

String Theory and Nonrelativistic Gravity

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work done in collaboration with

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Workshop Dualities and Generalized Geometries

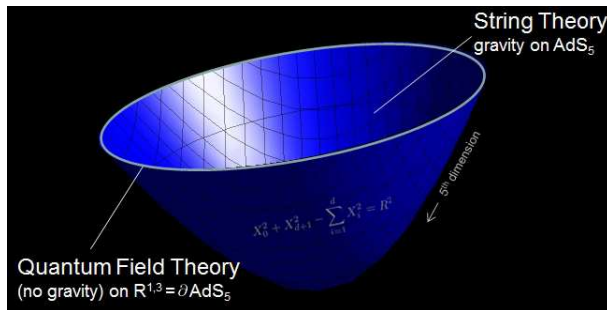
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Motivation

Holography



Gravity is not only used to describe the gravitational force!

Non-relativistic Holography

two approaches

- Keep general relativity in the bulk but take background geometry with **non-relativistic isometries**

Christensen, Hartong, Kiritsis, Obers and Rollier (2013-2015)

- Take **non-relativistic gravity** in the bulk

Gomis, Ooguri (2001); Gopakumar, Bagchi (2009)

Outline

String Newton-Cartan Gravity

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The Non-relativistic Polyakov String

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T-duality

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NC Gravity couples to **particles**

what about **strings**?

Comparing Limits

The NR limit of a particle coupled to general relativity is **finite** provided that the particle is coupled to a **(central charge) gauge field** B_μ which is expressed in terms of the **timelike NC Vierbein field** τ_μ as

$$B_\mu = \omega \tau_\mu$$

with $\omega \rightarrow \infty$ in the NR limit

Similarly, the string should be coupled to a **2-form gauge field** $B_{\mu\nu}$ with

$$B_{\mu\nu} = \omega^2 \epsilon_{AB} \tau_\mu^A \tau_\nu^B, \quad A = 0, 1$$

defining a **string NC geometry** with **generalized clock functions** τ_μ^A

see also Gomis, Ooguri (2001)

String Galilei Symmetries

Gomis, Ooguri (2001)

$$D + 1 \text{ flat indices} \rightarrow \begin{cases} 2 \text{ longitudinal indices } A \\ D - 1 \text{ transverse indices } A' \end{cases}$$

longitudinal translations H_A

transverse translations $P_{A'}$

string Galilei boosts $G_{AB'}$

longitudinal Lorentz rotations M_{AB}

transverse spatial rotations $J_{A'B'}$

Conventional versus Geometric Constraints

$$R_{\mu\nu}{}^A(H) = D_{[\mu}(\Omega)\tau_{\nu]}{}^A = 0 \quad \text{with} \quad \tau_{\mu}{}^A \text{ generalized clock function}$$

4D : 12 components

- 4 conventional constraints: solves for $\Omega_{\mu}{}^{AB} = \epsilon^{AB}\Sigma_{\mu}$
- 8 geometric constraints

Noncentral Extension

$$[G_{AA'}, P_{B'}] = 0 \quad \rightarrow \quad [G_{AA'}, P_{B'}] = \delta_{A'B'} Z_A$$

The independent string NC fields $\{\tau_\mu^A, e_\mu^{A'}, m_\mu^A\}$ transform as follows:

$$\delta \tau_\mu^A = \Lambda^A_{B'} \tau_\mu^{B'}$$

$$\delta E_\mu^{A'} = \Lambda^{A'}_{B'} E_\mu^{B'} - \Sigma^A_{A'} \tau_\mu^A$$

$$\delta m_\mu^A = D_\mu \sigma^A + \Sigma^A_{A'} E_\mu^{A'}$$

longitudinal metric:

$$\tau_{\mu\nu} \equiv \tau_\mu^A \tau_\nu^B \eta_{AB}$$

transverse 'metric':

$$H_{\mu\nu} \equiv E_\mu^{A'} E_\nu^{B'} \delta_{A'B'} + (\tau_\mu^A m_\nu^B + \tau_\nu^A m_\mu^B) \eta_{AB}$$

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NR Limit Polyakov Particle

$$S_{\text{Pol.}} = -\frac{1}{2} \int d\tau \left\{ -\frac{1}{e} E_{\mu}^A \dot{x}^{\mu} E_{\nu}^B \dot{x}^{\nu} \eta_{AB} + M^2 e - 2M B_{\mu} \dot{x}^{\mu} \right\}$$

$$\begin{aligned} S_{\text{Pol.}}(c^2) &= -\frac{1}{2} \int d\tau \frac{1}{e} c^2 [\tau_{\mu} \dot{x}^{\mu} - me]^2 \\ &= -\frac{1}{2} \int d\tau \frac{1}{e} \left\{ \lambda (\tau_{\mu} \dot{x}^{\mu} - me) - \frac{1}{4c^2} \lambda^2 \right\} \Rightarrow \end{aligned}$$

$$S_{\text{Pol.}}(\text{N.R.}) = -\frac{1}{2} \int d\tau \frac{1}{e} \left\{ \dot{x}^{\mu} \dot{x}^{\nu} H_{\mu\nu} + \lambda (\tau_{\mu} \dot{x}^{\mu} - me) \right\}$$

Nambu-Goto (NG)

- **relativistic NG** is non-linear in **longitudinal** and **transverse** embedding coordinates

- **nonrelativistic NG** is nonlinear in **longitudinal** embedding coordinates only

The Nonrelativistic Polyakov String

$$h_{\alpha\beta} = e_{\alpha}{}^a e_{\beta}{}^b \eta_{ab}$$

$$e_{\alpha} \equiv e_{\alpha}{}^0 + e_{\alpha}{}^1, \quad \bar{e}_{\alpha} \equiv e_{\alpha}{}^0 - e_{\alpha}{}^1$$

$$\tau_{\mu} \equiv \tau_{\mu}{}^0 + \tau_{\mu}{}^1, \quad \bar{\tau}_{\mu} \equiv \tau_{\mu}{}^0 - \tau_{\mu}{}^1$$

$$S_{\text{Pol.}} = -\frac{T}{2} \int d^2\sigma \left[\sqrt{-h} h^{\alpha\beta} \partial_{\alpha} x^{\mu} \partial_{\beta} x^{\nu} H_{\mu\nu} + \epsilon^{\alpha\beta} (\lambda e_{\alpha} \tau_{\mu} + \bar{\lambda} \bar{e}_{\alpha} \bar{\tau}_{\mu}) \partial_{\beta} x^{\mu} \right]$$

$$- \frac{T}{2} \int d^2\sigma \epsilon^{\alpha\beta} \partial_{\alpha} x^{\mu} \partial_{\beta} x^{\nu} B_{\mu\nu}$$

Nambu-Goto : $h_{\alpha\beta} = \partial_{\alpha} x^{\mu} \partial_{\beta} x^{\nu} \tau_{\mu\nu}$

World-sheet Analysis

Gomis, Ooguri (2001)

Conformal Gauge

$$\sqrt{-h} h^{\alpha\beta} = \eta^{\alpha\beta}$$

Flat Spacetime

$$\tau_{\mu}{}^A = \delta_{\mu}^A, \quad E_{\mu}{}^{A'} = \delta_{\mu}^{A'}, \quad m_{\mu}{}^A = 0$$

⇓

$$S = -\frac{T}{2} \int d^2\sigma \left(\partial_{X^{A'}} \bar{\partial}_{X^{B'}} \delta_{A'B'} + \lambda \bar{\partial} X + \bar{\lambda} \partial \bar{X} \right)$$

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Relativistic T-duality

Buscher (1987,1988); Roček, Verlinde (1992)

adapted coordinates: $x^\mu = (y, x^i)$ $k^\mu \partial_\mu = \partial_y$

$$S_{\text{parent}} = \underbrace{S_{\text{Pol.}}(\partial_\alpha y \rightarrow v_\alpha)}_{\text{quadratic in } v_\alpha!} - T \int d^2 \sigma \epsilon^{\alpha\beta} \tilde{y} \partial_\alpha v_\beta$$

$$\frac{\delta S_{\text{parent}}}{\delta v_\alpha} = 0 \quad \rightarrow \quad v_\alpha \text{ is solved for in terms of dual coordinate } \tilde{y}$$

$$\tilde{G}_{yy} = \frac{1}{G_{yy}} \quad \Rightarrow \quad R \Leftrightarrow \frac{1}{R}$$

Non-relativistic Longitudinal T-duality

$$\tau_\mu{}^0 k^\mu = 0, \quad \tau_\mu{}^1 k^\mu \neq 0, \quad E_\mu{}^{A'} k^\mu = 0$$

adapted coordinates: $x^\mu = (y, x^i)$ $k^\mu \partial_\mu = \partial_y$

$$S_{\text{parent}} = S_{\text{Pol.}}(\partial_\alpha y \rightarrow v_\alpha) - T \int d^2 \sigma \epsilon^{\alpha\beta} \tilde{y} \partial_\alpha v_\beta$$

$\frac{\delta S_{\text{parent}}}{\delta v_\alpha} = 0 \quad \rightarrow \quad v_\alpha$ is solved for in terms of **dual coordinate \tilde{y}** and

the Lagrange multipliers $\lambda, \bar{\lambda}$ if we dualize in the **longitudinal** direction

$\rightarrow \lambda, \bar{\lambda}$ are no Lagrange multipliers anymore!

Dual Action

$$\tilde{S}_{\text{long.}} = -\frac{T}{2} \int d^2\sigma \left(\sqrt{-h} h^{\alpha\beta} \partial_\alpha \tilde{x}^\mu \partial_\beta \tilde{x}^\nu \tilde{G}_{\mu\nu} + \epsilon^{\alpha\beta} \partial_\alpha \tilde{x}^\mu \partial_\beta \tilde{x}^\nu \tilde{B}_{\mu\nu} \right)$$

with $\tilde{x}^\mu = (\tilde{y}, x^i)$ and $\tilde{G}_{yy} = 0$: **lightlike direction**

- The **longitudinal spatial** T-dual of the NR string is the Polyakov string moving in a GR background with a **lightlike direction**
 - **DLCQ relativistic string**
 - **Rigid susy in Lorentzian curved spacetimes**

See, e.g., Cassani, Klare, Martelli, Tomasiello, Zaffaroni (2014)

- The **transverse spatial** T-dual of the NR string is again a NR string with a transverse spatial isometry direction à la Buscher

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3D Extended Bargmann Gravity

$$S = \frac{1}{2\kappa^2} \int d^3x (E R(\Omega) - \epsilon^{\mu\nu\rho} B_{\mu} \partial_{\nu} A_{\rho}) \Rightarrow$$

Chern-Simons action for **3D Extended Bargmann Algebra**

Rosseel + E.B. (2016); Hartong, Obers (2016)

- The **2 central charges** originate from the **2 vector fields**
- **3D Extended Bargmann Gravity** \neq **3D NC gravity**

4D Extended String Bargmann Gravity

$$S = \frac{1}{2\kappa^2} \int d^4x (E R(\Omega) - \epsilon^{\mu\nu\rho\sigma} B_{\mu\nu} \partial_\rho A_\sigma)$$

Grosvenor, Şimşek, Yan + E.B., work in progress

- The 2-form gives rise to the **2 noncentral extensions** Z_A ($A = 0, 1$) of the String Bargmann Algebra
- The 1-form gives rise to the **extra central extension** S of the **Extended String Bargmann Algebra**

A 4D First-Order Action

Grosvenor, Şimşek, Yan + E.B. (to appear)

$$S = \frac{1}{2\kappa^2} \int d^4x \epsilon^{\mu\nu\rho\sigma} \left[-\frac{1}{2} \epsilon_{A'B'} E_\mu^{A'} E_\nu^{B'} \mathcal{R}_{\rho\sigma}(M) - \epsilon_{AB} \epsilon_{A'B'} \tau_\mu^A E_\nu^{A'} \mathcal{R}_{\rho\sigma}{}^{BB'}(G) \right. \\ \left. + \epsilon_{AB} \tau_\mu^A m_\nu^B \mathcal{R}_{\rho\sigma}(J) + \frac{1}{2} \epsilon_{AB} \tau_\mu^A \tau_\nu^B \mathcal{R}_{\rho\sigma}(S) \right]$$

Relation to 3D Extended Bargmann Gravity

The 3D Chern-Simons action of **Extended Bargmann gravity** is obtained after a dimensional reduction over the **longitudinal** y -direction with $\mu = (y, i), i = 0, 1, 2$ followed by a **truncation**. For instance:

$$\tau_{\mu}^A = \begin{pmatrix} \tau_y^0 & \tau_y^1 \\ \tau_i^0 & \tau_i^1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ \tau_i & 0 \end{pmatrix} \quad \Rightarrow$$

$$S_{\text{CS}} = \frac{k}{4\pi} \int d^3x \epsilon^{ijk} \left[\epsilon_{A'B'} E_i^{A'} \mathcal{R}_{jk}{}^{B'}(G) - m_i \mathcal{R}_{jk}(J) - \tau_i \mathcal{R}_{jk}(S) \right]$$

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Nonrelativistic String Theory

- **Nonrelativistic String Theory** can be defined independent of any limit of **relativistic string theory**
 - T-duality
 - Target Space Actions

- **String NC Geometry** is to NR string theory what **Riemannian geometry** is to relativistic string theory

Generalization to $p + 3$ dimensions

In $p + 3$ dimensions the divergencies originating from the Einstein-Hilbert term are cancelled by introducing a $p + 1$ -form gauge field B and a 1-form gauge field A :

$$\epsilon^{\mu_1 \dots \mu_{p+3}} \mathcal{B}_{\mu_1 \dots \mu_{p+1}} \partial_{\mu_{p+2}} \mathcal{A}_{\mu_{p+3}}$$

The non-relativistic limit leads to an action for **extended p -brane Bargmann gravity** or **extended co-dimension 2 Bargmann gravity**

They are all related by dimensional reduction of a spatial direction of the p -brane followed by a truncation.

Open Issues

- Does β -function calculation leads to consistent backgrounds?

- Nonrelativistic holography?

Gopakumar, Bagchi (2009)

- Anyonic strings?

cp. to Duval, Horvathy (2000); Jackiw, Nair (2000); Mezincescu, Townsend (2010)

- Lie algebra expansions

Izquierdo, private communication

- Double Field Theory?

S. M. Ko, C. Melby-Thompson, R. Meyer and J.-H. Park (2015)

Take Home Message

Nonrelativistic String Theory can be studied!

Gomis, Ooguri (2001)