The EFT approach to torsional modified gravity and gravitational waves

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CANTATA
Cosmology and Astrophysics Network for Theoretical Advances and Training Actions

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Goal

- We construct and apply the EFT approach to torsional modified gravity, in order to investigate the propagation of gravitational waves (GW)

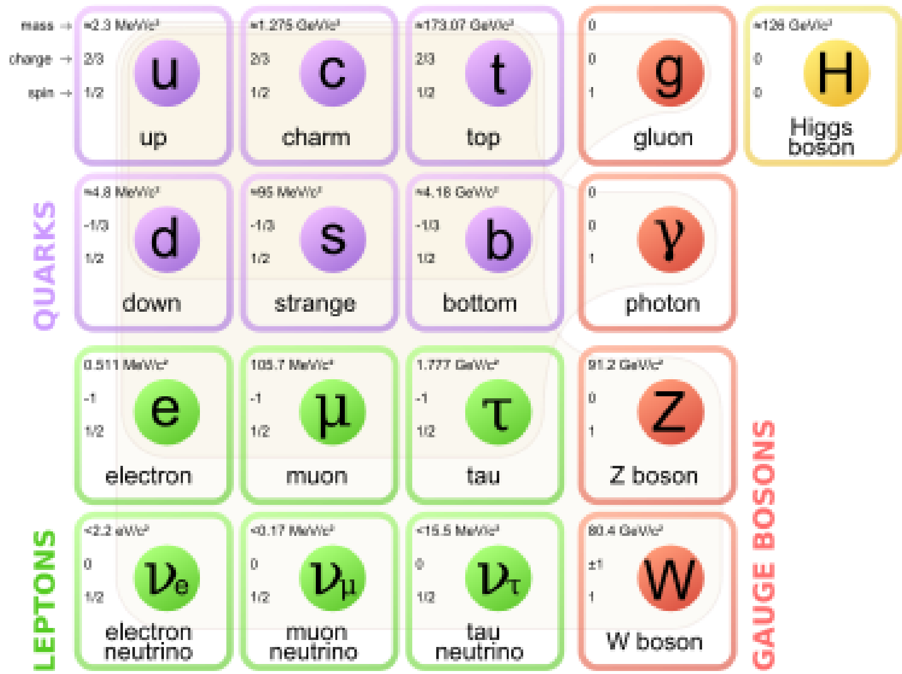
- High accuracy advancing GW astronomy offers a new window in testing Modified Gravity

Talk Plan

- 1) Introduction: Why Modified Gravity
- 2) Teleparallel Equivalent of General Relativity and f(T) modification
- 3) Non-minimal scalar-torsion theories
- 4) Teleparallel Equivalent of Gauss-Bonnet and f(T,T_G) modification
- 5) Solar system, growth-index, baryogenesis and BBN constraints
- 6) The EFT approach to torsional gravity
- 7) Background solutions
- 8) Gravitational Waves and observational signatures
- 9) Conclusions-Prospects

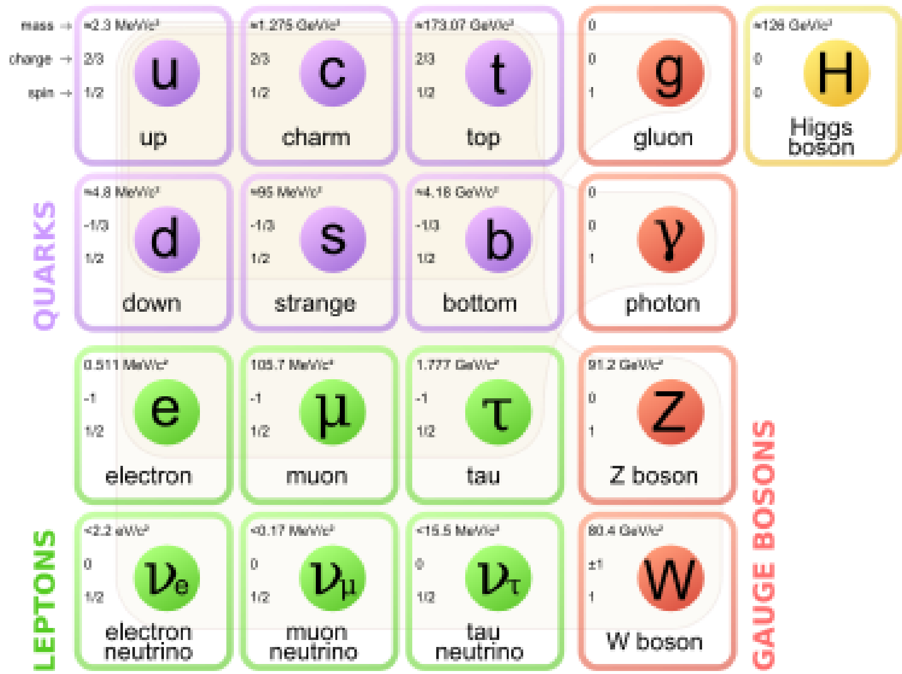
Why Modified Gravity?

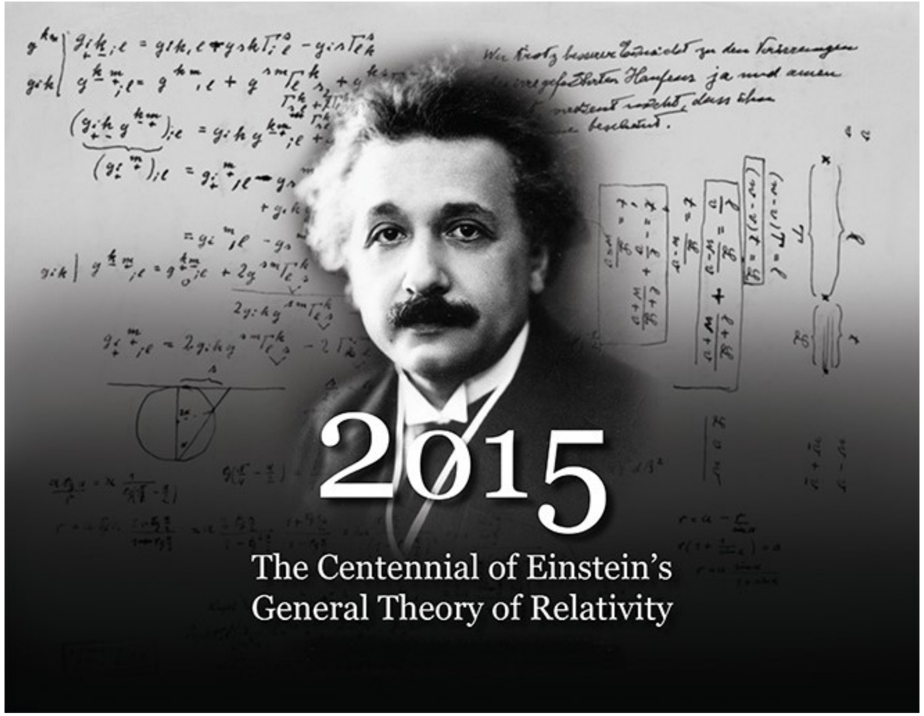
Knowledge of Physics: Standard Model



Why Modified Gravity?

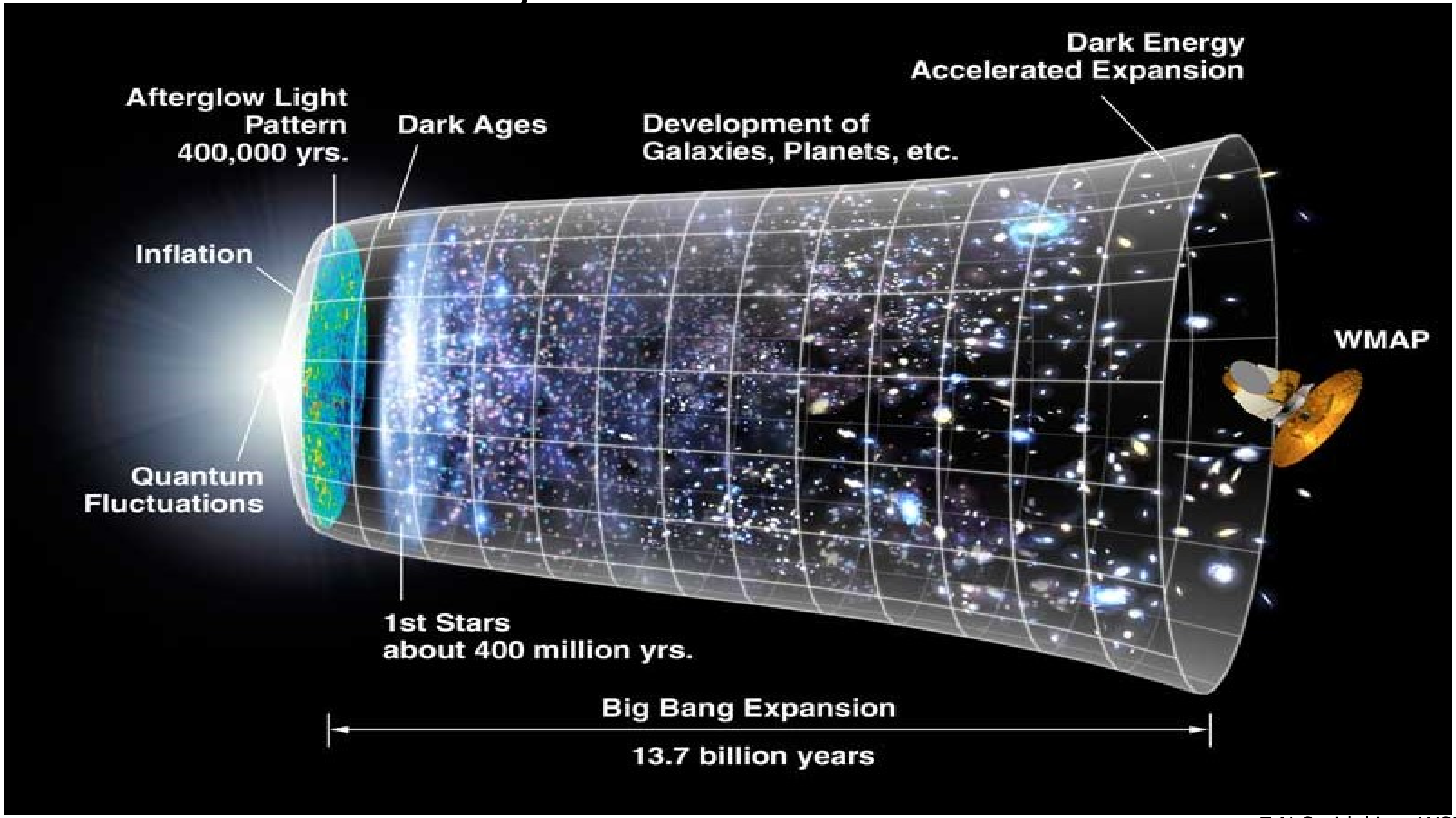
Knowledge of Physics: Standard Model + General Relativity





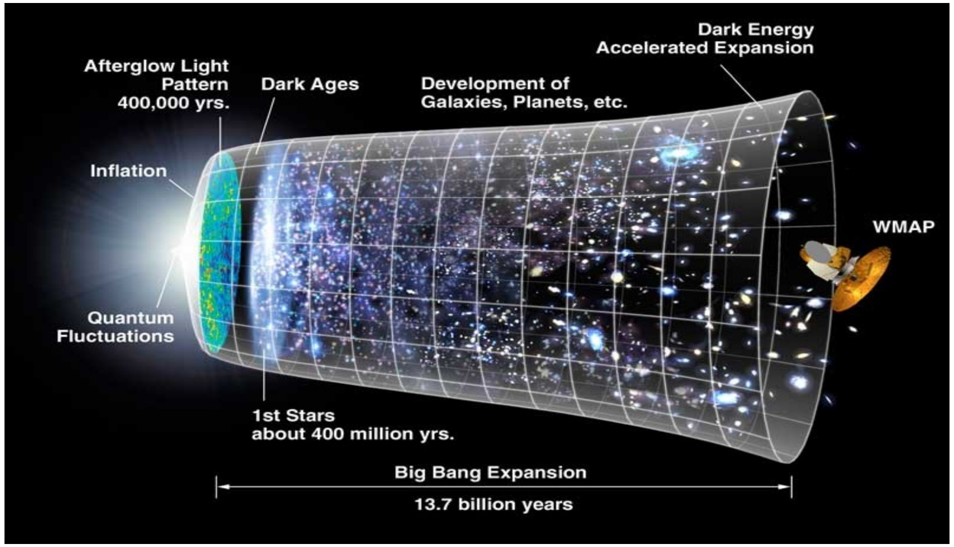
Why Modified Gravity?

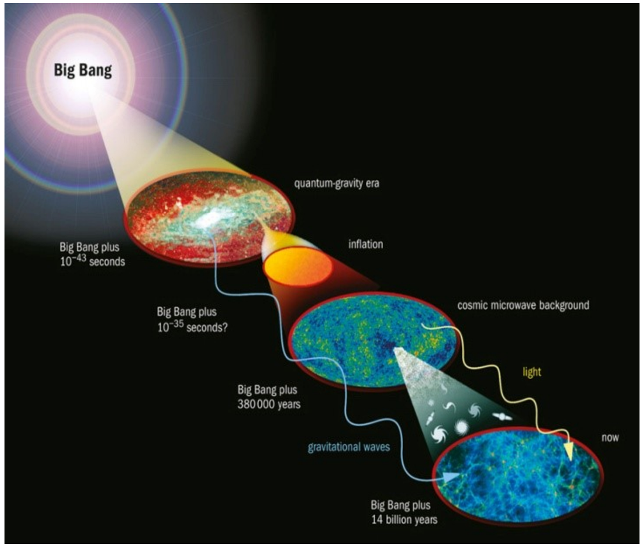
Universe History:



Why Modified Gravity?

So can our knowledge of Physics describes all these?





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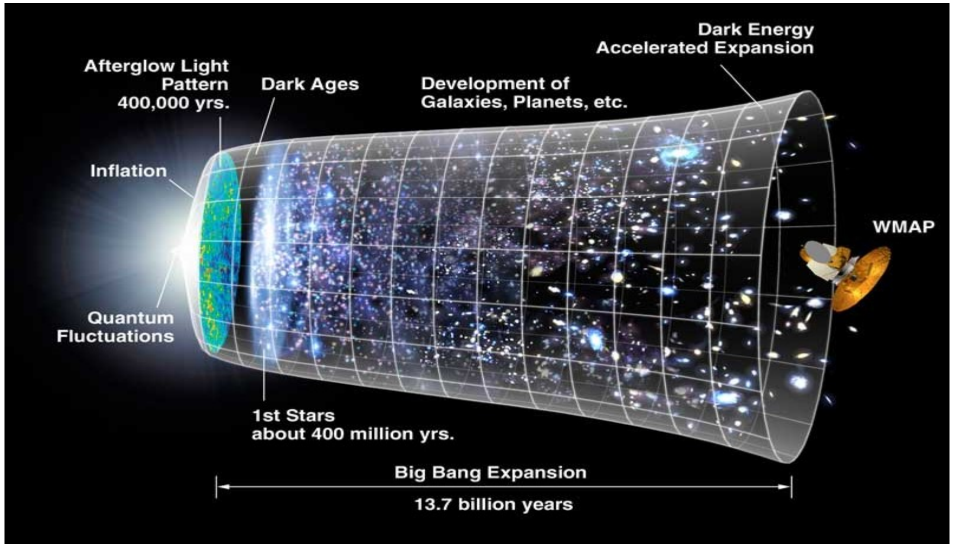
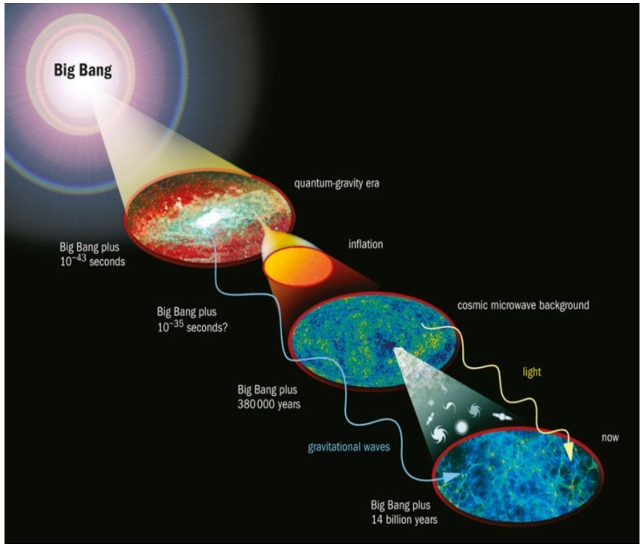




NO!

Why Modified Gravity?

- Einstein 1916: **General Relativity**:

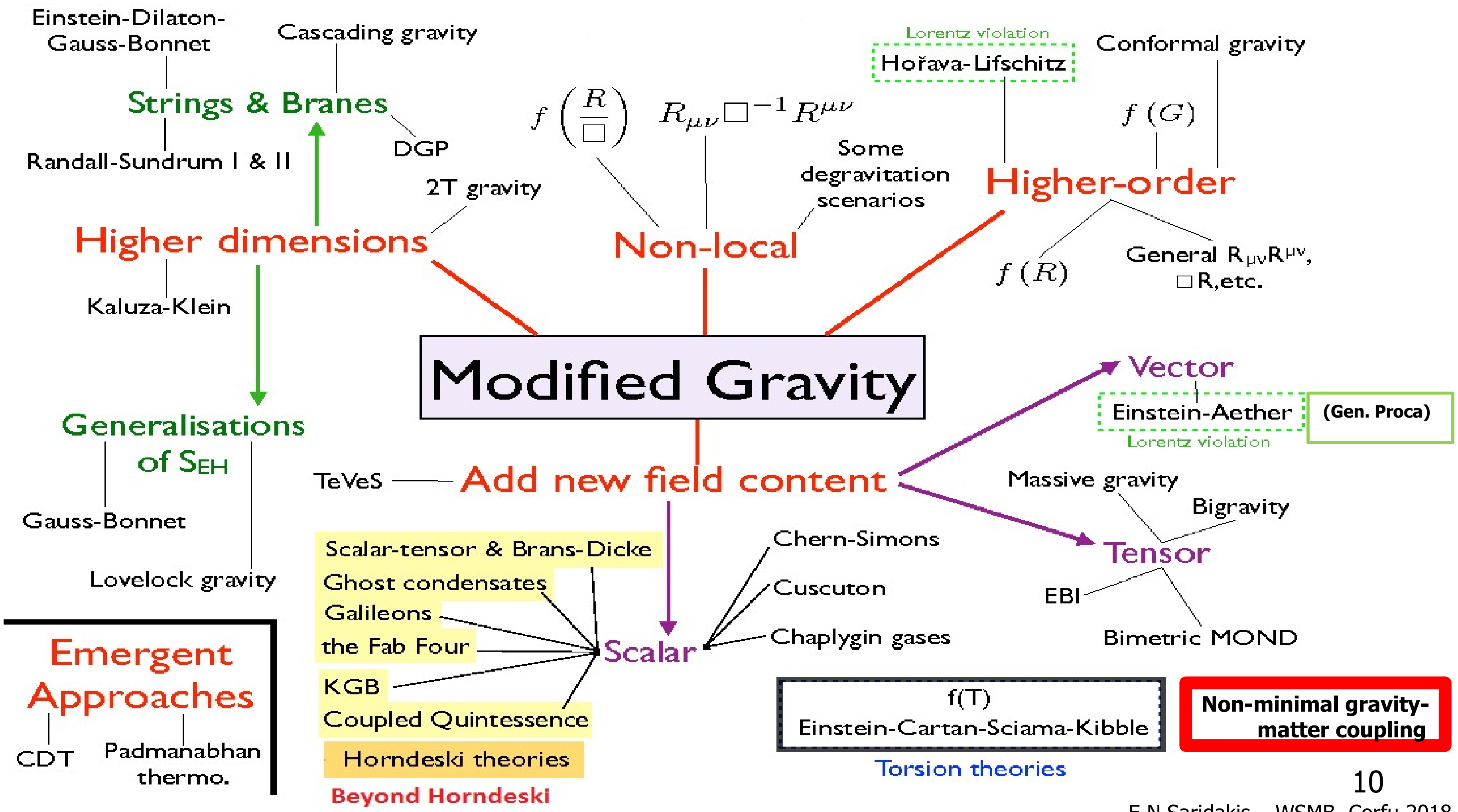
energy-momentum source of spacetime Curvature

$$\boxed{S=\frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[R-2\Lambda\right]+\int d^4x\; L_m\left(g_{\mu\nu},\psi\right)}$$

$$\Rightarrow\quad R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R+g_{\mu\nu}\Lambda=8\pi G\; T_{\mu\nu}$$

with $$T^{\mu\nu}\equiv\frac{2}{\sqrt{-g}}\frac{\delta L_m}{\delta g_{\mu\nu}}$$

Modified Gravity



Introduction

- Einstein 1916: **General Relativity**:

  energy-momentum source of spacetime Curvature

  Levi-Civita connection: Zero Torsion

- Einstein 1928: **Teleparallel Equivalent of GR:**

  Weitzenbock connection: Zero Curvature

[Cai, Capozziello, De Laurentis, Saridakis, Rept.Prog.Phys. 79]

Teleparallel Equivalent of General Relativity (TEGR)

- Let's start from the simplest tosion-based gravity formulation, namely TEGR:
- Vierbeins e_A^{μ} : four linearly independent fields in the tangent space

$$g_{\mu\nu}(x) = \eta_{AB}\ e_\mu^A(x)\ e_\nu^B(x)$$

- Use curvature-less Weitzenböck connection instead of torsion-less Levi-Civita one: $\Gamma_{\nu\mu}^{\lambda\{W\}} = e_A^{\lambda}\partial_\mu e_\nu^A$
- Torsion tensor:

$$T_{\mu\nu}^{\lambda} = \Gamma_{\nu\mu}^{\lambda\{W\}} - \Gamma_{\mu\nu}^{\lambda\{W\}} = e_A^{\lambda}\left(\partial_\mu e_\nu^A - \partial_\nu e_\mu^A\right)$$

[Einstein 1928], [Pereira: Introduction to TG]

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- Lagrangian (imposing coordinate, Lorentz, parity invariance, and up to 2nd order in torsion tensor)

$$L \equiv T = \frac{1}{4} T^{\rho\mu\nu} T_{\rho\mu\nu} + \frac{1}{2} T^{\rho\mu\nu} T_{\nu\mu\rho} - T_{\rho\mu}^{\rho} T_{\nu}^{\nu\mu}$$

- **Completely equivalent** with GR at the level of equations

[Einstein 1928], [Hayaski,Shirafuji PRD 19], [Pereira: Introduction to TG]

f(T) Gravity and f(T) Cosmology

- f(T) Gravity: Simplest torsion-based modified gravity
- Generalize T to f(T) (inspired by f(R))

$$S = \frac{1}{16\pi G}\int d^4x\, e\, \left[T + f(T)\right] + S_m$$

[Ferraro, Fiorini PRD 78], [Bengochea, Ferraro PRD 79]
[Linder PRD 82]

- Equations of motion:

$$e^{-1}\partial_\mu\left(e e_A^{\rho} S_\rho^{\mu\nu}\right)\left(1+f_T\right) - e_A^{\lambda} T_{\mu\lambda}^{\rho} S_\rho^{\nu\mu} + e_A^{\rho} S_\rho^{\mu\nu}\partial_\mu(T) f_{TT} - \frac{1}{4} e_A^{\nu}\left[T + f(T)\right] = 4\pi G e_A^{\rho} \mathrm{T}_\rho^{\nu\{\mathrm{EM}\}}$$

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- f(T) Cosmology: Apply in FRW geometry:

$$e_\mu^A = diag\,(1, a, a, a) \;\Rightarrow\; ds^2 = dt^2 - a^2(t)\delta_{ij}dx^i dx^j$$ (not unique choice)

- Friedmann equations:

$$H^2 = \frac{8\pi G}{3}\rho_m - \frac{f(T)}{6} - 2 f_T H^2$$

$$\dot{H} = -\frac{4\pi G(\rho_m + p_m)}{1 + f_T - 12 H^2 f_{TT}}$$

- Find easily

$$T = -6H^2$$

f(T) Cosmology: Background

- Effective Dark Energy sector:

$$\rho_{DE} = \frac{3}{8\pi G}\left[-\frac{f}{6}+\frac{T}{3}f_T\right]$$

$$w_{DE} = -\frac{f - Tf_T + 2T^2 f_{TT}}{[1+f_T+2Tf_{TT}][f-2Tf_T]}$$

[Linder PRD 82]

- Interesting cosmological behavior: Acceleration, Inflation etc
- At the background level indistinguishable from other dynamical DE models

Non-minimally coupled scalar-torsion theory

- In curvature-based gravity, apart from $R+f(R)$ one can use $R+\xi R\varphi$^2
- Let's do the same in torsion-based gravity:

$$S=\int d^4x\ e\left[\frac{T}{2\kappa^2}+\frac{1}{2}\left(\partial_\mu\varphi\partial^\mu\varphi+\xi T\varphi^2\right)-V(\varphi)+L_m\right]$$

[Geng, Lee, Saridakis, Wu PLB 704]

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- Friedmann equations in FRW universe:

$$H^2=\frac{\kappa^2}{3}(\rho_m+\rho_{DE})$$

$$\dot{H}=-\frac{\kappa^2}{2}(\rho_m+p_m+\rho_{DE}+p_{DE})$$

with effective Dark Energy sector: $\rho_{DE}=\frac{\dot{\varphi}^2}{2}+V(\varphi)-3\xi H^2\varphi^2$

$$p_{DE}=\frac{\dot{\varphi}^2}{2}-V(\varphi)+4\xi H\varphi\dot{\varphi}+\xi\left(3H^2+2\dot{H}\right)\varphi^2$$

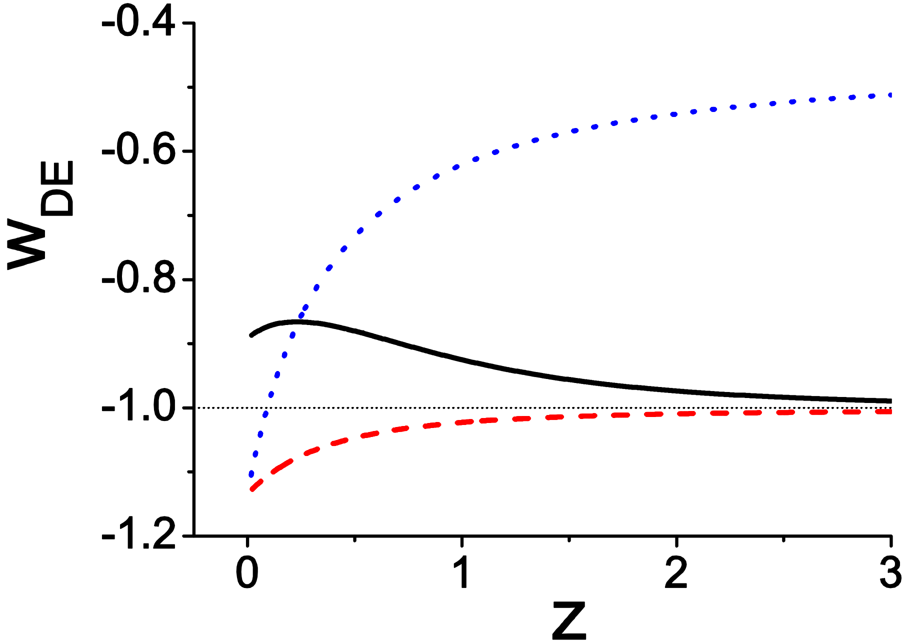
- **Different** than non-minimal quintessence!

[Geng, Lee, Saridakis,Wu PLB 704]

(no conformal transformation in the present case)

Non-minimally coupled scalar-torsion theory

- Main advantage: Dark Energy may lie in the phantom regime or/and experience the phantom-divide crossing
- Teleparallel Dark Energy:



[Geng, Lee, Saridakis, Wu PLB 704]

Non-minimally matter-torsion coupled theory

- In curvature-based gravity, one can use $f(R)L_m$ coupling
- Let's do the same in torsion-based gravity:

$$S = \frac{1}{2\kappa^2}\int d^4x\, e\, \left\{T + f_1(T) + \left[1 + \lambda\, f_2(T)\right] L_m\right\}$$ [Harko, Lobo, Otalora, Saridakis, PRD 89]

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with effective Dark Energy sector: $\rho_{DE} = -\frac{1}{2\kappa^2}\left(f_1 + 12H^2 f_1'\right) + \lambda\rho_m\left(f_2 + 12H^2 f_2'\right)$

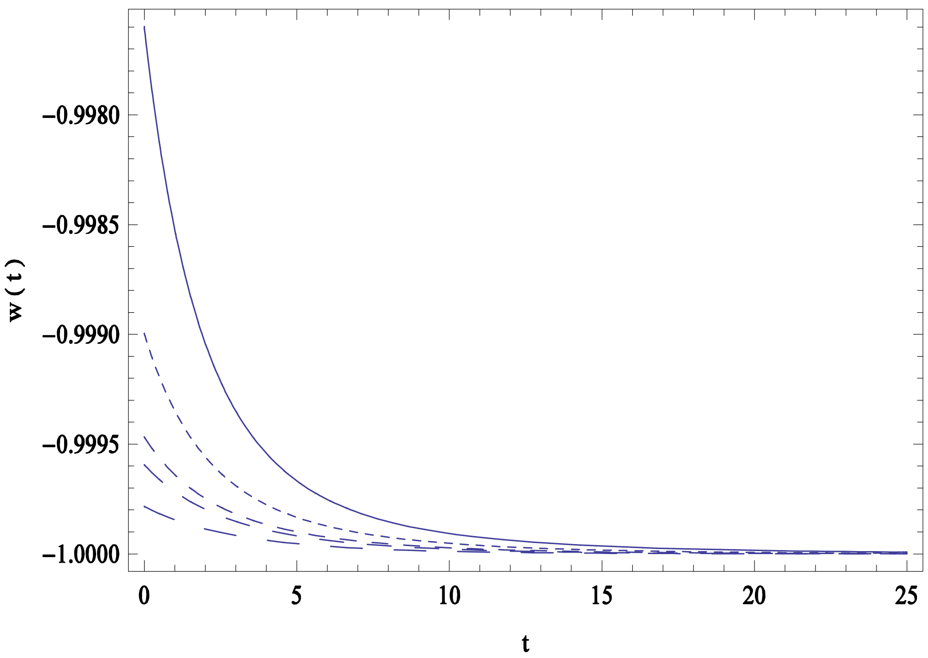
$$p_{DE} = \left(\rho_m + p_m\right)\left[\frac{1 + \lambda\left(f_2 + 12H^2 f_2'\right)}{1 + f_1' - 12H^2 f_1'' - 2\kappa^2\lambda\rho_m\left(f_2' - 12H^2 f_2''\right)}\right] + \frac{\lambda\left(f_1 + 12H^2 f_1'\right)}{2\kappa^2} - \lambda\rho_m\left(f_2 + 12H^2 f_2'\right)$$

- **Different** than non-minimal matter-curvature coupled theory

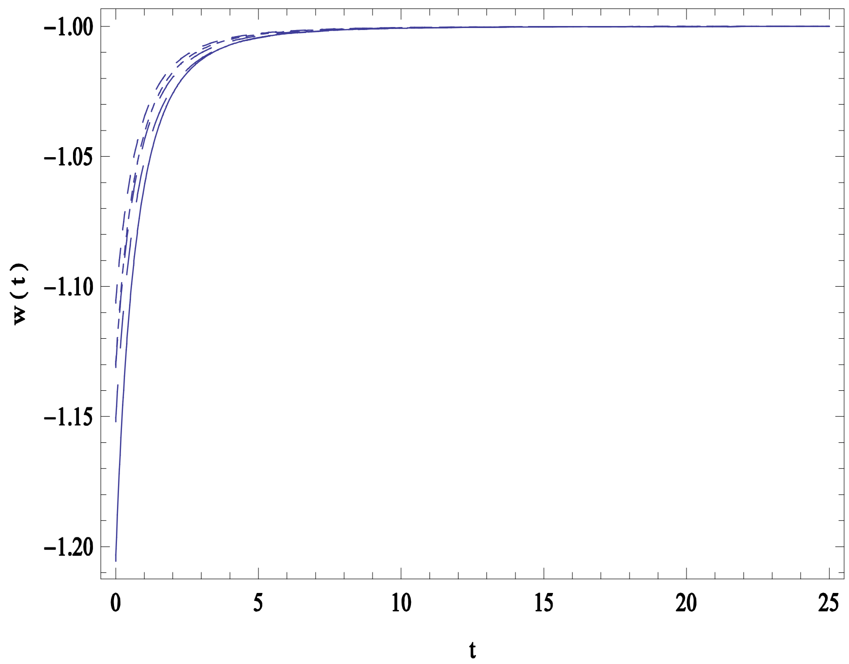
[Harko, Lobo, Otalora, Saridakis, PRD 89]

Non-minimally matter-torsion coupled theory

- Interesting phenomenology



$$f_1(T) = -\Lambda + \alpha_1 T^2, \quad f_2(T) = \beta_1 T^2$$



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- Friedmann equations in FRW universe ($\mathrm{T}=\rho_m-3p_m$):

$$H^2=\frac{\kappa^2}{3}(\rho_m+\rho_{DE})$$

$$\dot{H}=-\frac{\kappa^2}{2}(\rho_m+p_m+\rho_{DE}+p_{DE})$$

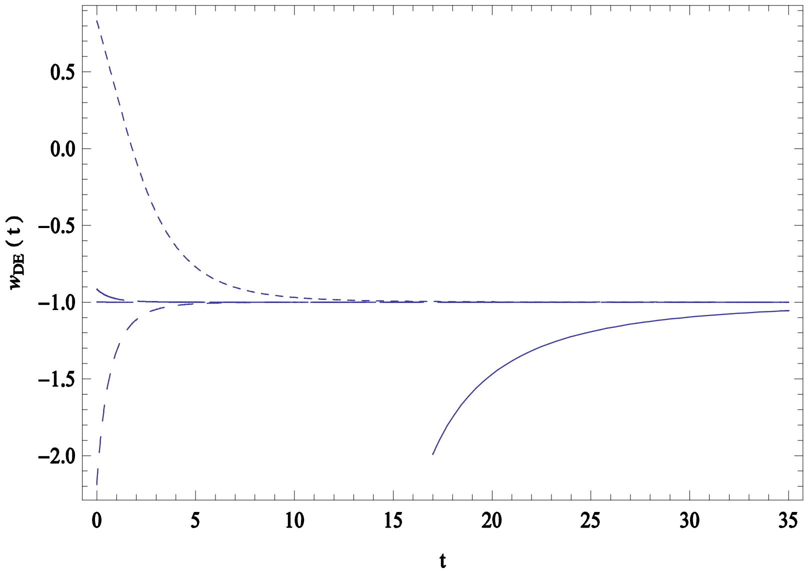
with effective Dark Energy sector: $\rho_{DE}=-\frac{1}{2\kappa^2}\left[f+12H^2f_T-2f_{\mathrm{T}}(\rho_m+p_m)\right]$

$$p_{DE}=(\rho_m+p_m)\left[\frac{1+f_{\mathrm{T}}/\kappa^2}{1+f_T-12H^2f_{TT}+H(d\rho_m/dH)(1-3dp_m/d\rho_m)f_{T\mathrm{T}}}-1\right]+\frac{1}{2\kappa^2}\left[f+12H^2f_T-2f_{\mathrm{T}}(\rho_m+p_m)\right]$$

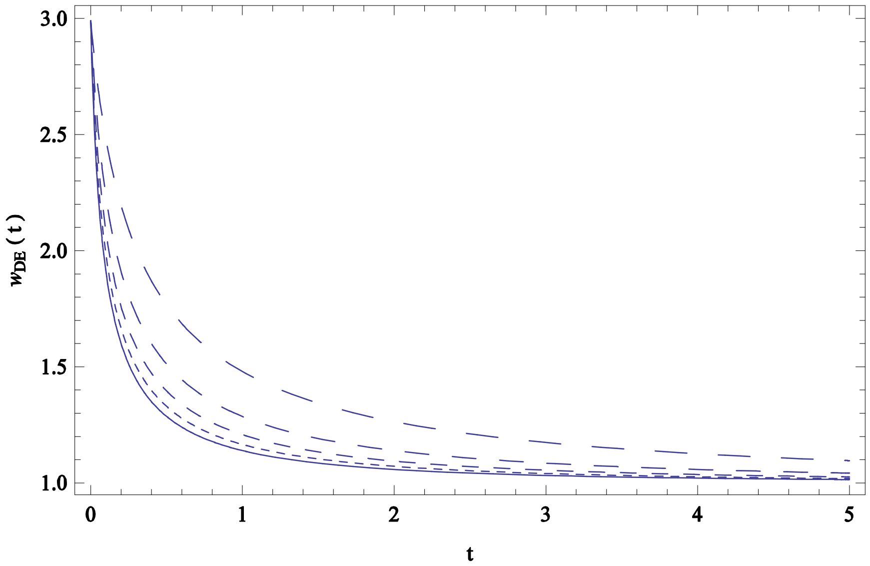
- **Different** from $f(R,\mathrm{T})$ gravity [Harko, Lobo, Otalora, Saridakis, JCAP 1412]

Non-minimally matter-torsion coupled theory

- Interesting phenomenology



$$f(T,\mathrm{T}) = \alpha \mathrm{T} T^n + \Lambda$$



$$f(T,\mathrm{T}) = \alpha \mathrm{T} + \beta T^2$$

[Harko, Lobo, Otalora, Saridakis, JCAP 1412]

Teleparallel Equivalent of Gauss-Bonnet and f(T,T_G) gravity

- In curvature-based gravity, one can use higher-order invariants like the Gauss-Bonnet one $G = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\kappa\lambda}R^{\mu\nu\kappa\lambda}$
- Let's do the same in torsion-based gravity:
- Similar to $e\overline{R} = -eT + 2\left(eT_{\nu}^{\nu\mu}\right)_{,\mu}$ we construct $e\overline{G} = eT_G + tot.diverg$ with

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$$T_G = \left(K^{a_1}_{\ ea_2}K^{ea_2}_{\ \ b}K^{a_3}_{\ fc}K^{fa_4}_{\ \ d} - 2K^{a_1a_2}_{\ \ a}K^{a_3}_{\ eb}K^{e}_{\ fc}K^{fa_4}_{\ \ d} + 2K^{a_1a_2}_{\ \ a}K^{a_3}_{\ eb}K^{ea_4}_{\ \ f}K^{f}_{\ cd} + +2K^{a_1a_2}_{\ \ a}K^{a_3}_{\ eb}K^{ea_4}_{\ \ f}K^{f}_{\ c,d}\right)\delta^{abcd}_{a_1a_2a_3a_4}$$

- $f(\mathrm{T},T_G)$ gravity:

$$S = \frac{1}{2\kappa^2}\int d^4x\, e\, \left\{T + f(T,T_G)\right\} + S_m$$

[Kofinas, Saridakis, PRD 90a]
[Kofinas, Saridakis, PRD 90b]
[Kofinas, Leon, Saridakis, CQG 31]

- **Different** from $f(R,G)$ and $f(T)$ gravities

Teleparallel Equivalent of Gauss-Bonnet and f(T,T_G) gravity

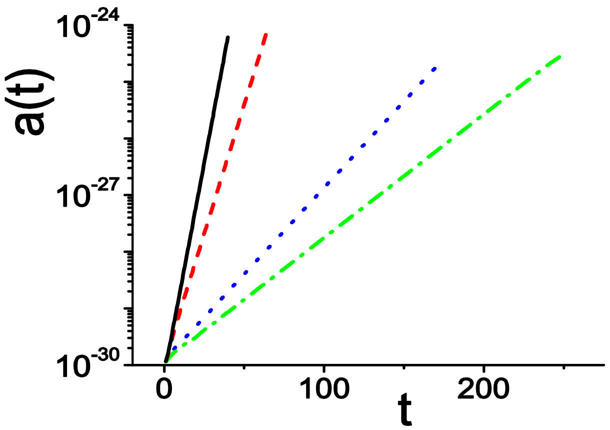
- Cosmological application:

$$\rho_{DE} = -\frac{1}{2\kappa^2}\left[f - 12H^2 f_T - T_G f_{T_G} + 24H^3 \dot{f}_{T_G}\right]$$

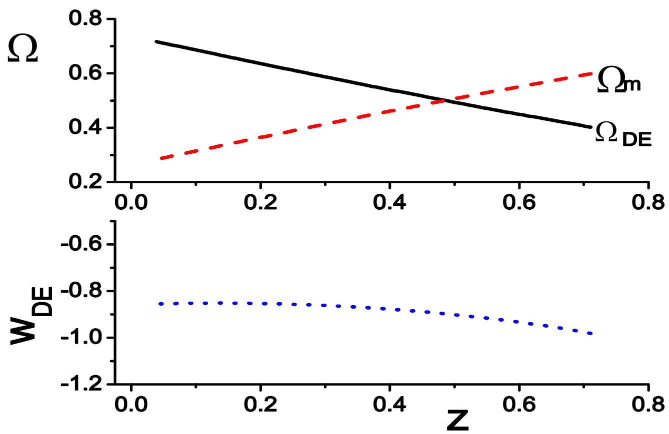
$$p_{DE} = \frac{1}{2\kappa^2}\left[f - 4\left(\dot{H} + 3H^2\right)f_T - 4H\dot{f}_T - T_G f_{T_G} + \frac{2}{3H}T_G \dot{f}_{T_G} + 8H^2 \ddot{f}_{T_G}\right]$$

$$T = 6H^2$$

$$T_G = 24H^2\left(\dot{H} + H^2\right)$$



$$f(T,T_G) = \alpha_1 T^2 + \alpha_2 T\sqrt{|T_G|}$$



$$f(T,T_G) = \beta_1\sqrt{T^2 + \beta_2 T_G}$$

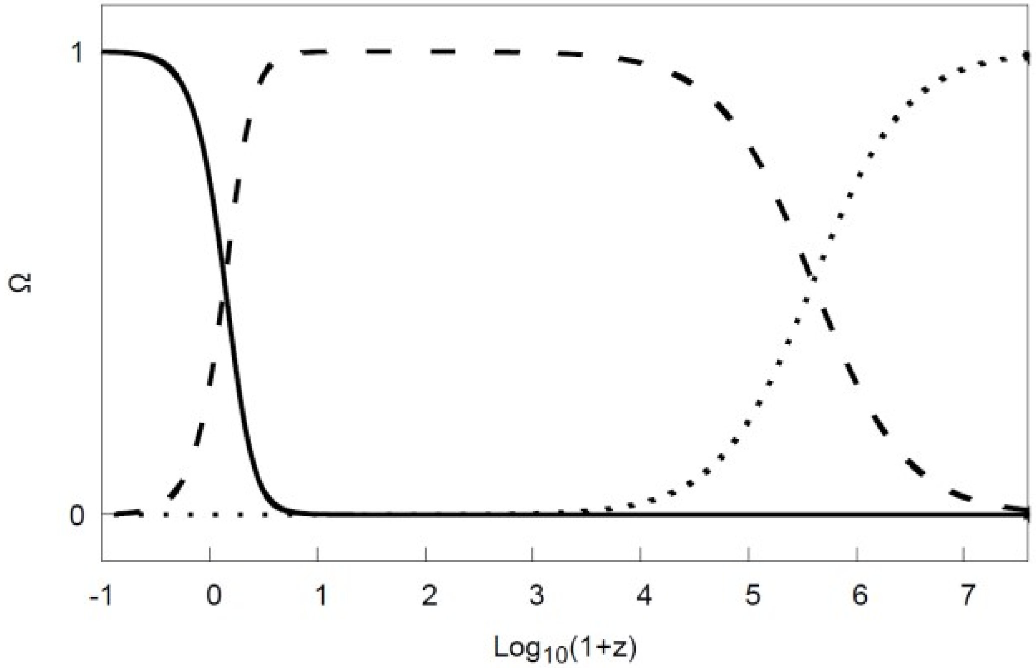
[Kofinas, Saridakis, PRD 90a]

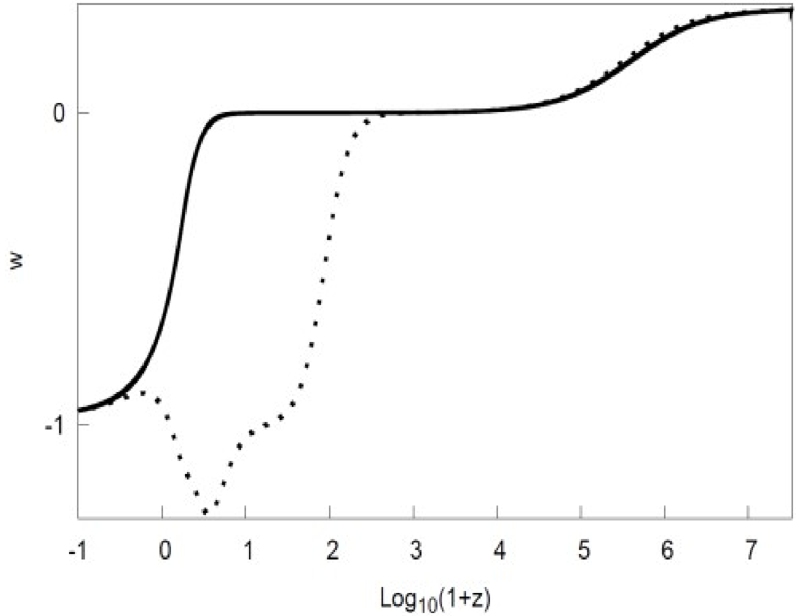
[Kofinas, Saridakis, PRD 90b]

[Kofinas, Leon, Saridakis, CQG 31]

Torsional Gravity with higher derivatives

$$S = \frac{1}{2\kappa^2}\int d^4x\, e\, F\big(T,(\nabla T)^2,\Diamond T\big) + S_m(e^A_\mu,\Psi_m)$$





[Otalora, Saridakis, PRD 94]

Torsional Modified Gravity

$$
\begin{tikzcd}[column sep=small]
 & & f(T,B,T_G,B_G,\mathcal{T}) \arrow[dll, "{f=f(-T+B,-T_G+B_G)}"'] \arrow[d, "{f=f(T,B,\mathcal{T})}"] \arrow[drr, "{f=f(T,T_G)}"] & & \\
\boxed{f(R,G)} \arrow[ddd, "{f=f(R)}"'] & & f(T,B,\mathcal{T}) \arrow[dl, "{f=f(-T+B,\mathcal{T})}"'] \arrow[ddd, "{f=f(T,B)}"] \arrow[dr, "{f=f(T,\mathcal{T})}"] & & \boxed{f(T,T_G)} \arrow[ddd, "{f=f(T)}"] \\
 & f(R,\mathcal{T}) \arrow[ddl, "{f=f(R)}"'] & & f(T,\mathcal{T}) \arrow[ddr, "{f=f(T)}"] & \\
 & & & & \\
\boxed{f(R)} \arrow[drr, "{f=R}"'] & & f(T,B) \arrow[ll, "{f=f(-T+B)}"'] \arrow[rr, "{f=f(T)}"] & & \boxed{f(T)} \arrow[dll, "{f=T}"] \\
 & & \text{GR \& TEGR} & &
\end{tikzcd}
$$

Growth-index constraints on f(T) gravity

- Perturbations: $\ddot{\delta}_m + 2H\dot{\delta}_m = 4\pi G_{eff}\,\rho_m \delta_m$, clustering growth rate: $\dfrac{d\ \ln \delta_m}{d\ \ln a} = \Omega_m^{\gamma}(a)$
- γ(z): Growth index. $G_{eff} = \dfrac{1}{1+f'(T)}$

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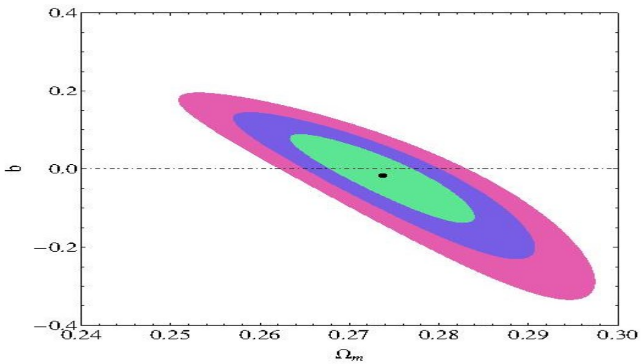
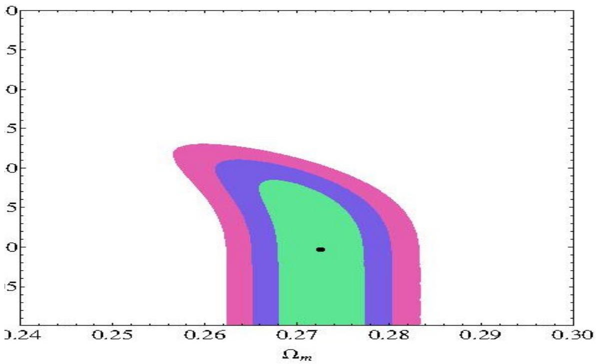
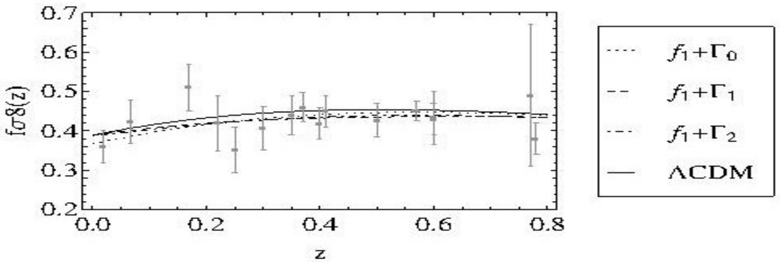
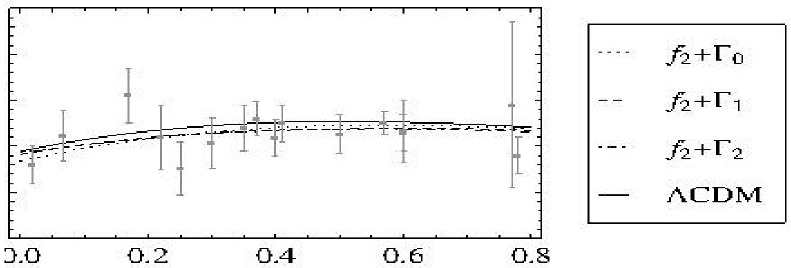
- γ(z): Growth index. $G_{eff} = \dfrac{1}{1+f'(T)}$









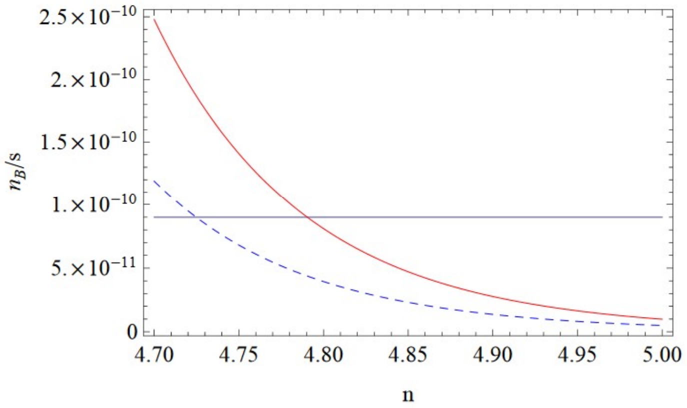
- Viable f(T) models are practically indistinguishable from ΛCDM.

[Nesseris, Basilakos, Saridakis, Perivolaropoulos, PRD 88]

[Nunes, Pan, Saridakis, JCAP 1608]

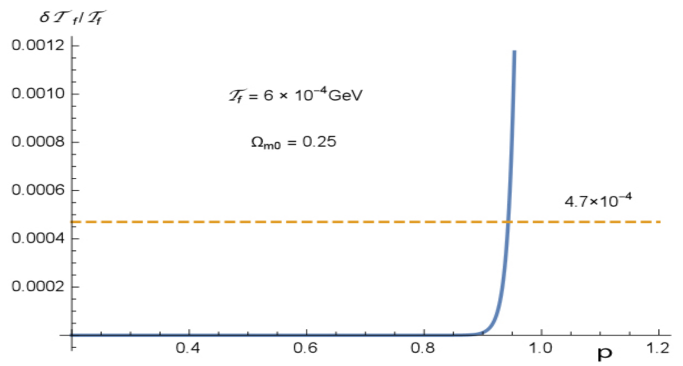
Baryogenesis and BBN constraints on f(T) gravity

- Baryon-anti-baryon asymmetry through CP violating term: $\frac{1}{M_*^2}\int d^4x\, e\left[\partial_\mu f(T)\right]J^\mu$



[Oikonomou, Saridakis, PRD 94]

- BBN constraints: $\frac{\delta T_f}{T_f} \approx \frac{\rho_T}{\rho}\frac{H_{GR}}{10\, q\, T_f^5}$



[Capozziello, Lambiase, Saridakis, EPJC77]

The Effective Field Theory (EFT) approach

- The EFT approach allows to ignore the details of the underlying theory and write an action for the perturbations around a time-dependent background solution.
- One can systematically analyze the perturbations separately from the background evolution. [Arkani-Hamed, Cheng JHEP0405 (2004)]

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$$S = \int d^4x \Big\{ \sqrt{-g} \Big[\frac{M_P^2}{2} \Psi(t) R - \Lambda(t) - b(t) g^{00}$$ <- background

$$+ M_2^4 (\delta g^{00})^2 - \bar{m}_1^3 \delta g^{00} \delta K - \bar{M}_2^2 \delta K^2 - \bar{M}_3^2 \delta K^\nu_\mu \delta K^\mu_\nu$$ <- linear evolution of perturbations

$$+ m_2^2 h^{\mu\nu} \partial_\mu g^{00} \partial_\nu g^{00} + \lambda_1 \delta R^2 + \lambda_2 \delta R_{\mu\nu} \delta R^{\mu\nu} + \mu_1^2 \delta g^{00} \delta R \Big]$$ <- linear evolution of perturbations

$$+ \gamma_1 C^{\mu\nu\rho\sigma} C_{\mu\nu\rho\sigma} + \gamma_2 \epsilon^{\mu\nu\rho\sigma} C_{\mu\nu}{}^{\kappa\lambda} C_{\rho\sigma\kappa\lambda}$$ <- linear evolution of perturbations

$$+ \sqrt{-g} \Big[\frac{M_3^4}{3} (\delta g^{00})^3 - \bar{m}_2^3 (\delta g^{00})^2 \delta K + \ldots \Big] \Big\} ,$$ <- 2nd-order evolution of perturbations

The functions Ψ(t), Λ(t), b(t), are determined by the background solution

[Gubitosi, Piazza, Vernizzi, JCAP1302]

The (EFT) approach to torsional gravity

- Application of the EFT approach to torsional gravity leads to include terms:
- i) Invariant under 4D diffeomorphisms: e.g. R,T multiplied by functions of time.
- ii) Invariant under spatial diffeomorphisms: e.g. g^{00}, R^{00} and T^{0}
- ii) Invariant under spatial diffeomorphisms: e.g. ${}^{(3)}R_{\mu\nu\rho\sigma}$, ${}^{(3)}T^{\rho}{}_{\mu\nu}$, $K_{\mu\nu}$, $\hat{K}_{\mu\nu}$

  the extrinsic torsion is defined as

$$\hat{K}_{\mu\nu} \equiv h^{\sigma}_{\mu}\hat{\nabla}_{\sigma}n_{\nu} = K_{\mu\nu} - \mathcal{K}^{\lambda}{}_{\nu\mu}n_{\lambda} + n_{\mu}\frac{1}{g^{00}}T^{00}{}_{\nu} \; ;$$

with n_{μ} the orthogonal to t=cont. surfaces unitary vector $n_{\mu} = \frac{\delta^{0}_{\mu}}{\sqrt{-g^{00}}}$

[Cai, Li, Saridakis, Xue, PRD 97], [Li, Cai, Cai, Saridakis, 1803.09818]

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Using the projection operator h^{μ}_{ν} we can express ${}^{(3)}R_{\mu\nu\rho\sigma} = h^{\alpha}_{\mu}h^{\beta}_{\nu}h^{\gamma}_{\rho}h^{\delta}_{\sigma}R_{\alpha\beta\gamma\delta} - K_{\mu\rho}K_{\nu\sigma} + K_{\nu\rho}K_{\mu\sigma}$

$$h^d_a h^c_b h^f_e T^e{}_{dc} = {}^{(3)}T^f{}_{ab}$$

[Cai, Li, Saridakis, Xue, PRD 97], [Li, Cai, Cai, Saridakis, 1803.09818]

The (EFT) approach to torsional gravity

- We perturb the previous tensors, and we finally obtain:

$$
\begin{aligned}
R^{(0)}_{\mu\nu\rho\sigma} &= f_1(t) g_{\mu\rho} g_{\nu\sigma} + f_2(t) g_{\mu\rho} n_\nu n_\sigma + f_3(t) g_{\mu\sigma} g_{\nu\rho} \\
&\quad + f_4(t) g_{\mu\sigma} n_\nu n_\rho + f_5(t) g_{\nu\sigma} n_\mu n_\rho \\
&\quad + f_6(t) g_{\nu\rho} n_\mu n_\sigma , \\
T^{(0)}_{\rho\mu\nu} &= g_1(t) g_{\rho\nu} n_\mu + g_2(t) g_{\rho\mu} n_\nu , \\
K^{(0)}_{\mu\nu} &= f_7(t) g_{\mu\nu} + f_8(t) n_\mu n_\nu , \\
\hat{K}^{(0)}_{\mu\nu} &= 0 .
\end{aligned}
$$

where the time-dependent functions are determined by the background solution.

[Cai, Li, Saridakis, Xue, PRD 97], [Li, Cai, Cai, Saridakis, 1803.09818]

The (EFT) approach to torsional gravity

- Finally, the EFT action of torsional gravity becomes:

$$S = \int d^4x\sqrt{-g}\Big[\frac{M_P^2}{2}\Psi(t)R - \Lambda(t) - b(t)g^{00} + \frac{M_P^2}{2}d(t)T^0\Big]$$
$$+S^{(2)} \ ,$$

- The perturbation part contains:

  i) Terms present in curvature EFT action

  ii) Pure torsion terms such as δT^2 , $\delta T^0\delta T^0$ and $\delta T^{\rho\mu\nu}\delta T_{\rho\mu\nu}$

  iii) Terms that mix curvature and torsion, such as $\delta T\delta R,\ \delta g^{00}\delta T$, $\delta g^{00}\delta T^0$ and $\delta K\delta T^0$

[Cai, Li, Saridakis, Xue, PRD 97], [Li, Cai, Cai, Saridakis, 1803.09818]

The (EFT) approach to f(T) gravity: Background

- For the case of f(T) gravity, at the background level, we have:

$$S = \frac{M_P^2}{2} \int d^4x \sqrt{-g} \big[- f_T(T^{(0)}) R + 2 \dot{f}_T(T^{(0)}) T^{(0)} - T^{(0)} f_T(T^{(0)}) + f(T^{(0)}) \big]$$

where by comparison:

$$\Psi(t) = -f_T(T^{(0)}) \; ,$$

$$\Lambda(t) = \frac{M_P^2}{2} \left[T^{(0)} f_T(T^{(0)}) - f(T^{(0)}) \right] \; ,$$

$$d(t) = -2 \dot{f}_T(T^{(0)}) \; ,$$

$$b(t) = 0 \; .$$

[Li, Cai, Cai, Saridakis, 1803.09818]

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$$d(t) = -2\dot{f}_T(T^{(0)}) \,,$$
$$b(t) = 0 \,.$$

- Performing variation we obtain the background equations of motion (Friedmann Eqs):

$$b(t) = M_P^2\Psi\left(-\dot{H} - \frac{\ddot{\Psi}}{2\Psi} + \frac{H\dot{\Psi}}{2\Psi} - \frac{\dot{d}}{4\Psi} + \frac{3Hd}{4\Psi}\right) - \frac{1}{2}(\rho_m + p_m),$$

$$\Lambda(t) = M_P^2\Psi\left(3H^2 + \frac{5H\dot{\Psi}}{2\Psi} + \dot{H} + \frac{\ddot{\Psi}}{2\Psi} + \frac{\dot{d}}{4\Psi} + \frac{3Hd}{4\Psi}\right) - \frac{1}{2}(\rho_m - p_m),$$

[Li, Cai, Cai, Saridakis, 1803.09818]

The (EFT) approach to f(T) gravity: Background

- These can be written as:

$$H^2 = \frac{1}{3M_P^2}(\rho_m + \rho_{DE}^{\rm eff}),$$

$$\dot{H} = -\frac{1}{2M_P^2}(\rho_m + \rho_{DE}^{\rm eff} + p_m + p_{DE}^{\rm eff})$$

with

$$\rho_{DE}^{\rm eff} = b + \Lambda - 3M_P^2\left[H\dot{\Psi} + \frac{dH}{2} + H^2(\Psi - 1)\right]$$

$$p_{DE}^{\rm eff} = b - \Lambda + M_P^2\Big[\ddot{\Psi} + 2H\dot{\Psi} + \frac{\dot{d}}{2} + (H^2 + 2\dot{H})(\Psi - 1)\Big].$$

and thus:

$$\rho_{DE}^{\rm eff} = \frac{M_P^2}{2}\left[T^{(0)} - f(T^{(0)}) + 2T^{(0)}f_T(T^{(0)})\right]$$

$$p_{DE}^{\rm eff} = -\frac{M_P^2}{2}\Big[4\dot{H}(1 + f_T(T^{(0)}) + 2T^{(0)}f_{TT}(T^{(0)})) - f(T^{(0)}) + T^{(0)} + 2T^{(0)}f_T(T^{(0)})\Big]$$

- The same equations with standard approach!

[Li, Cai, Cai, Saridakis, 1803.09818]

The (EFT) approach to f(T) gravity: Tensor Perturbations

- For tensor perturbations: $g_{00}=-1\ ,\quad g_{0i}=0\ ,$

$$g_{ij}=a^2\left(\delta_{ij}+h_{ij}+\frac{1}{2}h_{ik}h_{kj}\right)$$

i.e.

$$\bar{e}^0_\mu=\delta^0_\mu\ ,$$
$$\bar{e}^a_\mu=a\delta^a_\mu+\frac{a}{2}\delta^i_\mu\delta^{aj}h_{ij}+\frac{a}{8}\delta^i_\mu\delta^{ja}h_{ik}h_{kj}\ ,$$
$$\bar{e}^\mu_0=\delta^\mu_0\ ,$$
$$\bar{e}^\mu_a=\frac{1}{a}\delta^\mu_a-\frac{1}{2a}\delta^{\mu i}\delta^j_a h_{ij}+\frac{1}{8a}\delta^{i\mu}\delta^j_a h_{ik}h_{kj}$$

- We obtain:

$$^{(3)}R\approx-\frac{1}{4}a^{-2}(\partial_i h_{kl}\partial_i h_{kl})\ ,$$
$$K^{ij}K_{ij}\approx 3H^2+\frac{1}{4}\dot{h}_{ij}\dot{h}_{ij}\ ,$$
$$K\approx 3H\ ,$$
$$\boxed{T=T^{(0)}+O(h^2)=6H^2+O(h^2)}$$

- And finally:

$$S=\frac{M_P^2}{2}\int d^4x\sqrt{-g}\Big[\frac{f_T}{4}\big(a^{-2}\vec{\nabla}h_{ij}\cdot\vec{\nabla}h_{ij}-\dot{h}_{ij}\dot{h}_{ij}\big)+6H^2 f_T-12H\dot{f}_T-T^{(0)}f_T+f(T^{(0)})\Big]$$

[Cai, Li, Saridakis, Xue, PRD 97]

The (EFT) approach to f(T) gravity: Gravitational Waves

- Varying the action and going to Fourier space we get the equation for GWs:

$$\ddot{h}_{ij} + 3H(1-\beta_T)\dot{h}_{ij} + \frac{k^2}{a^2}h_{ij} = 0$$

with $\beta_T \equiv -\dfrac{\dot{f}_T}{3H f_T}$

- An immediate result: The speed of GWs is equal to the speed of light!
- GW170817 constraints that

$$|c_g/c - 1| \leq 4.5 \times 10^{-16}$$

are trivially satisfied.

[Cai, Li, Saridakis, Xue, PRD 97]

The (EFT) approach to f(T) gravity: Gravitational Waves

- We can express: $\beta_T = \frac{d \ln f_T}{d \ln T}(1 + w_{tot})$

- In GR and TEGR β_T is zero. Thus, if a non-zero $\boxed{\beta_T}$ is measured in future observations, it could be the smoking gun of modified gravity.

- Very important since f(T) gravity has the same polarization modes with GR.
- The effect of f(T) gravity on GWs comes through its effect on the background solutions itself, since at linear perturbation order f(T) gravity is effectively TEGR.

[Cai, Li, Saridakis, Xue, PRD 97]

The (EFT) approach to f(T) gravity: Scalar Perturbations

- For scalar perturbations:

$$g_{00} = -1 - 2\phi\ ,$$
$$g_{0i} = 0\ ,$$
$$g_{ij} = a^2[(1-2\psi)\delta_{ij} + \partial_i\partial_j F]$$

i.e

$$e^0_\mu = \delta^0_\mu + \delta^0_\mu \phi + a\delta^i_\mu \partial_i \chi\ ,$$
$$e^a_\mu = a\delta^i_\mu \delta^a_i + \delta^0_\mu \delta^a_i \partial^i \mathcal{E} + a\delta^i_\mu \delta^a_j \left[\epsilon_{ijk}\partial_k \sigma - \psi\delta_{ij} + \frac{1}{2}\partial_i\partial_j F\right]$$

- So $$T^0 = g^{0\mu}T^{\nu}{}_{\mu\nu} = -3H + 6H\phi + 3\dot{\psi} - 6H\phi^2 - 6\dot{\psi}\phi + \frac{1}{a}\partial_i\partial_i\chi - \frac{1}{2a}\partial_i\phi\partial_i\chi - \frac{3}{2a}\phi\partial_i\partial_i\chi - \frac{1}{2a}\partial_i\psi\partial_i\chi + \frac{1}{2a}\psi\partial_i\partial_i\chi$$

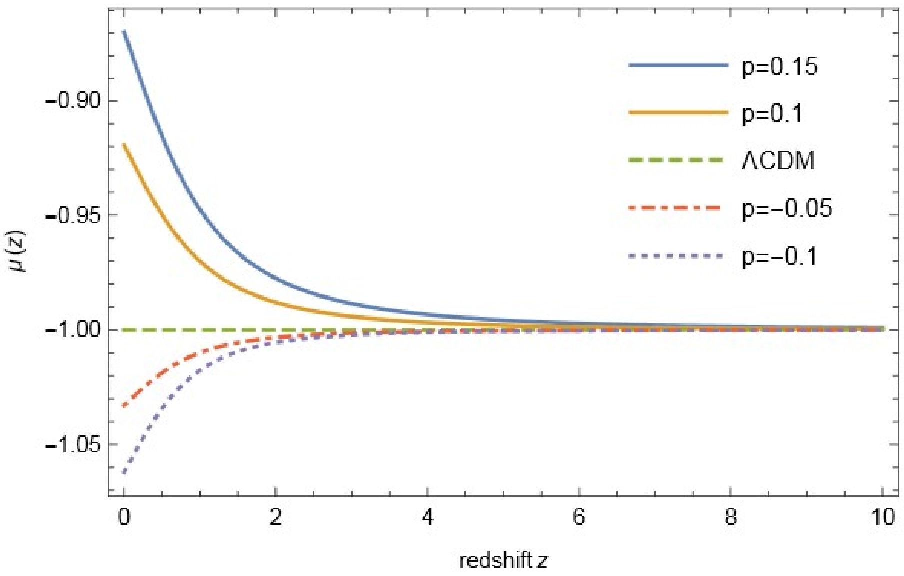
- Thus:

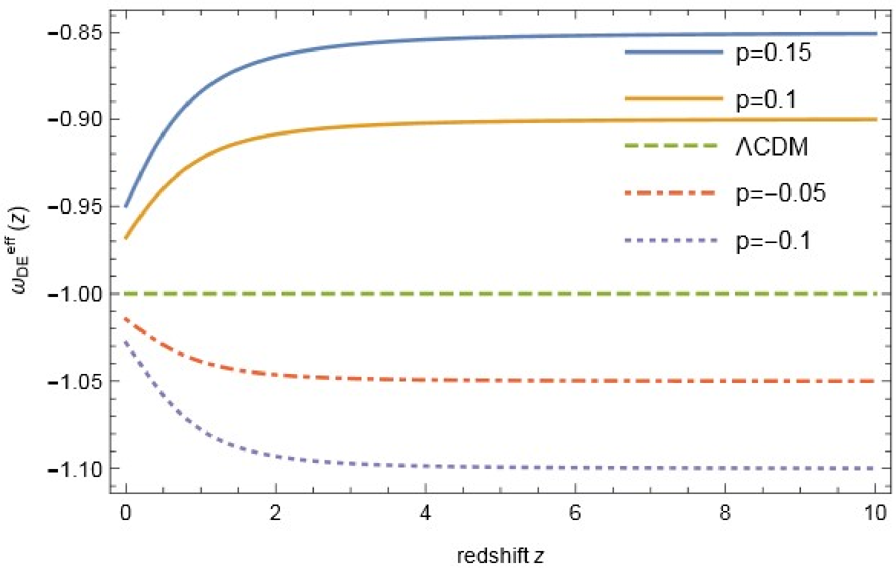
$$\boxed{S = \int d^4x \Big[\frac{M_P^2}{2}\left(-2af_T\partial_i\psi\partial_i\psi + 4af_T\partial_i\phi\partial_i\psi + 4a^2\dot{f}_T\partial_i\psi\partial_i\chi + 4\dot{f}_T a^2 H\partial_i\pi\partial_i\chi\right) + a^3M^2\pi^2 - a^3\phi\delta\rho_m\Big]}$$

[Li, Cai, Cai, Saridakis, 1803.09818]

The (EFT) approach to f(T) gravity: Tensor Perturbations

- Finally: $\mu(z) = \dfrac{2M_P^2 k^2 \phi (1+z)^2}{\delta \rho_m}$ with $\mu \equiv \dfrac{1}{f_T}$





$$f(T) = -T + \alpha T^p$$

$$\alpha = (6H_0^2)^{1-p} \frac{1-\Omega_{m0}}{2p-1}$$

[Li, Cai, Cai, Saridakis, 1803.09818]

Conclusions

- i) Many cosmological and theoretical arguments favor modified gravity.
- ii) Can we modify gravity based in its torsion formulation?
- iii) Simplest choice: f(T) gravity, i.e extension of TEGR
- iv) f(T) cosmology: Interesting phenomenology. Signatures in growth structure.
- v) Non-minimal coupled scalar-torsion theory: Quintessence, phantom or crossing behavior. Similarly in torsion-matter coupling and TEGB.
- vi) EFT approach allows for a systematic study of perturbations
- vii) Observational signatures in the dispersion relation of GWs
- viii) No further polarization modes.

Outlook

- Many subjects are open. Amongst them:

- i) Examine higher-order perturbations to look for further polarizations.
  See Jackson's talk.
  [Farugia, Gakis, Jackson, Saridakis, 1804.07365, to appear in PRD]

- ii) Extend the analysis to other torsional modified gravity.

- iii) Try to break the various degeneracies and find a signature of this particular class of modified gravity

- vi) Convince people to work on the subject!

- “There are the ones that invent occult fluids to understand the Laws of Nature. They come to conclusions, but they now run out into dreams and chimeras neglecting the true constitutions of the things...
  However there are those that from the simplest observation of Nature, they reproduce New Forces”…

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THANK YOU!

Covariant formulation of f(T) gravity

- In standard f(T) gravity spin connection is set to zero.
- However vierbein transformations must be accompanied by connection ones:

$$e'^{A}_{\mu} = \Lambda^{A}_{B} e^{B}_{\mu}$$

$$\omega'^{A}_{B\mu} = \Lambda^{A}_{C}\omega^{C}_{D\mu}\Lambda^{D}_{B} + \Lambda^{A}_{C}\partial_{\mu}\Lambda^{C}_{B}$$

[Krssak, Pereira EPJC 75]

- Example: FRW geometry

$e^{A}_{\mu} = diag\ (1, a, a, a)$ or $e^{A}_{\mu} = diag\ (1, a, ra, ra\ \sin\ \theta)$

$\omega^{A}_{B\mu} = 0$ $\quad \omega^{1}_{2\theta} = -1,\ \omega^{1}_{3\phi} = -\sin\theta,\ \omega^{2}_{3\phi} = -\cos\theta$

- On the other hand, if one assumes/imposes $\omega'^{A}_{B\mu} = 0$ then only "peculiar" forms of vierbeins will be allowed.

- $\Rightarrow$ Lorentz invariance has been restored in f(T) gravity

[Krssak, Saridakis CQG 33]

Curvature and Torsion

- Vierbeins e_A^μ : four linearly independent fields in the tangent space

  $g_{\mu\nu}(x) = \eta_{AB}\ e_\mu^A(x)\ e_\nu^B(x)$

- Connection: ω_{ABC}

- Curvature tensor: $R^A_{B\mu\nu} = \omega^A_{B\nu,\mu} - \omega^A_{B\mu,\nu} + \omega^A_{C\mu}\omega^C_{B\nu} - \omega^A_{C\nu}\omega^C_{B\mu}$

- Torsion tensor: $T^A_{\mu\nu} = e^A_{\nu,\mu} - e^A_{\mu,\nu} + \omega^A_{B\mu}e^B_\nu - \omega^A_{B\nu}e^B_\mu$

- Levi-Civita connection and Contorsion tensor: $\omega_{ABC} = \Gamma_{ABC} + K_{ABC}$

  $$K_{ABC} = \frac{1}{2}\left(T_{CAB} - T_{BCA} - T_{ABC}\right) = -K_{BAC}$$

- Curvature and Torsion Scalars: $R = \overline{R} + T - 2\left(T_\nu^{\ \nu\mu}\right)_{;\mu}$

$$R = g^{\mu\nu}R_{\mu\nu} = g^{\mu\nu}R^\rho_{\mu\rho\nu} \qquad T = \frac{1}{4}T^{\rho\mu\nu}T_{\rho\mu\nu} + \frac{1}{2}T^{\rho\mu\nu}T_{\nu\mu\rho} - \mathrm{T}^\rho_{\rho\mu}\mathrm{T}^{\nu\mu}_\nu$$