

CPT VIOLATION: From MATTER-ANTIMATTER ASYMMETRY in the EARLY UNIVERSE to ENTANGLED QUANTUM STATES



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Workshop on the Standard Model and Beyond
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OUTLINE

- I. Motivation – Insufficient CP Violation induced matter-antimatter asymmetry in Standard Model**
→ must go beyond to reproduce observed baryon asymmetry....
- II. Exotic scenarios: CPT Violation in early Universe?**
 - (a) Lorentz Violating Background (flux) fields & Baryogenesis through Leptogenesis**
→ matter-antimatter asymmetry of correct value:
→ evolution from early epochs to present day – current bounds
 - (b) Quantum Gravity Decoherence (QGD): strong(?) in early Universe → CPT generator ill-defined**
- III(a). Novel QGD-CPT Violating effects (ω -effect) in entangled states of particles, e.g. neutral mesons – bounds: Φ , B-factories searches**
- III(b). Connection to Baryogenesis (specific QGD models)**
- IV. Conclusions-Outlook**

IS THIS CPTV ROUTE WORTH FOLLOWING?

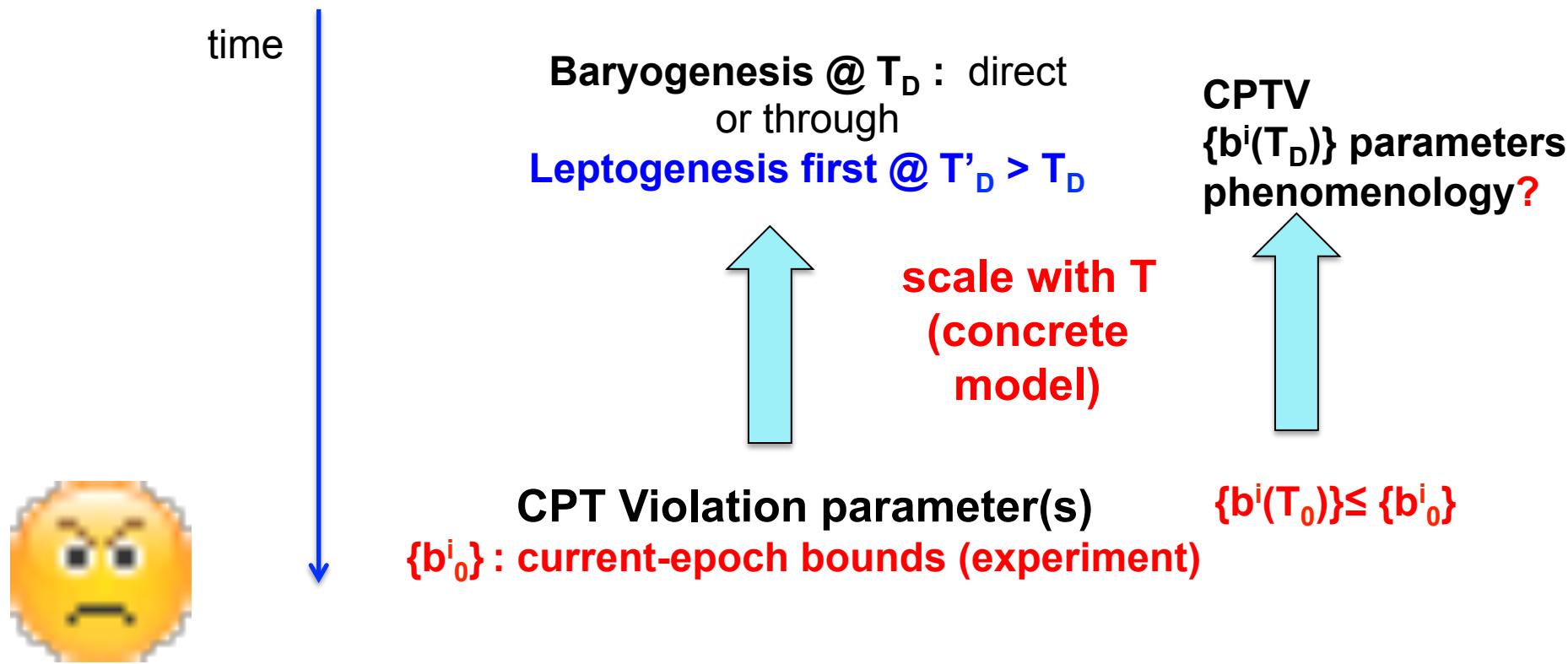


CPT Violation

Construct microscopic models with strong CPT Violation in Early Universe (due to background fields or quantum gravity), but weak today... Fit with all available data... in particular current stringent constraints → scale back in time Estimate in this way matter-antimatter asymmetry in Universe Does it agree with the expected phenomenological value ?



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time

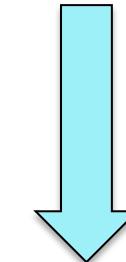
Baryogenesis @ T_D : direct
or through

Leptogenesis first @ $T'_D > T_D$

scale with T
(concrete
model)

CPT Violation parameter(s)
 $\{b^i_0\}$: current-epoch bounds (experiment)

CPTV
 $\{b^i(T_D)\}$ parameters
require correct
phenomenology



$\{b^i(T_0)\} \leq \{b^i_0\}$ check with expt
bounds

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in Early Universe (due to background fields or
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Part I

Motivation:

Matter-Antimatter Asymmetry

in

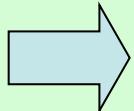
the Standard Model

STANDARD MODEL INCOMPATIBLE WITH BARYOGENESIS

- Matter-Antimatter asymmetry in the Universe \rightarrow Violation of Baryon # (B), C & CP
- Tiny CP violation ($O(10^{-3})$) in Labs: e.g. $K^0 \bar{K}^0$
- But Universe consists only of matter

$$\frac{n_B - \bar{n}_B}{n_B + \bar{n}_B} \sim \frac{n_B - \bar{n}_B}{s} = (8.4 - 8.9) \times 10^{-11} \quad T > 1 \text{ GeV}$$

Sakharov : Non-equilibrium physics of early Universe, **B, C, CP violation**



$$n_B - \bar{n}_B$$

but **not quantitatively in SM**, still a mystery

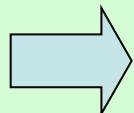
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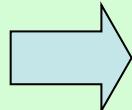
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OBSERVED CP VIOLATION UNLIKELY TO EXPLAIN BARYON ASYMMETRY IN THE UNIVERSE

Kuzmin, Rubakov, Shaposhnikov

Rate of B violation in Early Universe

$$\Gamma \sim \begin{cases} (\alpha_W T)^4 \left(\frac{M_{\text{sph}}}{T}\right)^7 \exp\left(-\frac{M_{\text{sph}}}{T}\right), & T \lesssim M_{\text{sph}}, \\ \alpha_W (\alpha_W T)^4 \log(1/\alpha_W), & T \gtrsim M_{\text{sph}}, \end{cases}$$

Thermal Equilibrium (i.e. $\Gamma > H$ (Hubble)) for B non conserv. occurs only for:

$$T_{\text{sph}}(m_H) < T < (\alpha_W)^5 M_{Pl} \sim 10^{12} \text{ GeV}$$

$$T_{\text{sph}}(m_H) \in [130, 190] \text{ GeV}$$

$$m_H \in [100, 300] \text{ GeV}$$

BAU could be produced this way only when sphaleron interactions freeze out, i.e.

$$T \simeq T_{\text{sph}}$$

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LHC Expts (2012) $m_H \approx 126$ GeV

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BAU COULD BE PRODUCED @

$$T \simeq T_{\text{sph}}$$

Compute CP
Violation Effects

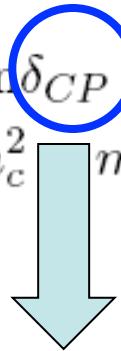
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Use CKM
Matrix for
 $T > T_{\text{sph}}$

Within the Standard Model, lowest CP Violating structures

$$d_{CP} = \sin(\theta_{12})\sin(\theta_{23})\sin(\theta_{13})\sin\delta_{CP} \cdot (m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2)(m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2)$$



Rubakov, Kuzmin, Shaposhnikov,
Gavela, Hernandez, Orloff, Pene

Kobayashi-Maskawa CP Violating phase

Shaposhnikov

$$D = \text{Im Tr} [\mathcal{M}_u^2 \mathcal{M}_d^2 \mathcal{M}_u \mathcal{M}_d]$$

$$\delta_{KM}^{CP} \sim \frac{D}{T^{12}} \sim 10^{-20}$$

$$<< \frac{n_B - \bar{n}_B}{n_B + \bar{n}_B} \sim \frac{n_B - \bar{n}_B}{s} = (8.4 - 8.9) \times 10^{-11}$$

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This CP Violation
Cannot be the
Source of Baryon
Asymmetry in
The Universe

Beyond the Standard Model

- Several Ideas to go beyond the SM (e.g. GUT models, Supersymmetry, extra dimensional models etc.) – to find **EXTRA SOURCES OF CP VIOLATION** within **CPT invariant** effective field theories

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- Several Ideas to go beyond the SM (e.g. GUT models, Supersymmetry, extra dimensional models etc.) – to find **EXTRA SOURCES OF CP VIOLATION** within **CPT invariant** effective field theories
 - **THIS TALK: TRY EXOTIC SCENARIOS WITH (SIMPLIFIED) MODELS OF CPT VIOLATION IN EARLY UNIVERSE ?**
- Consistency with stringent current constraints must be ensured**



Part II

CPT Violation

THEORY

CPT Theorem



Schwinger 1951



Lüders 1954



J S Bell 1954



Pauli 1955



Res Jost 1958

CPT Theorem

Conditions for the Validity of CPT Theorem

$$P : \vec{x} \rightarrow -\vec{x}, \quad T : t \rightarrow -t(T), \quad C\psi(q_i) = \psi(-q_i)$$

i \rightarrow f, T: f \rightarrow i

CPT Invariance Theorem :
A quantum field theory
lagrangian is invariant
under CPT if it satisfies

- (i) Flat space-times
- (ii) Lorentz invariance
- (iii) Locality
- (iv) Unitarity

Schwinger, Pauli,
Luders, Jost, Bell

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Schwinger, Pauli, Luders, Jost, Bell revisited by:
Greenberg,
Chaichian, Dolgov,
Novikov, Fujikawa,
Tureanu ...

(ii)-(iv) Independent reasons for violation

CPT VIOLATION

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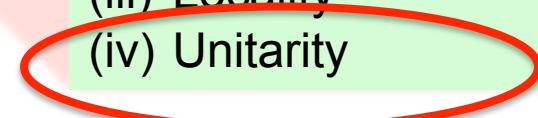
(ii)-(iii) CPT V well-defined as Operator Θ does not commute with Hamiltonian $[\Theta, H] \neq 0$

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**(ii)-(iii) CPT V well-defined as Operator Θ
does not commute with Hamiltonian
 $[\Theta, H] \neq 0$**

**(iv) e.g. Quantum Gravity decoherence →
CPT operator may not be well defined (Wald)
→ consequence for Entangled States (Bernabeu, NEM, Papavassiliou, ...)**



CPT VIOLATION

Conditions for the Validity of CPT Theorem

This Talk

CPT Invariance Theorem :

- (i) Flat space-times
- (ii) Lorentz invariance
- (iii) Locality
- (iv) Unitarity

Kostelecky, Bluhm, Colladay, Potting, Russell, Lehnert, Mewes, Diaz , Tasson....

Standard Model Extension (SME)

(ii)-(iv) Independent reasons for violation

$$\mathcal{L} \ni \dots + \bar{\psi}^f \left(i\gamma^\mu \nabla_\mu - m_f \right) \psi^f + a_\mu \bar{\psi}^f \gamma^\mu \psi^f + b_\mu \bar{\psi}^f \gamma^\mu \gamma^5 \psi^f + \dots$$



Lorentz & CPT
Violation



Lorentz & CPT
Violation

$[\Theta, H] \neq 0$

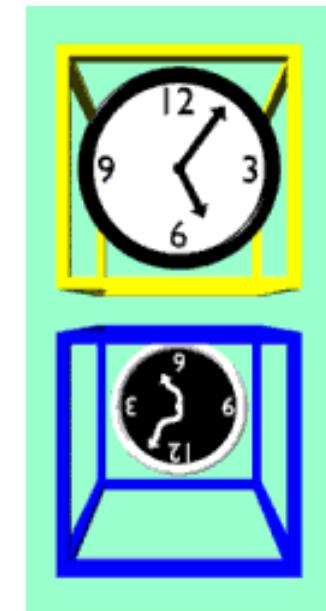
**Simplest ideas on
CPT Violation (CPTV)
do not work for
Baryogenesis**



CPT VIOLATION IN THE EARLY UNIVERSE

**GENERATE Baryon and/or Lepton ASYMMETRY
through CPT Violation**

Assume CPT Violation was
strong in the Early Universe



ONE POSSIBILITY:
particle-antiparticle mass differences

$$[\Theta, H] \neq 0 \quad \rightarrow \quad m \neq \bar{m}$$

physics.indiana.edu

$$0 \neq H\Theta|m\rangle - \Theta H|m\rangle = H\Theta|m\rangle - m\Theta|m\rangle$$

($|m\rangle$ = mass eigenstate
 $\Theta |m\rangle$ = antimatter state)

Equilibrium Distributions different between particle-antiparticles *Can these create the observed matter-antimatter asymmetry?*

$$f(E, \mu) = \frac{1}{\exp[(E - \mu)/T] \pm 1}$$

$$\begin{aligned} m &\neq \bar{m} \\ \delta m &= m - \bar{m} \end{aligned}$$

$$\delta n \equiv n - \bar{n} = g_d f \int \frac{d^3 p}{(2\pi)^3} [f(E, \mu) - f(\bar{E}, \bar{\mu})]$$

$$E = \sqrt{p^2 + m^2}, \bar{E} = \sqrt{p^2 + \bar{m}^2}$$

Dolgov, Zeldovich
Dolgov (2009)

Assume dominant contributions to Baryon asymmetry from quarks-antiquarks

$$m(T) \sim gT \quad \rightarrow$$

High-T quark mass >> Lepton mass

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Assuming dominant contributions to Baryon asymmetry from quarks-antiquarks

$$\beta_T = \frac{n_B}{n_\gamma} = -8.4 \cdot 10^{-3} (18m_u \delta m_u + 15m_d \delta m_d) / T^2$$

Dolgov, Zeldovich
Dolgov (2009)

$$n_\gamma = 0.24T^3 \quad \text{photon equilibrium density at temperature T}$$

Reasonable to take:

$$\delta m_q \sim \delta m_p$$

Dolgov (2009)

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$$n_\gamma = 0.24T^3$$

Current bound
for proton-anti
proton mass diff.

$$\delta m_p < 7 \times 10^{-10} m_p$$

C.L. 90%

ASACUSA Coll. (2016)

$$\delta m_q \sim \delta m_p \quad \longrightarrow$$

Too small
 $\beta^{T=0}$

NB: To reproduce the observed $\beta^{(T=0)} = 6 \cdot 10^{-10}$ need

$$\delta m_q(T = 100 \text{ GeV}) \sim 10^{-5} - 10^{-6} \text{ GeV} \gg \delta m_p$$

CPT Violating quark-antiquark Mass difference alone CANNOT REPRODUCE observed BAU



**But
CPT Violation (**CPTV**)
is associated
with **many more**
effects & parameters
to explore
in connection to
Baryogenesis...**



STANDARD MODEL EXTENSION

Kostelecky, Bluhm, Colladay, Lehnert, Potting, Russell *et al.*

$$\mathcal{L} = \frac{1}{2}i\bar{\psi}\Gamma^\nu\bar{\partial}_\nu\psi - \bar{\psi}M\psi, \quad M \equiv m + a_\mu\gamma^\mu + b_\mu\gamma_5\gamma^\mu + \frac{1}{2}H^{\mu\nu}\sigma_{\mu\nu}$$

LV & CPTV

$$\Gamma^\nu \equiv \gamma^\nu + c^{\mu\nu}\gamma_\mu + d^{\mu\nu}\gamma_5\gamma_\mu + e^\nu + if^\nu\gamma_5 + \frac{1}{2}g^{\lambda\mu\nu}\sigma_{\lambda\mu}$$

LV only



Spontaneous Violation of Lorentz Symmetry
(LV coefficients are v.e.v. of tensor-valued field quantities)

Microscopic Origin of SME coefficients?

Several ``Geometry-induced'' examples:

Microscopic Origin of SME coefficients?

$$[x^\mu, x^\nu] = \theta^{\mu\nu} \neq 0$$

Several ``Geometry-induced'' examples:

Non-Commutative Geometries **LV only ($H_{\mu\nu}$, $d_{\mu\nu}$, ...)**

Microscopic Origin of SME coefficients?

Several ``Geometry-induced'' examples:

Non-Commutative Geometries **LV only**

**LV &
CPTV**



Axisymmetric Background

Geometries of the Early Universe

Torsionful Geometries (including strings...)

Early Universe T-dependent effects:

Large @ high T, low values today
for coefficients of SME

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Kostelecky et al.

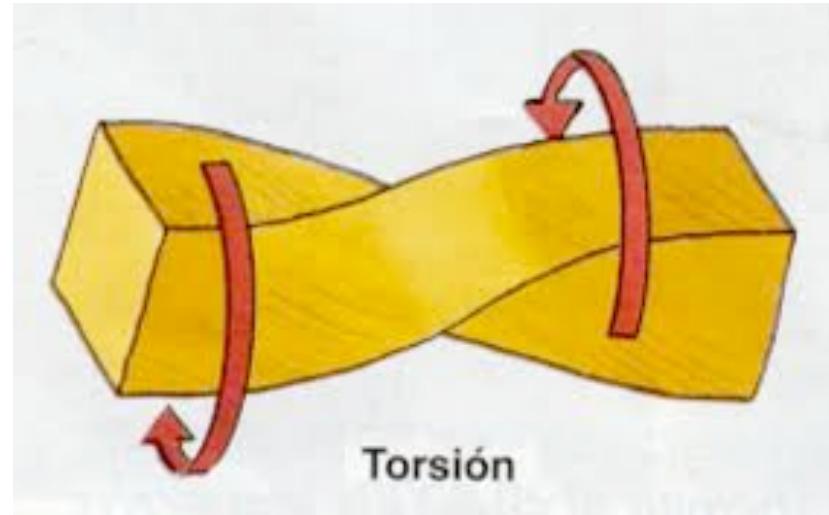
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In particular,
Space-times with



CPTV Effects of different Space-Time-Curvature/
Spin couplings between fermions/antifermions

B. Mukhopadhyay, U. Debnath, N. Dadhich, M. Sinha, Lambiase, Mohanty,
NEM, Ellis, Sarkar, de Cesare, Bossingham

Dirac Lagrangian (for concreteness, it can be extended to Majorana neutrinos)

$$\mathcal{L} = \sqrt{-g} (i \bar{\psi} \gamma^a D_a \psi - m \bar{\psi} \psi)$$

$$D_a = \left(\partial_a - \frac{i}{4} \omega_{bca} \sigma^{bc} \right),$$

$$g_{\mu\nu} = e_\mu^a \eta_{ab} e_\nu^b$$

$$\omega_{bca} = e_{b\lambda} (\partial_a e_c^\lambda + \Gamma_{\gamma\mu}^\lambda e_c^\gamma e_a^\mu).$$

Gravitational covariant derivative
including spin connection

$$\sigma^{ab} = \frac{i}{2} [\gamma^a, \gamma^b]$$

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Standard Model Extension
type Lorentz-violating
coupling
(Kostelecky et al.)



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For homogeneous and isotropic
Friedman-Robertson-Walker
geometries the resulting B^μ **vaniſh**

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Can be constant in a given local frame in Early Universe
axisymmetric (Bianchi) cosmologies or **near rotating Black holes**,



NEM & Sarben Sarkar, [arXiv:1211.0968](#)

John Ellis, NEM & Sarkar, [arXiv:1304.5433](#)

De Cesare, NEM & Sarkar [arXiv:1412.7077](#)

Bossingham, NEM & Sarkar, [arXiv:1712.03312](#)

$$\mathcal{L} = \sqrt{-g} (i \bar{\psi} \gamma^a D_a \psi - m \bar{\psi} \psi)$$

$$D_a = \left(\partial_a - \frac{i}{4} \omega_{bca} \sigma^{bc} \right),$$

$$g_{\mu\nu} = e_\mu^a \eta_{ab} e_\nu^b$$

$$\omega_{bca} = e_{b\lambda} (\partial_a e_c^\lambda + \Gamma_{\gamma\mu}^\lambda e_c^\gamma e_a^\mu).$$

Gravitational covariant derivative including spin connection

$$\sigma^{ab} = \frac{i}{2} [\gamma^a, \gamma^b]$$

$$\mathcal{L} = \mathcal{L}_f + \mathcal{L}_I = \sqrt{-g} \bar{\psi} [(i\gamma^a \partial_a - m) + \gamma^a \gamma^5 B_a] \psi,$$

$$B^d = \epsilon^{abcd} e_{b\lambda} (\partial_a e_c^\lambda + \Gamma_{\alpha\mu}^\lambda e_c^\alpha e_a^\mu)$$

If **torsion** then $\Gamma_{\mu\nu} \neq \Gamma_{\nu\mu}$
antisymmetric part is the
contorsion tensor, contributes



NEM & Sarben Sarkar, [arXiv:1211.0968](#)

John Ellis, NEM & Sarkar, [arXiv:1304.5433](#)

De Cesare, NEM & Sarkar [arXiv:1412.7077](#)

Bossingham, NEM & Sarkar, [arXiv:1712.03312](#)

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in string theory models
antisymmetric tensor
field-strength (H-torsion)
cosmological backgrounds lead to
constant B^0 in FRW frame



Part IIa

CPT Violation in a String-Inspired Model of the Early Universe

A non-trivial example of Torsion: String Theories with Antisymmetric Tensor Backgrounds

NEM & Sarben Sarkar, [arXiv:1211.0968](#)

John Ellis, NEM & Sarkar, [arXiv:1304.5433](#)

De Cesare, NEM & Sarkar [arXiv:1412.7077](#)

Bossingham, NEM & Sarkar, [arXiv:1712.03312](#)

Massless Gravitational multiplet of (closed) strings: **spin 0 scalar (dilaton)**

spin 2 traceless symmetric rank 2 tensor (graviton)

spin 1 antisymmetric rank 2 tensor

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spin 2 traceless symmetric rank 2 tensor (graviton)

spin 1 antisymmetric rank 2 tensor

KALB-RAMOND FIELD

$$B_{\mu\nu} = -B_{\nu\mu}$$

Effective field theories (low energy scale $E \ll M_s$) ``**gauge**'' invariant

$$B_{\mu\nu} \rightarrow B_{\mu\nu} + \partial_{[\mu} \theta(x)_{\nu]}$$

Depend only on field strength :

$$H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]}$$

Bianchi identity :

$$\partial_{[\sigma} H_{\mu\nu\rho]} = 0 \rightarrow d \star H = 0$$

ROLE OF H-FIELD AS TORSION

EFFECTIVE GRAVITATIONAL ACTION IN STRING LOW-ENERGY LIMIT

4-DIM
PART

$$\kappa^2 = 8\pi G$$

$$\begin{aligned} S^{(4)} &= \int d^4x \sqrt{-g} \left(\frac{1}{2\kappa^2} R - \frac{1}{6} H_{\mu\nu\rho} H^{\mu\nu\rho} \right) \\ &= \int d^4x \sqrt{-g} \left(\frac{1}{2\kappa^2} \bar{R} \right) \end{aligned}$$

$$\bar{R}(\bar{\Gamma})$$

generalised
curvature

$$\bar{\Gamma}_{\nu\rho}^\mu = \Gamma_{\nu\rho}^\mu + \frac{\kappa}{\sqrt{3}} H_{\nu\rho}^\mu \neq \bar{\Gamma}_{\rho\nu}^\mu$$

Contorsion

ROLE OF H-FIELD AS TORSION – AXION FIELD

EFFECTIVE GRAVITATIONAL ACTION IN STRING LOW-ENERGY LIMIT

4-DIM
PART

$$\begin{aligned} S^{(4)} &= \int d^4x \sqrt{-g} \left(\frac{1}{2\kappa^2} R - \frac{1}{6} H_{\mu\nu\rho} H^{\mu\nu\rho} \right) \\ &= \int d^4x \sqrt{-g} \left(\frac{1}{2\kappa^2} \bar{R} \right) \end{aligned}$$

$\kappa^2 = 8\pi G$

$$\bar{R}(\bar{\Gamma}) \quad \bar{\Gamma}_{\nu\rho}^\mu = \Gamma_{\nu\rho}^\mu + \frac{\kappa}{\sqrt{3}} H_{\nu\rho}^\mu \neq \bar{\Gamma}_{\rho\nu}^\mu$$

IN 4-DIM DEFINE DUAL OF H AS :

$$-3\sqrt{2}\partial_\sigma b = \sqrt{-g} \epsilon_{\mu\nu\rho\sigma} H^{\mu\nu\rho}$$

$b(x)$ = Pseudoscalar
(Kalb-Ramond (KR) axion)

$$\sim \frac{1}{2} \partial^\mu b \partial_\mu b$$

FERMIONS COUPLE TO H -TORSION VIA GRAVITATIONAL COVARIANT DERIVATIVE

$$S_\psi = \frac{i}{2} \int d^4x \sqrt{-g} \left(\bar{\psi} \gamma^\mu \bar{\mathcal{D}}_\mu \psi - (\bar{\mathcal{D}}_\mu \bar{\psi}) \gamma^\mu \psi \right)$$

TORSIONFUL CONNECTION, FIRST-ORDER FORMALISM

$$\bar{\mathcal{D}}_a = \partial_a - \frac{i}{4} \bar{\omega}_{bca} \sigma^{bc}$$

$$\bar{\omega}_{ab\mu} = \omega_{ab\mu} + K_{ab\mu}$$

contorsion

$$K_{abc} = \frac{1}{2} \left(T_{cab} - T_{abc} - T_{bca} \right)$$

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Non-trivial contributions to B^μ

$$B^d = \epsilon^{abcd} e_{b\lambda} \left(\partial_a e_c^\lambda + \Gamma_{\alpha\mu}^\lambda e_c^\alpha e_a^\mu \right)$$

$$H_{cab}$$

$$\bar{\Gamma}_{\nu\rho}^\mu = \Gamma_{\nu\rho}^\mu + \frac{\kappa}{\sqrt{3}} H_{\nu\rho}^\mu \neq \bar{\Gamma}_{\rho\nu}^\mu$$

FERMIONS COUPLE TO H -TORSION VIA GRAVITATIONAL COVARIANT DERIVATIVE

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TORSIONFUL CONNECTION, FIRST-ORDER FORMALISM

$$S_\psi \ni \int d^4x B_a \bar{\psi} \gamma^a \gamma^5 \psi$$

$$\bar{\omega}_{ab\mu} = \omega_{ab\mu} + K_{ab\mu}$$

contorsion

$$B^d \sim \epsilon^{abcd} H_{bca}$$

$$K_{abc} = \frac{1}{2} (T_{cab} - T_{abc} - T_{bca})$$

Non-trivial contributions to B^μ

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When **db/dt = constant** → Torsion is constant

Covariant Torsion tensor

$$\bar{\Gamma}^\lambda_{\mu\nu} = \Gamma^\lambda_{\mu\nu} + e^{-2\Phi} H^\lambda_{\mu\nu} \equiv \Gamma^\lambda_{\mu\nu} + T^\lambda_{\mu\nu}$$

$$T_{ijk} \sim \epsilon_{ijk} \dot{b}$$

Constant



$$S_\psi \ni \int d^4x B_a \bar{\psi} \gamma^a \gamma^5 \psi$$

$$\text{constant } \mathbf{B}^0 \propto \dot{b}$$

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Covariant Torsion tensor

$$\bar{\Gamma}^\lambda_{\mu\nu} = \Gamma^\lambda_{\mu\nu} + e^{-2\Phi} H^\lambda_{\mu\nu} \equiv \Gamma^\lambda_{\mu\nu} +$$

Antoniadis, Bachas,
Ellis, Nanopoulos

In string theory a constant B^0 background is guaranteed by **exact solutions** with linear in FRW time $b = (\text{const}) t$

$$T_{ijk} \sim \epsilon_{ijk} \dot{b}$$

Constant



constant $B^0 \propto \dot{b}$

$$S_\psi \ni \int d^4x B_a \bar{\psi} \gamma^a \gamma^5 \psi$$

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$$S_\psi \ni \int d^4x B_a \bar{\psi} \gamma^a \gamma^5 \psi$$

$$\text{constant } \mathbf{B}^0 \propto \dot{b}$$

LV & CPTV

$$\mathcal{L} = \frac{1}{2} i \bar{\psi}^\nu \bar{\partial}_\nu \psi - \bar{\psi} M \psi, \quad M \equiv m + b_\mu \gamma^5 \gamma^\mu$$



Standard Model Extension type with CPT and Lorentz Violating background $b^0 = B^0$

NB:

Perturbatively we may also have such a constant B^0 background in the presence of **Lorentz-violating condensates** of fermion axial current temporal component

$$\langle 0 | J^{05} | 0 \rangle \neq 0$$

at the high-density, high-temperature Early Universe epochs

Lagrangian :

$$\mathcal{L} = \sqrt{-g} \left[\frac{1}{2\kappa^2} R - \frac{1}{2} (\partial_\mu b)^2 - \Omega + \sum_i [\bar{\psi}_i (i\gamma^\mu \partial_\mu - m_i) \psi_i + \frac{\kappa}{3\sqrt{6}} \partial_\mu b \bar{\psi}_i \gamma^\mu \gamma^5 \psi_i] + \dots \right]$$

i = Standard Model
fermionic species

vacuum
energy

$$\mathcal{O}((\partial b)^4)$$

higher derivative
terms in strings

NB:

Perturbatively we may also have such a constant B^0 background in the presence of **Lorentz-violating condensates** of fermion axial current temporal component

$$\langle 0 | J^{05} | 0 \rangle \neq 0$$

at the high-density, high-temperature Early Universe epochs

Eqs of motion for pseudoscalar:

$$\partial^\mu \left(\sqrt{-g} [\epsilon_{\mu\nu\rho\sigma} (\partial^\sigma \bar{b} - \tilde{c} J^{5\sigma}) + \mathcal{O}((\partial \bar{b})^3)] \right) = 0$$

$$\dot{\bar{b}} = \tilde{c} \langle J_0^5 \rangle = \tilde{c} \langle \psi_i^\dagger \gamma^5 \psi_i \rangle = \text{constant} \neq 0 \quad i \neq \text{Majorana neutrinos}$$

Condensate may be **subsequently destroyed** at a temperature T_c , $\langle 0 | J^{05} | 0 \rangle \rightarrow 0$ by relevant operators so eventually in an expanding FRW Universe **for $T < T_c$**

$$B^0 \sim \dot{\bar{b}} \sim 1/a^3(t) \sim T^3$$

weak torsion today,
compatible
with stringent
experimental limits



B⁰ : (string) theory underwent a **phase transition**
@ T ≈ T_d = 10⁵ GeV, from B⁰ = const = 1 MeV **to** :

(i) **either B⁰ = 0**

(ii) **or B⁰ small today but non zero**

$$B^0 \sim \dot{\bar{b}} \sim 1/a^3(t) \sim T^3$$

$$B_0 = c_0 T^3$$

$$c_0 = 10^{-42} \text{ meV}^{-2}$$

$$B_{0 \text{ today}} = \mathcal{O}(10^{-44}) \text{ meV}$$

Quite safe from stringent
Experimental Bounds

$$|B^0| < 10^{-2} \text{ eV}$$
$$B_i \equiv b_i < 10^{-31} \text{ GeV}$$



If Fermions are **DIRAC** (e.g. quarks, electrons)

DISPERSION RELATIONS OF FERMIONS ARE *DIFFERENT* FROM THOSE OF ANTI-FERMIONS IN *SUCH* GEOMETRIES



CPTV Dispersion relations ($B_0 = b_0$)

$$E = \sqrt{\vec{p}^2 + m^2 + B_0^2 + 2|\vec{p}| B_0}$$

$$\overline{E} = \sqrt{\vec{p}^2 + m^2 + B_0^2 - 2|\vec{p}| B_0}$$

but (bare) **masses** are equal between **particle/anti-particle** sectors

Abundances of fermions in Early Universe, then, **different** from those of antifermions, if B_0 is **non-trivial**, **ALREADY IN THERMAL EQUILIBRIUM**

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$$n - \bar{n} = \frac{g}{(2\pi)^3} \int d^3p \left(\frac{1}{1 + e^{E/T}} - \frac{1}{1 + e^{\bar{E}/T}} \right) \neq 0$$
$$E \neq \bar{E}$$

If Fermions are **DIRAC** (e.g. quarks, electrons)

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Abundances of fermions in Early Universe, then, **different** from those of antifermions, if B_0 is **non-trivial**, **ALREADY IN THERMAL EQUILIBRIUM**

But for Majorana fermions (their own antiparticles. situation is different...

cf below...

CPT VIOLATION IN THE EARLY UNIVERSE

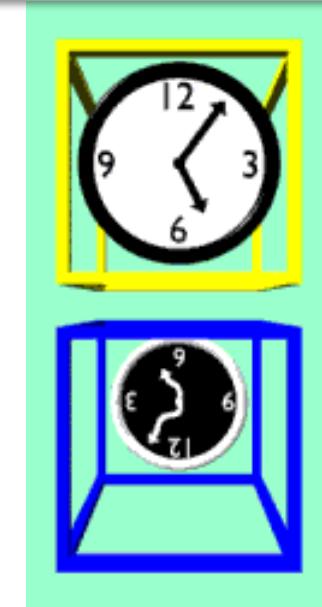
De Cesare, NEM & Sarkar [arXiv:1412.7077](https://arxiv.org/abs/1412.7077)
(Eur.Phys.J. C75 (2015), 514)

Bossingham, NEM & Sarkar [arXiv:1712.03312](https://arxiv.org/abs/1712.03312)
(Eur.Phys.J. C78 (2018) 10, 113)

Right-Handed Heavy Majorana Neutrinos

Mechanism
For Torsion-Background-
Induced tree-level
Leptogenesis → Baryogenesis

Through B-L conserving
Sphaleron processes
In the standard model



physics.indiana.edu

SM Extension with N extra right-handed neutrinos

Non SUSY

ν MSM

Boyarski, Ruchayskiy, Shaposhnikov

$$L = L_{SM} + \bar{N}_I i\partial_\mu \gamma^\mu N_I - F_{\alpha I} \bar{L}_\alpha N_I \tilde{\phi} - \frac{M_I}{2} \bar{N}_I^c N_I + \text{h.c.}$$

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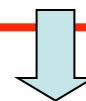
Yukawa couplings
Matrix (N=2 or 3)

$$F = \tilde{K}_L f_d \tilde{K}_R^\dagger$$

SM Extension with N extra right-handed neutrinos

ν MSM

$$L = L_{SM} + \bar{N}_I i\partial_\mu \gamma^\mu N_I - F_{\alpha I} \bar{L}_\alpha N_I \tilde{\phi} - \frac{M_I}{2} \bar{N}_I^c N_I + \text{h.c.}$$



Majorana masses
to (2 or 3) active
neutrinos via **seesaw**

**Yukawa couplings
Matrix (N=2 or 3)**

Minkowski, Yanagida,
Mohapatra, Senjanovic
Sechter, Valle ...

$$m_\nu = -M^D \frac{1}{M_I} [M^D]^T .$$

$$M_D = F_{\alpha I} v \quad M_D \ll M_I$$
$$v = \langle \phi \rangle \sim 175 \text{ GeV}$$



CPTV Thermal Leptogenesis

Early Universe
 $T > 10^5$ GeV

CPT Violation



Lepton number & CP Violations @ tree-level
due to Lorentz/CPTV Background

$$N_I \rightarrow H\nu, \bar{H}\bar{\nu}$$

CPTV Thermal

$$\mathcal{L} = i\bar{N}\partial N - \frac{m}{2}(\bar{N^c}N + \bar{N}N^c) - \bar{N}\not{B}\gamma^5 N - Y_k \bar{L}_k \not{\phi} N + h.c.$$

Early Universe
 $T > 10^5$ GeV

CPT Violation



Lepton number & CP Violations @ tree-level
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Early Universe
 $T > 10^5$ GeV

CPT Violation



One generation of massive neutrinos N suffices for generating CPTV Leptogenesis;



$$N_I \rightarrow H\nu, \bar{H}\bar{\nu}$$

Lepton number & CP Violations @ tree-level
due to Lorentz/CPTV Background

CPTV Thermal

$$\mathcal{L} = i\bar{N}\partial N - \frac{m}{2}(\bar{N^c}N + \bar{N}N^c) - \bar{N}\not{B}\gamma^5 N - Y_k \bar{L}_k \not{\phi} N + h.c.$$

Early Universe
 $T > 10^5$ GeV

CPT Violation



One generation of massive neutrinos N suffices for generating CPTV Leptogenesis; mass m free to be fixed



Lepton number & CP Violations @ tree-level
due to Lorentz/CPTV Background

$$N_I \rightarrow H\nu, \bar{H}\bar{\nu}$$

CPTV Thermal

$$\mathcal{L} = i\bar{N}\partial N - \frac{m}{2}(\bar{N}^c N + \bar{N} N^c) - \bar{N} \not{B} \gamma^5 N - Y_k \bar{L}_k \not{\phi} N + h.c.$$

Early Universe
 $T > 10^5$ GeV

CPT Violation

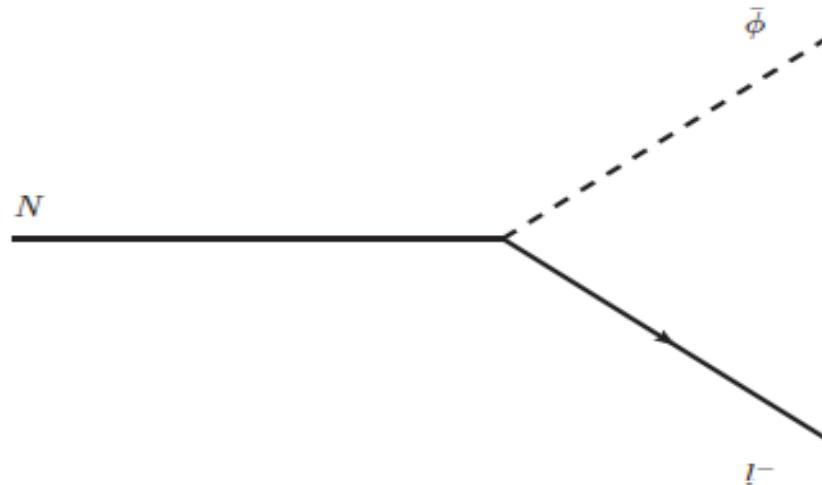


Constant H-torsion
(antisymmetric
tensor field strength
in string models)

Lepton number & CP Violations @ tree-level
due to Lorentz/CPTV Background

$$N_I \rightarrow H\nu, \bar{H}\bar{\nu}$$

Produce Lepton asymmetry



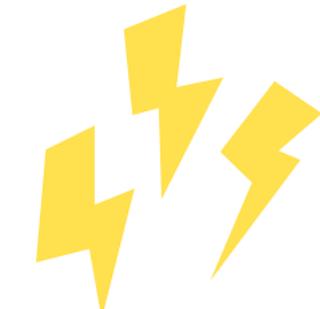
CPTV Thermal

$$\mathcal{L} = i\bar{N}\phi N - \frac{m}{2}(\bar{N^c}N + \bar{N}N^c) - \bar{N}X\gamma^5 N - Y_k\bar{L}_k\bar{\phi}N + h.c.$$

Early Universe
 $T > 10^5$ GeV

CPT Violation

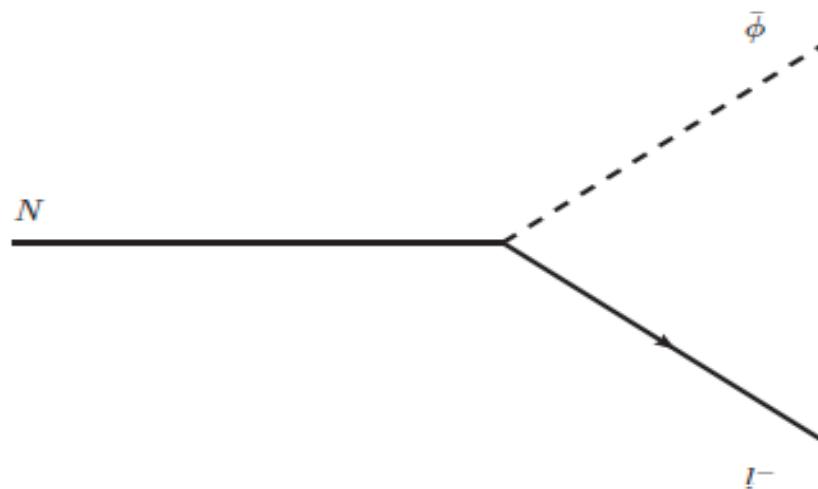
Constant H-torsion



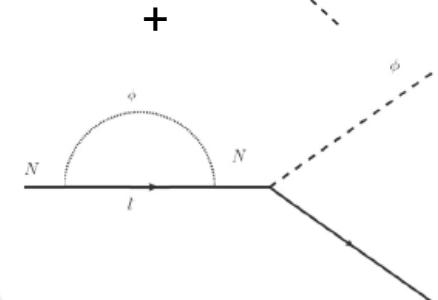
Lepton number & CP Violations @ tree-level
 due to Lorentz/CPTV Background

$$N_I \rightarrow H\nu, \bar{H}\bar{\nu}$$

Produce Lepton asymmetry



Contrast with one-loop conventional CPV Leptogenesis (in absence of H-torsion)



Fukugita, Yanagida,

CPTV Thermal

$$\mathcal{L} = i\bar{N}\partial N - \frac{m}{2}(\bar{N^c}N + \bar{N}N^c) - \bar{N}\not{B}\gamma^5 N - Y_k \bar{L}_k \not{\phi} N + h.c.$$

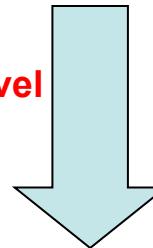
Early Universe
 $T > 10^5$ GeV

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CPT Violation

Constant H-torsion



Produce Lepton asymmetry

CPTV Thermal

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Early Universe
 $T > 10^5$ GeV

Lepton number & CP Violations @ tree-level
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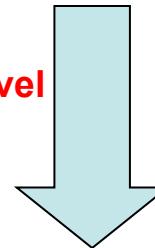
CPT Violation



Constant H-torsion
 $B^0 \neq 0$ background



Solving system
of Boltzmann eqs



$$\frac{\Delta L}{n_\gamma} \simeq 10^{-10},$$

Produce Lepton asymmetry



Decoupling Temperature T_D : **decay process out of equilibrium**
@ which Lepton asymmetry is evaluated

$$\Gamma \simeq H = 1,66 T_D^2 \mathcal{N}^{1/2} m_P^{-1}$$

d.o.f.

assume standard cosmology

$$T_D \simeq 6.2 \cdot 10^{-2} \frac{|Y|}{\mathcal{N}^{1/4}} \sqrt{\frac{m_P(\Omega^2 + B_0^2)}{\Omega}}$$

for one generation
of RH heavy neutrino

$$\Omega = \sqrt{B_0^2 + m_N^2} .$$

Estimate: Total Lepton number asymmetry

$$(N \rightarrow \ell^- \phi^+, \nu \phi^0) - (N \rightarrow \ell^+ \phi^-, \bar{\nu} \phi^0)$$

via solving the appropriate system of **Boltzmann equations**:

CPTV Thermal

$$\mathcal{L} = i\bar{N}\partial N - \frac{m}{2}(\bar{N^c}N + \bar{N}N^c) - \bar{N}\not{B}\gamma^5 N - Y_k \bar{L}_k \not{\phi} N + h.c.$$

Early Universe
T > 10⁵ GeV

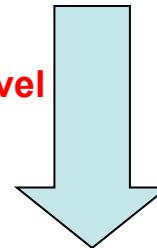
Lepton number & CP Violations @ tree-level
due to Lorentz/CPTV Background

$$N_I \rightarrow H\nu, \bar{H}\bar{\nu}$$

CPT Violation



Constant H-torsion
 $B^0 \neq 0$ background



$$\frac{\Delta L}{n_\gamma} \simeq 10^{-10},$$



$$\frac{B_0}{m} \simeq 10^{-8}$$

Produce Lepton asymmetry

$$Y_k \sim 10^{-5}$$

$$m \geq 100 \text{TeV} \rightarrow$$

$$B^0 \sim 1 \text{MeV}$$

$$T_D \simeq m \sim 100 \text{ TeV}$$

CPTV Thermal

$$\mathcal{L} = i\bar{N}\partial N - \frac{m}{2}(\bar{N^c}N + \bar{N}N^c) - \bar{N}\not{B}\gamma^5 N - Y_k \bar{L}_k \not{\phi} N + h.c.$$

Early Universe
 $T > 10^5$ GeV

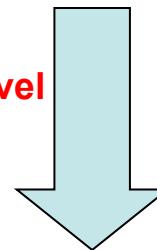
Lepton number & CP Violations @ tree-level
 due to Lorentz/CPTV Background

$$N_I \rightarrow H\nu, \bar{H}\bar{\nu}$$

CPT Violation



Constant H-torsion
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Early Universe
T > 10⁵ GeV

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Equilibrated electroweak
B+L violating sphaleron interactions

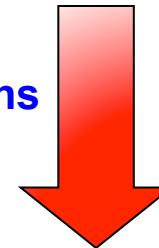
B-L conserved

$$L = \frac{2}{M} l_L l_L \phi \phi + \text{H.c.}$$

where

$$l_L = \begin{bmatrix} \nu_e \\ e \end{bmatrix}_L, \begin{bmatrix} \nu_\mu \\ \mu \end{bmatrix}_L, \begin{bmatrix} \nu_\tau \\ \tau \end{bmatrix}_L$$

*Environmental
Conditions Dependent*



Observed Baryon Asymmetry
In the Universe (BAU)

Fukugita, Yanagida,



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Observed Baryon Asymmetry
In the Universe (BAU)

Estimate BAU by fixing CPTV background parameters
In some models this means fine tuning

B⁰ : (string) theory underwent a **phase transition**
@ T ≈ T_d = 10⁵ GeV, from B⁰ = const = 1 MeV **to** :

(i) **either B⁰ = 0**

(ii) **or B⁰ small today but non zero**

$$B^0 \sim \dot{\bar{b}} \sim 1/a^3(t) \sim T^3$$

$$B_0 = c_0 T^3$$

$$c_0 = 10^{-42} \text{ meV}^{-2}$$

$$B_{0 \text{ today}} = \mathcal{O}(10^{-44}) \text{ meV}$$

Quite safe from stringent
Experimental Bounds

$$|B^0| < 10^{-2} \text{ eV}$$
$$B_i \equiv b_i < 10^{-31} \text{ GeV}$$



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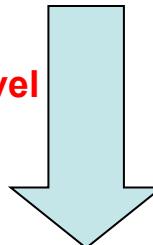
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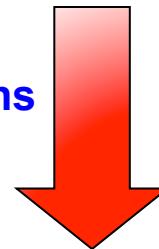
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e.g. May Require
Fine tuning of
Vacuum energy

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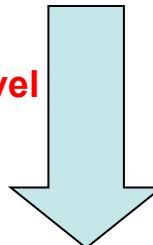
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ROLE OF H-FIELD AS TORSION – AXION FIELD

EFFECTIVE GRAVITATIONAL ACTION IN STRING LOW-ENERGY LIMIT

4-DIM
PART

$$\begin{aligned} S^{(4)} &= \int d^4x \sqrt{-g} \left(\frac{1}{2\kappa^2} R - \frac{1}{6} H_{\mu\nu\rho} H^{\mu\nu\rho} \right) \\ &= \int d^4x \sqrt{-g} \left(\frac{1}{2\kappa^2} \bar{R} \right) \end{aligned}$$

$$\kappa^2 = 8\pi G$$

IN 4-DIM DEFINE DUAL OF H AS :

$$-3\sqrt{2}\partial_\sigma b = \sqrt{-g} \epsilon_{\mu\nu\rho\sigma} H^{\mu\nu\rho}$$

$b(x)$ = Pseudoscalar
(Kalb-Ramond (KR) axion)

$$\sim \frac{1}{2} \partial^\mu b \partial_\mu b$$

e.g. May Require
Fine tuning of
Vacuum energy

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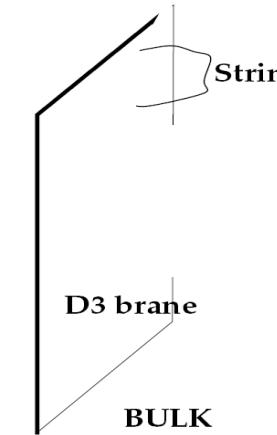
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$$\sim \frac{1}{2} \partial^\mu b \partial_\mu b$$



constant if
 $\dot{b} = \text{const}$

need to be
cancelled by
bulk contrib.
in brane models

$b(x)$ = Pseudoscalar
(Kalb-Ramond (KR) axion)

Alternatively: B^0 approximately constant

Add chemical potential μ of fermions (e.g. quarks)

Eqs of motion for pseudoscalar:

$$\partial^\mu \left(\sqrt{-g} [\epsilon_{\mu\nu\rho\sigma} (\partial^\sigma \bar{b} - \tilde{c} J^{5\sigma}) + \mathcal{O}((\partial \bar{b})^3)] \right) = 0$$

Thermal (high $T > m_N/10$) average $\langle J_0^5 \rangle \simeq -\frac{2.722 g_q}{\pi^2} \mu B_0 T$

$$B_0(z) = B_0(z_D) \frac{\sqrt{z_D}(z_D + 3\epsilon)}{\sqrt{z}(z + 3\epsilon)}$$

$$0 < z = m_N/T \leq 10$$

$$\epsilon = \frac{2.722 g_q}{\pi} \frac{\mu m_N}{m_{pl}^2} \simeq 1.0062 \times 10^{-29}$$

Boltzmann equation solutions for **Leptogenesis**

$$B_0(z) \simeq (0.1399 \text{ MeV}) z^{-3/2}$$



Consistent with current bounds

$$|B^0|(\text{today}) < 10^{-2} \text{ eV}$$



Part II(b)

Quantum-Gravity induced Decoherence & intrinsic CPT Violation

CPT VIOLATION

Conditions for the Validity of CPT Theorem

CPT Invariance Theorem :

- (i) Flat space-times
- (ii) Lorentz invariance
- (iii) Locality
- (iv) Unitarity

This Talk

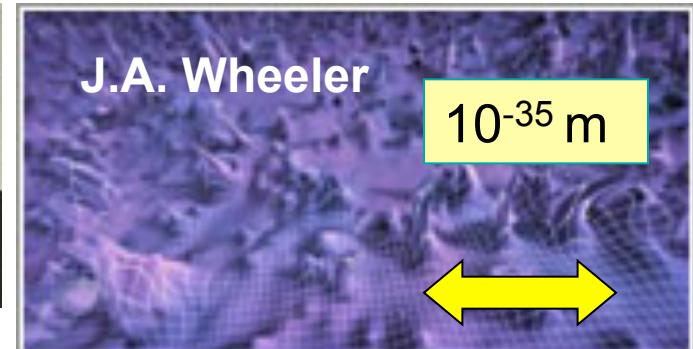
(ii)-(iv) Independent reasons for violation

e.g. **QUANTUM SPACE-TIME
FOAM AT PLANCK SCALES**



J.A. Wheeler

10^{-35} m



CPT VIOLATION

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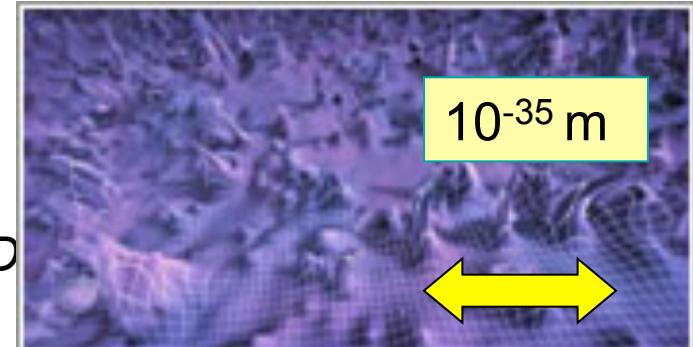
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Hawking,
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QUANTUM GRAVITY INDUCED DECOHERENCE
EVOLUTION OF PURE QM STATES TO MIXED
AT LOW ENERGIES

LOW ENERGY **CPT OPERATOR NOT WELL DEFINED**



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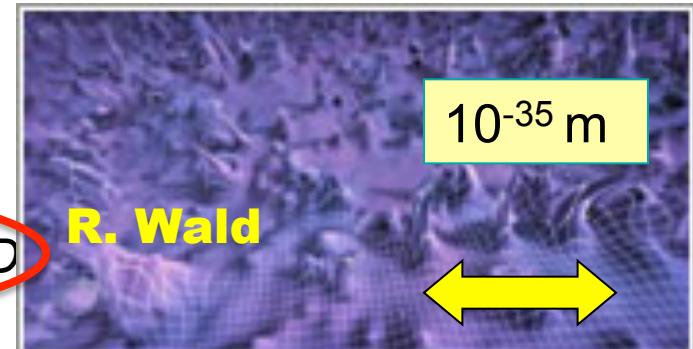
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Decoherence & ill-defined CPT

Decoherence implies
that
asymptotic density
matrix of
low-energy matter :

$$\rho = \text{Tr}|\psi\rangle\langle\psi|$$

$$\rho_{\text{out}} = \$\rho_{\text{in}}$$

$$\$ \neq S S^\dagger$$

$$S = e^{i \int H dt}$$

QG May induce **quantum decoherence** of propagating matter and **intrinsic CPT Violation** in the sense that the CPT operator Θ is **not well-defined**



$$\Theta \rho_{\text{in}} = \bar{\rho}_{\text{out}}$$

If Θ well-defined
can show that $\$^{-1} = \Theta^{-1} \$ \Theta^{-1}$
exists !

INCOMPATIBLE WITH DECOHERENCE !

Hence Θ ill-defined at low-energies in QG foam models

Wald (79)

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beyond Local Effective Field theory

$$\rho = \text{Tr} |\psi\rangle\langle\psi|$$

$$|i\rangle = \mathcal{N} \left[|M_0(\vec{k})\rangle |\overline{M}_0(-\vec{k})\rangle - |\overline{M}_0(\vec{k})\rangle |M_0(-\vec{k})\rangle + \omega \left(|M_0(\vec{k})\rangle |\overline{M}_0(-\vec{k})\rangle + |\overline{M}_0(\vec{k})\rangle |M_0(-\vec{k})\rangle \right) \right]$$

$$\omega = |\omega| e^{i\Omega}$$

May contaminate initially antisymmetric neutral meson M state by symmetric parts (ω -effect)

Bernabeu, NEM,
Papavassiliou (04),...

Hence Θ ill-defined at low-energies in
QG foam models → **may affect EPR**

Wald (79)

NB: Including conventional CPTV (θ) in the Hamiltonian

Bernabeu, Botella, NEM, Nebot (2018)

$$\mathbf{H}|B_H\rangle = \mu_H|B_H\rangle, \quad |B_H\rangle = p_H|B_d^0\rangle + q_H|\bar{B}_d^0\rangle,$$

$$\mathbf{H}|B_L\rangle = \mu_L|B_L\rangle, \quad |B_L\rangle = p_L|B_d^0\rangle - q_L|\bar{B}_d^0\rangle.$$

H (L) = (High (Low) mass states



$$|\Psi_0\rangle \propto |B_L\rangle|B_H\rangle - |B_H\rangle|B_L\rangle$$

$$+ \omega \left\{ \theta [|B_H\rangle|B_L\rangle + |B_L\rangle|B_H\rangle] + (1 - \theta) \frac{p_L}{p_H} |B_H\rangle|B_H\rangle - (1 + \theta) \frac{p_H}{p_L} |B_L\rangle|B_L\rangle \right\}$$

ω -effect

CPTV in Hamiltonian

$$\theta = \frac{\mathbf{H}_{22} - \mathbf{H}_{11}}{\mu_H - \mu_L}$$

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Wald (79)

If CPT ill-defined →
tiny effect (if due to **Quantum Gravity decoherence**) → concept of antiparticle still well-defined,
but...



- (i) **observable effects in entangled (neutral) meson-states**
 ω -effect

Part III(a)

ω -effect searches

in

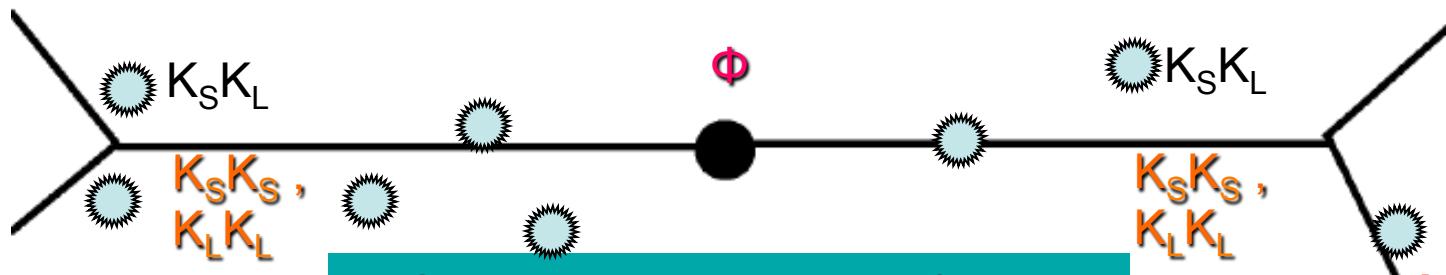
Entangled Neutral Meson

Systems

- Neutral mesons **no** longer **indistinguishable** particles, initial entangled state:

$$|i\rangle = \mathcal{N} \left[(|K_S(\vec{k}), K_L(-\vec{k})\rangle - |K_L(\vec{k}), K_S(-\vec{k})\rangle) + \omega (|K_S(\vec{k}), K_S(-\vec{k})\rangle - |K_L(\vec{k}), K_L(-\vec{k})\rangle) \right]$$

$$\omega = |\omega| e^{i\Omega}$$



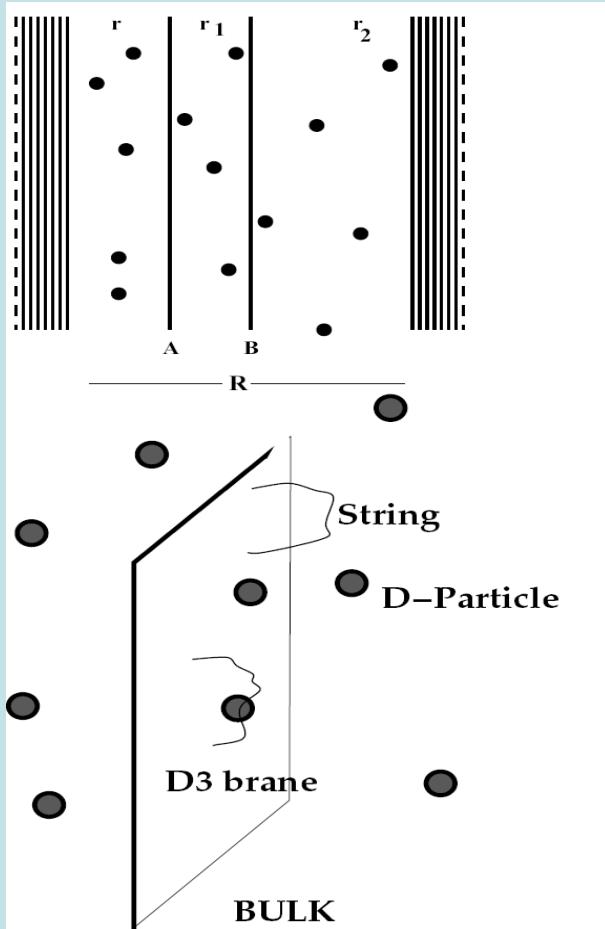
IF CPT ILL-DEFINED (e.g. flavour violating (FV) D-particle Foam)

In a concrete model of space-time foam

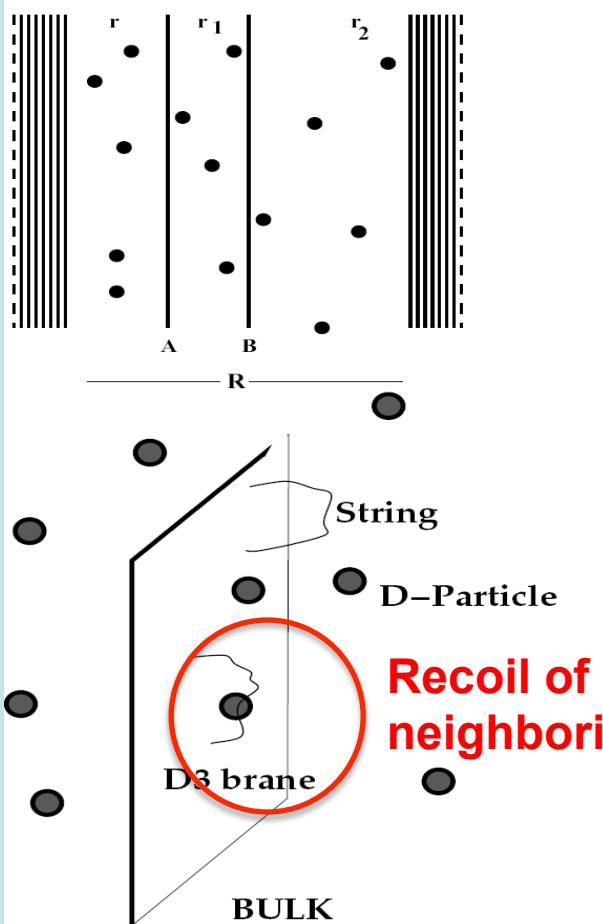
$$|\omega|^2 \sim \frac{\zeta^2 k^4}{M_{QG}^2 (m_1 - m_2)^2}, \Delta k = \zeta k \text{ (particle momentum transfer)}$$

If QCD effects, sub-structure in neutral mesons ignored, and D-foam acts as if they were structureless particles, then for $M_{QG} \sim 10^{18} \text{ GeV}$
the estimate for ω : $|\omega| \sim 10^{-4} |\zeta|$, for $1 > |\zeta| > 10^{-2}$ (natural)
Not far from sensitivity of upgraded meson factories (e.g. **KLOE2**)

D-particle recoil and entangled Meson States



D-particle recoil and entangled Meson States

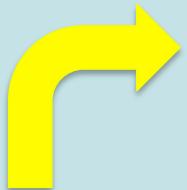


Gravitationally dressed quantum states viewed as quantum mechanically perturbed



propagation of matter string in curved background interaction with metric

Recoil of massive Defect distorts neighboring space-time on brane world



D-particle recoil and entangled Meson States

- Apply non-degenerate perturbation theory to construct “gravitationally dressed” states from $|k, \uparrow\rangle^{(i)}, |k, \downarrow\rangle^{(i)}, i = 1, 2$

$$|k^{(i)}, \downarrow\rangle_{QG}^{(i)} = |k^{(i)}, \downarrow\rangle^{(i)} + |k^{(i)}, \uparrow\rangle^{(i)} \alpha^{(i)}$$

$$\alpha^{(i)} = \frac{^{(i)}\langle \uparrow, k^{(i)} | \widehat{H}_I | k^{(i)}, \downarrow \rangle^{(i)}}{E_2 - E_1}$$

$$\widehat{H} = - (r_1 \sigma_1 + r_2 \sigma_2) \widehat{k}$$

$$(g^{ab} \nabla_a \nabla_b - m^2) \Phi = 0$$

FLAVOUR FLIP
Perturbation due to
recoil distortion of space-time

$$g_{0i} \propto \Delta k_i / M_P \otimes (\text{flavour} - \text{flip})$$

$$\Delta k_i = r_i k, \langle\langle r_i \rangle\rangle = 0, \langle\langle r_i r_j \rangle\rangle = \Delta \delta_{ij}$$

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Similarly for $|k^{(i)}, \uparrow\rangle^{(i)}$

the dressed state

$|\downarrow\rangle \leftrightarrow |\uparrow\rangle$ and $\alpha \rightarrow \beta$

$$\beta^{(i)} = \frac{^{(i)}\langle \downarrow, k^{(i)} | \hat{H}_I | k^{(i)}, \uparrow \rangle^{(i)}}{E_1 - E_2}$$

$$\begin{aligned} & |k, \uparrow\rangle_{QG}^{(1)} | -k, \downarrow \rangle_{QG}^{(2)} - |k, \downarrow\rangle_{QG}^{(1)} | -k, \uparrow \rangle_{QG}^{(2)} = \\ & |k, \uparrow\rangle^{(1)} | -k, \downarrow \rangle^{(2)} - |k, \downarrow\rangle^{(1)} | -k, \uparrow \rangle^{(2)} \\ & + |k, \downarrow\rangle^{(1)} | -k, \downarrow \rangle^{(2)} (\beta^{(1)} - \beta^{(2)}) + |k, \uparrow\rangle^{(1)} | -k, \uparrow \rangle^{(2)} (\alpha^{(2)} - \alpha^{(1)}) \\ & + \beta^{(1)} \alpha^{(2)} |k, \downarrow\rangle^{(1)} | -k, \uparrow \rangle^{(2)} - \alpha^{(1)} \beta^{(2)} |k, \uparrow\rangle^{(1)} | -k, \downarrow \rangle^{(2)} \end{aligned}$$

D-particle recoil and entangled Meson States

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$$\alpha^{(i)} = \frac{^{(i)}\langle \uparrow, k^{(i)} | \hat{H}_I | k^{(i)}, \downarrow \rangle^{(i)}}{E_2 - E_1}$$

Similarly for $|k^{(i)}, \uparrow\rangle^{(i)}$

the dressed state

$|\downarrow\rangle \leftrightarrow |\uparrow\rangle$ and $\alpha \rightarrow \beta$

$$\beta^{(i)} = \frac{^{(i)}\langle \downarrow, k^{(i)} | \hat{H}_I | k^{(i)}, \uparrow \rangle^{(i)}}{E_1 - E_2}$$

$$\begin{aligned}
 & |k, \uparrow\rangle_{QG}^{(1)} | -k, \downarrow \rangle_{QG}^{(2)} - |k, \downarrow\rangle_{QG}^{(1)} | -k, \uparrow \rangle_{QG}^{(2)} = \\
 & |k, \uparrow\rangle^{(1)} | -k, \downarrow \rangle^{(2)} - |k, \downarrow\rangle^{(1)} | -k, \uparrow \rangle^{(2)} \\
 & + |k, \downarrow\rangle^{(1)} | -k, \downarrow \rangle^{(2)} (\beta^{(1)} - \beta^{(2)}) + |k, \uparrow\rangle^{(1)} | -k, \uparrow \rangle^{(2)} (\alpha^{(2)} - \alpha^{(1)}) \\
 & + \beta^{(1)} \alpha^{(2)} |k, \downarrow\rangle^{(1)} | -k, \uparrow \rangle^{(2)} - \alpha^{(1)} \beta^{(2)} |k, \uparrow\rangle^{(1)} | -k, \downarrow \rangle^{(2)}
 \end{aligned}$$



ω-effect

D-particle recoil and entangled Meson States

- Apply non-degenerate perturbation theory to construct “gravitationally dressed” states from $|k, \uparrow\rangle^{(i)}, |k, \downarrow\rangle^{(i)}, i = 1, 2$

$$|k^{(i)}, \downarrow\rangle_{QG}^{(i)} = |k^{(i)}, \downarrow\rangle^{(i)} + |k^{(i)}, \uparrow\rangle^{(i)} \alpha^{(i)}$$

$$\alpha^{(i)} = \frac{^{(i)}\langle \uparrow, k^{(i)} | \hat{H}_I | k^{(i)}, \downarrow \rangle^{(i)}}{E_2 - E_1}$$

Similarly for $|k^{(i)}, \uparrow\rangle^{(i)}$

the dressed state

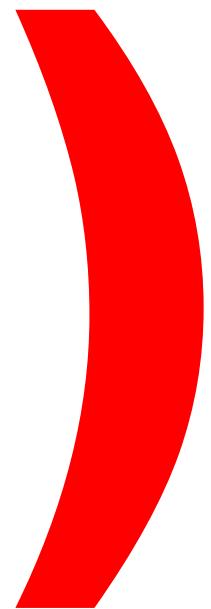
$|\downarrow\rangle \leftrightarrow |\uparrow\rangle$ and $\alpha \rightarrow \beta$

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 & |k, \uparrow\rangle_{QG}^{(1)} | -k, \downarrow \rangle_{QG}^{(2)} - |k, \downarrow\rangle_{QG}^{(1)} | -k, \uparrow \rangle_{QG}^{(2)} = \\
 & |k, \uparrow\rangle^{(1)} | -k, \downarrow \rangle^{(2)} - |k, \downarrow\rangle^{(1)} | -k, \uparrow \rangle^{(2)} \\
 & + |k, \downarrow\rangle^{(1)} | -k, \downarrow \rangle^{(2)} (\beta^{(1)} - \beta^{(2)}) + |k, \uparrow\rangle^{(1)} | -k, \uparrow \rangle^{(2)} (\alpha^{(2)} - \alpha^{(1)}) \\
 & + \beta^{(1)} \alpha^{(2)} |k, \downarrow\rangle^{(1)} | -k, \uparrow \rangle^{(2)} - \alpha^{(1)} \beta^{(2)} |k, \uparrow\rangle^{(1)} | -k, \downarrow \rangle^{(2)}
 \end{aligned}$$



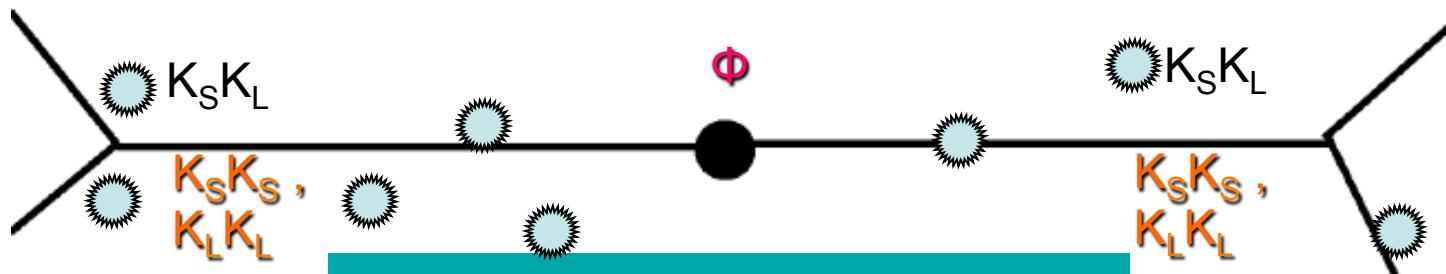
ω -effect



- Neutral mesons **no** longer **indistinguishable** particles, initial entangled state:

$$|i\rangle = \mathcal{N} \left[(|K_S(\vec{k}), K_L(-\vec{k})\rangle - |K_L(\vec{k}), K_S(-\vec{k})\rangle) + \omega (|K_S(\vec{k}), K_S(-\vec{k})\rangle - |K_L(\vec{k}), K_L(-\vec{k})\rangle) \right]$$

$$\omega = |\omega| e^{i\Omega}$$



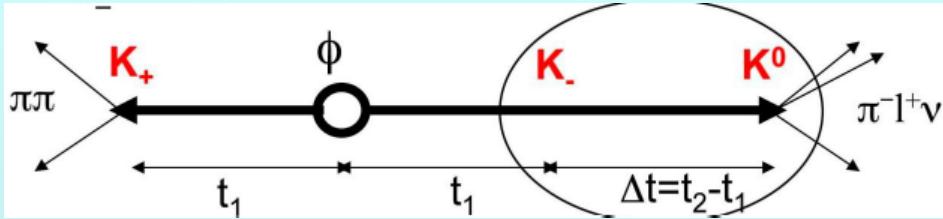
IF CPT ILL-DEFINED (e.g. flavour violating (FV) D-particle Foam)

In a concrete model of space-time foam

$$|\omega|^2 \sim \frac{\zeta^2 k^4}{M_{QG}^2 (m_1 - m_2)^2}, \Delta k = \zeta k \text{ (particle momentum transfer)}$$

If QCD effects, sub-structure in neutral mesons ignored, and D-foam acts as if they were structureless particles, then for $M_{QG} \sim 10^{18} \text{ GeV}$ the estimate for ω : $|\omega| \sim 10^{-4} |\zeta|$, for $1 > |\zeta| > 10^{-2}$ (natural)
Not far from sensitivity of upgraded meson factories (e.g. **KLOE2**)

Current Measurement Status of ω -effect



KLOE result: PLB 642(2006) 315
Found. Phys. 40 (2010) 852

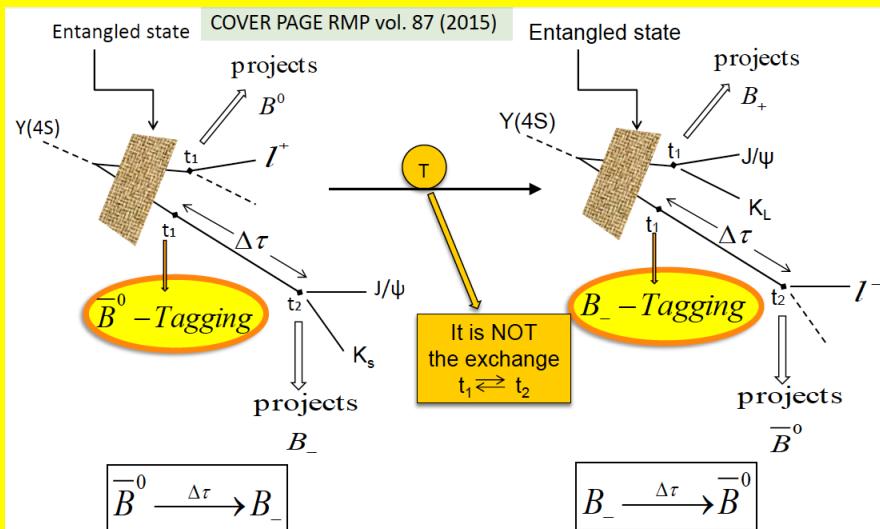
$$\Re \omega = (-1.6_{-2.1}^{+3.0} \text{STAT} \pm 0.4 \text{SYST}) \times 10^{-4}$$

$$\Im \omega = (-1.7_{-3.0}^{+3.3} \text{STAT} \pm 1.2 \text{SYST}) \times 10^{-4}$$

$$|\omega| < 1.0 \times 10^{-3} \text{ at } 95\% \text{ C.L.}$$

Neutral Kaons

Prospects KLOE-2 $\text{Re}(\omega), \text{Im}(\omega) \rightarrow 2 \times 10^{-5}$



Natural B-mesons

Equal Sign Dilepton Asymmetry

(Alvarez, Bernabeu, Nebot, JHEP 0611 (2006) 087)

$$-0.0084 \leq \text{Re}(\omega) \leq 0.0100 \quad 95\% \text{C.L.}$$

Novel signal from (f,g) \leftrightarrow (g,f)
(Bernabeu, Botella, NEM, Nebot
EPJC 77 (2017) 865)

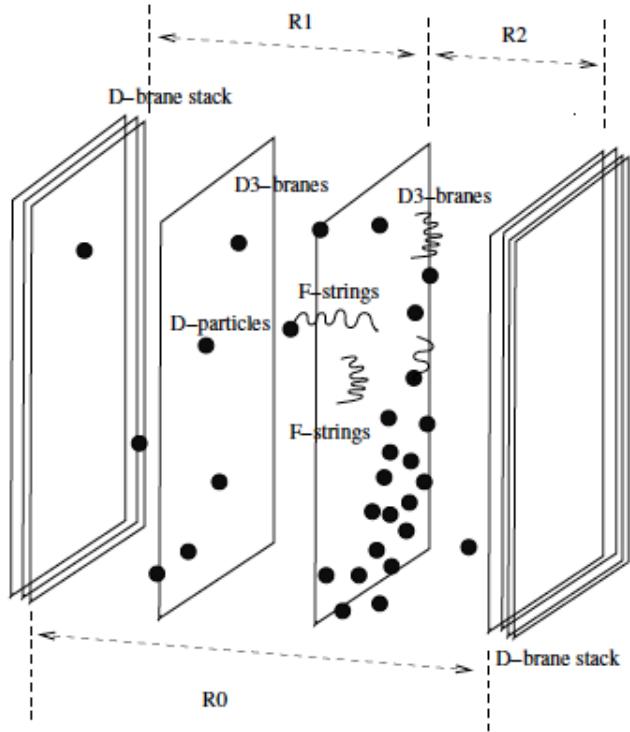
$\text{Im}(\theta)$	$(0.99 \pm 1.98)10^{-2}$
$\text{Im}(\omega)$	$\pm(6.40 \pm 2.80)10^{-2}$

Part III(b)

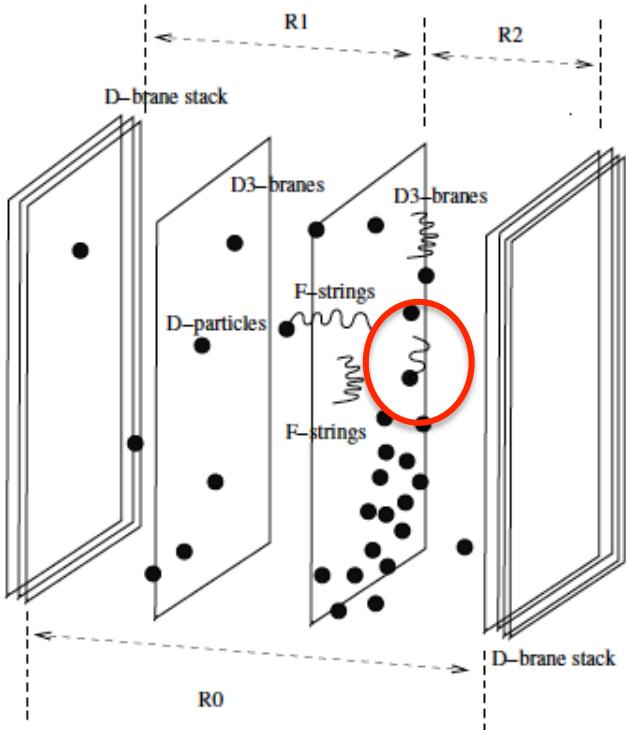
Connection to

Baryogenesis

Connection to Baryogenesis



Connection to Baryogenesis



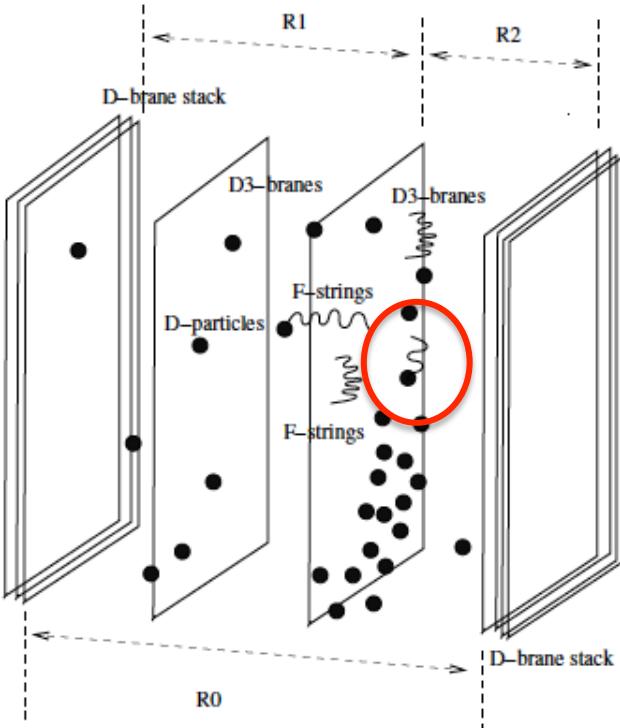
Effective metric due to LV foam medium

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = (\eta_{\mu\nu} + h_{\mu\nu}) dx^\mu dx^\nu , \quad h_{0i} = (u_i^a \parallel \sigma_a)$$

Stochastic media

$$\ll u_i \parallel \gg = 0 , \quad \ll u_i \parallel u_j \parallel \gg = \sigma^2 \delta_{ij} .$$

Connection to Baryogenesis



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Energy differences between matter-antimatter

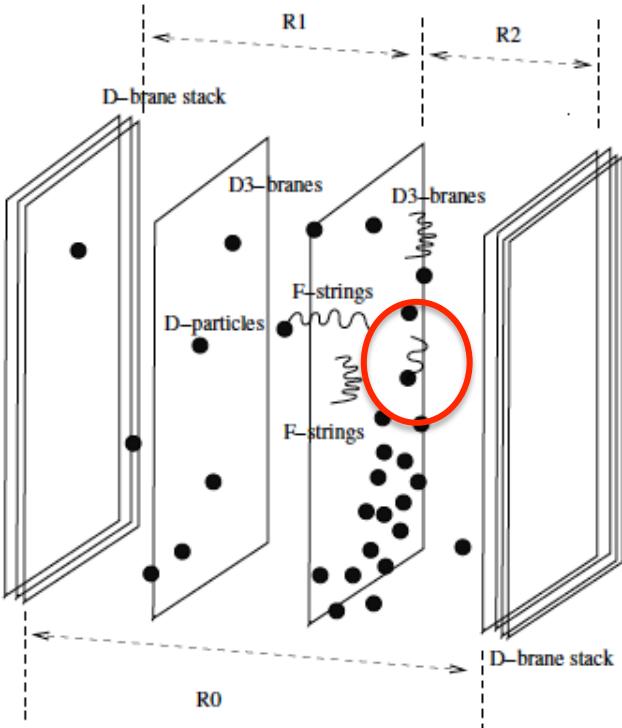
$$\ll E \gg = \sqrt{p^2 + m^2} + \frac{p^2 \sigma^2}{2 \sqrt{p^2 + m^2}} - \frac{1}{2} \frac{M_s}{g_s} \sigma^2$$

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situation equivalent to an induced chemical potential

$$\frac{1}{2} \frac{M_s}{g_s} \sigma^2 \equiv \mu_{\text{CPTV}} > 0$$

Connection to Baryogenesis



Effective metric due to LV foam medium

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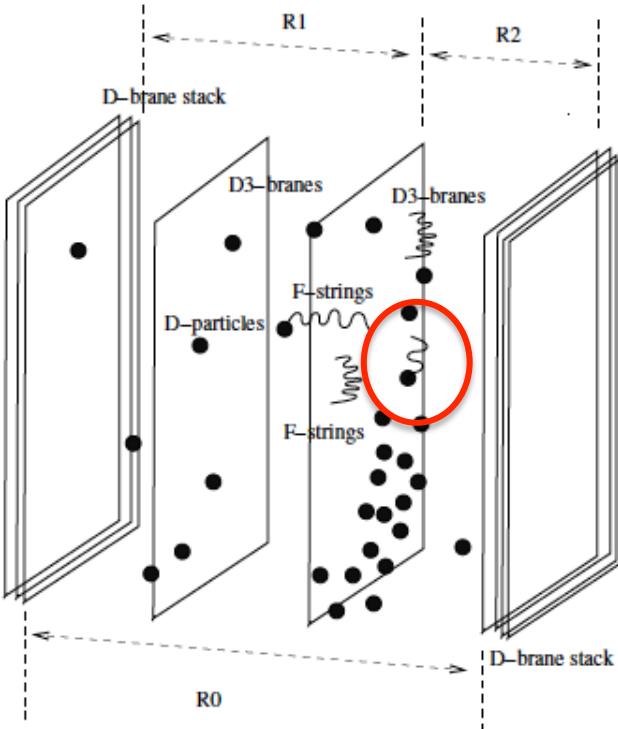
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**Baryon asymmetry
in Universe**

$$\begin{aligned} \Delta n_q &\equiv n - \bar{n} = g_{d.o.f.} \int \frac{d^3 p}{(2\pi)^3} [f(E, \mu) - f(\bar{E}, \bar{\mu})] \\ &\sim \frac{T^2}{2} \left(\frac{M_s \sigma^2}{g_s} \right) \end{aligned}$$

Connection to Baryogenesis



Effective metric due to LV foam medium

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = (\eta_{\mu\nu} + h_{\mu\nu}) dx^\mu dx^\nu , \quad h_{0i} = (u_i^a) \sigma_a$$

Stochastic media

$$\ll u_i \parallel \gg = 0 , \quad \ll u_i \parallel u_j \parallel \gg = \sigma^2 \delta_{ij} .$$

Energy differences between matter-antimatter

$$\ll E \gg = \sqrt{p^2 + m^2} + \frac{p^2 \sigma^2}{2 \sqrt{p^2 + m^2}} - \frac{1}{2} \frac{M_s}{g_s} \sigma^2$$

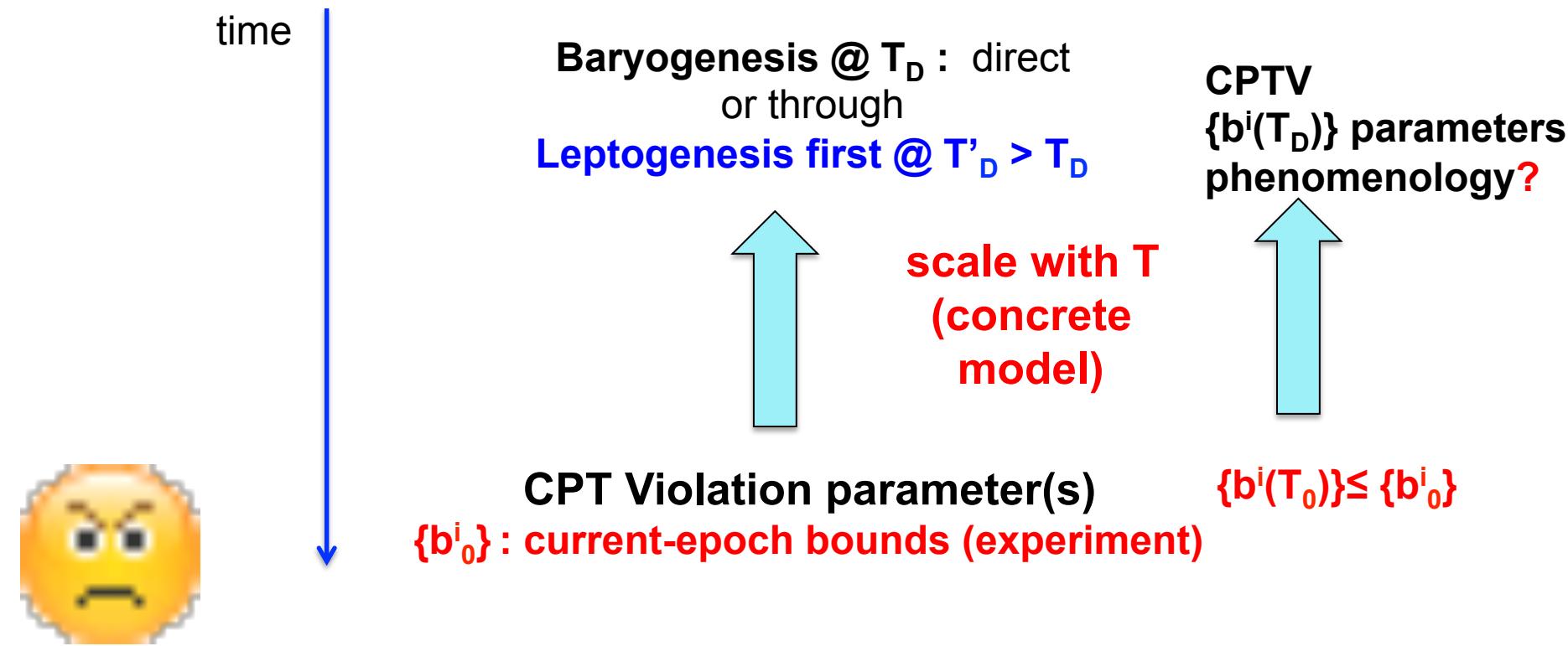
$$\ll \bar{E} \gg = \sqrt{p^2 + m^2} + \frac{p^2 \sigma^2}{2 \sqrt{p^2 + m^2}} + \frac{1}{2} \frac{M_s}{g_s} \sigma^2$$

**Baryon asymmetry
in Universe @ T_D**

$$s \sim \frac{2\pi^2}{45} g_\star(T) T^3$$

$$\Delta n(T < T_D) = \frac{\Delta n_q}{s} \sim \frac{M_s}{g_s} \frac{45 \sigma^2}{g_\star(T_D) T_D}$$

IS THIS CPTV ROUTE WORTH FOLLOWING?



Construct microscopic models with strong CPT Violation in Early Universe (due to background fields or quantum gravity), but weak today... Fit with all available data... in particular current stringent constraints → scale back in time Estimate in this way matter-antimatter asymmetry in Universe Does it agree with the expected phenomenological value ?



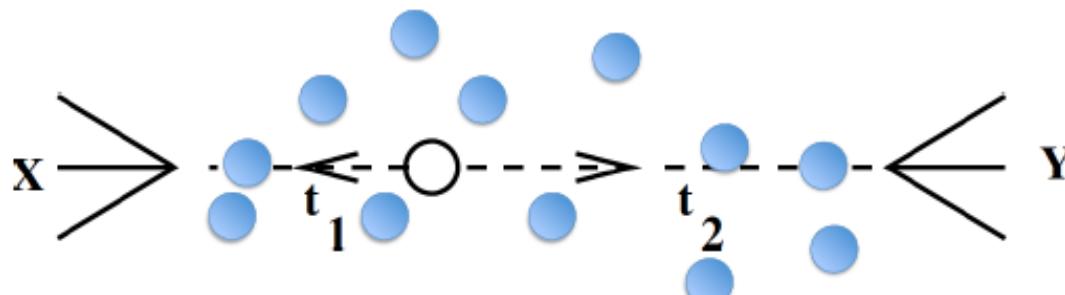
Φ-factories Current Limits → Baryogenesis?

$$|\omega|^2 = \frac{k^2 \Delta_2}{(m_1 - m_2)^2} \quad \Delta_2 = \Delta_L - \Delta_R = \xi^2 \frac{k^2}{M_P^2}, \quad 0 < \Delta_{R(L)} < 1$$

$$|\omega|^2 = \xi^2 \frac{k^4}{(m_1 - m_2)^2 M_P^2}$$

$$\langle r_i \rangle = 0, \langle r_i r_j \rangle = \Delta_{L(R)} \delta_{ij},$$

$$u_i = g_s r_i \frac{k_i}{M_s}$$

$$\sigma^2 = g_s^2 \Delta_{R(L)} \frac{k^2}{M_s^2}$$

Evolution of σ^2 with the cosmic temperature T can lead to interesting phenomenology on Baryogenesis models through such QG effects by tests of QGD CPTV ω -effect in current era :

Scaling with Temperature T $k \sim \sqrt{3T m_b}$

$$\sigma^2 \sim 3g_s^2 \frac{m_b T}{M_s^2} \Delta \quad \begin{aligned} \langle r^2 \rangle &= \Delta \\ \langle r \rangle &= 0 \end{aligned}$$

for low
 $T < m_b \sim \text{GeV}$

$$0 < \Delta < 1$$

T-dependence of Δ depends on details of QG model

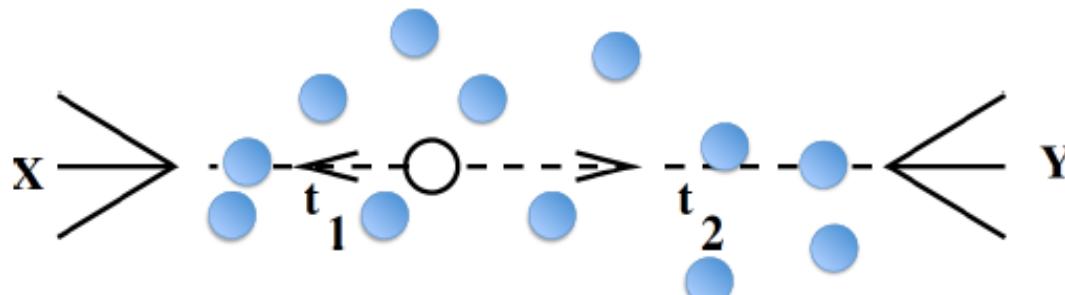
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Evolution of σ^2 with the cosmic temperature T can lead to interesting phenomenology on Baryogenesis models through such QG effects by tests of QGD CPTV ω -effect in current era :

Scaling with Temperature T

$k \sim T$
for high
 $T > m_b \sim \text{GeV}$

$$\sigma^2 = g_s^2 \frac{T^2}{M_s^2} \Delta \quad \begin{aligned} \langle r^2 \rangle &= \Delta \\ \langle r \rangle &= 0 \end{aligned}$$

$$0 < \Delta < 1$$

T-dependence of Δ depends on details of QG model

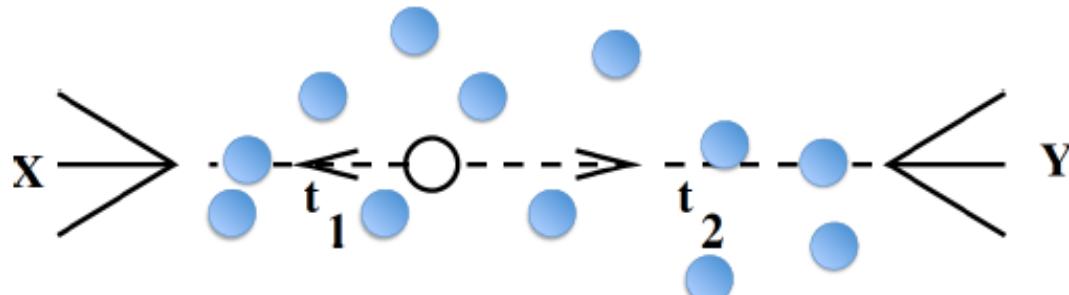
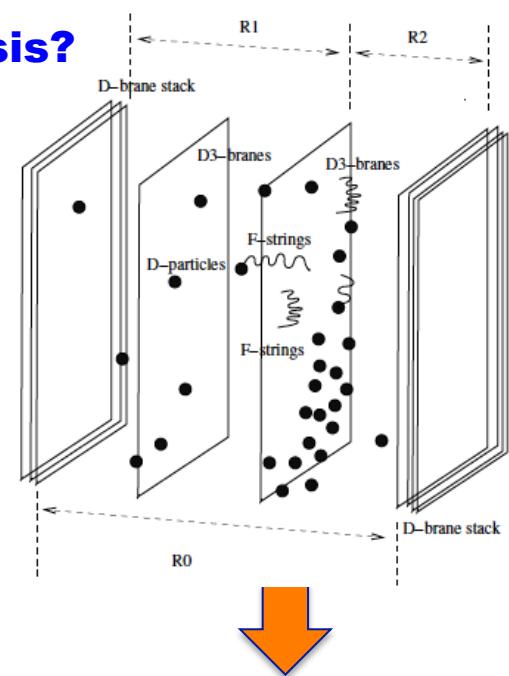
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D-particles in brane models if dark-matter like:

$$\Delta \propto n_{d-particles-density} \sim T^3$$

Neutral Kaons in Φ -factories

$$\text{Re}(\omega) = (-1.6_{-2.1 \text{ stat}}^{+3.0} \pm 0.4_{\text{syst}}) \times 10^{-4}$$

$$\text{Im}(\omega) = (-1.7_{-3.0 \text{ stat}}^{+3.3} \pm 1.2_{\text{syst}}) \times 10^{-4}$$

$$|m_1 - m_2| \sim 10^{-15} \text{ GeV} \quad k \sim 1 \text{ GeV}$$

$$\Delta_{\text{today}} < 10^{-38}$$

$$|\omega|^2 \sim \frac{k^2}{|m_1 - m_2|^2} \Delta$$

$$\langle r^2 \rangle = \Delta \quad \langle r \rangle = \hat{0}$$
$$0 < \Delta < 1$$

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D-particle induced Baryogenesis

**Scaling with Temperature T
in D-particle foam model**

D-particles = dark-matter like

$$\Delta n(T < T_D) = \frac{\Delta n_q}{s} \sim \frac{M_s}{g_s} \frac{45 \sigma^2}{g_\star(T_D) T_D}$$

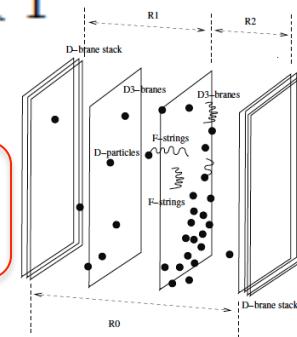
$$\sigma^2 = g_s^2 \frac{T^2}{M_s^2} \Delta \quad \rightarrow \quad \Delta n \sim 0.45 \frac{g_s}{M_s} T_D \Delta(T_D)$$

$$\begin{aligned} \frac{\Delta_{100 \text{ GeV}}}{\Delta_{\text{today}}} &\sim \frac{M_s}{g_s} \frac{1}{1.3 \text{ GeV}} \times 10^{28} \\ &\sim \left(\frac{T = 100 \text{ GeV}}{T_{\text{CMB}}^0 = 0.2 \text{ meV}} \right)^3 \sim 10^{44} \end{aligned}$$

$$|\omega|^2 \sim \frac{k^2}{|m_1 - m_2|^2} \Delta$$

$$\langle r^2 \rangle = \Delta \quad \langle r \rangle = \hat{0}$$

$$0 < \Delta < 1$$



e.g. for quarks $T_D \approx 100 \text{ GeV}$
 $\Delta n(T = 100 \text{ GeV}) \sim 10^{-10}$

$$T_D \sim 100 \text{ GeV}$$

$$\frac{M_s}{g_s} \sim 4.5 \times 10^{17} \text{ GeV}$$

**Consistent perturbative
string model !**

Neutral Kaons in Φ -factories

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$$\Delta_{\text{today}} < 10^{-38}$$

D-particle induced Baryogenesis

**Scaling with Temperature T
in D-particle foam model**

D-particles = dark-matter

$$\Delta n(T < T_D) \sim \frac{45 \sigma^2}{g_s \frac{g_\star(T_D)}{T_D}}$$

$$\sigma^2 \frac{M_s^2}{M_s^2} \Delta \rightarrow \Delta n \sim 0.45 \frac{g_s}{M_s} T_D \Delta(T_D)$$

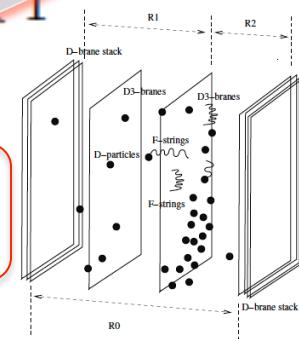
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$$|\omega|^2 \sim \frac{k^2}{|m_1 - m_2|^2} \Delta$$

$$\langle r^2 \rangle$$

$$\Delta \propto \langle r^2 \rangle \sim T^3$$



e.g. for quarks $T_D \approx 100 \text{ GeV}$
 $\Delta n(T = 100 \text{ GeV}) \sim 10^{-10}$

$$T_D \sim 100 \text{ GeV}$$

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**Consistent perturbative
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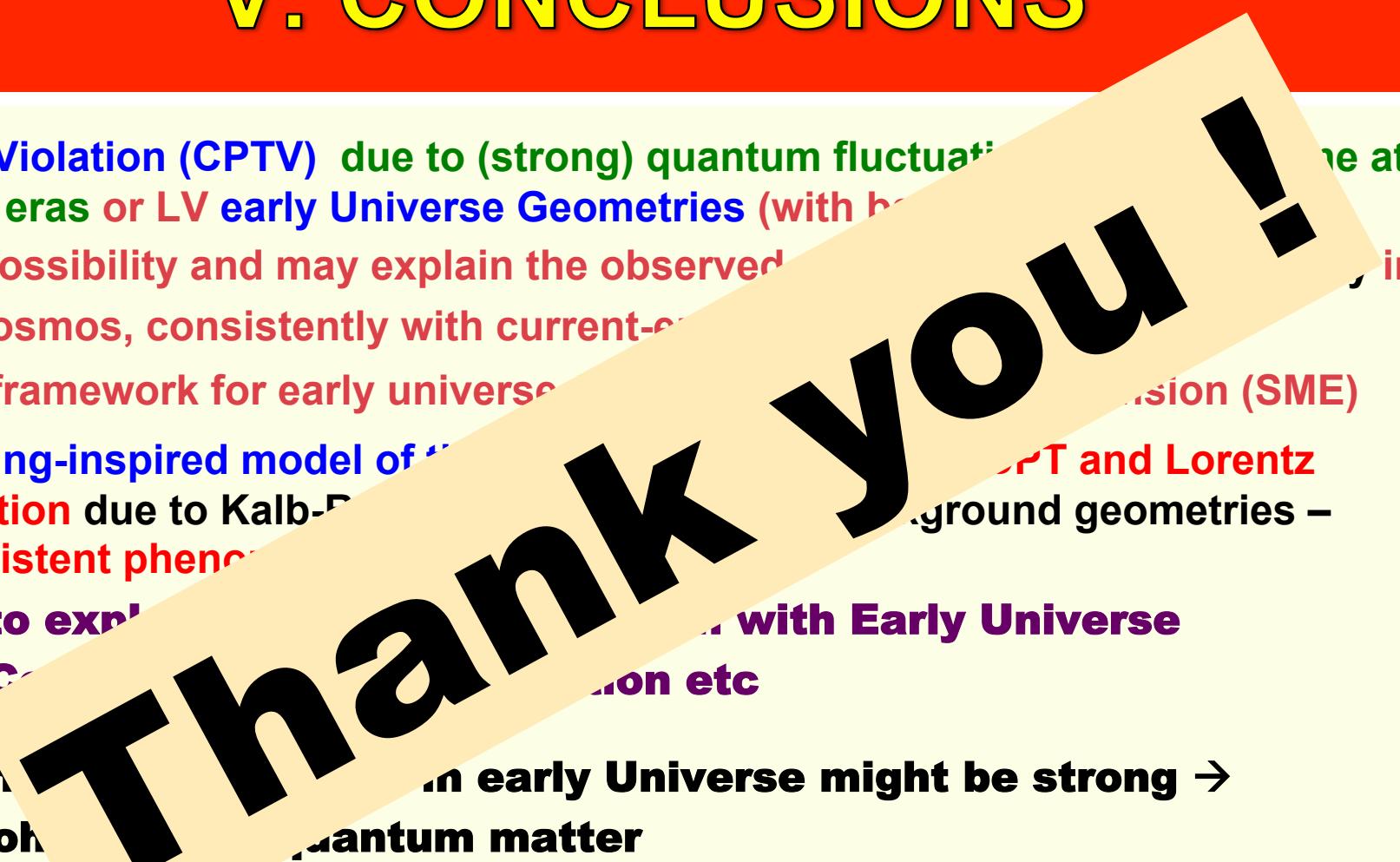
NB: RHN leptogenesis also consistent

V. CONCLUSIONS

- CPT Violation (CPTV) due to (strong) quantum fluctuations in space-time at early eras or LV early Universe Geometries (with background flux fields) is a possibility and may explain the observed matter-antimatter asymmetry in the cosmos, consistently with current-era bounds of CPTV
- One framework for early universe CPTV: Standard Model Extension (SME)
- A string-inspired model of the Early Universe entailing CPT and Lorentz Violation due to Kalb-Ramond-axion- modified background geometries – Consistent phenomenology in current era

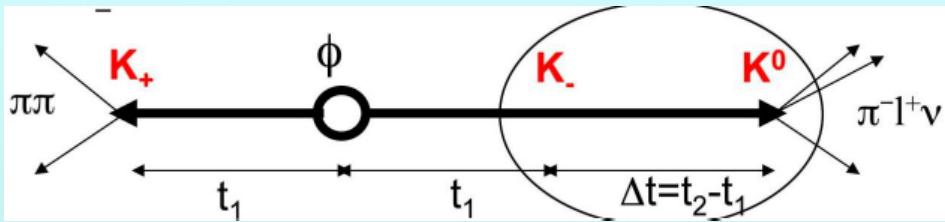
...to explore further, in connection with Early Universe Cosmology – CMB polarization etc
- Quantum Gravity Flcts in early Universe might be strong → Decoherence of quantum matter
CPT operator in effective theory ill-defined
→ affects EPR entangled states → current bounds from meson factories → explore link with early Universe in concrete models of space-time foam from string/brane theory

V. CONCLUSIONS

- CPT Violation (CPTV) due to (strong) quantum fluctuations at early eras or LV early Universe Geometries (with background geometries – in the cosmos, consistently with current observations)
 - One framework for early universe phenomenology – Standard Model Extension (SME)
 - A string-inspired model of CPT Violation due to Kalb-Ramond fields in curved backgrounds – Consistent phenomena ...to explore with Early Universe phenomenology etc
 - Quantum fluctuations in early Universe might be strong → Decoherence of quantum matter
CPT operator in effective theory ill-defined
→ affects EPR entangled states → current bounds from meson factories → explore link with early Universe in concrete models of space-time foam from string/brane theory
- 

SPARES

Current Measurement Status of ω -effect



Neutral Kaons

KLOE result: PLB 642(2006) 315
Found. Phys. 40 (2010) 852

$$\Re \omega = (-1.6_{-2.1}^{+3.0} \text{STAT} \pm 0.4_{\text{SYST}}) \times 10^{-4}$$
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$$|\omega| < 1.0 \times 10^{-3} \quad \text{at } 95\% \text{ C.L.}$$

Prospects KLOE-2 $\text{Re}(\omega), \text{Im}(\omega) \rightarrow 2 \times 10^{-5}$

ω -Effect & Intensities

$$I(\Delta t) \equiv \frac{1}{2} \int_{|\Delta t|}^{\infty} dt |A(\pi^+ \pi^-, \pi^+ \pi^-)|^2 = |\langle \pi^+ \pi^- | K_S \rangle|^4 |\mathcal{N}|^2 |\eta_{+-}|^2 [I_1 + I_2 + I_{12}]$$

$$I_1(\Delta t) = \frac{e^{-\Gamma_S \Delta t} + e^{-\Gamma_L \Delta t} - 2e^{-(\Gamma_S + \Gamma_L) \Delta t/2} \cos(\Delta M \Delta t)}{\Gamma_L + \Gamma_S}$$

$$I_2(\Delta t) = \frac{|\omega|^2}{|\eta_{+-}|^2} \frac{e^{-\Gamma_S \Delta t}}{2\Gamma_S}$$

enhancement factor due to CP violation
compared with, eg, B-mesons

$$I_{12}(\Delta t) = -\frac{4}{4(\Delta M)^2 + (3\Gamma_S + \Gamma_L)^2} \frac{|\omega|}{|\eta_{+-}|} \times$$

$$\left[2\Delta M \left(e^{-\Gamma_S \Delta t} \sin(\phi_{+-} - \Omega) - e^{-(\Gamma_S + \Gamma_L) \Delta t/2} \sin(\phi_{+-} - \Omega + \Delta M \Delta t) \right) \right.$$

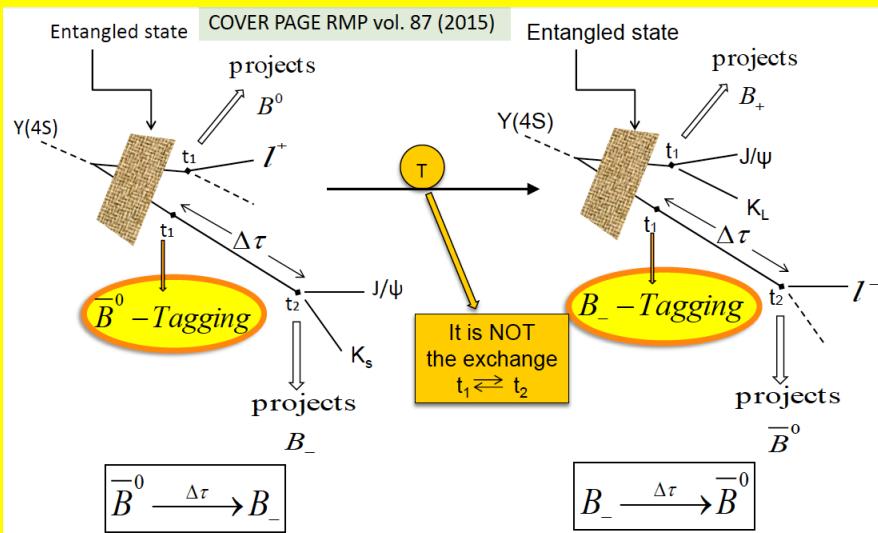
$$\left. - (3\Gamma_S + \Gamma_L) \left(e^{-\Gamma_S \Delta t} \cos(\phi_{+-} - \Omega) - e^{-(\Gamma_S + \Gamma_L) \Delta t/2} \cos(\phi_{+-} - \Omega + \Delta M \Delta t) \right) \right]$$

$$\Delta M = M_S - M_L \text{ and } \eta_{+-} = |\eta_{+-}| e^{i\phi_{+-}}.$$

NB: sensitivities up to $|\omega| \sim 10^{-6}$ in ϕ factories, due to enhancement by $|\eta_{+-}| \sim 10^{-3}$ factor.



Current Measurement Status of ω -effect



Neutral B-mesons

Equal Sign Dilepton Asymmetry
 (Alvarez, Bernabeu, Nebot, JHEP 0611 (2006)
 see)

$$-0.0084 \leq \text{Re}(\omega) \leq 0.0100 \quad 95\% \text{C.L}$$

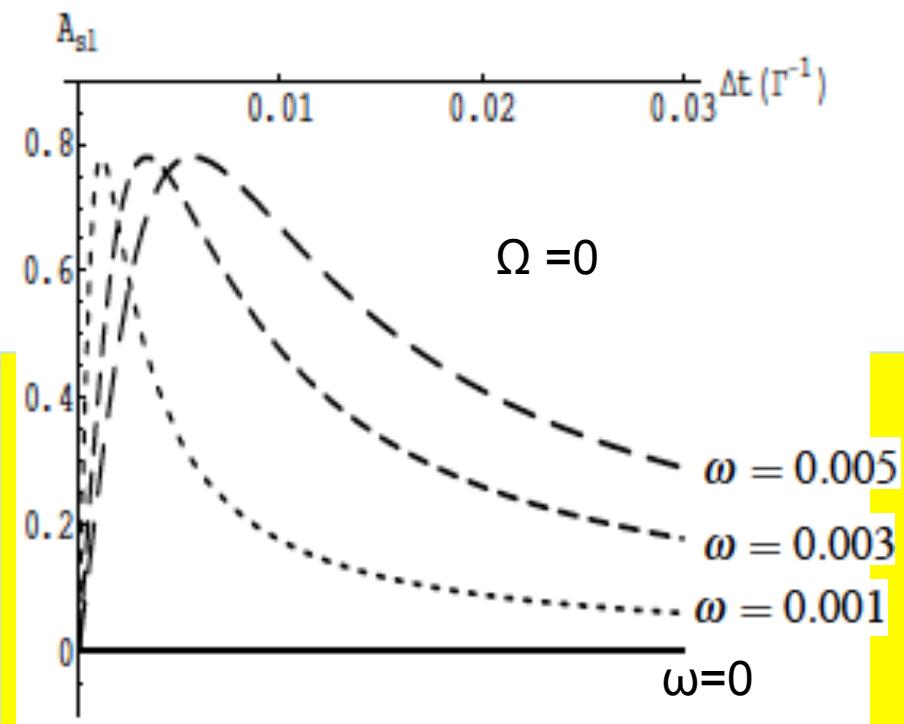
Current Measurement Status of ω -effect

$$A_{sl} = \frac{I(\ell^+, \ell^+, \Delta t) - I(\ell^-, \ell^-, \Delta t)}{I(\ell^+, \ell^+, \Delta t) + I(\ell^-, \ell^-, \Delta t)} \Big|_{\omega=0} = 4 \frac{\text{Re}(\varepsilon)}{1 + |\varepsilon|^2} + \mathcal{O}((\text{Re } \varepsilon)^2) \rightarrow \omega = |\omega| e^{i\Omega} \neq 0$$



$$\Delta t_{peak} = \frac{1}{\Gamma} 1.12 |\omega|$$

$$A_{sl}(\Delta t_{peak}) = 0.77 \cos(\Omega)$$

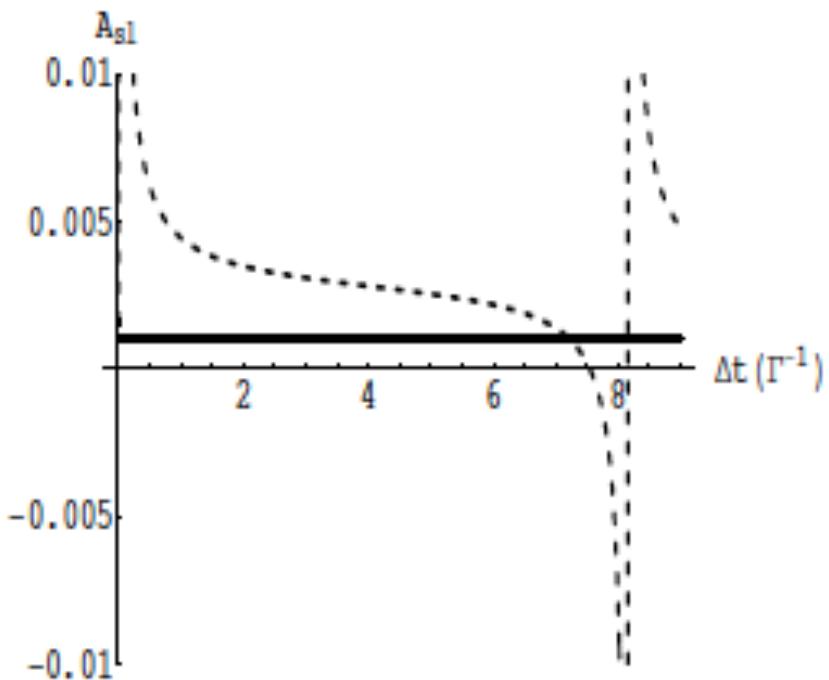


Equal Sign Dilepton Asymmetry
 (Alvarez, Bernabeu, Nebot, JHEP 0611 (2006)
arXiv:hep-ph/0605075)

$$-0.0084 \leq \text{Re}(\omega) \leq 0.0100 \quad 95\% \text{ C.L}$$

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$$\Delta m \Delta t \approx 2\pi \quad (\Delta t \approx 8.2 \Gamma^{-1}),$$

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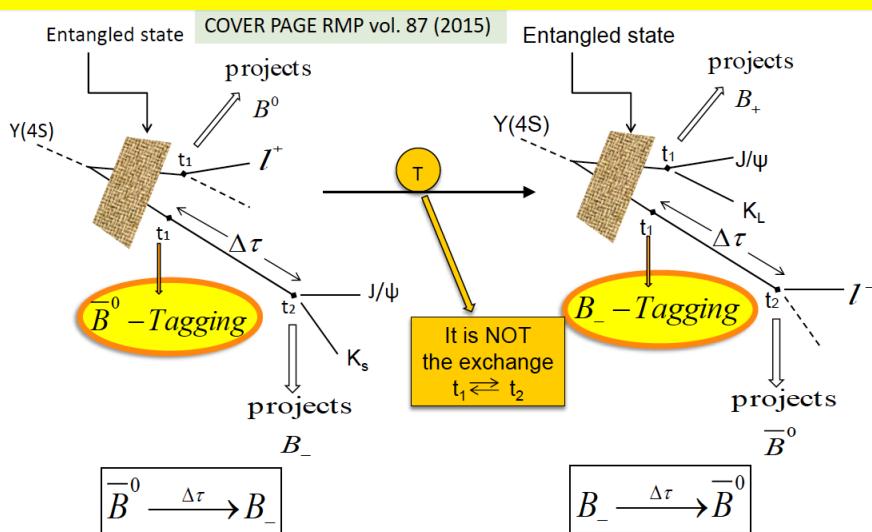
$$\Delta\Gamma = 0$$

$$I(f, g; t) = \int_0^\infty dt_0 |\langle f, t_0; g, t + t_0 | T | \Psi_0 \rangle|^2 = \frac{\langle \Gamma_f \rangle \langle \Gamma_g \rangle}{\Gamma} e^{-\Gamma t} \{ \mathcal{C}_h^\omega[f, g] \\ + \mathcal{C}_c^\omega[f, g] \cos(\Delta M t) + \mathcal{S}_c^\omega[f, g] \sin(\Delta M t) \}$$

Observables

$$C[f, g] = \frac{\mathcal{C}_c^\omega[f, g]}{\mathcal{C}_h^\omega[f, g]} \quad \text{and} \quad S[f, g] = \frac{\mathcal{S}_c^\omega[f, g]}{\mathcal{C}_h^\omega[f, g]},$$

$$C[\ell^\pm, g] - C[g, \ell^\pm] = \frac{1}{1 + (x/2)^2} \{ [x S_g \mp 2(C_g^2 - 1) \mp x C_g S_g] \operatorname{Re}(\omega) \\ + x R_g [C_g \pm 1] \operatorname{Im}(\omega) \} \quad x = \frac{\Delta M}{\Gamma} \simeq 0.77$$



Neutral B-mesons

Equal Sign Dilepton Asymmetry
(Alvarez, Bernabeu, Nebot, JHEP 0611 (2006)
see)

$$-0.0084 \leq \operatorname{Re}(\omega) \leq 0.0100 \quad 95\% \text{C.L}$$

Novel signal from $(f,g) \leftrightarrow (g,f)$
(Bernabeu, Botella, NEM, Nebot (2018))

$\operatorname{Im}(\theta)$	$(0.99 \pm 1.98)10^{-2}$
$\operatorname{Im}(\omega)$	$\pm(6.40 \pm 2.80)10^{-2}$

Current Measurement Status of ω -effect

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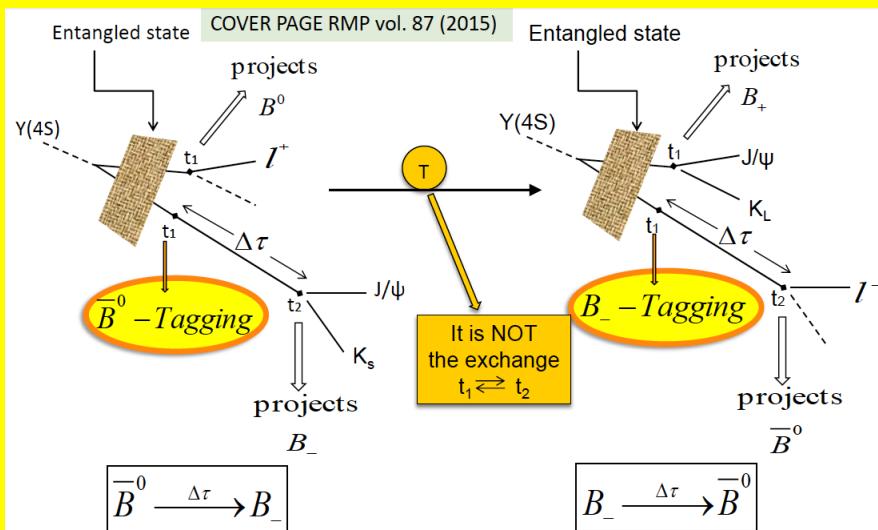
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$$S[\ell^\pm, g] + S[g, \ell^\pm] = \frac{1}{1 + (x/2)^2} \{ [xC_g \pm x(1 - S_g^2) \mp 2C_g S_g] \text{Re}(\omega) \\ + R_g[xS_g \mp 2] \text{Im}(\omega) \}.$$

$$x = \frac{\Delta M}{\Gamma} \simeq 0.77$$



Neutral B-mesons

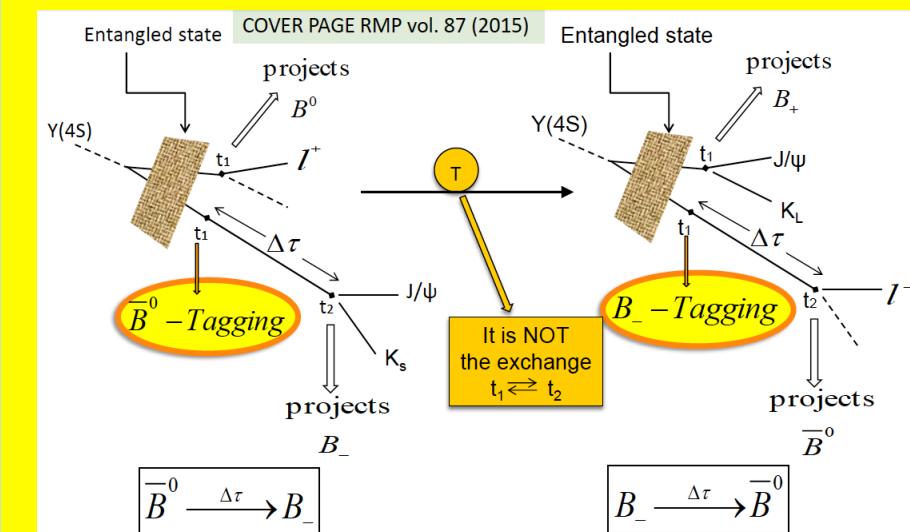
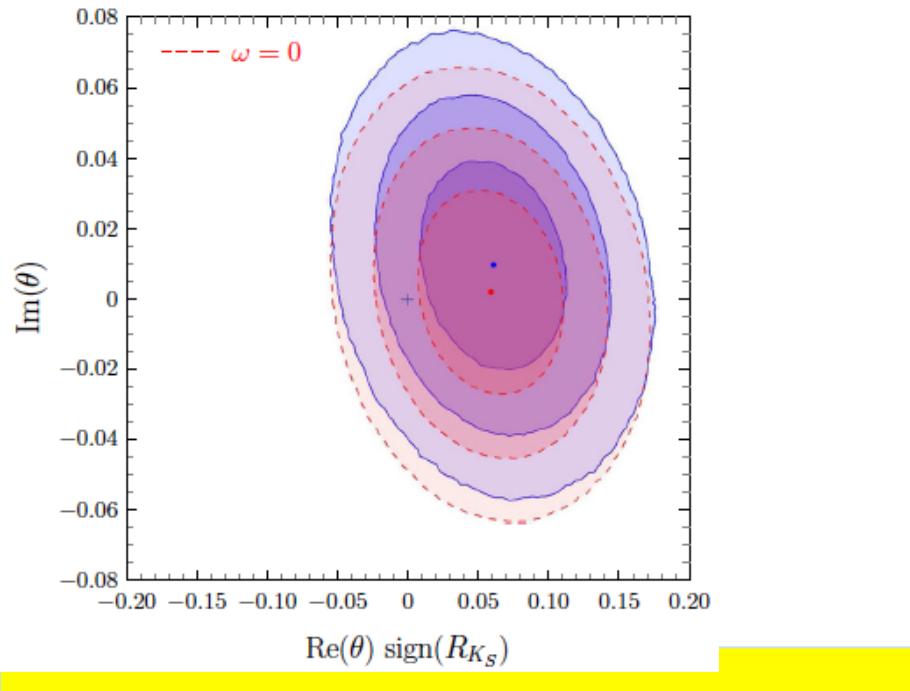
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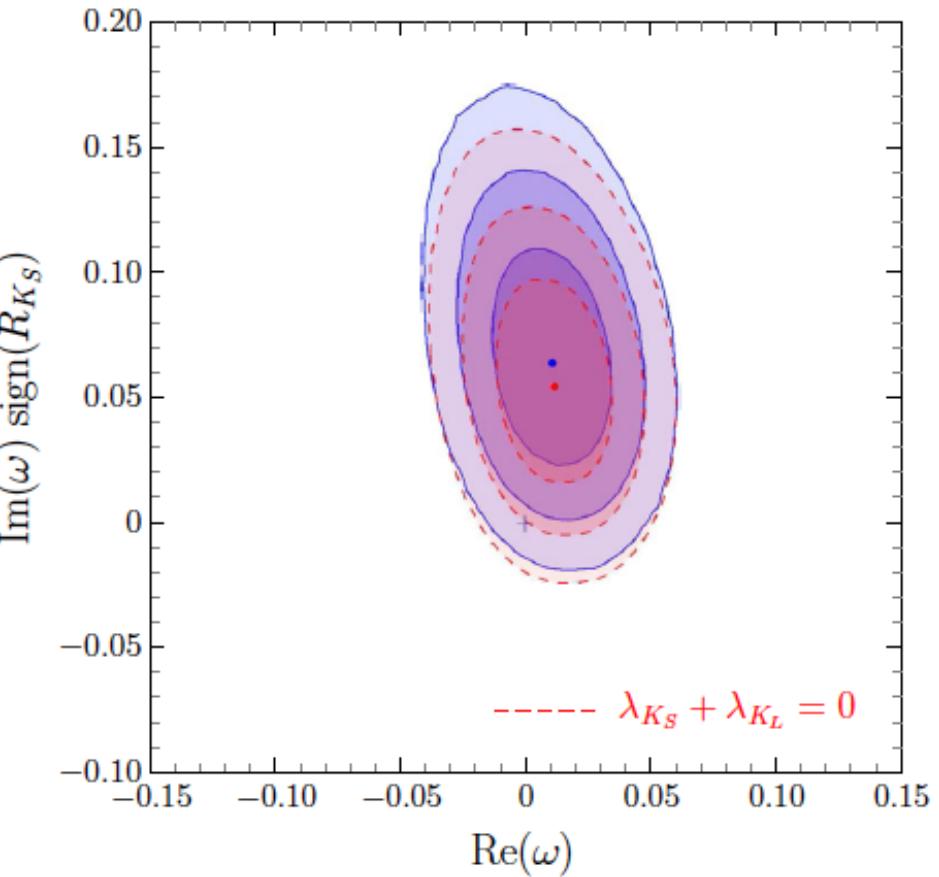
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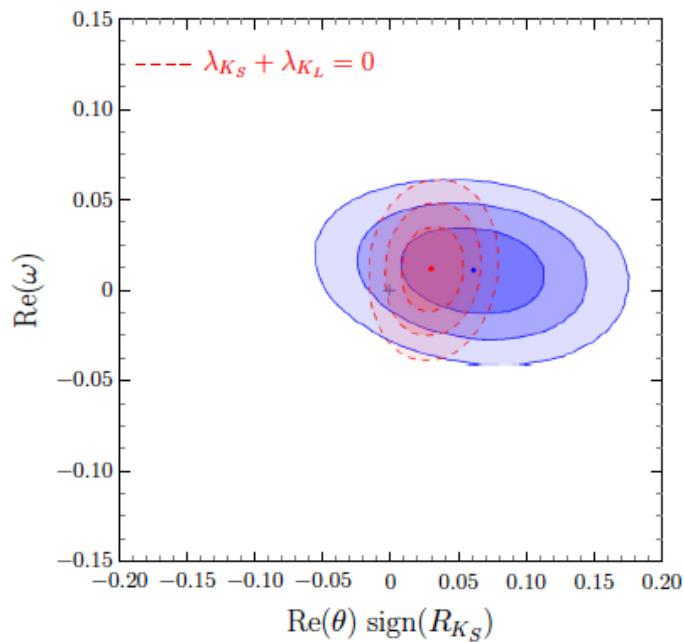
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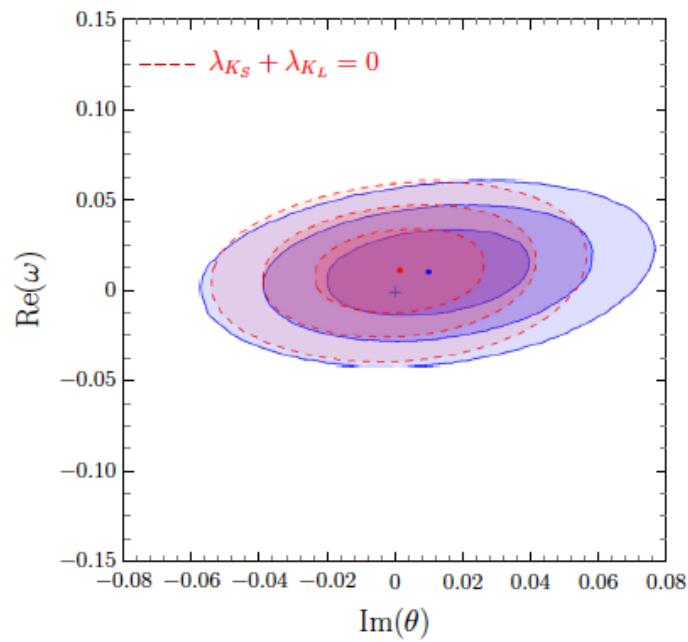
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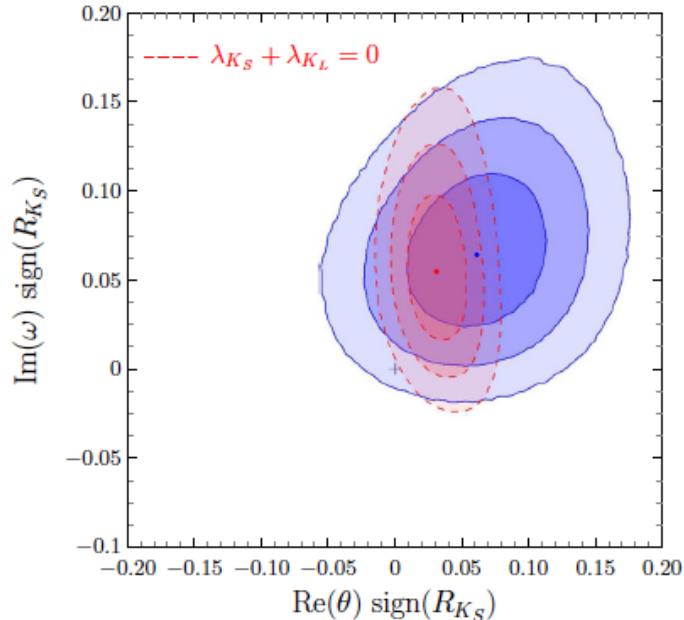
Correlations among ω & θ



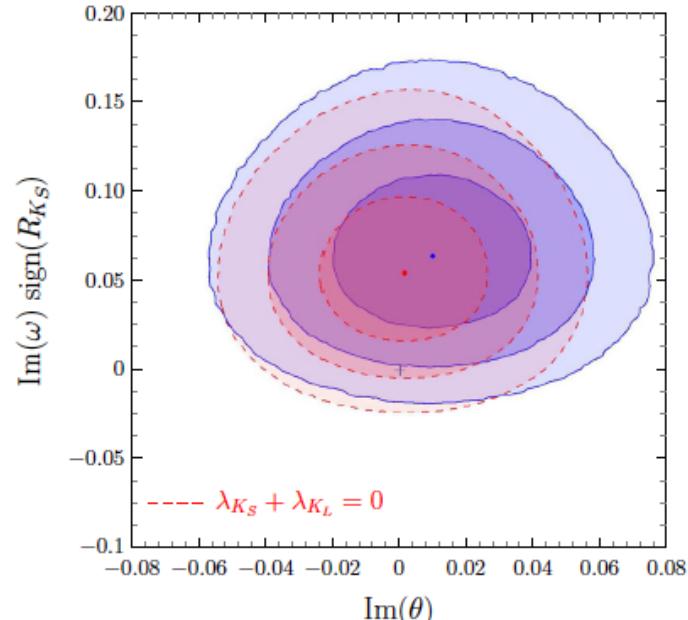
(a) $\text{Re}(\omega)$ vs. $\text{Re}(\theta)$.



(b) $\text{Re}(\omega)$ vs. $\text{Im}(\theta)$.



(c) $\text{Im}(\omega)$ vs. $\text{Re}(\theta)$.



(d) $\text{Im}(\omega)$ vs. $\text{Im}(\theta)$.



Proper Treatment through solving Boltzmann Eqs.

T. Bossingham, N.E.M., Sarkar

Boltzmann equation in presence of CPTV & LV Background B_0

RHN Helicity specific λ_r :

$$\begin{aligned} \frac{dn_r}{dt} + 3Hn_r - \frac{g}{2\pi^2} 2\lambda_r \frac{B_0}{T} T^3 \int du u f(E(B_0 = 0), u) \\ = \frac{g}{8\pi^3} \int \frac{d^3 p}{E(B_0 \neq 0)} C[f] + \mathcal{O}(B_0^2) \end{aligned}$$

Summing over RHN Helicities $\sum_r \lambda_r = 0$ (**for small $B_0/T \ll 1$**) :

$$\frac{dn_N}{dt} + 3Hn_N = \frac{g}{8\pi^3} \int \frac{d^3 p}{E} \tilde{C}[f] + \mathcal{O}(B_0^2)$$



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$$E = \sqrt{\vec{p}^2 + m^2 + B_0^2 + 2|\vec{p}| B_0}$$

$$\bar{E} = \sqrt{\vec{p}^2 + m^2 + B_0^2 - 2|\vec{p}| B_0}$$

But still modified due to B_0 -Dependence
of Energy-Momentum dispersion $E(p, B_0)$





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λI

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Proper Treatment through solving Boltzmann Eqs.

T. Bossingham, N.E.M., Sarkar

$$Y_x = n_x/s$$

$$z = m_N/T$$

Y^{eq} : Thermal Equilibrium
Distributions (at high T)

Standard Cosmology at early eras (radiation era) $T \sim a^{-1} \Rightarrow s \sim T^3$, $a \sim t^{1/2}$

Hubble rate $H \sim T^2/2 = m_N^2/2z^2$.

Averaged (over helicities) heavy (right-handed)
neutrino abundance

$$\bar{Y}_N \equiv \frac{Y_N^{(-)} + Y_N^{(+)}}{2},$$

Lepton Asymmetry: $\mathcal{L} = \bar{Y}_{l^-} - \bar{Y}_{l^+}$

$$\mathcal{L} \equiv Y_{l^-}^{(-)} - Y_{l^+}^{(+)} = 2[\bar{Y}_{l^-} - \bar{Y}_{l^+}],$$

$$\bar{Y}_l \equiv \frac{Y_l^{(-)} + Y_l^{(+)}}{2} = \frac{Y_N^{(-)} + Y_N^{(+)}}{2} = \bar{Y}_N$$

Proper Treatment through solving Boltzmann Eqs.

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$$\mathcal{L} \equiv Y_{l^-}^{(-)} - Y_{l^+}^{(+)} = 2[\bar{Y}_{l^-} - \bar{Y}_{l^+}],$$

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Proper Treatment through solving Boltzmann Eqs.

Heavy (right-handed) neutrino abundance

$$\begin{aligned}
 z H s \frac{dY_N^{(\lambda)}}{dz} - \lambda I = & - \left\{ \left[\gamma^{eq,(\lambda)}(N \rightarrow l^- h^+) \frac{Y_N^{(\lambda)}}{Y_{N,eq}} - \gamma^{eq,(\lambda)}(l^- h^+ \rightarrow N) \frac{Y_{l^-}^{(\lambda)}}{Y_{l^-,eq}} \frac{Y_{h^+}}{Y_{h^+,eq}} \right] \right. \\
 & + \left[\gamma^{eq,(\lambda)}(N \rightarrow l^+ h^-) \frac{Y_N^{(\lambda)}}{Y_{N,eq}} - \gamma^{eq,(\lambda)}(l^+ h^- \rightarrow N) \frac{Y_{l^+}^{(\lambda)}}{Y_{l^+,eq}} \frac{Y_{h^-}}{Y_{h^-,eq}} \right] \\
 & + \left[\gamma^{eq,(\lambda)}(N \rightarrow \nu h^0) \frac{Y_N^{(\lambda)}}{Y_{N,eq}} - \gamma^{eq,(\lambda)}(\nu h^0 \rightarrow N) \frac{Y_\nu^{(\lambda)}}{Y_{\nu,eq}} \frac{Y_{h^0}}{Y_{h^0,eq}} \right] \\
 & \left. + \left[\gamma^{eq,(\lambda)}(N \rightarrow \bar{\nu} h^0) \frac{Y_N^{(\lambda)}}{Y_{N,eq}} - \gamma^{eq,(\lambda)}(\bar{\nu} h^0 \rightarrow N) \frac{Y_{\bar{\nu}}^{(\lambda)}}{Y_{\bar{\nu},eq}} \frac{Y_{h^0}}{Y_{h^0,eq}} \right] \right\}.
 \end{aligned}$$

Proper Treatment through solving Boltzmann Eqs.

Heavy (right-handed) neutrino abundance

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$\gamma^{eq,(-)}(N \leftrightarrow l^- h^+) = \gamma^{eq,(-)}(N \leftrightarrow \nu h^0) \frac{Y_{h^-}}{Y_{h^-,eq} Y_{h^+,eq}}$

$\gamma^{eq,(+)}(N \leftrightarrow l^+ h^-) = \gamma^{eq,(+)}(N \leftrightarrow \bar{\nu} h^0) \frac{Y_{h^0}}{Y_{h^0,eq}}$

$$\left. + \left[\gamma^{eq,(\lambda)}(N \rightarrow \bar{\nu} h^0) \frac{Y_N^{(\lambda)}}{Y_{N,eq}} - \gamma^{eq,(\lambda)}(\bar{\nu} h^0 \rightarrow N) \frac{Y_{\bar{\nu}}^{(\lambda)}}{Y_{\bar{\nu},eq}} \frac{Y_{h^0}}{Y_{h^0,eq}} \right] \right\}.$$

Proper Treatment through solving Boltzmann Eqs.

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Lepton Asymmetry:

$$2zHs \frac{d\mathcal{L}}{dz} + 4I = -2 \left[\gamma^{eq,(-)}(N \leftrightarrow l^- h^+) \left\{ \frac{Y_{l^-}^{(-)}}{Y_{l^-,eq}} - \frac{Y_N^{(-)}}{Y_{N,eq}} \right\} \right. \\ \left. - \gamma^{eq,(+)}(N \leftrightarrow l^+ h^-) \left\{ \frac{Y_{l^+}^{(+)}}{Y_{l^+,eq}} - \frac{Y_N^{(+)}}{Y_{N,eq}} \right\} \right]$$



$$\frac{\Delta L^{TOT, \text{complete}}}{s} \simeq (0.016 - 0.019) \frac{B_0}{m_N}$$

at freezeout temperature

$$T = T_D : m_N/T_D \simeq (1.44 - 1.77)$$

$$\Gamma \simeq H = 1,66 T^2 \mathcal{N}^{1/2} m_P^{-1} \quad \text{assume standard cosmology}$$

$$T_D \simeq 6.2 \cdot 10^{-2} \frac{|Y|}{\mathcal{N}^{1/4}} \sqrt{\frac{m_P(\Omega^2 + B_0^2)}{\Omega}}$$

for one generation
of RH heavy neutrino

$$\Omega = \sqrt{B_0^2 + m_N^2} . \quad Y_k \sim 10^{-5}$$

*

$$\frac{\Delta L^{TOT, \text{complete}}}{s} \simeq (0.016 - 0.019) \frac{B_0}{m_N}$$

at freezeout temperature

$$T = T_D : m_N/T_D \simeq (1.44 - 1.77)$$

$$\frac{\Delta L^{TOT, \text{complete}}}{s} \simeq \mathcal{O}(8 \times 10^{-11})$$

Pheno

$$\frac{B_0}{m_N} \sim 5.0 \times 10^{-9} - 4.2 \times 10^{-9},$$

at freezeout temperature

