrust (τ), C-parameter and heavy jet mass (ρ)
- N^2LL for broadenings and EEC

Matching NNLO with N^{3}LL



Works for τ > 0.1, fails in peak regions

How to improve?

- ✓ Match to approximate predictions that resum large logarithms of the event shapes

  precise predictions are available, e.g.:

  - N^3LL for thrust (τ), C-parameter and heavy jet mass (ρ)
  - N^2LL for broadenings and EEC

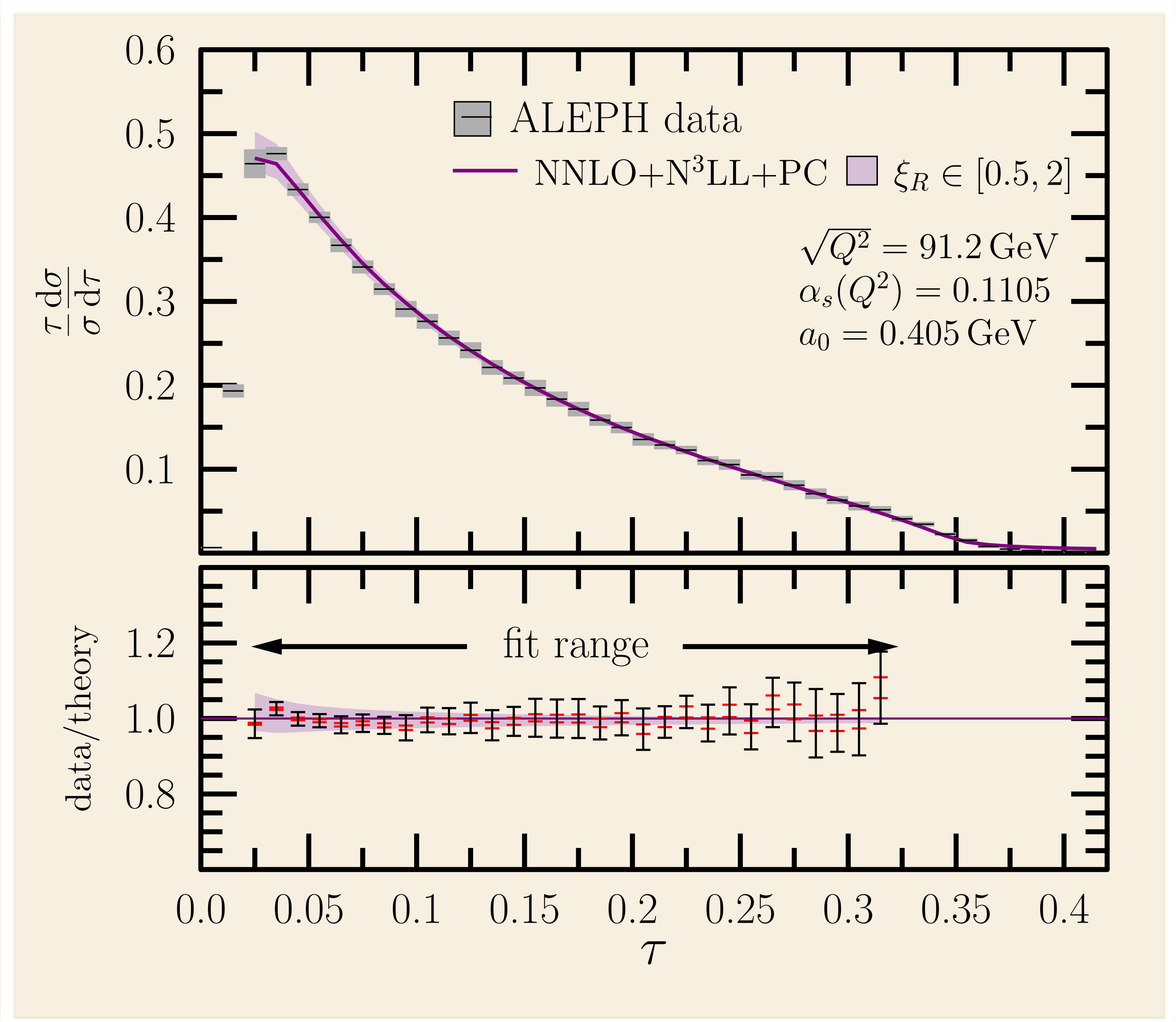
- ✓ Correct for hadronisation

  two options:

  - estimate of hadronisation using modern MC tools
  - use analytic model for power corrections, e.g.:

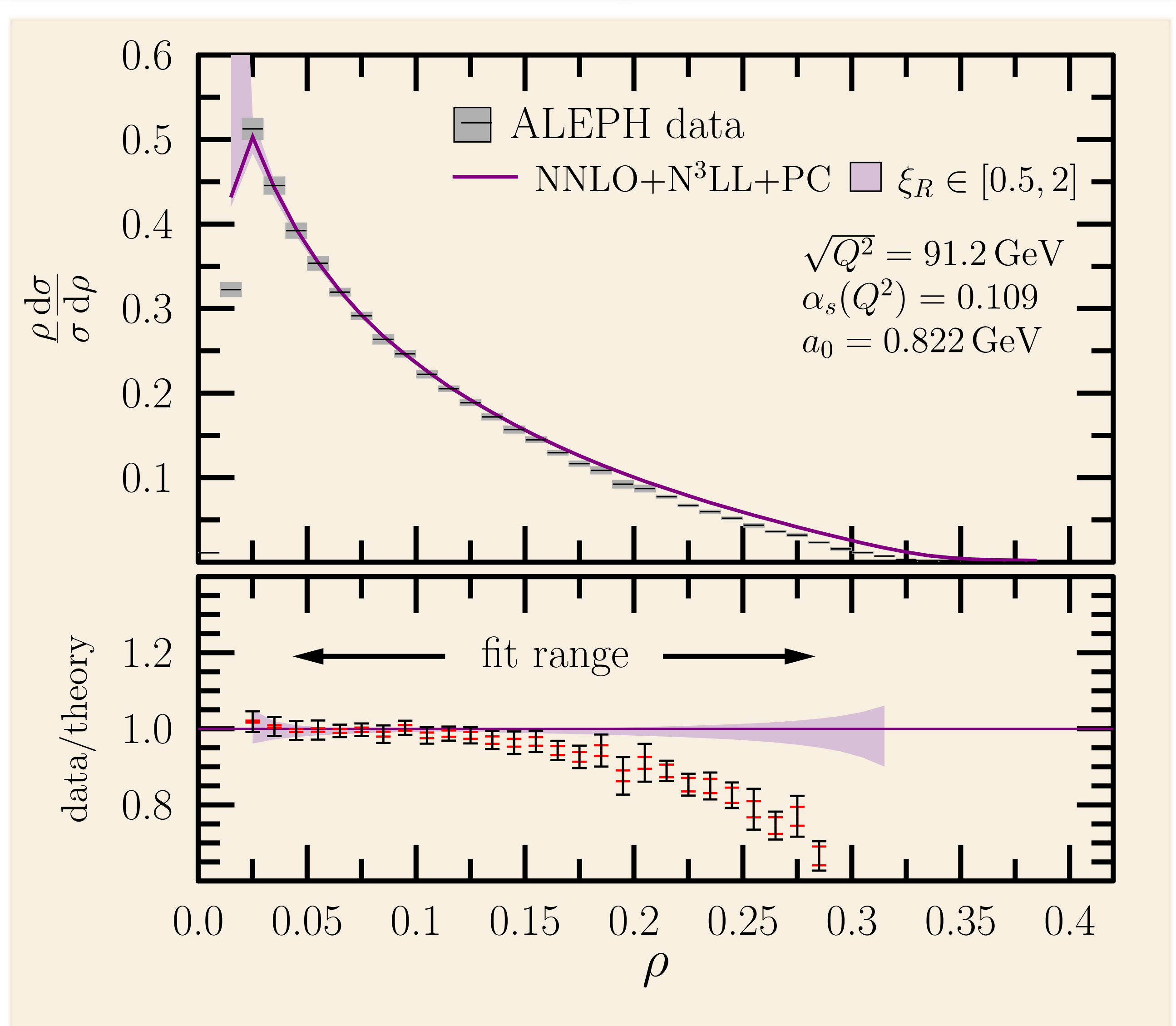
$$\frac{\tau}{\sigma}\frac{d\sigma}{d\tau}(\tau) \to \frac{\tau}{\sigma}\frac{d\sigma}{d\tau}(\tau - 2a_0)$$

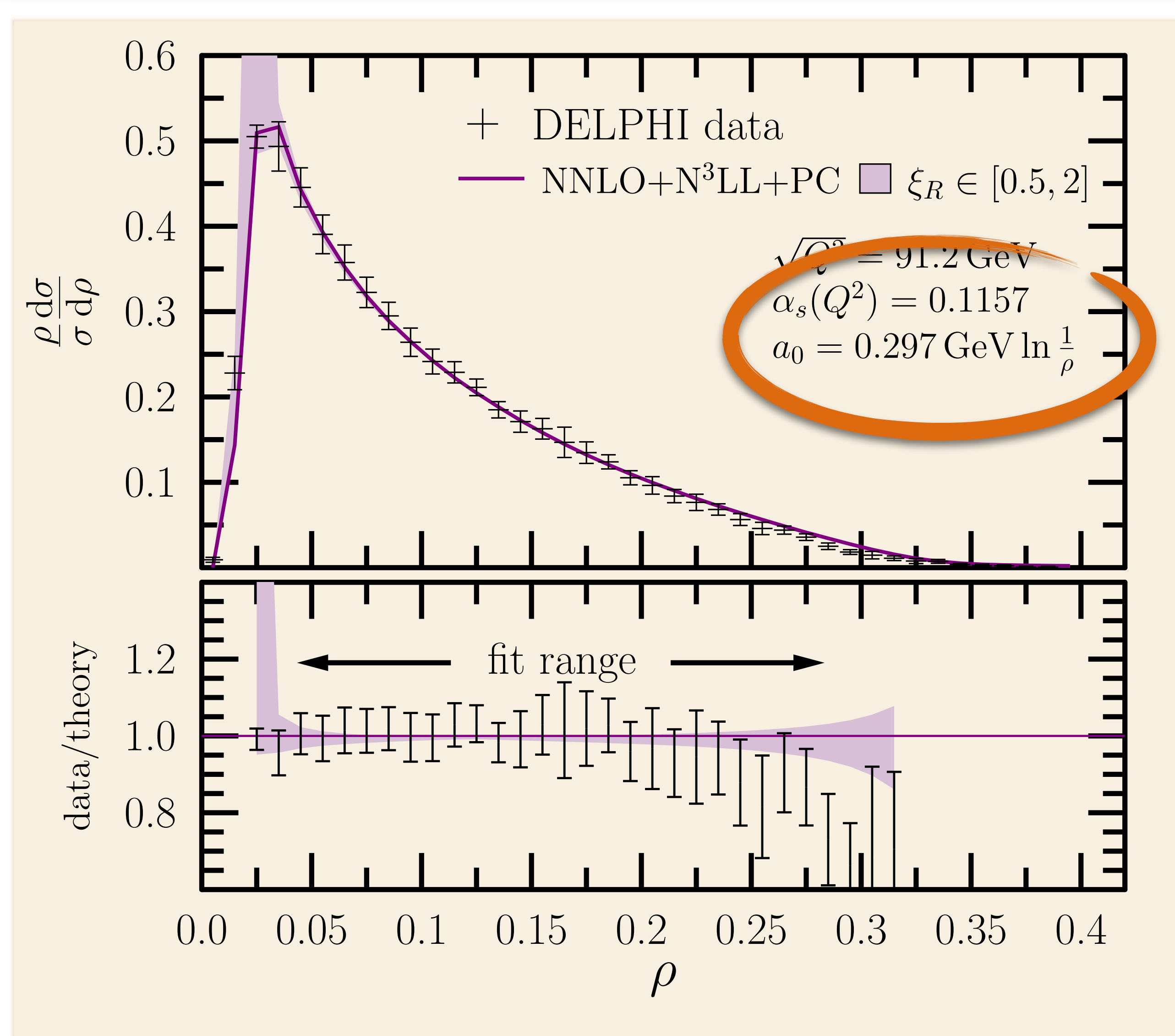
Fit to data with NNLO+N^{3}LL+PC



Works down to the peak, but

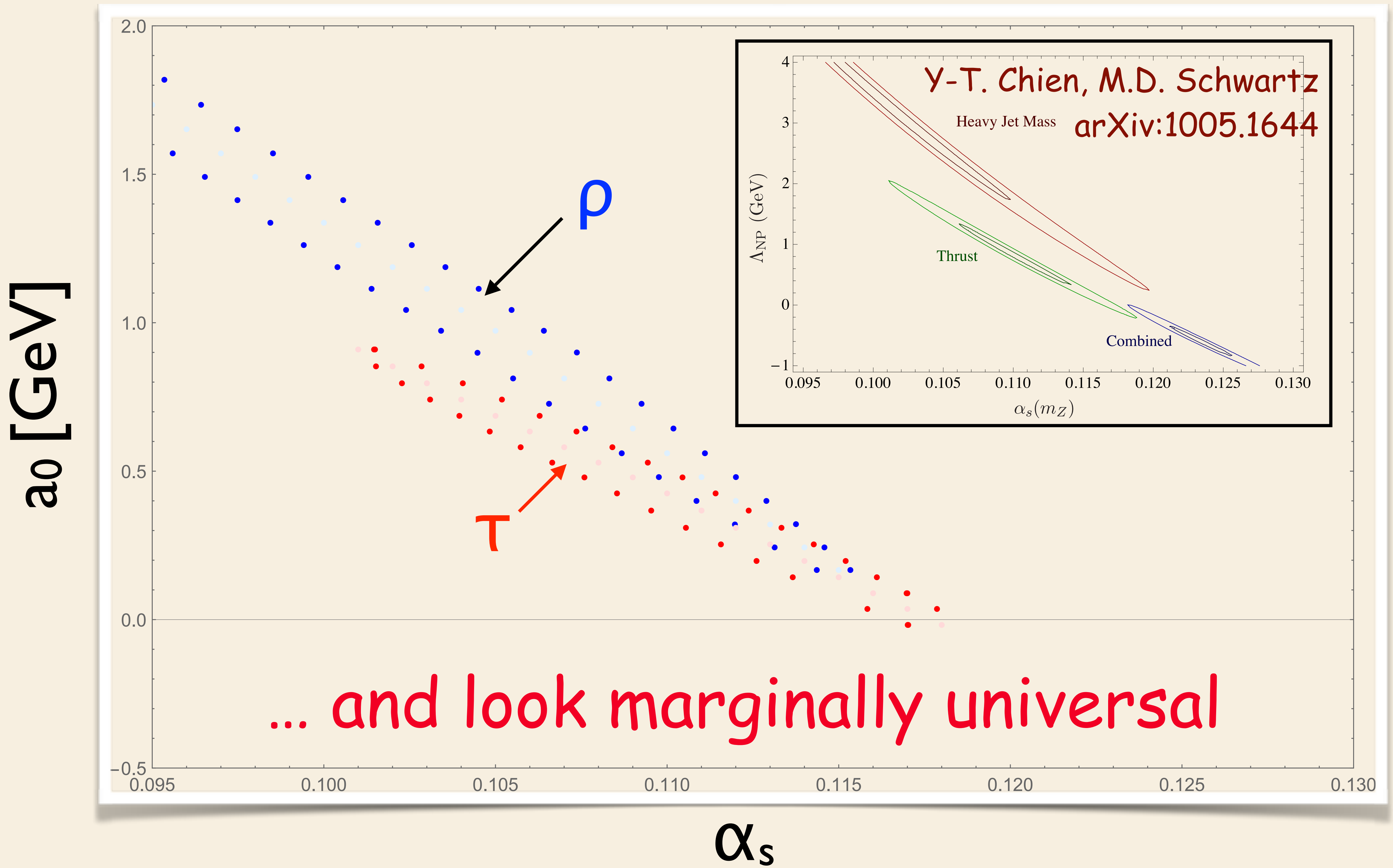
Fit data on heavy jet mass with NNLO+N^{3}LL+PC





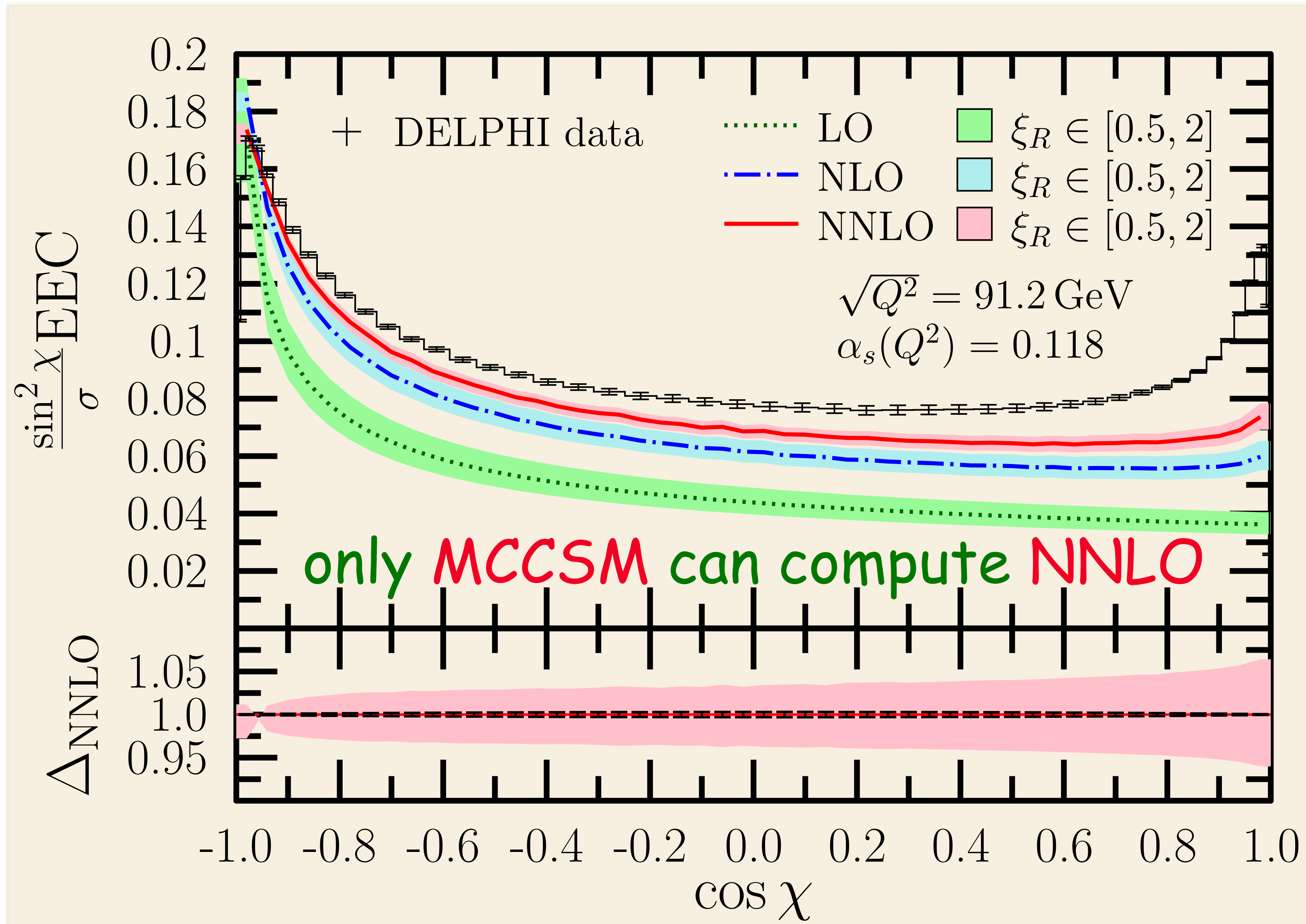
... not exactly as expected

Fit to data with PC



but a_0 and α_s are strongly anticorrelated

EEC @ fixed orders

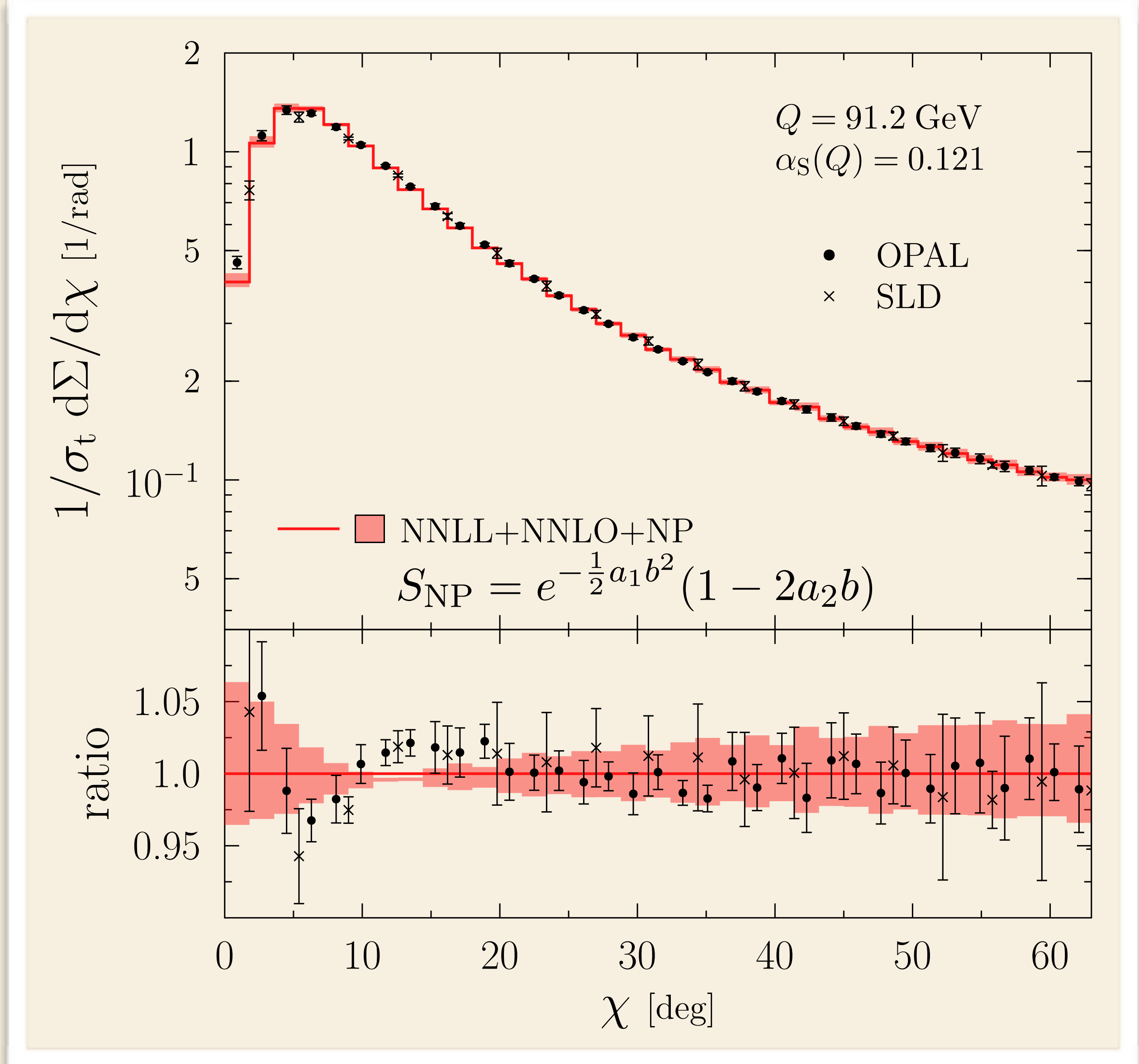


large corrections

$$\mathrm{EEC}(\chi) = \frac{1}{\sigma_{\mathrm{had}}} \sum_{i,j} \int \frac{E_i E_j}{Q^2} \times \mathrm{d}\sigma_{e^+e^- \to i\,j+X} \delta(\cos\chi + \cos\theta_{ij})$$

V. Del Duca et al, arXiv:1603.08927

EEC @ NNLO+NNLL+NP



$$\alpha_S(M_Z) = 0.121^{+0.001}_{-0.003} \qquad a_1 = 2.47^{+0.48}_{-2.38}\ \text{GeV}^2 \qquad a_2 = 0.31^{+0.27}_{-0.05}\ \text{GeV}$$

$$\text{corr}(\alpha_S, a_1, a_2) = \begin{pmatrix} 1 & 0.05 & -0.97 \\ 0.05 & 1 & -0.07 \\ -0.97 & -0.07 & 1 \end{pmatrix}$$

☞ parameters are strongly anticorrelated

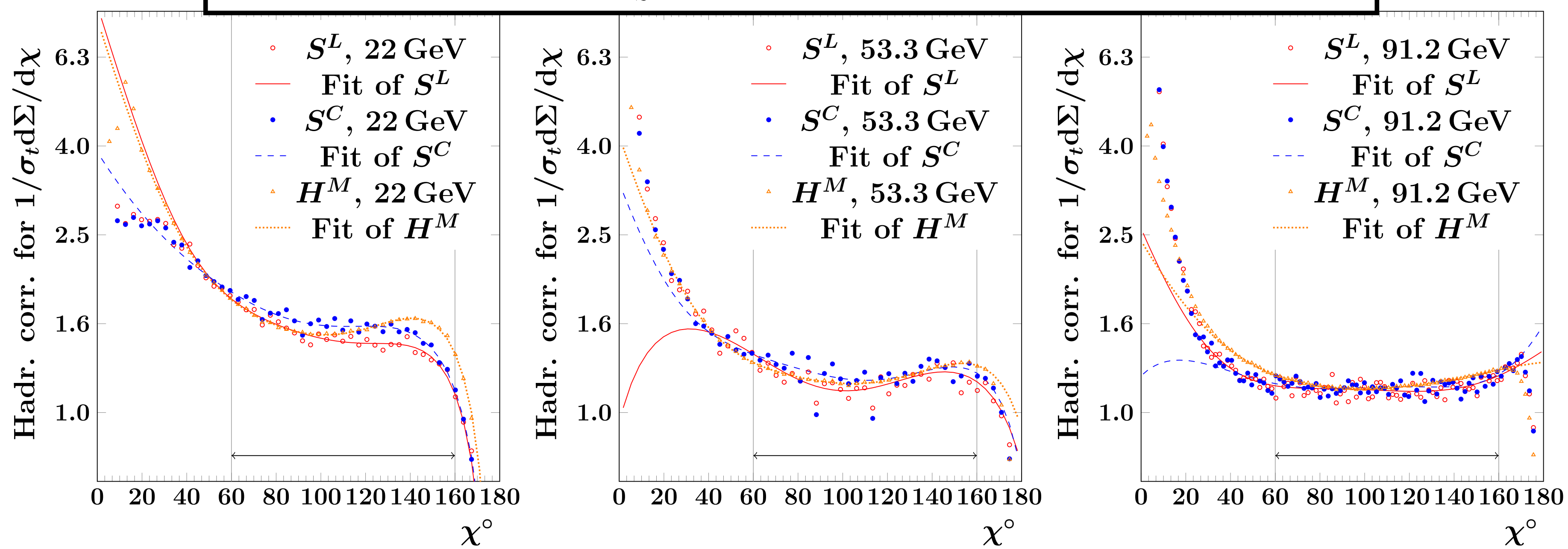
Z. Tulipánt et al, arXiv:1708.04093

How to improve?

✓ Correct for hadronisation, 2nd option:

– estimate of hadronisation using modern MC tools

$$\frac{1}{\sigma_t}\frac{d\Sigma}{d\cos\chi} = \frac{1}{\sigma_t}\int\sum_{i,j}\frac{E_i\,E_j}{Q^2}d\sigma_{e^+e^-\to ij+X}\delta(\cos\chi+\cos\theta_{ij})$$



Hadronization corrections are parametrized using smooth functions to tame statistical fluctuations

Parametrization is valid only in the fit range

Z. Tulipánt et al, arXiv: 1804.09146

Fit results

Global fit at NNLL+NLO:

$\alpha_S(M_Z) = 0.12200 \pm 0.00023(exp.) \pm 0.00113(hadr.) \pm 0.00433(ren.) \pm 0.00293(res.)$

with combined uncertainty: $\alpha_S(M_Z) = 0.12200 \pm 0.00535$

Global fit at NNLL+NNLO:

$\alpha_S(M_Z) = 0.11750 \pm 0.00018(exp.) \pm 0.00102(hadr.) \pm 0.00257(ren.) \pm 0.00078(res.)$

with combined uncertainty: $\alpha_S(M_Z) = 0.11750 \pm 0.00287$

The effect of NNLO on central value is moderate but not negligible, *ren.* uncertainty down by a factor of 2, *res.* uncertainty down by a factor of 3

The overall uncertainty is dominated by theoretical uncertainty (*ren.* and *res.*)

Z. Tulipánt et al, arXiv: 1804.09146

How to improve?

✓ Correct for hadronisation, 2nd option:

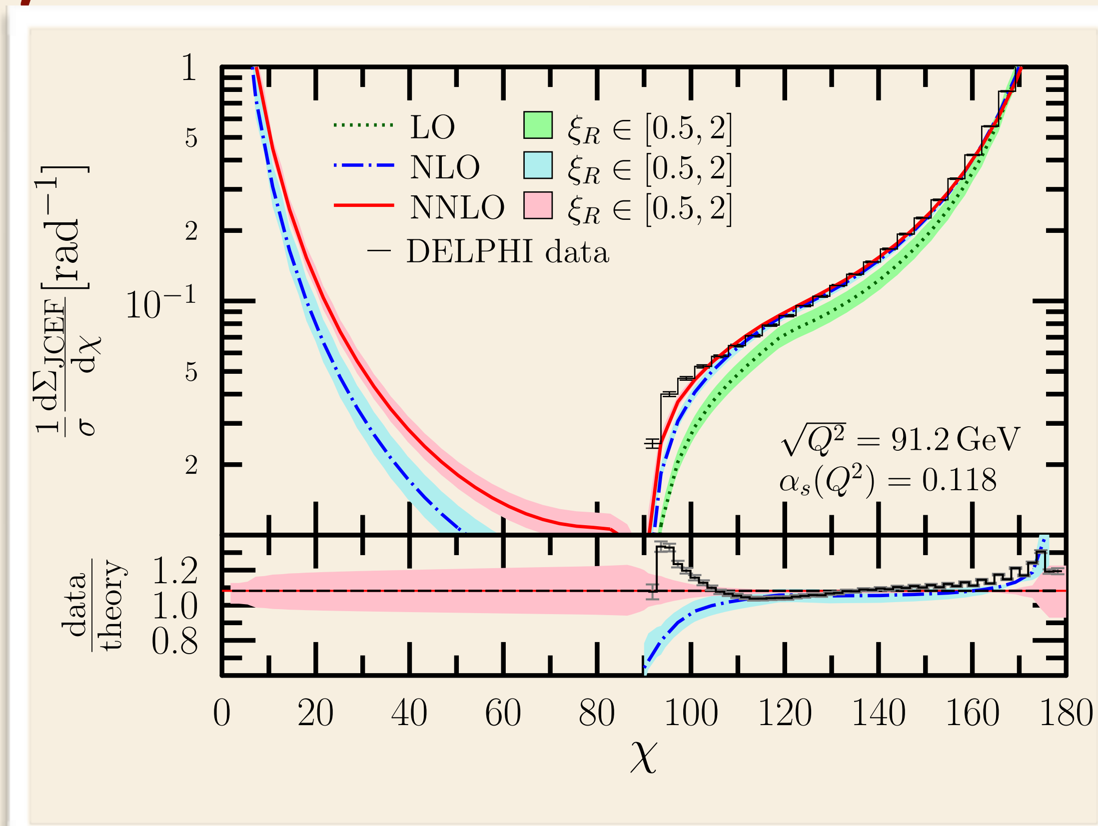
– estimate of hadronisation using modern MC tools

✓ Find observable quantities with small perturbative and hadronisation corrections:

motto: "large uncertainty in small quantity is small uncertainty"

jet cone energy fraction:

$$\frac{d\Sigma_{\mathrm{JCEF}}}{d\cos\chi} = \sum_i \int \frac{E_i}{Q} d\sigma_{e^+e^-\to i+X}\,\delta\left(\cos\chi - \frac{\vec{p}_i\cdot\vec{n}_T}{|\vec{p}_i|}\right)$$



V. Del Duca et al, arXiv:1606.03453

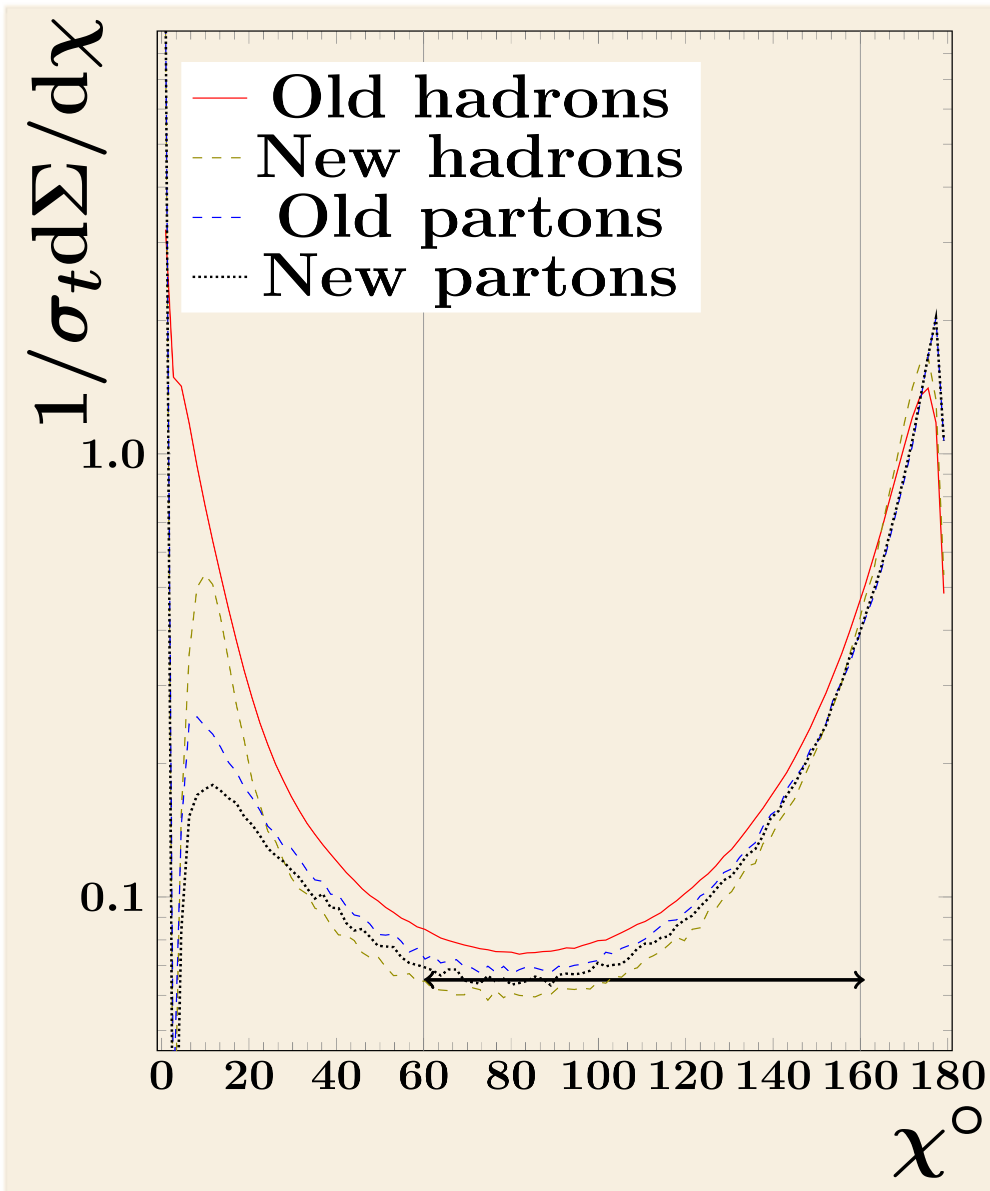
How to improve?

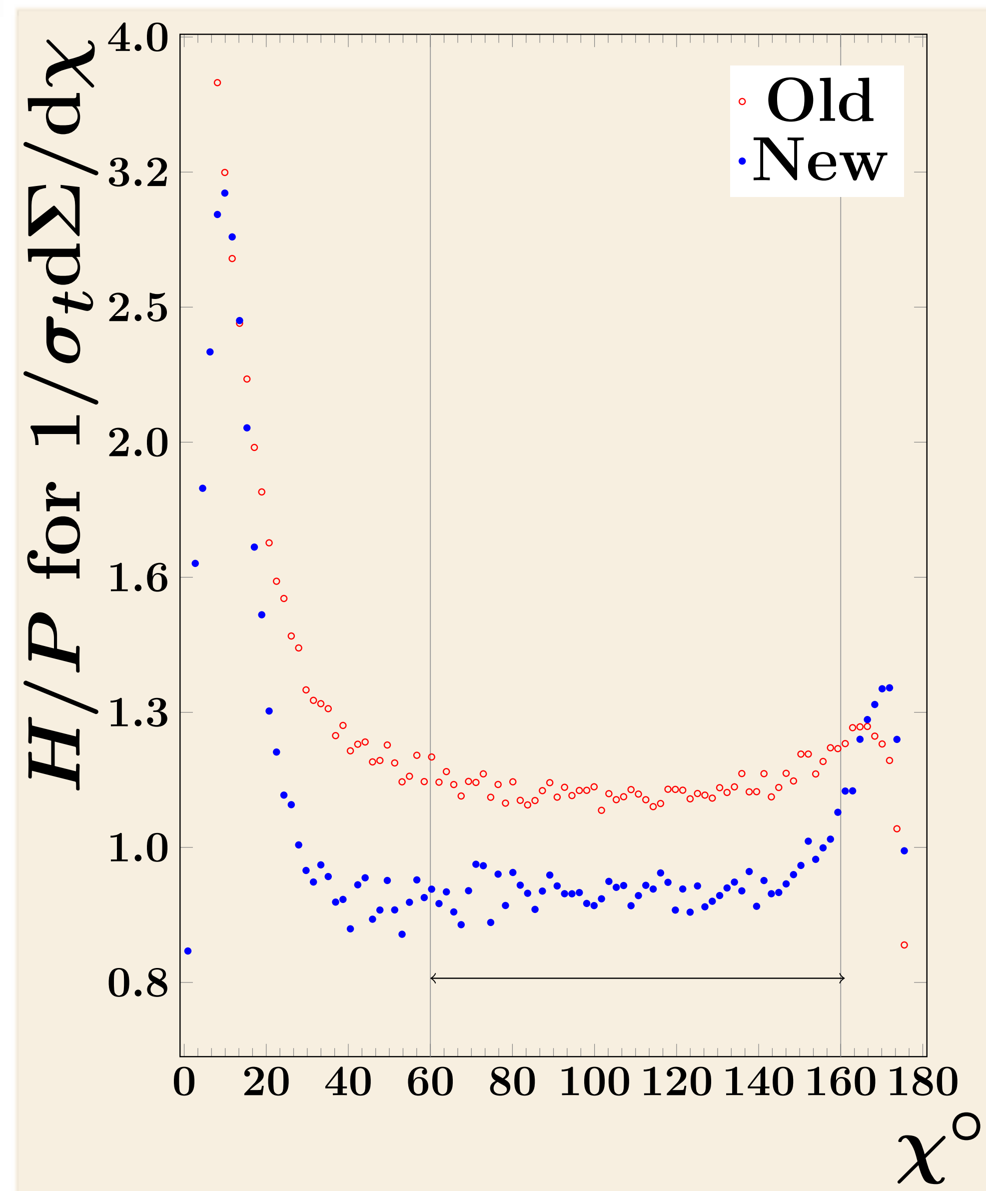
- ✓ Correct for hadronisation, 2nd option:
  - estimate of hadronisation using modern MC tools
- ✓ Find observable quantities with small perturbative and hadronisation corrections:

motto: “large uncertainty in small quantity is small uncertainty”

  - precluster hadrons and compute shapes from jets Decamp et al [ALEPH], Phys.Lett. B257 (1991) 479-491

Preclustering reduces hadronization corrections





Old: without, New: with plecustering (requiring 5 jets)

A. Verbytskyi, private communication

How to improve?

- ✓ Correct for hadronisation, 2nd option:
  - estimate of hadronisation using modern MC tools
- ✓ Find observable quantities with small perturbative and hadronisation corrections:

motto: "large uncertainty in small quantity is small uncertainty"

  - precluster hadrons and compute shapes from jets Decamp et al [ALEPH], Phys.Lett. B257 (1991) 479-491
  - groomed (soft drop) event shapes, designed to reduce contamination from non-perturbative effects

Soft drop event shapes

Soft drop grooming is defined

for a jet with radius R using Cambridge-Aachen clustering as:

1. Undo the last step of the clustering for the jet J, and split it into two sub-jets.
2. Check if these sub-jets pass the soft drop condition, which is defined for e^+e^- collisions as:

$$\frac{\min[E_i, E_j]}{E_i + E_j} > z_{\rm cut}(1 - \cos\theta_{ij})^{\beta/2} \quad \text{or} \quad z_{\rm cut}\left(\frac{1-\cos\theta_{ij}}{1-\cos R}\right)^{\beta/2}$$

   where E_i and E_j are the energies of the two sub-jets and θ_{ij} is the angle between them.

3. If the splitting fails this condition, the softer sub-jet is dropped and the groomer continues to the next step in the clustering. In other words the jet J is set to be the harder of the two sub-jets.
4. If the splitting passes this condition the procedure ends and the jet J is the soft-drop jet.

Soft drop thrust

Special kind of grooming

(a) the thrust axis n_T is calculated, thus dividing the event into two hemispheres;

(b) the soft-drop algorithm is applied in each hemisphere;

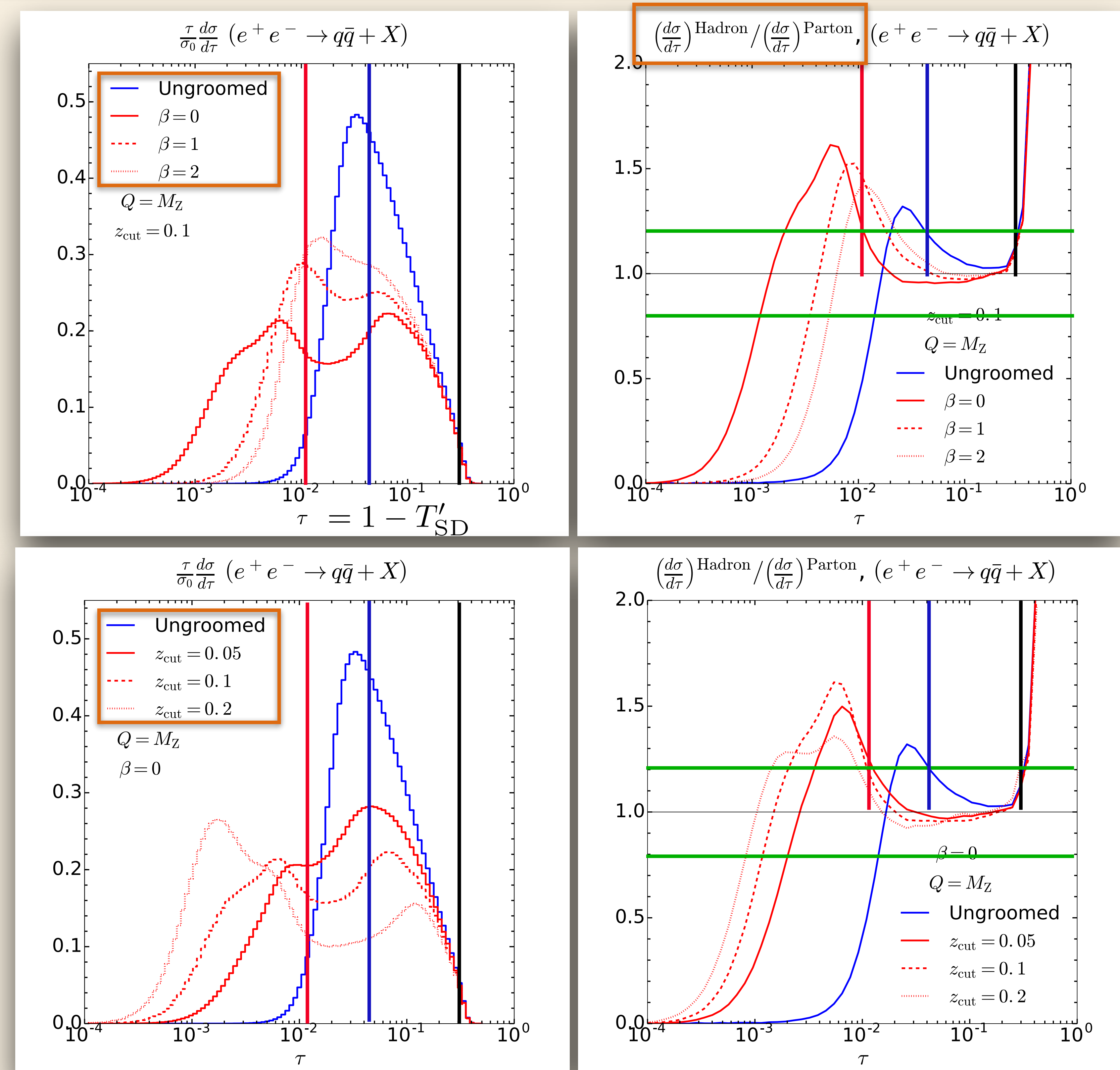
(c) the sets of particles left in the two hemispheres after soft drop constitute the soft-drop hemispheres H^L and H^R, on which the soft-drop thrust T'_{SD} is defined as

$$T'_{\mathrm{SD}} = \frac{\sum_{i\in\mathcal{H}^L_{\mathrm{SD}}} |\vec{n_L}\cdot\vec{p}_i|}{\sum_{i\in\mathcal{E}_{\mathrm{SD}}} |\vec{p}_i|} + \frac{\sum_{i\in\mathcal{H}^R_{\mathrm{SD}}} |\vec{n_R}\cdot\vec{p}_i|}{\sum_{i\in\mathcal{E}_{\mathrm{SD}}} |\vec{p}_i|}$$

Numerically challenging computation

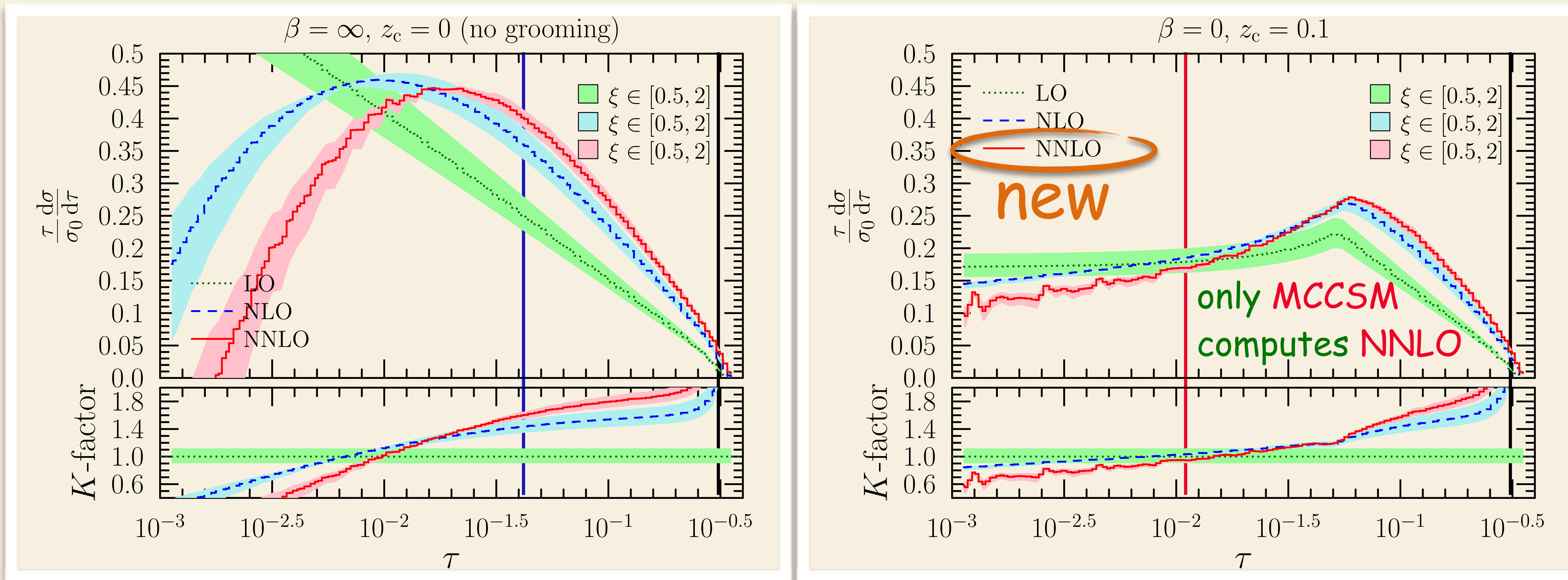
J. Baron et al, arXiv: 1803.04719

Soft drop thrust in PYTHIA



J. Baron et al, arXiv: 1803.04719

Soft drop thrust



K-factors are significantly smaller for soft drop thrust in the possible fit range

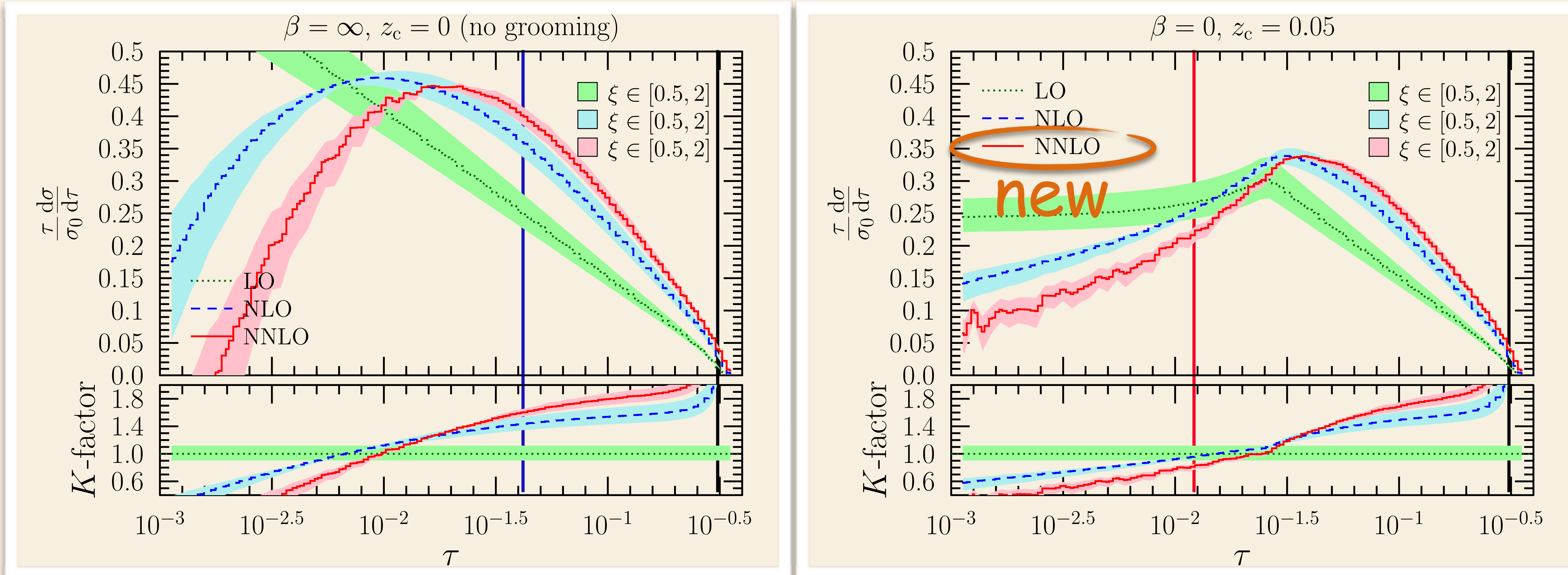
$$K_{\rm NLO}(\mu) = \frac{d\sigma_{\rm NLO}(\mu)}{dO} \Big/ \frac{d\sigma_{\rm LO}(Q)}{dO}$$

$$K_{\rm NNLO}(\mu) = \frac{d\sigma_{\rm NNLO}(\mu)}{dO} \Big/ \frac{d\sigma_{\rm LO}(Q)}{dO}$$

expect smaller dependence on reg. scale

A. Kardos et al, arXiv: 1807.11472

Soft drop thrust



K-factors are significantly smaller
for soft drop thrust
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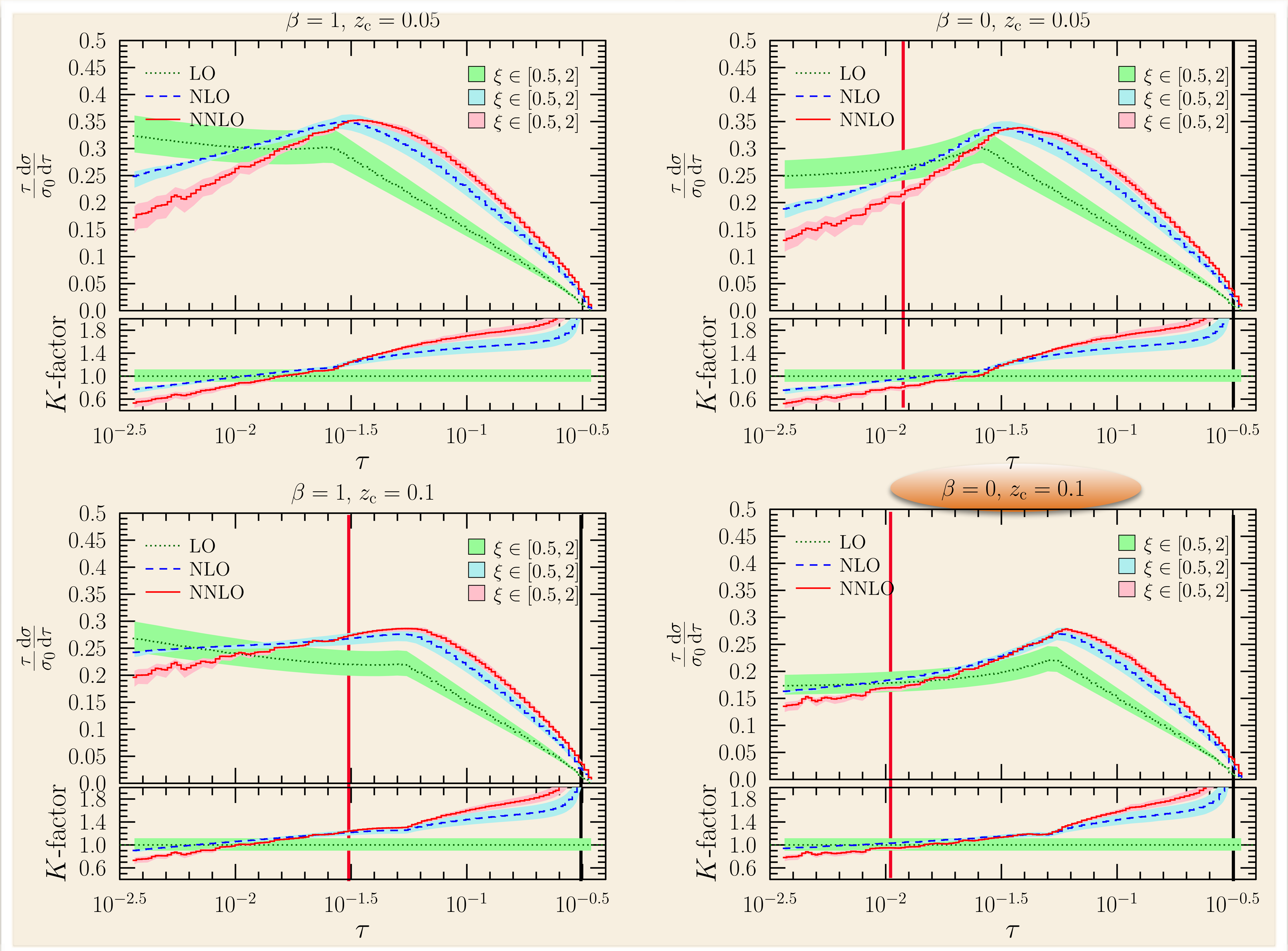
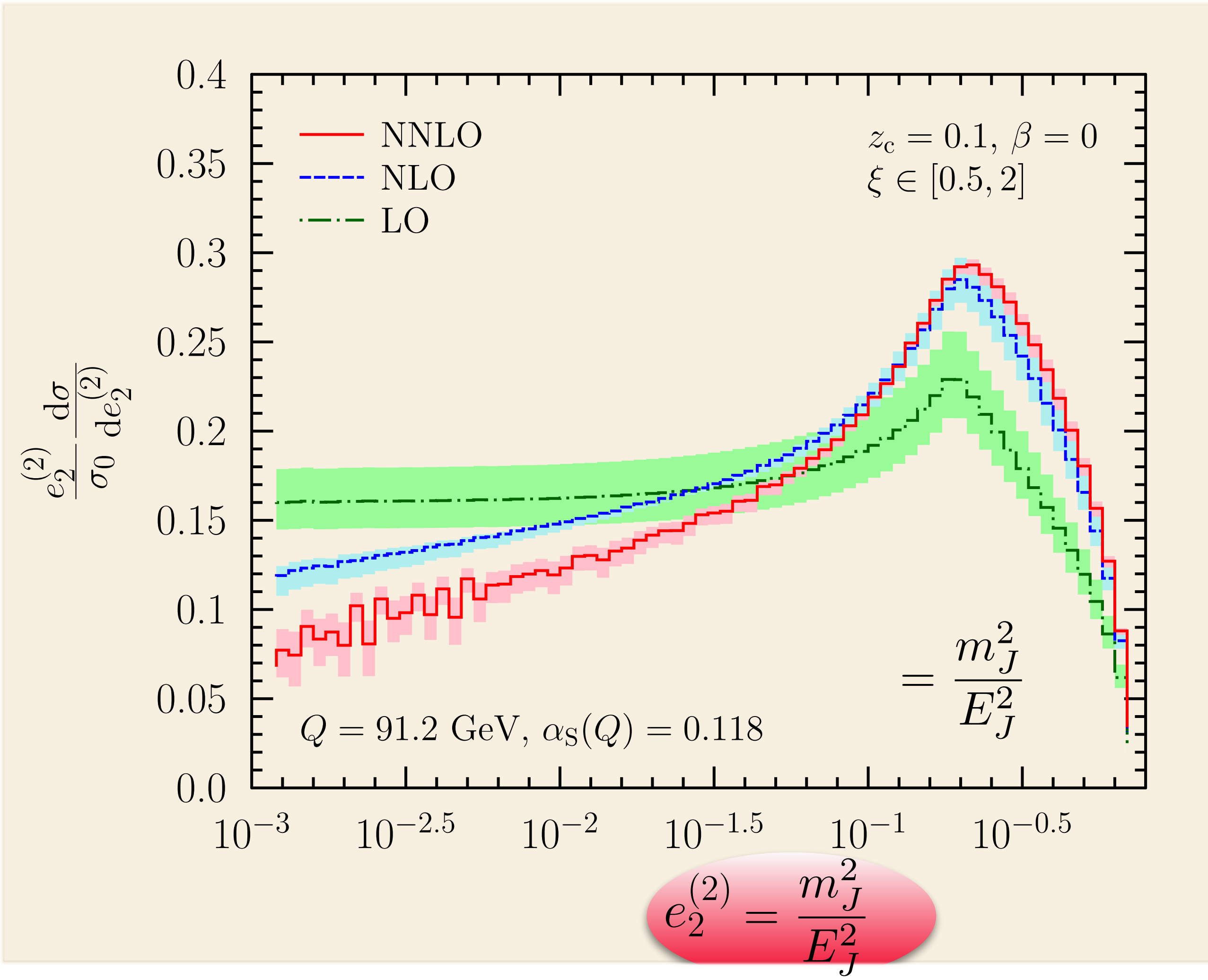
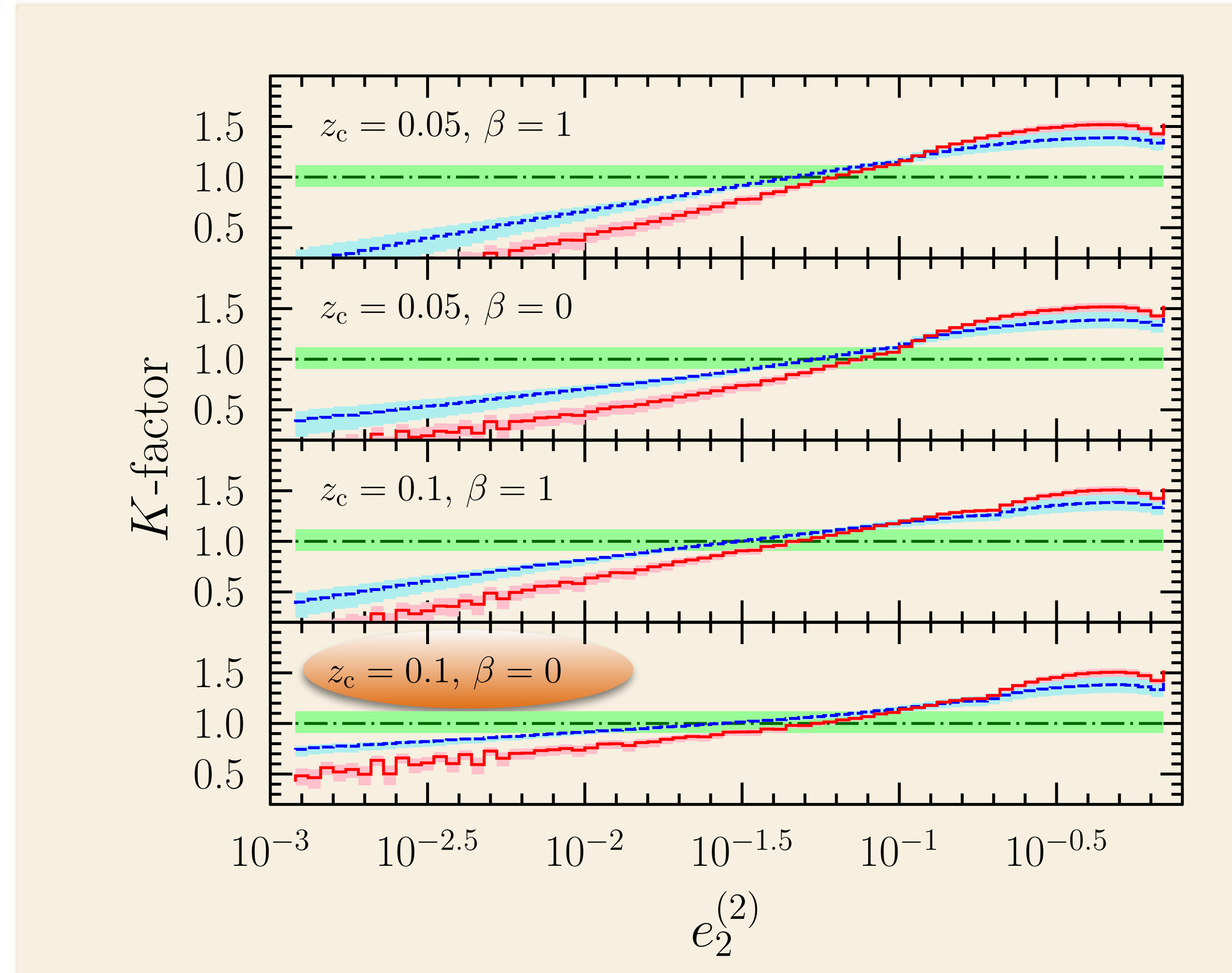
expect smaller dependence on renormalization scale

Soft drop thrust



Soft drop hemisphere mass





cluster an event into exactly two jets, J is the heavier

A. Kardos et al, arXiv: 1807.11472

Conclusions

Conclusions

- ✓ Precise determination of the strong coupling using hadronic final states in electron-positron annihilation requires
  - careful selection of observables (and data — not discussed here)
  - methods to reduce hadronisation corrections
  - estimation of the hadronisation corrections with modern MCs
- ✓ MCCSM was used to compute differential distributions for groomed (soft drop) event shapes:
  - thrust
  - hemisphere invariant mass
  - narrow jet invariant mass (not shown here)
- ✓ Our predictions
  - are stable numerically
  - show better perturbative stability (smaller scale dependence) than un-groomed event shapes

Outlook: prospects for α_s

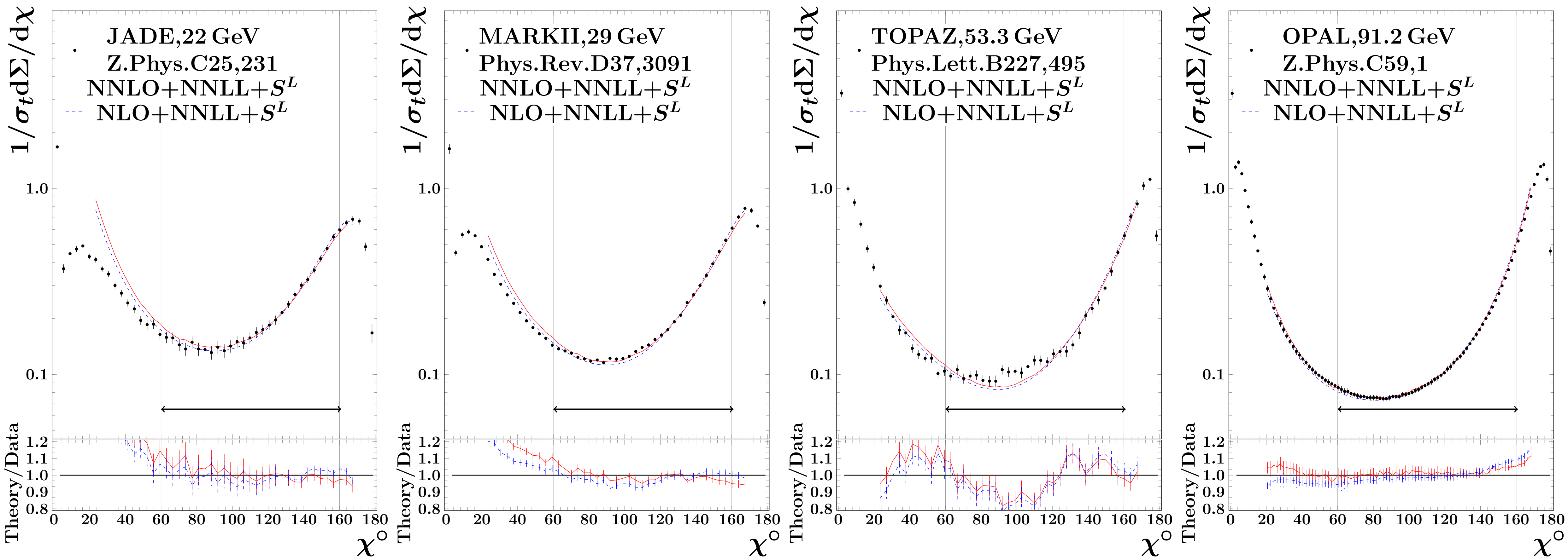
Method	Current relative precision		Future relative precision
e^+e^- evt shapes	expt $\sim 1\%$ (LEP) thry ~ 1–3% (NNLO+up to N^{3}LL, $\boxed{\text{n.p. signif.}}$)	[27]	$< 1\%$ possible (ILC/TLEP) $\sim 1\%$ (control n.p. via Q^2-dep.)
e^+e^- jet rates	expt $\sim 2\%$ (LEP) thry $\sim 1\%$ (NNLO, n.p. moderate)	[28]	$< 1\%$ possible (ILC/TLEP) $\sim 0.5\%$ (NLL missing)
precision EW	expt $\sim 3\%$ (R_Z, LEP) thry $\sim 0.5\%$ (N^{3}LO, n.p. small)	[9, 29]	0.1% (TLEP [10]), 0.5% (ILC [11]) $\sim 0.3\%$ (N^{4}LO feasible, $\sim$ 10 yrs)
τ decays	expt $\sim 0.5\%$ (LEP, B-factories) thry $\sim 2\%$ (N^{3}LO, n.p. small)	[8]	$< 0.2\%$ possible (ILC/TLEP) $\sim 1\%$ (N^{4}LO feasible, $\sim$ 10 yrs)
ep colliders	~ 1–2% (pdf fit dependent) (mostly theory, NNLO)	[30, 31], [32, 33]	0.1% (LHeC + HERA [23]) $\sim 0.5\%$ (at least N^{3}LO required)
hadron colliders	$\sim 4\%$ (Tev. jets), $\sim 3\%$ (LHC $t\bar{t}$) (NLO jets, NNLO $t\bar{t}$, gluon uncert.)	[17, 21, 34]	$< 1\%$ challenging (NNLO jets imminent [22])
lattice	$\sim 0.5\%$ (Wilson loops, correlators, ...) (limited by accuracy of pert. th.)	[35–37]	$\sim 0.3\%$ ($\sim$ 5 yrs [38])

Determination of strong coupling from e^+e^- data with decreased theoretical uncertainty might be possible

J.M. Campbell et al [Snowmass], arXiv: 1310.5189

Appendix

Fit prediction for EEC to data



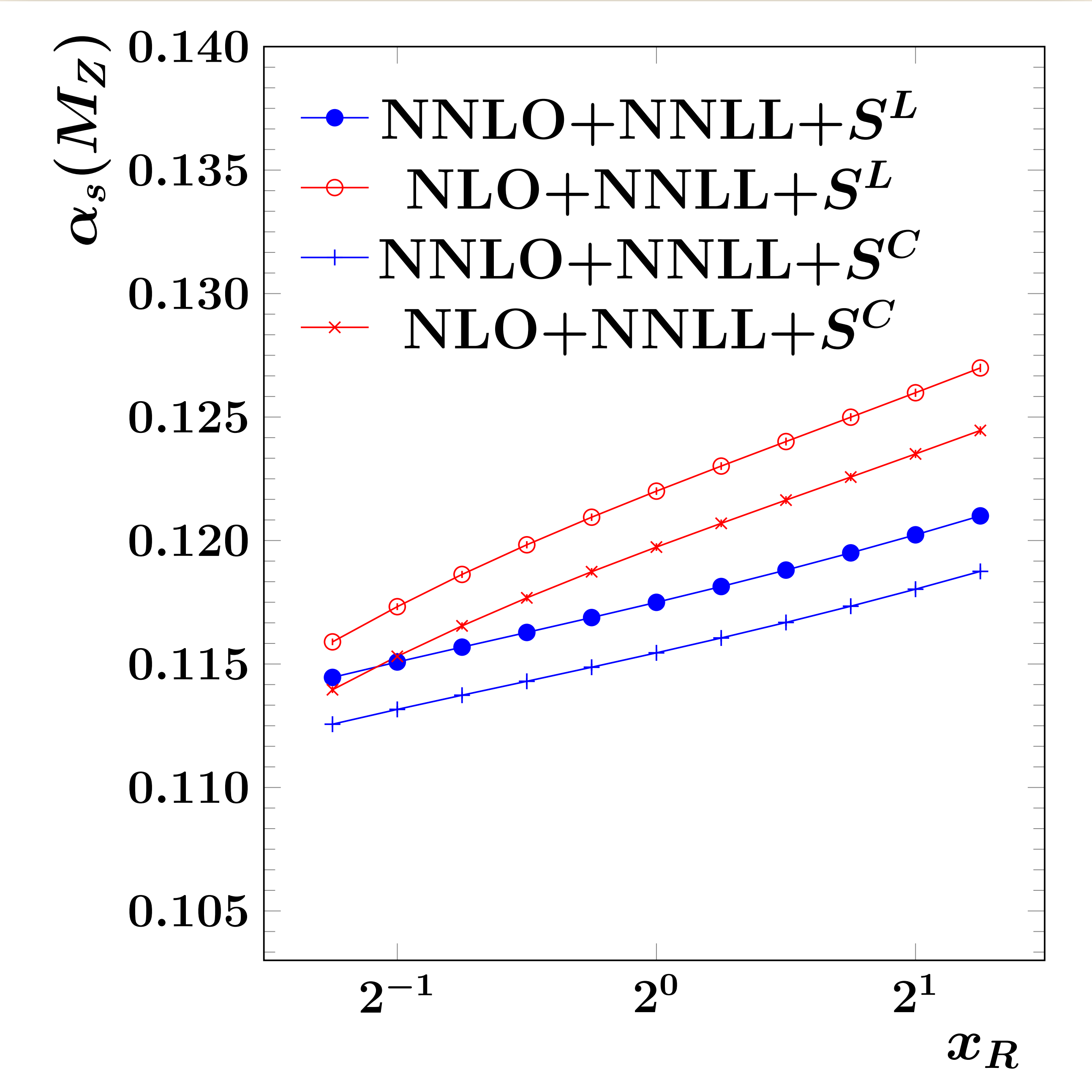
Fit range: $[60^\circ, 160^\circ]$ ($[117^\circ, 165^\circ]$ for DMW setup)

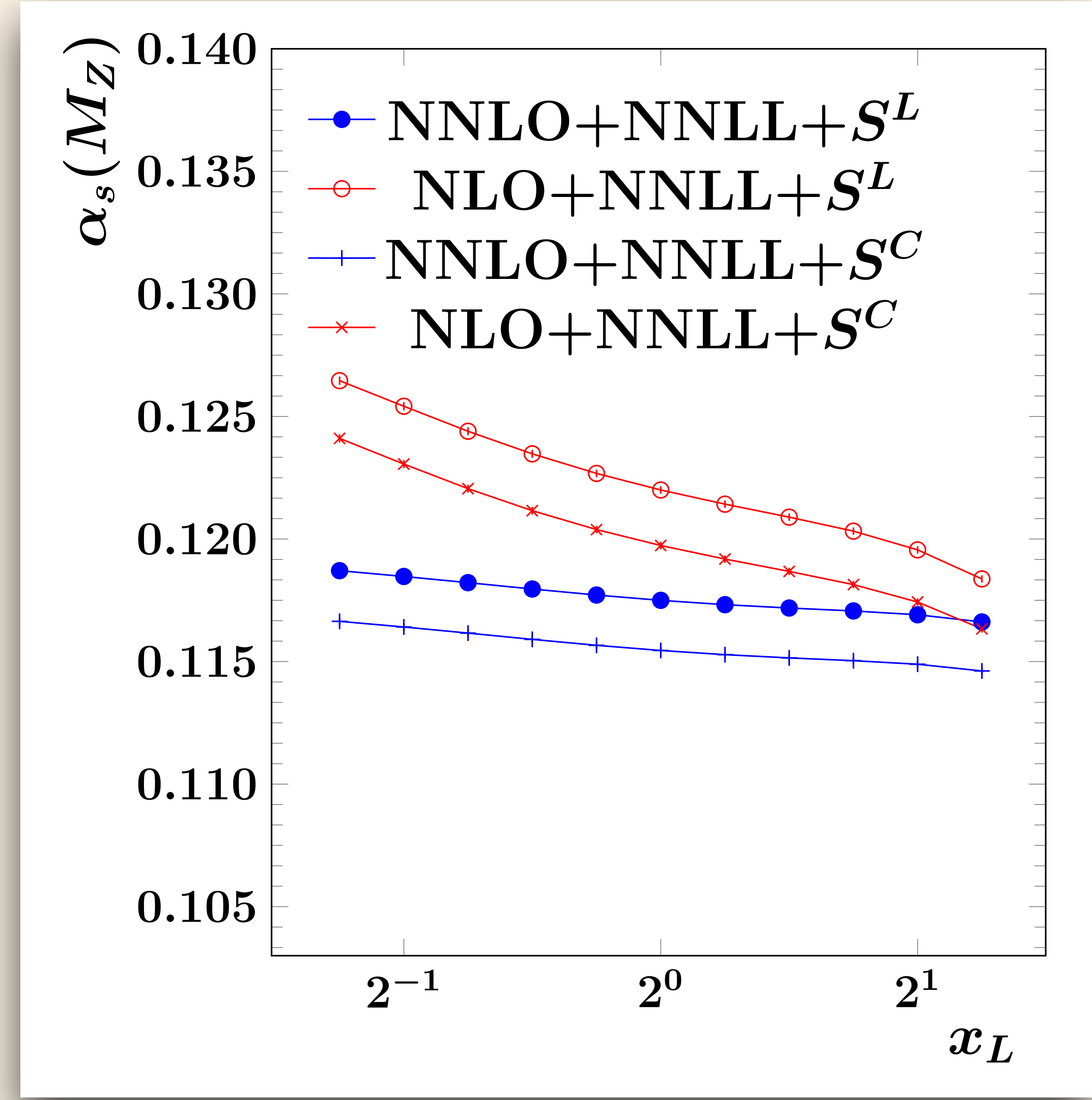
Fit range was chosen to avoid regions where the theoretical prediction or hadronization corrections become unreliable

The result is insensitive to a $\pm 5^\circ$ change in the fit range

Z. Tulipánt et al, arXiv: 1804.09146

Dependence on unphysical scales





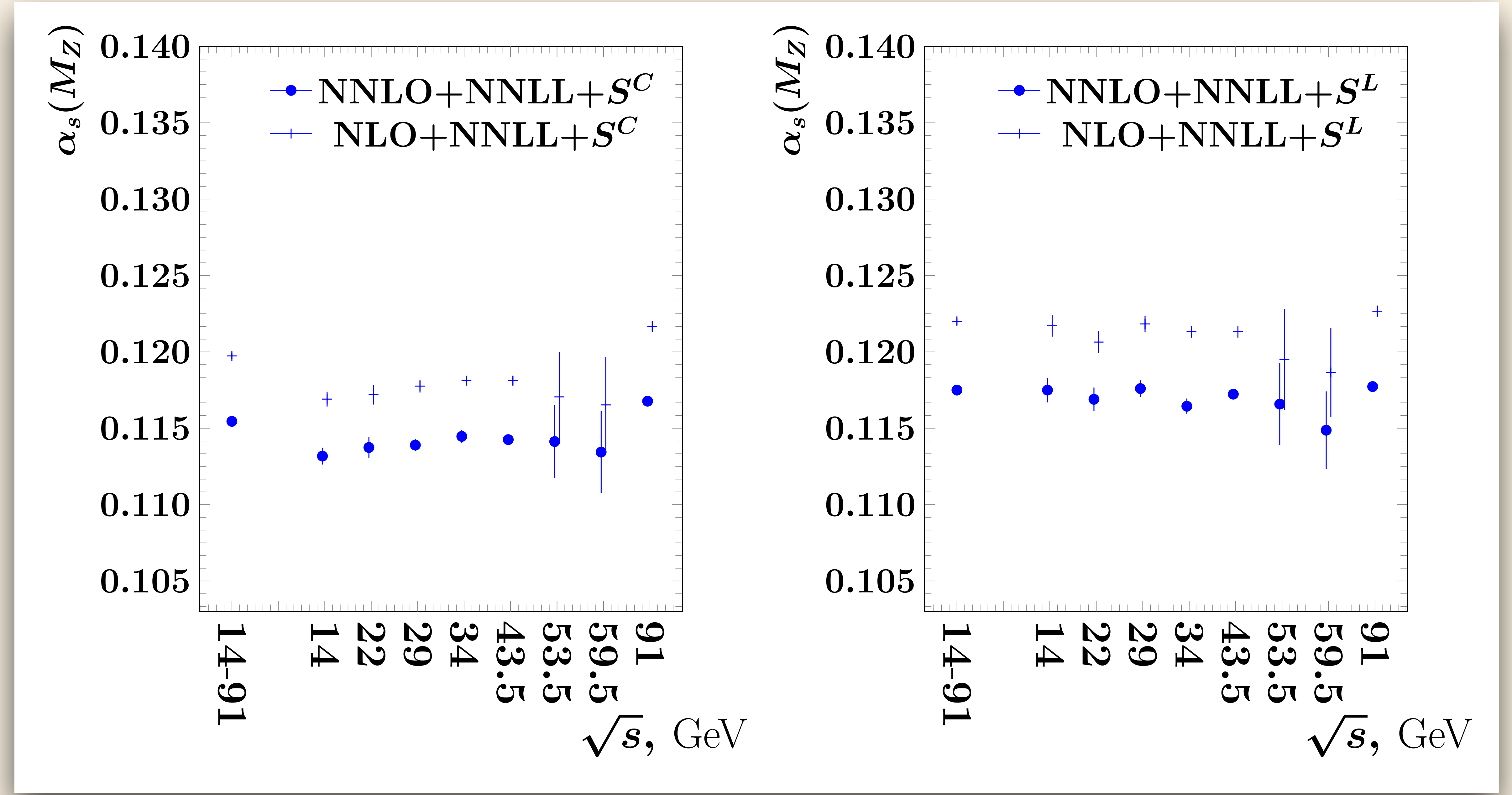
variation of

renormalization scale

resummation scale

Z. Tulipánt et al, arXiv: 1804.09146

Dependence on energy



fitted values

Z. Tulipánt et al, arXiv: 1804.09146

CoLoRFulNNLO

Problem

$$\sigma_m^{\mathrm{NNLO}} = \sigma_{m+2}^{\mathrm{RR}} + \sigma_{m+1}^{\mathrm{RV}} + \sigma_m^{\mathrm{VV}}$$

$$\equiv \int_{m+2} \mathrm{d}\sigma_{m+2}^{\mathrm{RR}} J_{m+2} + \int_{m+1} \mathrm{d}\sigma_{m+1}^{\mathrm{RV}} J_{m+1} + \int_m \mathrm{d}\sigma_m^{\mathrm{VV}} J_m$$

- matrix elements are known for σ^{RR} and σ^{RV} for many processes
- σ^{VV} is known for many $0 \to 4$ parton, V+3 parton, VV+2 parton processes – higher multiplicities are on the horizon
- the three contributions are separately divergent in d = 4 dimensions:
  - in σ^{RR} kinematical singularities as one or two partons become unresolved yielding ϵ-poles at $O(\epsilon^{-4}, \epsilon^{-3}, \epsilon^{-2}, \epsilon^{-1})$ after integration over phase space, no explicit ϵ-poles
  - in σ^{RV} kinematical singularities as one parton becomes unresolved yielding ϵ-poles at $O(\epsilon^{-2}, \epsilon^{-1})$ after integration over phase space + explicit ϵ-poles at $O(\epsilon^{-2}, \epsilon^{-1})$
  - in σ^{VV} explicit ϵ-poles at $O(\epsilon^{-4}, \epsilon^{-3}, \epsilon^{-2}, \epsilon^{-1})$

How to combine to obtain finite cross section?

Approaches

- Sector decomposition

  Anastasiou, Melnikov, Petriallo et al 2004-

- Antennae subtraction

  Gehrmann, Gehrmann-De Ridder, Glover et al 2004-

- q_T-slicing

  S. Catani, M. Grazzini et al 2007-

- SecToR-Improved Phase sPacE for Real radiation (STRIPPER)

  Czakon et al 2010-

- τ_N-slicing

  Boughezal et al 2015-
  Gaunt et al 2015-

- Completely Local SubtRactions for Fully Differential Predictions at NNLO (CoLoRFulNNLO)

  ZT, Somogyi et al 2005-

Several options available - why a new one?

Our goal is to devise a subtraction scheme with

- ✓ fully local counter-terms
  (efficiency and mathematically well-defined)
- ✓ fully differential predictions
  (with jet functions defined in d = 4)
- ✓ explicit expressions including flavor and color
  (color space notation is used)
- ✓ completely general construction
  (valid in any order of perturbation theory)
- ✓ option to constrain subtraction near singular
  regions (important check)

such schemes are known at NLO (CS-dipoles, FKS etc)

How to build a local subtraction scheme?

Steps used at NLO:

S. Catani, S. Dittmaier, M.H. Seymour, ZT
hep-ph/0201036

- ✓ compute QCD factorization formulae
  (universal)
- ✓ construct local subtractions on whole phase space
  (explicit and universal in d = 4)
- ✓ integrate subtractions over unresolved phase space
  (once and for all)
- ✓ cancel IR poles
  (analytically, universal)
- ✓ implement integration of finite part in partonic MC
  (simple user interface defines observables)

steps proven to be too difficult at NNLO:

given up by many, used here

Structure

of subtractions is governed by the jet functions

$$\sigma^{\mathrm{NNLO}} = \sigma^{\mathrm{RR}}_{m+2} + \sigma^{\mathrm{RV}}_{m+1} + \sigma^{\mathrm{VV}}_{m} = \sigma^{\mathrm{NNLO}}_{m+2} + \sigma^{\mathrm{NNLO}}_{m+1} + \sigma^{\mathrm{NNLO}}_{m}$$

$$\sigma^{\mathrm{NNLO}}_{m+2} = \int_{m+2} \left\{ \mathrm{d}\sigma^{\mathrm{RR}}_{m+2} J_{m+2} - \mathrm{d}\sigma^{\mathrm{RR,A}_2}_{m+2} J_m - \left(\mathrm{d}\sigma^{\mathrm{RR,A}_1}_{m+2} J_{m+1} - \mathrm{d}\sigma^{\mathrm{RR,A}_{12}}_{m+2} J_m \right) \right\}$$

$$\sigma^{\mathrm{NNLO}}_{m+1} = \int_{m+1} \left\{ \left(\mathrm{d}\sigma^{\mathrm{RV}}_{m+1} + \int_1 \mathrm{d}\sigma^{\mathrm{RR,A}_1}_{m+2} \right) J_{m+1} - \left[\mathrm{d}\sigma^{\mathrm{RV,A}_1}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR,A}_1}_{m+2} \right)^{\mathrm{A}_1} \right] J_m \right\}$$

$$\sigma^{\mathrm{NNLO}}_{m} = \int_{m} \left\{ \mathrm{d}\sigma^{\mathrm{VV}}_{m} + \int_2 \left(\mathrm{d}\sigma^{\mathrm{RR,A}_2}_{m+2} - \mathrm{d}\sigma^{\mathrm{RR,A}_{12}}_{m+2} \right) + \int_1 \left[\mathrm{d}\sigma^{\mathrm{RV,A}_1}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR,A}_1}_{m+2} \right)^{\mathrm{A}_1} \right] \right\} J_m$$

Structure

of subtractions is governed by the jet functions

$$\sigma^{\rm NNLO} = \sigma^{\rm RR}_{m+2} + \sigma^{\rm RV}_{m+1} + \sigma^{\rm VV}_{m} = \sigma^{\rm NNLO}_{m+2} + \sigma^{\rm NNLO}_{m+1} + \sigma^{\rm NNLO}_{m}$$

$$\sigma^{\rm NNLO}_{m+2} = \int_{m+2} \left\{ {\rm d}\sigma^{\rm RR}_{m+2} J_{m+2} - {\rm d}\sigma^{\rm RR,A_2}_{m+2} J_m - \left({\rm d}\sigma^{\rm RR,A_1}_{m+2} J_{m+1} - {\rm d}\sigma^{\rm RR,A_{12}}_{m+2} J_m \right) \right\}$$

$$\sigma^{\rm NNLO}_{m+1} = \int_{m+1} \left\{ \left({\rm d}\sigma^{\rm RV}_{m+1} + \int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \right) J_{m+1} - \left[{\rm d}\sigma^{\rm RV,A_1}_{m+1} + \left(\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \right)^{\rm A_1} \right] J_m \right\}$$

$$\sigma^{\rm NNLO}_{m} = \int_{m} \left\{ {\rm d}\sigma^{\rm VV}_{m} + \int_2 \left({\rm d}\sigma^{\rm RR,A_2}_{m+2} - {\rm d}\sigma^{\rm RR,A_{12}}_{m+2} \right) + \int_1 \left[{\rm d}\sigma^{\rm RV,A_1}_{m+1} + \left(\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \right)^{\rm A_1} \right] \right\} J_m$$

RR,A_2 regularizes doubly-unresolved limits

G. Somogyi, ZT hep-ph/0609041, hep-ph/0609043
G. Somogyi, ZT, V. Del Duca hep-ph/0502226, hep-ph/0609042
Z. Nagy, G. Somogyi, ZT hep-ph/0702273

Structure

of subtractions is governed by the jet functions

$$\sigma^{\mathrm{NNLO}} = \sigma^{\mathrm{RR}}_{m+2} + \sigma^{\mathrm{RV}}_{m+1} + \sigma^{\mathrm{VV}}_{m} = \sigma^{\mathrm{NNLO}}_{m+2} + \sigma^{\mathrm{NNLO}}_{m+1} + \sigma^{\mathrm{NNLO}}_{m}$$

$$\sigma^{\mathrm{NNLO}}_{m+2} = \int_{m+2} \left\{ \mathrm{d}\sigma^{\mathrm{RR}}_{m+2} J_{m+2} - \mathrm{d}\sigma^{\mathrm{RR,A_2}}_{m+2} J_m - \left(\mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} J_{m+1} - \mathrm{d}\sigma^{\mathrm{RR,A_{12}}}_{m+2} J_m \right) \right\}$$

$$\sigma^{\mathrm{NNLO}}_{m+1} = \int_{m+1} \left\{ \left(\mathrm{d}\sigma^{\mathrm{RV}}_{m+1} + \int_1 \mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} \right) J_{m+1} - \left[\mathrm{d}\sigma^{\mathrm{RV,A_1}}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} \right)^{\mathrm{A_1}} \right] J_m \right\}$$

$$\sigma^{\mathrm{NNLO}}_{m} = \int_{m} \left\{ \mathrm{d}\sigma^{\mathrm{VV}}_{m} + \int_2 \left(\mathrm{d}\sigma^{\mathrm{RR,A_2}}_{m+2} - \mathrm{d}\sigma^{\mathrm{RR,A_{12}}}_{m+2} \right) + \int_1 \left[\mathrm{d}\sigma^{\mathrm{RV,A_1}}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} \right)^{\mathrm{A_1}} \right] \right\} J_m$$

RR,A_1 regularizes singly-unresolved limits

G. Somogyi, ZT hep-ph/0609041, hep-ph/0609043
G. Somogyi, ZT, V. Del Duca hep-ph/0502226, hep-ph/0609042
Z. Nagy, G. Somogyi, ZT hep-ph/0702273

Structure

of subtractions is governed by the jet functions

$$\sigma^{\mathrm{NNLO}} = \sigma^{\mathrm{RR}}_{m+2} + \sigma^{\mathrm{RV}}_{m+1} + \sigma^{\mathrm{VV}}_{m} = \sigma^{\mathrm{NNLO}}_{m+2} + \sigma^{\mathrm{NNLO}}_{m+1} + \sigma^{\mathrm{NNLO}}_{m}$$

$$\sigma^{\mathrm{NNLO}}_{m+2} = \int_{m+2}\left\{\mathrm{d}\sigma^{\mathrm{RR}}_{m+2} J_{m+2} - \mathrm{d}\sigma^{\mathrm{RR,A}_2}_{m+2} J_m - \left(\mathrm{d}\sigma^{\mathrm{RR,A}_1}_{m+2} J_{m+1} - \mathrm{d}\sigma^{\mathrm{RR,A}_{12}}_{m+2} J_m\right)\right\}$$

$$\sigma^{\mathrm{NNLO}}_{m+1} = \int_{m+1}\left\{\left(\mathrm{d}\sigma^{\mathrm{RV}}_{m+1} + \int_1 \mathrm{d}\sigma^{\mathrm{RR,A}_1}_{m+2}\right) J_{m+1} - \left[\mathrm{d}\sigma^{\mathrm{RV,A}_1}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR,A}_1}_{m+2}\right)^{\mathrm{A}_1}\right] J_m\right\}$$

$$\sigma^{\mathrm{NNLO}}_{m} = \int_{m}\left\{\mathrm{d}\sigma^{\mathrm{VV}}_{m} + \int_2\left(\mathrm{d}\sigma^{\mathrm{RR,A}_2}_{m+2} - \mathrm{d}\sigma^{\mathrm{RR,A}_{12}}_{m+2}\right) + \int_1\left[\mathrm{d}\sigma^{\mathrm{RV,A}_1}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR,A}_1}_{m+2}\right)^{\mathrm{A}_1}\right]\right\} J_m$$

RR,A12 removes overlapping subtractions

G. Somogyi, ZT hep-ph/0609041, hep-ph/0609043
G. Somogyi, ZT, V. Del Duca hep-ph/0502226, hep-ph/0609042
Z. Nagy, G. Somogyi, ZT hep-ph/0702273

Structure

of subtractions is governed by the jet functions

$$\sigma^{\rm NNLO} = \sigma^{\rm RR}_{m+2} + \sigma^{\rm RV}_{m+1} + \sigma^{\rm VV}_{m} = \sigma^{\rm NNLO}_{m+2} + \sigma^{\rm NNLO}_{m+1} + \sigma^{\rm NNLO}_{m}$$

$$\sigma^{\rm NNLO}_{m+2} = \int_{m+2} \left\{ {\rm d}\sigma^{\rm RR}_{m+2} J_{m+2} - {\rm d}\sigma^{\rm RR,A_2}_{m+2} J_m - \left({\rm d}\sigma^{\rm RR,A_1}_{m+2} J_{m+1} - {\rm d}\sigma^{\rm RR,A_{12}}_{m+2} J_m \right) \right\}$$

$$\sigma^{\rm NNLO}_{m+1} = \int_{m+1} \left\{ \left({\rm d}\sigma^{\rm RV}_{m+1} + \int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \right) J_{m+1} - \left[{\rm d}\sigma^{\rm RV,A_1}_{m+1} + \left(\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \right)^{\rm A_1} \right] J_m \right\}$$

$$\sigma^{\rm NNLO}_{m} = \int_{m} \left\{ {\rm d}\sigma^{\rm VV}_{m} + \int_2 \left({\rm d}\sigma^{\rm RR,A_2}_{m+2} - {\rm d}\sigma^{\rm RR,A_{12}}_{m+2} \right) + \int_1 \left[{\rm d}\sigma^{\rm RV,A_1}_{m+1} + \left(\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \right)^{\rm A_1} \right] \right\} J_m$$

RV,A_1 regularizes singly-unresolved limits

G. Somogyi, ZT hep-ph/0609041, hep-ph/0609043
G. Somogyi, ZT, V. Del Duca hep-ph/0502226, hep-ph/0609042
Z. Nagy, G. Somogyi, ZT hep-ph/0702273

CoLoRFulNNLO method

can now be computed by numerical Monte Carlo integrations

$$\sigma^{\rm NNLO} = \sigma^{\rm RR}_{m+2} + \sigma^{\rm RV}_{m+1} + \sigma^{\rm VV}_{m} = \sigma^{\rm NNLO}_{m+2} + \sigma^{\rm NNLO}_{m+1} + \sigma^{\rm NNLO}_{m}$$

$$\sigma^{\rm NNLO}_{m+2} = \int_{m+2} \left\{ {\rm d}\sigma^{\rm RR}_{m+2} J_{m+2} - {\rm d}\sigma^{\rm RR,A_2}_{m+2} J_m - \left({\rm d}\sigma^{\rm RR,A_1}_{m+2} J_{m+1} - {\rm d}\sigma^{\rm RR,A_{12}}_{m+2} J_m \right) \right\}$$

$$\sigma^{\rm NNLO}_{m+1} = \int_{m+1} \left\{ \left({\rm d}\sigma^{\rm RV}_{m+1} + \int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \right) J_{m+1} - \left[{\rm d}\sigma^{\rm RV,A_1}_{m+1} + \left(\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \right)^{\rm A_1} \right] J_m \right\}$$

$$\sigma^{\rm NNLO}_{m} = \int_{m} \left\{ {\rm d}\sigma^{\rm VV}_{m} + \int_2 \left({\rm d}\sigma^{\rm RR,A_2}_{m+2} - {\rm d}\sigma^{\rm RR,A_{12}}_{m+2} \right) + \int_1 \left[{\rm d}\sigma^{\rm RV,A_1}_{m+1} + \left(\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \right)^{\rm A_1} \right] \right\} J_m$$

G. Somogyi, ZT hep-ph/0609041, hep-ph/0609043
G. Somogyi, ZT, V. Del Duca hep-ph/0502226, hep-ph/0609042
Z. Nagy, G. Somogyi, ZT hep-ph/0702273

implementation for general m in MCCSM code

Adam Kardos 2015

MCCSM built in checks

Checking finiteness in singular regions, e.g. regularized RR:

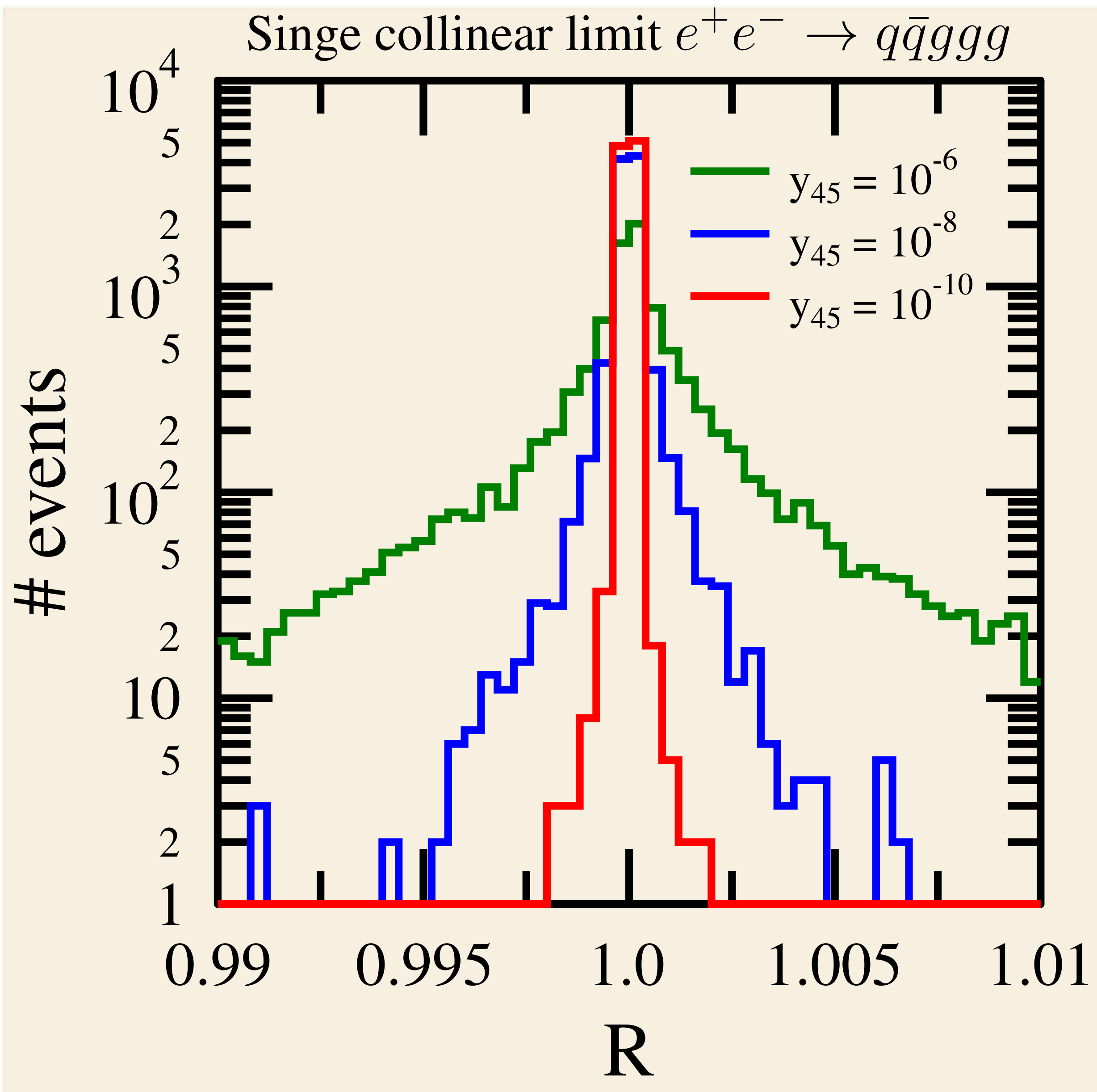
```
CSirs: g (6) -> g (6) || g (7) , g (5) -> 0  VALID
iter no.  1 scale no.  1 1.0626663494874406131036910247582 5 *-WARN-*
iter no.  2 scale no.  1 .99933339118756664131317235085510 9
iter no.  3 scale no.  1 .99993605671620667930196132866217 9
iter no.  4 scale no.  1 .99999321715885735308166967682532 0
iter no.  5 scale no.  1 .99999928952733456236747237157707 3
iter no.  6 scale no.  1 .99999992795555748046415914784189 5
iter no.  7 scale no.  1 .99999999276423133274830626094779 4
iter no.  8 scale no.  1 .99999999927543467248458956328478 1
iter no.  9 scale no.  1 .99999999999275122293185044066694 79
iter no. 10 scale no.  1 .99999999999927502353279967356633 20
iter no. 11 scale no.  1 .99999999999992749923043113272822 04
iter no. 12 scale no.  1 .99999999999999274982428947529107 29
iter no. 13 scale no.  1 .99999999999999927497944747092755 27
iter no. 14 scale no.  1 .99999999999999992750038439839119 18
iter no. 15 scale no.  1 .99999999999999999276754146625355 21
```

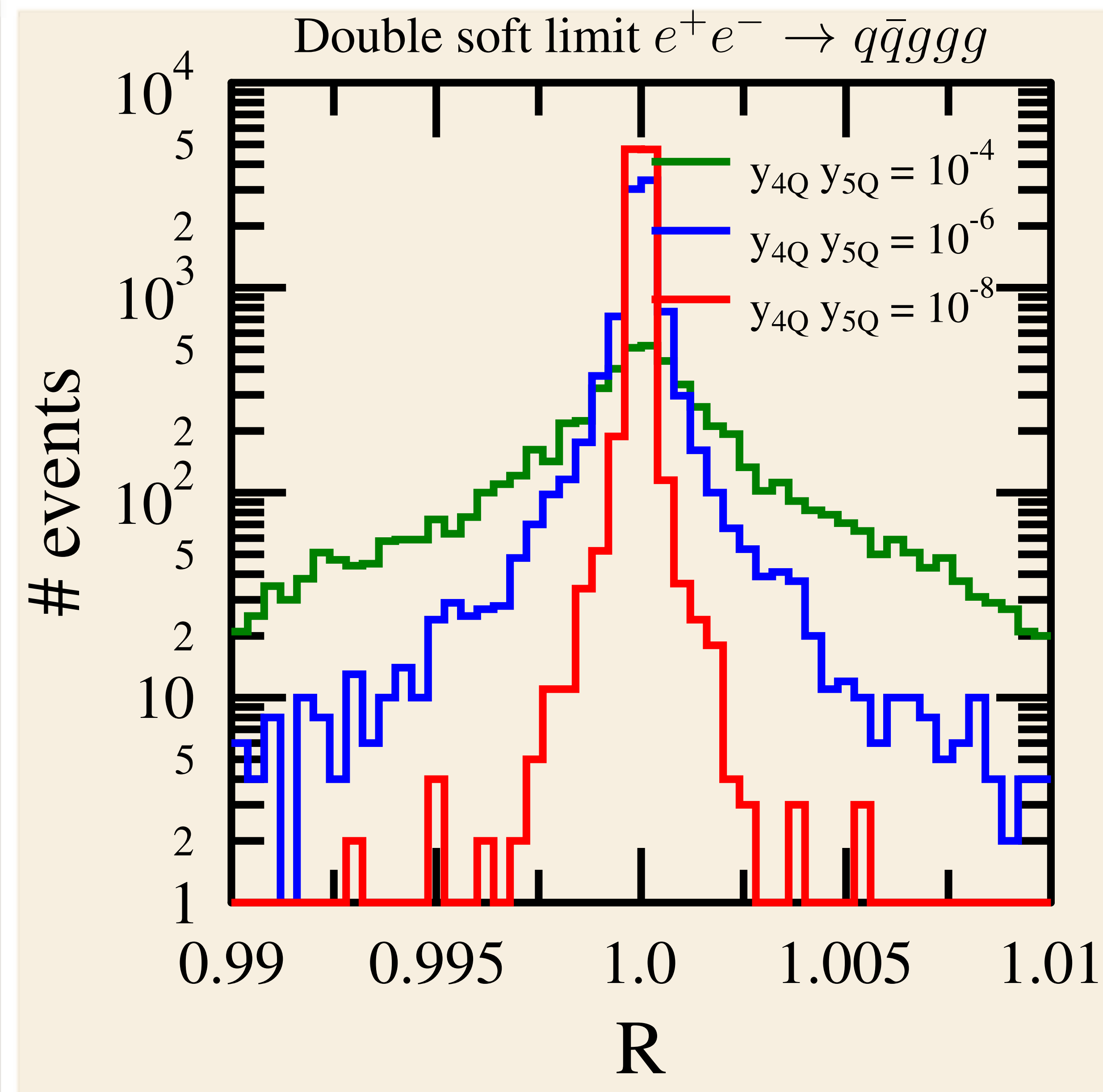
double unresolved

```
Cir: b (3) -> b (3) || g (7)  VALID
iter no.  1 scale no.  1 .96148670801871865442260647152993 8 *-WARN-*
iter no.  2 scale no.  1 1.0060295920978622083723511280477 7
iter no.  3 scale no.  1 1.0006658004717423478286812819735 6
iter no.  4 scale no.  1 1.0000674992486446447146088537433 2
iter no.  5 scale no.  1 1.0000067595112341689215862256272 2
iter no.  6 scale no.  1 1.0000006760473957286239347671044 7
iter no.  7 scale no.  1 1.0000000676057027060685822559986 9
iter no.  8 scale no.  1 1.0000000067605799023491568994038 8
iter no.  9 scale no.  1 1.0000000006760580865527488728314 1
iter no. 10 scale no.  1 1.0000000000676058096184534061560 2
iter no. 11 scale no.  1 1.0000000000067605809714718350712 7
iter no. 12 scale no.  1 1.0000000000006760580972580247363 1
iter no. 13 scale no.  1 1.0000000000000676058092182273659 7
iter no. 14 scale no.  1 1.0000000000000067605779495431716 5
iter no. 15 scale no.  1 1.0000000000000006761539666111960 2
```

single unresolved

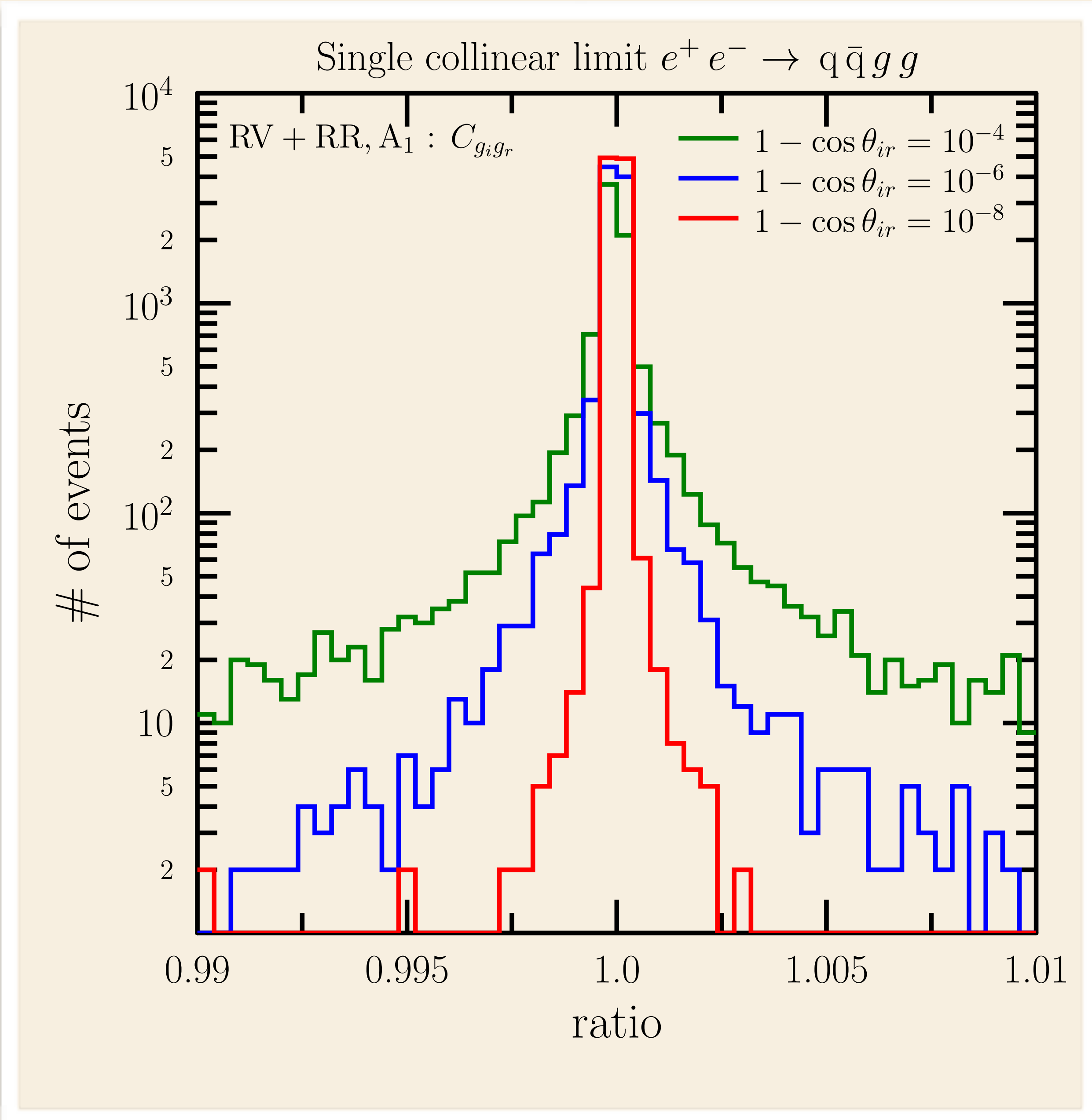
Kinematic singularities cancel in RR

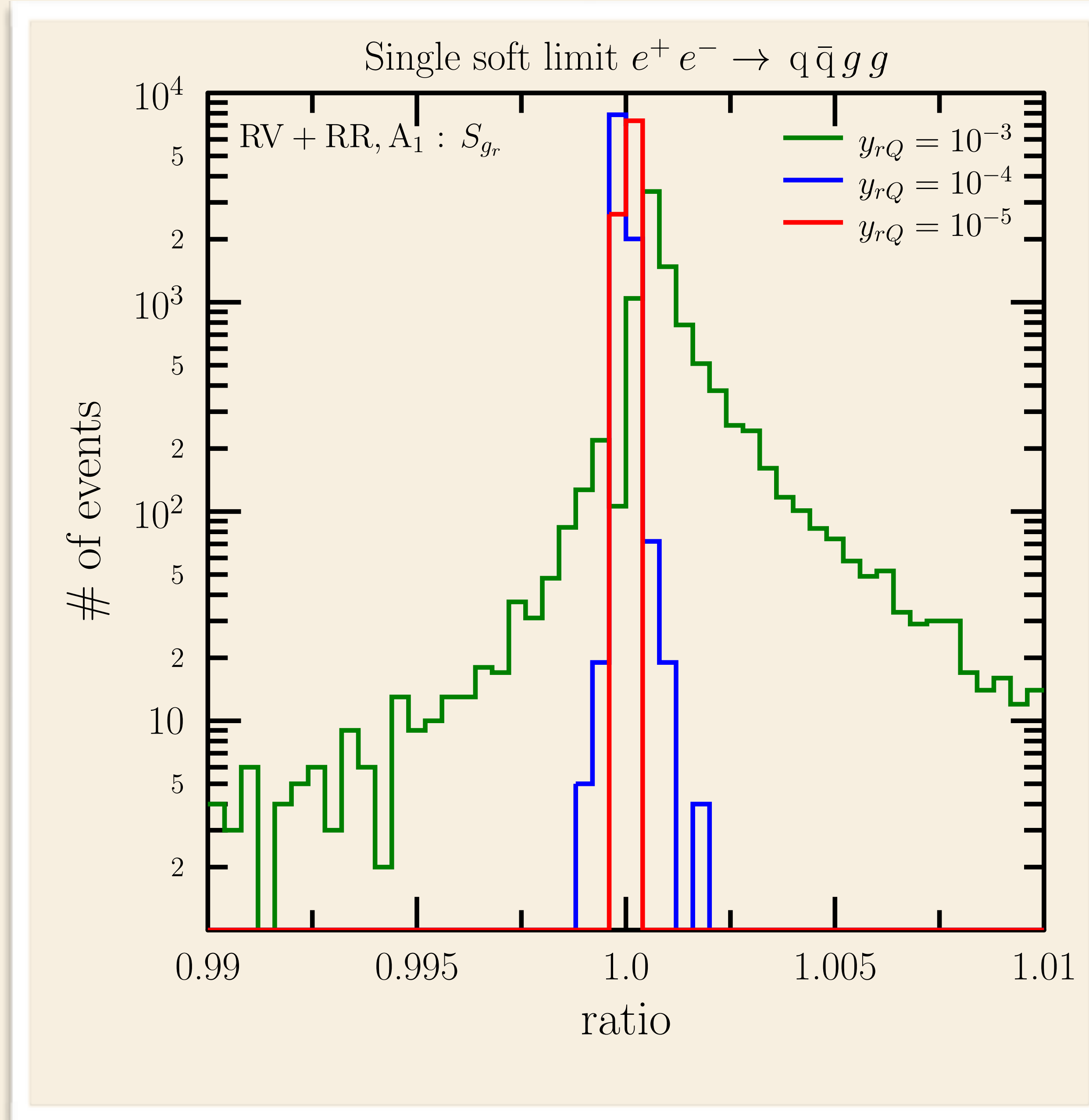




R = subtraction/RR

Kinematic singularities cancel in RV





R = subtraction/(RV+RR,A_1)

Pole-cancelation: $H \to b\bar{b}$ at $\mu = m_H$

$$\sigma_m^{\mathrm{NNLO}} = \int_m \left\{ \mathrm{d}\sigma_m^{\mathrm{VV}} + \int_2 \left[\mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_2} - \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_{12}} \right] + \int_1 \left[\mathrm{d}\sigma_{m+1}^{\mathrm{RV},\mathrm{A}_1} + \left(\int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_1} \right)^{\mathrm{A}_1} \right] \right\} J_m$$

$$\begin{aligned}
\mathrm{d}\sigma_{H\to b\bar{b}}^{\mathrm{VV}} = \left(\frac{\alpha_{\mathrm{s}}(\mu^2)}{2\pi}\right)^2 \mathrm{d}\sigma_{H\to b\bar{b}}^{\mathrm{B}} \Bigg\{ &+ \frac{2C_{\mathrm{F}}^2}{\epsilon^4} + \left(\frac{11 C_{\mathrm{A}} C_{\mathrm{F}}}{4} + 6 C_{\mathrm{F}}^2 - \frac{C_{\mathrm{F}} n_{\mathrm{f}}}{2} \right) \frac{1}{\epsilon^3} \\
&+ \left[\left(\frac{8}{9} + \frac{\pi^2}{12} \right) C_{\mathrm{A}} C_{\mathrm{F}} + \left(\frac{17}{2} - 2\pi^2 \right) C_{\mathrm{F}}^2 - \frac{2 C_{\mathrm{F}} n_{\mathrm{f}}}{9} \right] \frac{1}{\epsilon^2} \\
&+ \left[\left(-\frac{961}{216} + \frac{13\zeta_3}{2} \right) C_{\mathrm{A}} C_{\mathrm{F}} + \left(\frac{109}{8} - 2\pi^2 - 14\zeta_3 \right) C_{\mathrm{F}}^2 + \frac{65 C_{\mathrm{F}} n_{\mathrm{f}}}{108} \right] \frac{1}{\epsilon} \Bigg\}
\end{aligned}$$

C. Anastasiou, F. Herzog, A. Lazopoulos, arXiv:0111.2368

$$\begin{aligned}
\sum \int \mathrm{d}\sigma^{\mathrm{A}} = \left(\frac{\alpha_{\mathrm{s}}(\mu^2)}{2\pi}\right)^2 \mathrm{d}\sigma_{H\to b\bar{b}}^{\mathrm{B}} \Bigg\{ &- \frac{2C_{\mathrm{F}}^2}{\epsilon^4} - \left(\frac{11 C_{\mathrm{A}} C_{\mathrm{F}}}{4} + 6 C_{\mathrm{F}}^2 + \frac{C_{\mathrm{F}} n_{\mathrm{f}}}{2} \right) \frac{1}{\epsilon^3} \\
&- \left[\left(\frac{8}{9} + \frac{\pi^2}{12} \right) C_{\mathrm{A}} C_{\mathrm{F}} + \left(\frac{17}{2} - 2\pi^2 \right) C_{\mathrm{F}}^2 - \frac{2 C_{\mathrm{F}} n_{\mathrm{f}}}{9} \right] \frac{1}{\epsilon^2} \\
&- \left[\left(-\frac{961}{216} + \frac{13\zeta_3}{2} \right) C_{\mathrm{A}} C_{\mathrm{F}} + \left(\frac{109}{8} - 2\pi^2 - 14\zeta_3 \right) C_{\mathrm{F}}^2 + \frac{65 C_{\mathrm{F}} n_{\mathrm{f}}}{108} \right] \frac{1}{\epsilon} \Bigg\}
\end{aligned}$$

V. Del Duca, C. Duhr, G. Somogyi, F. Tramontano, Z. Trócsányi, arXiv:1501.07226

Poles cancel: $e^+e^- \longrightarrow$ m(=3) jets at μ^2

$$\sigma_m^{\mathrm{NNLO}} = \int_m \left\{ \mathrm{d}\sigma_m^{\mathrm{VV}} + \int_2 \left[\mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_2}} - \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_{12}}} \right] + \int_1 \left[\mathrm{d}\sigma_{m+1}^{\mathrm{RV,A_1}} + \left(\int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_1}} \right)^{\mathrm{A_1}} \right] \right\} J_m$$

$$\mathrm{d}\sigma_3^{\mathrm{VV}} = \mathcal{P}oles\left(A_3^{(2\times0)} + A_3^{(1\times1)}\right) + \mathcal{F}inite\left(A_3^{(2\times0)} + A_3^{(1\times1)}\right)$$

$$\mathcal{P}oles\left(A_3^{(2\times0)} + A_3^{(1\times1)}\right) + \mathcal{P}oles \sum \int \mathrm{d}\sigma^{\mathrm{A}} =$$ 200k Mathematica lines

= zero numerically in any phase space point:

```
              0.          2    0. nf
        0. + --- + 0. Nc  + ----- + 0. Nc nf
               2                Nc
              Nc
Out[1]= ---------------------------------- +
                        2
                       e

         0.          2   0. nf
    0. + --- + 0. Nc  + ----- + 0. Nc nf
          2              Nc
         Nc                                        0
------------------------------------------ + O[e]
                  e
```

Poles cancel: $e^+e^- \longrightarrow$ m(=3) jets at μ^2

$$\sigma_m^{\mathrm{NNLO}} = \int_m \left\{ \mathrm{d}\sigma_m^{\mathrm{VV}} + \int_2 \left[\mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_2} - \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_{12}} \right] + \int_1 \left[\mathrm{d}\sigma_{m+1}^{\mathrm{RV},\mathrm{A}_1} + \left(\int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_1} \right)^{\mathrm{A}_1} \right] \right\} J_m$$

$$\mathrm{d}\sigma_3^{\mathrm{VV}} = \mathcal{P}oles\left(A_3^{(2\times 0)} + A_3^{(1\times 1)}\right) + \mathcal{F}inite\left(A_3^{(2\times 0)} + A_3^{(1\times 1)}\right)$$

$\mathcal{P}oles\left(A_3^{(2\times 0)} + A_3^{(1\times 1)}\right) + \mathcal{P}oles \sum \int \mathrm{d}\sigma^{\mathrm{A}} =$ 200k Mathematica lines

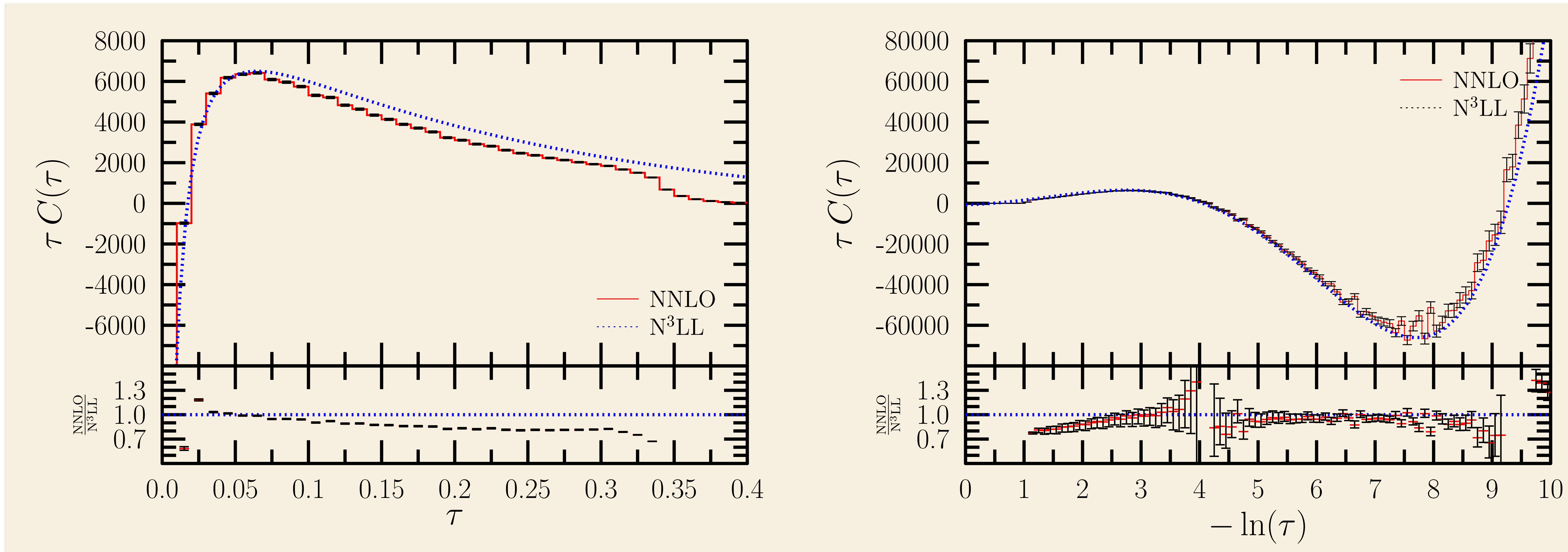
= zero analytically using symbol techniques (C. Duhr)

Message:

$$\sigma_3^{\mathrm{NNLO}} = \int_3 \left\{ \mathrm{d}\sigma_3^{\mathrm{VV}} + \sum \int \mathrm{d}\sigma^{\mathrm{A}} \right\}_{\epsilon=0} J_3$$

indeed finite in d=4 dimensions

Thrust at NNLO vs N^{3}LL



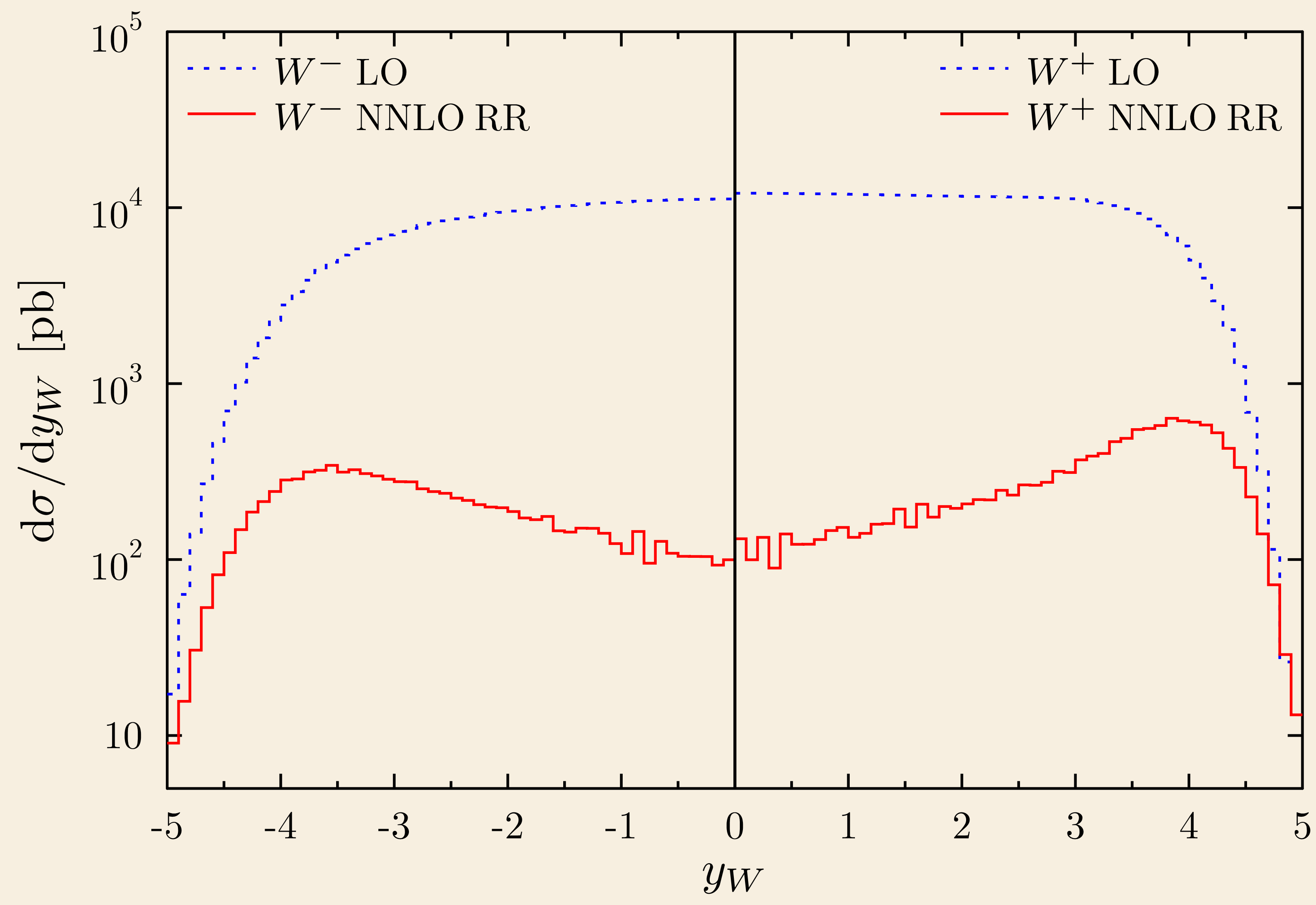
$\tau = 1 - T$

$$T = \max_{\vec{n}} \left(\frac{\sum_i |\vec{n} \cdot \vec{p}_i|}{\sum_i |\vec{p}_i|} \right)$$

N^{3}LL resummation from
T. Becher, M.D. Schwartz arXiv:0803.0343

Outlook: prospects for NNLO

✓ Extension of MCCSM to hadron collisions: Drell-Yan pair



Outlook: prospects for NNLO

✓ Extension of MCCSM to hadron collisions: H production