Half-supersymmetric consistent truncations with vector multiplets in AdS_6 and AdS_7 from ExFT

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with H. Samtleben and E. Malek

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Motivation and Summary

- ExFT treats gauge and metric fields in the same footing $\Rightarrow$ Natural language for flux compactifications
- Previous talk by E. Malek: Half-supersymmetric vacua in ExFT
  - Spin$(d-1)$ structures (analogous to G-structures in CY compactifications)
  - Method successfully used to obtain SUSY $\mathrm{AdS}_{7\,\&\,6}$ vacua.
- HERE: We generalise this method to include truncations with vector multiplets around these vacua.
- These 10d configurations are valuable in the context of holography.

Structure of the talk

1. Consistent truncations with vector multiplets in ExFT
2. Vector multiplets around AdS_7 in mIIA (No-go theorems for $m \neq 0$)
3. Vector multiplets around AdS_6 in IIB (Restrictive necessary conditions for existence)

Half-supersymmetric AdS vacua in ExFT

[From **Emanuel Malek**'s talk]

Half-SUSY $\Leftrightarrow$ Spin$(d-1)$ structure in the internal extended space

Spin$(d-1)$ structures

An Spin$(d-1)$ structure is defined in the extended space by the sections

$$J_u \in \mathcal{R}_1\,, \qquad \hat{K} \in \mathcal{R}_2\,, \qquad u = 1, \dots d-1$$

with the constraints:

$$\begin{aligned} J_u \wedge J_v &= \frac{1}{d-1}\delta_{uv} J_w \wedge J^w \\ \hat{K} \otimes \hat{K} &= 0 \\ J_w \wedge J^w \wedge \hat{K} &> 0 \end{aligned}$$

AdS vacua

"Weakly integrable" structure: $\quad \mathcal{L}_{J_u} J_v = R_{uvw} J^w\,, \qquad \mathcal{L}_{J_u} \hat{K} = 0\,, \qquad$ and

$$d\hat{K} = \epsilon^{uvw} R_{uvw} J_x \wedge J^x \;\; (AdS_7), \qquad d\hat{K} = \epsilon^{uvwx} R_{uvw} J_x \;\; (AdS_6)$$

R_{uvw} encodes the cosmological const. and breaks the R-symmetry to $SU(2)$.

Consistent truncations ansätze around AdS vacua in ExFT

Minimal truncation (SUGRA multiplet)

See [1707.00714] by E. Malek

$$\begin{aligned} \langle \mathcal{J}_u \rangle(x, Y) &= X^{-1}(x) J_u(Y)\,, \qquad \langle \hat{\mathcal{K}} \rangle(x, Y) = X^2(x) \hat{K}(Y) \\ \langle \mathcal{A}_\mu \rangle(x, Y) &= A_\mu{}^u(x) J_u(Y)\,, \qquad \ldots \end{aligned}$$

$X(x) \to$ scalar. $A_\mu{}^u \to$ vector fields of the grav. multiplet.

Truncation with N vector multiplets

See [1707.00714] by E. Malek

$$\begin{aligned} \langle \mathcal{J}_u \rangle(x, Y) &= X^{-1}(x) b_u{}^A(x) J_A(Y)\,, \qquad \langle \hat{\mathcal{K}} \rangle(x, Y) = X^2(x) \hat{K}(Y) \\ \langle \mathcal{A}_\mu \rangle(x, Y) &= A_\mu{}^A(x) J_A(Y)\,, \qquad \ldots \end{aligned}$$

with $A = 1, \ldots, d-1+N$. Split $A = (u, \bar{u})$. $A_\mu{}^{\bar{u}} \to N$ extra vector fields.

- $b_u{}^A$ is constrained by $b_u{}^A b_v{}^B \eta_{AB} = \delta_{uv}$, $\quad \eta_{AB} \to SO(d-1, N)$ metric
- $\{b_u{}^A, X\} \in \dfrac{SO(d-1,N)}{SO(d-1) \times SO(N)} \times \mathbb{R}^+$, $\quad$ scalar manifold.
- $J_A = (J_u, J_{\bar{u}})$ define an $\mathrm{Spin}(d-1-N) \subset \mathrm{Spin}(d-1)$ structure. ($N \leq d-1$)

Spin$(d-1-N)$ structures

An Spin$(d-1)$ structure is defined in the extended space by the sections

$$J_A \in \mathcal{R}_1\,, \qquad \hat{K} \in \mathcal{R}_2\,, \qquad u = 1, \dots d-1$$

with the constraints:
$$\begin{aligned} (d-1-N) J_A \wedge J_B &= \eta_{AB} J_C \wedge J^C \\ \hat{K} \otimes \hat{K} &= 0 \\ J_C \wedge J^C \wedge \hat{K} &> 0 \end{aligned}$$

Differential constraints

$$\mathcal{L}_{J_A} J_B = f_{AB}{}^C J_C\,, \qquad \mathcal{L}_{J_A} \hat{K} = 0$$

with $f_{AB}{}^C J_C$ constant and $f_{[ABC]} = f_{AB}{}^C J_D \eta_{DC}$, $\qquad (f_{ABC} \Leftarrow$ embedding tensor)

Spin$(d-1-N)$ structure + differential constraints $\Rightarrow$ consistency

AdS vacua:

$$f_{uvw} = R_{uvw}\,, \qquad f_{uv\bar{w}} = 0$$

$\Rightarrow \mathcal{L}_{J_u} J_v = R_{uvw} J^w, \qquad \mathcal{L}_{J_u} J_{\bar{v}} = f_{u\bar{v}}{}^{\bar{w}} J_{\bar{w}}$

$\Rightarrow J_{\bar{u}}$ organise into some representation of $SO(3)_{\text{R-symmetry}}$.

AdS$_7$ in mIIA: vector multiplets

AdS$_7 \times I \times S^2$ vacua in mIIA. [See E. Malek's talk]

$$J_u = \sqrt{2} v_u - \frac{1}{2} d(y_u z) + \ddot{t}(z) y_u + \frac{1}{2\sqrt{2}} \ddot{t}(z) (y_u z Vol_{S^2} - \epsilon_{uvw} y^v dy^w \wedge dz)\,,$$

$$\hat{K} = -\dot{t}(z) + \frac{1}{2\sqrt{2}} \Big(z\dot{t}(z) - t(z) \Big) Vol_{S^2}$$

Solutions classified by $t(z) > 0$ satisfying $\dddot{t}(z) = -m/2$. $m \to$ Romans mass.

Spin$(d-1-N)$ structure and vector multiplets

We add $J_{\bar{u}}$, $\bar{u} = 1, \ldots, N$ satisfying

$$J_{\bar{u}} \wedge J_{\bar{v}} = -\frac{1}{3} \delta_{\bar{u}\bar{v}} J_u \wedge J^u\,, \qquad J_{\bar{u}} \wedge J_v = 0$$

$$\mathcal{L}_{J_u} J_{\bar{v}} = f_{u\bar{v}}{}^{\bar{w}} J_{\bar{w}}\,, \qquad \mathcal{L}_{J_{\bar{u}}} J_{\bar{v}} = f_{\bar{u}\bar{v}}{}^{w} J_w + f_{\bar{u}\bar{v}}{}^{\bar{w}} J_{\bar{w}}\,, \qquad \mathcal{L}_{J_{\bar{u}}} \hat{K} = 0$$

Two possibilities: ($N \leq 3$)

- Vector multiplets are **1**'s under $SO(3)_R$
- $N = 3$ vector multiplets are **3** under $SO(3)_R$

$N = 1$ vector multiplet, **1** under $SO(3)_R$

Algebraic constraints uniquely fix $(g(z),\, q(z),\, \ldots \rightarrow$ related to $t(z))$

$$J_{\bar{1}} = -q(z) + \frac{1}{2\sqrt{2}}\, g(z)\, q(z) vol_2$$

However, we find that $\mathcal{L}_{J_{\bar{1}}} \hat{K} = -\frac{m\, t(z)}{2\sqrt{2}} vol_2 \wedge dz\,.$

Only possible when $m = 0$ (Extra vector from M-theory circle)

$N = 3$ vector multiplet, **3** under $SO(3)_R$

Algebraic and differential constraints uniquely fix

$$J_{\bar{u}} = \sqrt{2} v_{\bar{u}} + \frac{1}{2}\left(g(z) dy_{\bar{u}} - y_{\bar{u}}\, dz\right) + \frac{1}{2\sqrt{2}} q(z)\left(-g(z)\, y_{\bar{u}} vol_2 - \theta_{\bar{u}} \wedge dz\right) + q(z) y_{\bar{u}}\,,$$

With $\mathcal{L}_{J_{\bar{u}}} \hat{K} = \frac{1}{2\sqrt{2}}\left(2\, \ddot{t}\, \dot{t} + m\, t(z)\right) y_{\bar{u}}\, vol_2 \wedge dz\,.$

Only possible if $2\ddot{t}\, \dot{t} + m\, t = 0 \quad \overset{d_z^2}{\Longrightarrow} \quad \ddot{t} = 0$ (using $\dddot{t} = -\frac{m}{2}$)

This can be only satisfied if $m = 0$ (since $\dddot{t} = -\frac{m}{2}$).

Consistent truncations with vector multiplets around AdS_6 in IIB

AdS_6 vacua in IIB. [See E. Malek's talk]

$$\begin{aligned} J_I &= v_I + d\left(k^\alpha\, y_I\right) + \frac{1}{2} d\left(k^\alpha\, \theta_I \wedge dk_\alpha\right) \\ J_4 &= dp^\alpha - k_\alpha\, dp^\alpha \wedge Vol_{S^2} \\ \hat{K} &= p_\alpha - \left(r + p_\alpha k^\alpha\right) Vol_{S^2} \end{aligned}$$

[Notation: $u = (I, 4)$, $I = 1, 2, 3$]

where $f^\alpha = k^\alpha + ip^\alpha$ is an $SL(2)$-doublet of **holomorphic** functions.

$\mathrm{Spin}(d - 1 - N)$ structure and vector multiplets

We add $J_{\bar{u}}$, $\bar{u} = 1, \ldots, N$ satisfying conditions analogous to the AdS_7 case.

Three possibilities: ($N \leq 4$)

- Vector multiplets are $\mathbf{1}$'s under $SO(3)_R$
- $N = 3$ vector multiplets are $\mathbf{3}$ under $SO(3)_R$
- $N = 4$ vector multiplets are $\mathbf{3} \oplus \mathbf{1}$ under $SO(3)_R$

$N = 1$ vector multiplet, $\mathbf{1}$ under $SO(3)_R$

Algebraic and differential constraints require

$$J_{\bar{1}} = \rho \left(k_\beta M^\beta \wedge Vol_{S^2} + M^\alpha \right)$$

with 1-forms $M^\alpha = (e^{i\chi(z,\bar{z})}\partial f^\alpha d\bar{z} + c.c.)$, satisfying $dM^\alpha = 0$.

- $\chi(z,\bar{z})$ is a real function, fixed by $dM^\alpha = 0$:

$$\partial(e^{i\chi(z,\bar{z})}\partial f^\alpha) \in \text{Real functions on } \Sigma$$

- The existence of solutions depend on f^α.
- An example: Solutions exist when f^α quadratic in z, as in the T-dual of D4-D8 Brandhuber-Oz solution. In this case: $\chi =$ const.

$N = 2$ vector multiplets, $\mathbf{1}$'s under $SO(3)_R$

We need two real functions $\chi_1(z,\bar{z})$, $\chi_2(z,\bar{z})$ parametrising $M^\alpha_{\bar{1}}$ and $M^\alpha_{\bar{2}}$

- Algebraic constraint $J_{\bar{1}} \wedge J_{\bar{2}} = 0 \Rightarrow e^{i\chi_1} = i\, e^{i\chi_2}$. ($\Rightarrow N > 2$ **forbidden**)
- $dM^\alpha_{\bar{1}} = dM^\alpha_{\bar{2}} = 0$ can only be solved by $\partial(e^{i\chi_1}\partial f^\alpha) = 0$.
  This equation has solutions only when $\partial f^1 = \lambda \partial f^2$ for some $\lambda =$const.
- No global solutions can be found with $\partial f^1 = \lambda \partial f^2 \Rightarrow$ **No-go**.

$N = 3$ vector multiplets, $\mathbf{3}$ under $SO(3)_R$

Algebraic and differential constraints uniquely fix $\quad (\bar{I} = 1, 2, 3)$

$$J_{\bar{I}} = v_I + \Pi^\alpha y_I + k^\alpha dy_I - y_I k_\beta \Pi^\beta \wedge \, Vol_{S^2} + |\Pi| \theta_I \wedge Vol_\Sigma$$

with $\Pi^\alpha = \dfrac{i}{2}\left(\dfrac{p_\alpha \bar{\partial} \bar{f}^\alpha}{p_\beta \partial f^\beta}\right) \partial f^\alpha d\bar{z} + c.c.$ together with the differential condition

$$r\, d\Pi^\alpha = p^\alpha \Pi^\beta \wedge \Pi_\beta \qquad (\text{with } dr = p_\alpha dk^\alpha)$$

This condition imposes restrictions on the possible f^α's.

$N = 4$ vector multiplets, $\mathbf{3} \oplus \mathbf{1}$ under $SO(3)_R$

The following objects are needed on the Riemann surface:

- One doublet of 1-forms Π^α satisfying the above conditions
- One doublet of 1-forms M^α characterised by $\chi = \dfrac{1}{2}\left(\dfrac{p_\alpha \bar{\partial} \bar{f}^\alpha}{p_\beta \partial f^\beta}\right)$

  (Fixed by the algebraic condition $J_{\bar{I}} \wedge J_{\bar{4}} = 0$).
- This χ should satisfy the condition $\partial(e^{i\chi} \partial f^\alpha) \in$ Real functions on Σ

These conditions impose further restrictions on the possible f^α's.

From the structures one can obtain the 10d uplift.

Example: AdS$_6$ uplift formulae for 1 vector multiplet

Scalar fields $\in \frac{SO(4,1)}{SO(4)} \times \mathbb{R}^+$: Σ and $m_A = (m_I,\, m_4,\, m_5)\,,\qquad$ s.t. $\eta^{AB} m_A m_B = -1\,,$
$\eta_{AB} = \mathrm{diag}(1,1,1,1,-1)\,.$

$$ds^2 = \frac{r^{5/4}\,\lambda^{3/2}}{2^{5/4}\bar{\Delta}^{3/4}}\left[\frac{2\sqrt{2}}{r\,\lambda^2}ds_6^2 + \Sigma^2\left(ds_{S^2}^2 + w\otimes w - \frac{1}{r^2}p_\alpha p_\beta M^\alpha \otimes M^\beta\right) + \frac{2}{\Sigma^2\, r}M_\alpha \otimes \sigma^\alpha \right.$$
$$\left. + \frac{4\tilde{\Delta}}{\Sigma^2 r^2\,\lambda^2}dk^\alpha \otimes dp_\alpha\right]\,,$$

$$C^\alpha_{(2)} = -vol_{S^2}\left(k^\alpha + \frac{\Sigma^4\, r\,\lambda}{2\bar{\Delta}}p_\beta\left[m_5\partial_\gamma k^\beta \partial^\gamma p^\alpha + M^{\beta\gamma}\sigma^\alpha{}_\gamma\right]\right)$$
$$+ \frac{\lambda^2}{4\bar{\Delta}}\left(2\,r\left[m_5 M^\alpha - \omega^\alpha\right] - \Sigma^4\, p^\alpha\, p_\beta \star M^\beta\right)\wedge dy^I y^J m^K \epsilon_{IJK}\,,$$

$$H^{\alpha\beta} = \frac{\Sigma^4\, p^\alpha\, p^\beta\,\lambda + 4\,r\left(m_5\,\partial_\gamma k^\alpha \partial^\gamma p^\beta + M^{\alpha\gamma}\sigma^\beta{}_\gamma\right)}{2\sqrt{2\,r\,\bar{\Delta}}}\,,$$

$$\omega^\alpha = m_I y^I dk^\alpha + m_4 dp^\alpha\,,\qquad \sigma^\alpha = m_I y^I dp^\alpha - m_4 dk^\alpha\,,\qquad w = m_I dy^I + \frac{p_\alpha}{r}\left(m_5 M^\alpha - \omega^\alpha\right)\,,$$

$$\bar{\Delta} = \frac{1}{2}r\,\lambda^2\left(m_5^2 - m_4^2 - \left(m_I y^I\right)^2\right) + \frac{1}{2}\Sigma^4\lambda\, p_\alpha\, p_\beta\left(m_5\partial_\gamma k^\alpha \partial^\gamma p_\alpha + M^{\alpha\gamma}\sigma^\beta{}_\gamma\right)\,,$$

$$\tilde{\Delta} = \frac{1}{2}r\, m_5\lambda^2 + \frac{1}{2}\Sigma^4\,\lambda\, p_\alpha p_\beta \partial_\gamma k^\alpha \partial^\gamma p^\beta\,.$$

Conclusions and Outlook

- We successfully used the analyis of generalised structures in ExFT to study the most general consistent truncations with vector multiplets around half-supersymmetric AdS_7 and AdS_6 vacua.
- Our results:
  - AdS_7 in mIIA: **NO** possible truncations with vector multiplets for $m \neq 0$
  - AdS_6 in IIB:

N	$SO(3)_R$ rep.	Consistent truncation?
1	$\mathbf{1}$	**Only if** $\exists\chi$: $\partial(e^{i\chi}\partial f^\alpha) \in$ Real functions on Σ
2	$\mathbf{1}\oplus\mathbf{1}$	**NO** (due to global issues)
3	$\mathbf{1}\oplus\mathbf{1}\oplus\mathbf{1}$	**NO**
3	$\mathbf{3}$	**Only if** $r\, d\Pi^\alpha = p^\alpha \Pi^\beta \wedge \Pi_\beta$
4	$\mathbf{1}\oplus\mathbf{1}\oplus\mathbf{1}\oplus\mathbf{1}$	**NO**
4	$\mathbf{3}\ \oplus\mathbf{1}$	**Only if** $\exists\,\mathbf{3}$ and $\exists\,\mathbf{1}$ with $\chi = \frac{1}{2}\left(\frac{p_\alpha \bar{\partial}\bar{f}^\alpha}{p_\beta \partial f^\beta}\right)$

- Using ExFT-10d dictionaries $\Rightarrow$ uplifts for the consistent truncations.
- Open question: find more examples or general solutions.

Future work

Generalise to AdS_5, AdS_4,... (richer structure, moduli,...)
More general setups, non-geometric compactifications? ...