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Invariants and CP violation in the 2HDM and 3HDM

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Motivations for 2HDM, 3HDM (and NHDM):

- Simple extensions of the SM that allow for CP violation.
- Possibility for Dark Matter
- CP conserving 2HDM is part of SUSY-models
- Rich (but not too rich) particle zoo
- Large portions of parameter space testable at LHC.

The general 2HDM potential

$$
\begin{aligned}
V(\Phi_1,\Phi_2) &= -\frac{1}{2}\left\{m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 + \left[m_{12}^2\Phi_1^\dagger\Phi_2 + \text{h.c.}\right]\right\} \\
&\quad + \frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\
&\quad + \frac{1}{2}\left[\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \text{h.c.}\right] + \left\{\left[\lambda_6(\Phi_1^\dagger\Phi_1) + \lambda_7(\Phi_2^\dagger\Phi_2)\right](\Phi_1^\dagger\Phi_2) + \text{h.c.}\right\} \\
&\equiv Y_{ab}\Phi_a^\dagger\Phi_b + \frac{1}{2}Z_{abcd}(\Phi_a^\dagger\Phi_b)(\Phi_c^\dagger\Phi_d)
\end{aligned}
$$

$$
Y_{11} = -\frac{m_{11}^2}{2}, \quad Y_{12} = -\frac{m_{12}^2}{2},
$$
$$
Y_{21} = -\frac{(m_{12}^2)^*}{2}, \quad Y_{22} = -\frac{m_{22}^2}{2},
$$

$$
\begin{aligned}
&Z_{1111} = \lambda_1, \quad Z_{2222} = \lambda_2, \quad Z_{1122} = Z_{2211} = \lambda_3, \\
&Z_{1221} = Z_{2112} = \lambda_4, \quad Z_{1212} = \lambda_5, \quad Z_{2121} = (\lambda_5)^*, \\
&Z_{1112} = Z_{1211} = \lambda_6, \quad Z_{1121} = Z_{2111} = (\lambda_6)^*, \\
&Z_{1222} = Z_{2212} = \lambda_7, \quad Z_{2122} = Z_{2221} = (\lambda_7)^*.
\end{aligned}
$$

- Standard parametrization(s) of the general 2HDM potential.
- Second form most convenient in the study of invariants.

Choice of basis is not unique

- Initial expression of potential is defined with respect to doublets Φ_1 and Φ_2.
- We may rotate to a new basis by the following transformation

$$\bar{\Phi}_i = U_{ij}\Phi_j$$

  where *U* is any U(2) matrix.
- Potential parameters change under change of basis.
- Physics is the same regardless of our choice of basis.
- **Observables cannot depend on choice of basis – they should be basis-independent, i.e. invariant under a change of basis**.
- Most general U(2) matrix:

$$U = e^{i\psi}\begin{pmatrix} \cos\theta & e^{-i\xi}\sin\theta \\ -e^{i\chi}\sin\theta & e^{i(\chi-\xi)}\cos\theta \end{pmatrix}$$

Parameters transform under change of basis

- All the parameters of the potential change under a U(2) basis transformation.
- Meaning: None of the parameters represent physical observables.
- Combinations of parameters can remain unchanged, for instance

$$
\begin{aligned}
\bar{m}_{11}^2 + \bar{m}_{22}^2 &= m_{11}^2 + m_{22}^2, \\
\bar{\lambda}_1 + \bar{\lambda}_2 + 2\bar{\lambda}_3 &= \lambda_1 + \lambda_2 + 2\lambda_3, \\
\bar{\lambda}_1 + \bar{\lambda}_2 + 2\bar{\lambda}_4 &= \lambda_1 + \lambda_2 + 2\lambda_4.
\end{aligned}
$$

- Meaning: These combinations represent physical observables.

$$
\begin{aligned}
\bar{m}_{11}^2 &= m_{11}^2 \cos^2\theta + m_{22}^2 \sin^2\theta + \mathrm{Re}\,(m_{12}^2 e^{i\xi}) \sin 2\theta \\
\bar{m}_{22}^2 &= m_{11}^2 \sin^2\theta + m_{22}^2 \cos^2\theta - \mathrm{Re}\,(m_{12}^2 e^{i\xi}) \sin 2\theta \\
\bar{m}_{12}^2 &= \left[\tfrac{1}{2}(-m_{11}^2 + m_{22}^2) \sin 2\theta + \mathrm{Re}\,(m_{12}^2 e^{i\xi}) \cos 2\theta + i\mathrm{Im}\,(m_{12}^2 e^{i\xi})\right] e^{-i\chi} \\
\bar{\lambda}_1 &= \lambda_1 \cos^4\theta + \lambda_2 \sin^4\theta + \tfrac{1}{2}\lambda_{345} \sin^2 2\theta + 2\sin 2\theta[\cos^2\theta \mathrm{Re}\,(\lambda_6 e^{i\xi}) + \sin^2\theta \mathrm{Re}\,(\lambda_7 e^{i\xi})] \\
\bar{\lambda}_2 &= \lambda_1 \sin^4\theta + \lambda_2 \cos^4\theta + \tfrac{1}{2}\lambda_{345} \sin^2 2\theta - 2\sin 2\theta[\sin^2\theta \mathrm{Re}\,(\lambda_6 e^{i\xi}) + \cos^2\theta \mathrm{Re}\,(\lambda_7 e^{i\xi})] \\
\bar{\lambda}_3 &= \frac{1}{4}\sin^2 2\theta(\lambda_1 + \lambda_2 - 2\lambda_{345}) + \lambda_3 - \sin 2\theta \cos 2\theta \mathrm{Re}\,[(\lambda_6 - \lambda_7)e^{i\xi}] \\
\bar{\lambda}_4 &= \frac{1}{4}\sin^2 2\theta(\lambda_1 + \lambda_2 - 2\lambda_{345}) + \lambda_4 - \sin 2\theta \cos 2\theta \mathrm{Re}\,[(\lambda_6 - \lambda_7)e^{i\xi}] \\
\bar{\lambda}_5 &= \Bigg(\frac{1}{4}\sin^2 2\theta(\lambda_1 + \lambda_2 - 2\lambda_{345}) + \mathrm{Re}\,(\lambda_5 e^{2i\xi}) + i\cos 2\theta \mathrm{Im}\,(\lambda_5 e^{2i\xi}) \\
&\qquad - \sin 2\theta \cos 2\theta \mathrm{Re}\,[(\lambda_6 - \lambda_7)e^{i\xi}] - i \sin 2\theta \mathrm{Im}\,[(\lambda_6 - \lambda_7)e^{i\xi}]\Bigg) e^{-2i\chi} \\
\bar{\lambda}_6 &= \left(-\tfrac{1}{2}\sin 2\theta[\lambda_1 \cos^2\theta - \lambda_2 \sin^2\theta - \lambda_{345}\cos 2\theta - i\mathrm{Im}\,(\lambda_5 e^{2i\xi})]\right. \\
&\qquad + \cos\theta \cos 3\theta \mathrm{Re}\,(\lambda_6 e^{i\xi}) + \sin\theta \sin 3\theta \mathrm{Re}\,(\lambda_7 e^{i\xi}) \\
&\qquad \left. + i\cos^2\theta \mathrm{Im}\,(\lambda_6 e^{i\xi}) + i\sin^2\theta \mathrm{Im}\,(\lambda_7 e^{i\xi})\right) e^{-i\chi} \\
\bar{\lambda}_7 &= \left(-\tfrac{1}{2}\sin 2\theta[\lambda_1 \sin^2\theta - \lambda_2 \cos^2\theta + \lambda_{345}\cos 2\theta + i\mathrm{Im}\,(\lambda_5 e^{2i\xi})]\right. \\
&\qquad + \sin\theta \sin 3\theta \mathrm{Re}\,(\lambda_6 e^{i\xi}) + \cos\theta \cos 3\theta \mathrm{Re}\,(\lambda_7 e^{i\xi}) \\
&\qquad \left. + i\sin^2\theta \mathrm{Im}\,(\lambda_6 e^{i\xi}) + i\cos^2\theta \mathrm{Im}\,(\lambda_7 e^{i\xi})\right) e^{-i\chi}.
\end{aligned}
$$

$$\lambda_{345} = \lambda_3 + \lambda_4 + \mathrm{Re}\,\left(\lambda_5 e^{2i\xi}\right)$$

Vacuum expectation values (VEVs)

Most general form that conserves electric charge:

$$\langle\Phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 e^{i\xi_1}\end{pmatrix}, \quad \langle\Phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 e^{i\xi_2}\end{pmatrix}$$

$$v_1^2 + v_2^2 = v^2 = (246\,\mathrm{GeV})^2$$

- We demand that the VEVs should represent a minimum of the potential
- Electroweak Symmetry Breaking: Work out stationary-point equations by differentiating the potential with respect to the fields and put these to zero. [JHEP11(2014)084].
- Minimum enforced by demanding all physical scalars have positive squared masses (later).

- VEVs also change under basis transformations:

$$\begin{aligned}\bar{v}_1 &= \sqrt{v_1^2\cos^2\theta + v_2^2\sin^2\theta + v_1 v_2 \sin 2\theta\cos(\xi_{21}-\xi)},\\ \bar{v}_2 &= \sqrt{v_1^2\sin^2\theta + v_2^2\cos^2\theta - v_1 v_2 \sin 2\theta\cos(\xi_{21}-\xi)}.\end{aligned} \qquad \xi_{21} \equiv \xi_2 - \xi_1$$

$$\begin{aligned}\cos\bar{\xi}_{21} &= \frac{\bar{v}_1(2v_1v_2(\cos 2\theta\cos(\xi_{21}-\xi)\cos\chi - \sin(\xi_{21}-\xi)\sin\chi) + (v_2^2-v_1^2)\sin 2\theta\cos\chi)}{\bar{v}_2(v_1^2+v_2^2-(v_2^2-v_1^2)\cos 2\theta + 2v_1v_2\cos(\xi_{21}-\xi)\sin 2\theta)},\\ \sin\bar{\xi}_{21} &= \frac{\bar{v}_1(2v_1v_2(\cos 2\theta\cos(\xi_{21}-\xi)\sin\chi + \sin(\xi_{21}-\xi)\cos\chi) + (v_2^2-v_1^2)\sin 2\theta\sin\chi)}{\bar{v}_2(v_1^2+v_2^2-(v_2^2-v_1^2)\cos 2\theta + 2v_1v_2\cos(\xi_{21}-\xi)\sin 2\theta)}.\end{aligned}$$

- It is easy to show that

$$\bar{v}_1^2 + \bar{v}_2^2 = v_1^2 + v_2^2$$

- Meaning: $v_1^2 + v_2^2 = v^2$ is a basis-invariant quantity, hence a physical observable.

Parametrization of the doublets and the charged fields

- Each doublet is parametrized as:

$$\Phi_j = e^{i\xi_j} \begin{pmatrix} \varphi_j^+ \\ (v_j + \eta_j + i\chi_j)/\sqrt{2} \end{pmatrix}, \quad j = 1, 2.$$

- Massless charged goldstone fields $G^\pm$ are extracted by introducing orthogonal states:

$$\begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = \begin{pmatrix} v_1/v & v_2/v \\ -v_2/v & v_1/v \end{pmatrix} \begin{pmatrix} \varphi_1^\pm \\ \varphi_2^\pm \end{pmatrix}$$

- $H^\pm$ represent the massive charged scalars

- We work out the mass of the charged scalars:

$$M_{H^\pm}^2 = \frac{v^2}{2v_1 v_2 \cos\xi_{21}} \mathrm{Re}\left(m_{12}^2 - v_1^2\lambda_6 - v_2^2\lambda_7 - v_1 v_2\left[\lambda_4 \cos\xi_{21} + \lambda_5 e^{i\xi_{21}}\right]\right)$$

- Performing a change of basis we find that

$$\bar{M}_{H^\pm}^2 = M_{H^\pm}^2$$

- telling us that $M_{H^\pm}^2$ is a basis invariant and therefore a physical observable (as it must be).

Parametrization of the doublets and the neutral fields

- Massless neutral goldstone field G^0 is also extracted by introducing orthogonal states:

$$\begin{pmatrix} G_0 \\ \eta_3 \end{pmatrix} = \begin{pmatrix} v_1/v & v_2/v \\ -v_2/v & v_1/v \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}$$

- We are left with three massive fields: η_1, η_2 and η_3, but these are not mass eigenstates.
- Mass terms given as

$$\frac{1}{2} \begin{pmatrix} \eta_1 & \eta_2 & \eta_3 \end{pmatrix} \mathcal{M}^2 \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix} .$$

- Matrix elements are given in [JHEP11(2014)084].

- We rotate into the physical fields by diagonalizing $\mathcal{M}^2$ using an orthogonal matrix R:

$$R = \begin{pmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{pmatrix}$$

$$R\mathcal{M}^2 R^{\mathrm{T}} = \mathrm{diag}(M_1^2, M_2^2, M_3^2)$$

- Physical fields are now given as

$$\begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix} = R \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix}$$

Transformations of mass matrix elements and rotation matrix elements under change of basis

$$
\begin{aligned}
\bar{R} &= RP, \\
\bar{\mathcal{M}}^2 &= P^T \mathcal{M}^2 P
\end{aligned}
$$

None of the squared mass matrix elements or rotation matrix elements are invariants, and therefore they are not observables:

$$
\begin{aligned}
P_{11} &= \frac{\cos\theta(v_1\cos\theta + v_2\sin\theta\cos(\xi_{21}-\xi))}{\bar{v}_1}, \\
P_{12} &= -\frac{\sin\theta(v_2\cos\theta\cos(\xi_{21}-\xi) - v_1\sin\theta)}{\bar{v}_2}, \\
P_{13} &= \frac{vv_2\sin 2\theta\sin(\xi_{21}-\xi)}{2\bar{v}_1\bar{v}_2}, \\
P_{21} &= \frac{\sin\theta(v_1\cos\theta\cos(\xi_{21}-\xi) + v_2\sin\theta)}{\bar{v}_1}, \\
P_{22} &= \frac{\cos\theta(v_2\cos\theta - v_1\sin\theta\cos(\xi_{21}-\xi))}{\bar{v}_2}, \\
P_{23} &= -\frac{vv_1\sin 2\theta\sin(\xi_{21}-\xi)}{2\bar{v}_1\bar{v}_2}, \\
P_{31} &= -\frac{v\sin 2\theta\sin(\xi_{21}-\xi)}{2\bar{v}_1}, \\
P_{32} &= \frac{v\sin 2\theta\sin(\xi_{21}-\xi)}{2\bar{v}_2}, \\
P_{33} &= \frac{2v_1v_2\cos 2\theta + \left(v_2^2 - v_1^2\right)\sin 2\theta\cos(\xi_{21}-\xi)}{2\bar{v}_1\bar{v}_2}.
\end{aligned}
$$

Invariance of the neutral masses

- Combinations of squared mass matrix elements that are invariant are the trace, the sum of principal cofactors and the determinant, i.e.

$$
\begin{aligned}
b &= -(\mathcal{M}_{11}^2 + \mathcal{M}_{22}^2 + \mathcal{M}_{33}^2), \\
c &= \mathcal{M}_{11}^2\mathcal{M}_{22}^2 + \mathcal{M}_{11}^2\mathcal{M}_{33}^2 + \mathcal{M}_{22}^2\mathcal{M}_{33}^2 \\
&\quad -(\mathcal{M}_{12}^2)^2 - (\mathcal{M}_{13}^2)^2 - (\mathcal{M}_{23}^2)^2, \\
d &= \mathcal{M}_{11}^2(\mathcal{M}_{23}^2)^2 + \mathcal{M}_{22}^2(\mathcal{M}_{13}^2)^2 + \mathcal{M}_{33}^2(\mathcal{M}_{12}^2)^2 \\
&\quad -\mathcal{M}_{11}^2\mathcal{M}_{22}^2\mathcal{M}_{33}^2 - 2\mathcal{M}_{12}^2\mathcal{M}_{13}^2\mathcal{M}_{23}^2.
\end{aligned}
$$

- Are all found to be basis invariant, hence observable

- The eigenvalues of the squared mass matrix gives us the three neutral masses.
- Characteristic equation for eigenvalues:

$$\lambda^3 + b\lambda^2 + c\lambda + d = 0$$

- Eigenvalues (masses) are found to be

$$
\begin{aligned}
M_1^2 &= \frac{-b}{3} + 2\sqrt{\frac{-p}{3}}\cos\left[\frac{1}{3}\arccos\left(\frac{3q}{2p}\sqrt{\frac{-3}{p}}\right) + \frac{2\pi}{3}\right], \\
M_2^2 &= \frac{-b}{3} + 2\sqrt{\frac{-p}{3}}\cos\left[\frac{1}{3}\arccos\left(\frac{3q}{2p}\sqrt{\frac{-3}{p}}\right) - \frac{2\pi}{3}\right], \\
M_3^2 &= \frac{-b}{3} + 2\sqrt{\frac{-p}{3}}\cos\left[\frac{1}{3}\arccos\left(\frac{3q}{2p}\sqrt{\frac{-3}{p}}\right)\right].
\end{aligned}
$$

All neutral masses are basis invariant, hence observable

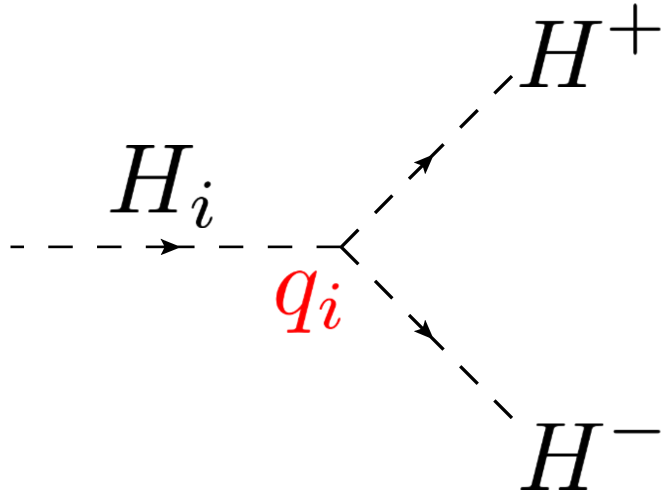
$$
\begin{aligned}
p &= c - b^2/3 \\
q &= \frac{2b^3 - 9bc + 27d}{27}
\end{aligned}
$$

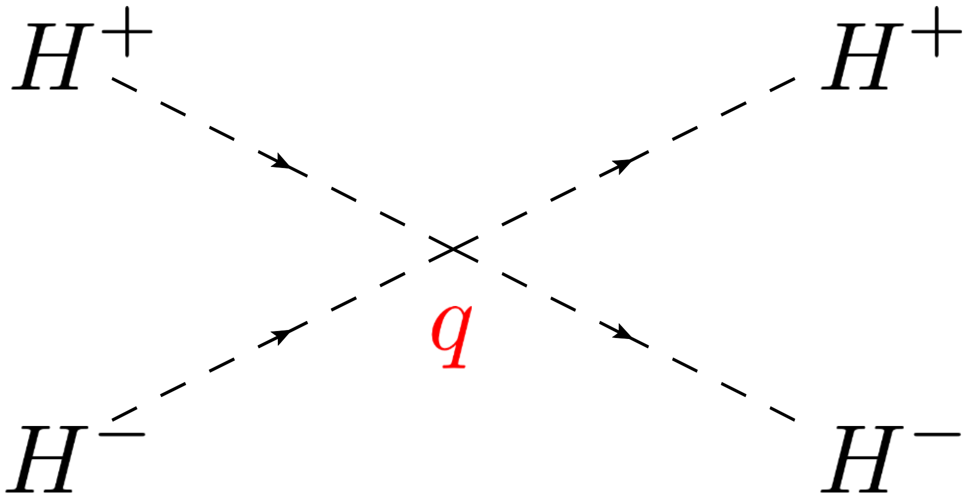
Invariance of scalar couplings

› Some important scalar couplings

$$
\begin{aligned}
q_i &\equiv \text{Coefficient}(V, H_i H^- H^+) \\
&= \frac{v_1 v_2^2}{v^2} R_{i1}\lambda_1 + \frac{v_1^2 v_2}{v^2} R_{i2}\lambda_2 + \frac{v_1^3 R_{i1} + v_2^3 R_{i2}}{v^2}\lambda_3 \\
&\quad - \frac{v_1 v_2 (v_2 R_{i1} + v_1 R_{i2})}{v^2}(\lambda_4 + \text{Re}\,\lambda_5) + \frac{v_1 v_2}{v} R_{i3}\text{Im}\,\lambda_5 \\
&\quad + \frac{v_2(v_2^2 - 2v_1^2)R_{i1} + v_1 v_2 R_{i2}}{v^2}\text{Re}\,\lambda_6 - \frac{v_2^2}{v} R_{i3}\text{Im}\,\lambda_6 \\
&\quad + \frac{v_1(v_1^2 - 2v_2^2)R_{i2} + v_1 v_2 R_{i1}}{v^2}\text{Re}\,\lambda_7 - \frac{v_1^2}{v} R_{i3}\text{Im}\,\lambda_7, \\
q &\equiv \text{Coefficient}(V, H^- H^- H^+ H^+) \\
&= \frac{v_2^4}{2v^4}\lambda_1 + \frac{v_1^4}{2v^4}\lambda_2 + \frac{v_1^2 v_2^2}{v^4}(\lambda_3 + \lambda_4 + \text{Re}\,\lambda_5) - \frac{2v_1 v_2^3}{v^4}\text{Re}\,\lambda_6 - \frac{2v_1^3 v_2}{v^4}\text{Re}\,\lambda_7.
\end{aligned}
$$

› Couplings also turn out to be basis invariant, hence observables.





Invariance of gauge couplings

› Gauge couplings

$$H_iH_jZ_\mu: \quad \frac{g}{2v\cos\theta_W}\epsilon_{ijk}\boxed{e_k}(p_i-p_j)_\mu,$$

$$H_iZ_\mu Z_\nu: \quad \frac{ig^2}{2\cos^2\theta_W}\boxed{e_i}g_{\mu\nu},$$

$$H_iW_\mu^+W_\nu^-: \quad \frac{ig^2}{2}\boxed{e_i}g_{\mu\nu}.$$

$$e_i \equiv v_1R_{i1}+v_2R_{i2}$$

$$e_1^2+e_2^2+e_3^2=v^2=(246\,\mathrm{GeV})^2$$

$$\bar{v}_1\bar{R}_{i1}+\bar{v}_2\bar{R}_{i2}=v_1R_{i1}+v_2R_{i2}$$

› Showing that these gauge couplings are invariant under a change of basis, hence they are observables.

› Most couplings are invariants. Some (the complex ones) are pseudo-invariants (their absolute value is invariant).

› **No surprise**: Masses and couplings are invariants and possible to measure in experiments.

Systematic construction of invariants by use of tensors.

- Y_{ab}, Z_{abcd} tensors already known.
- Introduce V_{ab} tensor as:

$$
\begin{aligned}
V_{ab} &= \frac{v_a v_b^*}{v^2} \\
&= \frac{1}{v^2}\begin{pmatrix} v_1^2 & v_1 v_2 e^{-i\xi_{21}} \\ v_1 v_2 e^{i\xi_{21}} & v_2^2 \end{pmatrix}
\end{aligned}
$$

- Transformation rules of V_{ab}, Y_{ab}, and Z_{abcd} tensors under change of basis:

$$
\begin{aligned}
\bar{V} &= UVU^\dagger, \\
\bar{Y} &= UYU^\dagger, \\
\bar{Z}_{abcd} &= U_{ae}U_{cg}Z_{efgh}U^\dagger_{fb}U^\dagger_{hd}
\end{aligned}
$$

- We may now put together an arbitrary number of *Y*-, *Z*- and *V*-tensors and contract the odd-numbered indices with the even-numbered indices to get an invariant quantity.
- Simple examples

$$
\begin{aligned}
V_{aa} &= 1, \\
Y_{aa} &= -\frac{1}{2}(m_{11}^2 + m_{22}^2), \\
Z_{aabb} &= \lambda_1 + \lambda_2 + 2\lambda_3, \\
Z_{abba} &= \lambda_1 + \lambda_2 + 2\lambda_4.
\end{aligned}
$$

- **We already know these to be invariant!**

Systematic construction of CP-violating invariants by use of tensors

- The real part of invariants constructed this way will be a CP-even invariant.
- The imaginary part of invariants constructed this way will be a CP-odd invariant.
- To find conditions for CP-violation, we systematically construct invariants and check if they have imaginary parts.
- Many invariants exist, but only three are needed to check for CP violation:

$$\mathrm{Im}\, J_1 = -\frac{2}{v^2}\mathrm{Im}\left[V_{da}Y_{ab}Z_{bccd}\right],$$
$$\mathrm{Im}\, J_2 = \frac{4}{v^4}\mathrm{Im}\left[V_{ab}V_{dc}Y_{be}Y_{cf}Z_{eafd}\right],$$
$$\mathrm{Im}\, J_3 = \mathrm{Im}\left[V_{ab}V_{dc}Z_{bgge}Z_{chhf}Z_{eafd}\right].$$

- These invariants are observables and so they must be expressible in terms of observable couplings and masses
- How do we translate from potential parameters/VEVs to masses and couplings?
- Choose to work in a particular basis (the Higgs-basis) and establish identities between invariant quantities in this basis.
- The identities established must then be valid in any basis.

From parameters to masses and couplings in the Higgs-basis

- Only one VEV is non-zero.

$$v_1 = v, \quad v_2 = 0, \quad \xi_1 = 0$$

$$\langle \Phi_1 \rangle_{\rm HB} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \langle \Phi_2 \rangle_{\rm HB} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

- Not unique, as one may still perform a U(1) transform on Φ_2 without giving Φ_2 a non-zero VEV.
- Stationary-point equations

$$\begin{aligned} Y_{11} &= -\frac{v^2}{2} Z_{1111}, \\ \mathrm{Re}\, Y_{12} &= -\frac{v^2}{2} \mathrm{Re}\, Z_{1112}, \\ \mathrm{Im}\, Y_{12} &= -\frac{v^2}{2} \mathrm{Im}\, Z_{1112}, \end{aligned}$$

$$\begin{aligned} m_{11}^2 &= v^2 \lambda_1, \\ \mathrm{Re}\, m_{12}^2 &= v^2 \mathrm{Re}\, \lambda_6, \\ \mathrm{Im}\, m_{12}^2 &= v^2 \mathrm{Im}\, \lambda_6, \end{aligned}$$

- Charged scalar mass:

$$M_{H^\pm}^2 = Y_{22} + \frac{v^2}{2} Z_{1122}$$

- Neutral mass matrix:

$$\mathcal{M}^2 = R^T \mathrm{diag}(M_1^2, M_2^2, M_3^2) R$$
$$= v^2 \begin{pmatrix} Z_{1111} & \mathrm{Re}\, Z_{1112} & -\mathrm{Im}\, Z_{1112} \\ \mathrm{Re}\, Z_{1112} & Z_{1122} + Z_{1221} + \mathrm{Re}\, Z_{1212} + \frac{Y_{22}}{v^2} & -\frac{1}{2}\mathrm{Im}\, Z_{1212} \\ -\mathrm{Im}\, Z_{1112} & -\frac{1}{2}\mathrm{Im}\, Z_{1212} & Z_{1122} + Z_{1221} - \mathrm{Re}\, Z_{1212} + \frac{Y_{22}}{v^2} \end{pmatrix}$$

- Treat the above as seven equations and solve to get

$$\begin{aligned} Y_{22} &= M_{H^\pm}^2 - \frac{v^2}{2} Z_{1122}, \\ Z_{1111} &= \frac{R_{11}^2 M_1^2 + R_{21}^2 M_2^2 + R_{31}^2 M_3^2}{v^2}, \\ Z_{1221} &= \frac{-2M_{H^\pm}^2 + (R_{12}^2 + R_{13}^2) M_1^2 + (R_{22}^2 + R_{23}^2) M_2^2 + (R_{32}^2 + R_{33}^2) M_3^2}{v^2}, \\ \mathrm{Re}\, Z_{1112} &= \frac{R_{11}R_{12} M_1^2 + R_{21}R_{22} M_2^2 + R_{31}R_{32} M_3^2}{v^2}, \\ \mathrm{Im}\, Z_{1112} &= -\frac{R_{11}R_{13} M_1^2 + R_{21}R_{23} M_2^2 + R_{31}R_{33} M_3^2}{v^2}, \\ \mathrm{Re}\, Z_{1212} &= \frac{(R_{12}^2 - R_{13}^2) M_1^2 + (R_{22}^2 - R_{23}^2) M_2^2 + (R_{32}^2 - R_{33}^2) M_3^2}{v^2}, \\ \mathrm{Im}\, Z_{1212} &= -2\frac{R_{12}R_{13} M_1^2 + R_{22}R_{23} M_2^2 + R_{32}R_{33} M_3^2}{v^2}. \end{aligned}$$

From parameters to masses and couplings

- Scalar couplings in the Higgs-basis.

$$\begin{aligned} q_i &= v(R_{i1}Z_{1122} + R_{i2}\mathrm{Re}\,Z_{1222} - R_{i3}\mathrm{Im}\,Z_{1222}), \\ q &= \frac{1}{2}Z_{2222}. \end{aligned}$$

- Treat as four equations and solve to get

$$\begin{aligned} Z_{1122} &= \frac{R_{11}q_1 + R_{21}q_2 + R_{31}q_3}{v}, \\ \mathrm{Re}\,Z_{1222} &= \frac{R_{12}q_1 + R_{22}q_2 + R_{32}q_3}{v}, \\ \mathrm{Im}\,Z_{1222} &= -\frac{R_{13}q_1 + R_{23}q_2 + R_{33}q_3}{v}, \\ Z_{2222} &= 2q. \end{aligned}$$

- All parameters of the potential has now been replaced by scalar couplings and masses (and elements from the rotation matrix).

- Gauge couplings in the Higgs-basis

$$e_i = vR_{i1}$$

- Combinations of rotation matrix elements appearing in the invariants can all be expressed in terms of the three e_i by utilizing the orthogonality of R.

- One immediately finds

$$\begin{aligned} \mathrm{Im}\,J_1 &= \frac{1}{v^5}[e_1e_3q_2(M_1^2 - M_3^2) + e_2e_1q_3(M_2^2 - M_1^2) \\ &\quad + e_3e_2q_1(M_3^2 - M_2^2)], \\ \mathrm{Im}\,J_2 &= 2\frac{e_1e_2e_3}{v^9}(M_1^2 - M_2^2)(M_2^2 - M_3^2)(M_3^2 - M_1^2). \end{aligned}$$

From parameters to masses and couplings

- Im J_3 is a little more complicated:

$$\mathrm{Im}\,J_3 = c_1 \mathrm{Im}\,J_1 + c_2 \mathrm{Im}\,J_2 + c_{11} \mathrm{Im}\,J_{11} + c_{30} \mathrm{Im}\,J_{30}$$

Vanishes when
Im J_1 = Im J_2 = 0

- Only independent part is Im J_{30} :

$$\mathrm{Im}\,J_{30} = \frac{1}{v^5}[e_2 q_1 q_3 (M_1^2 - M_3^2) + e_3 q_2 q_1 (M_2^2 - M_1^2) + e_1 e_3 q_2 (M_3^2 - M_2^2)],$$

- Put Im J_1 = Im J_2 = Im J_{30} = 0 and solve

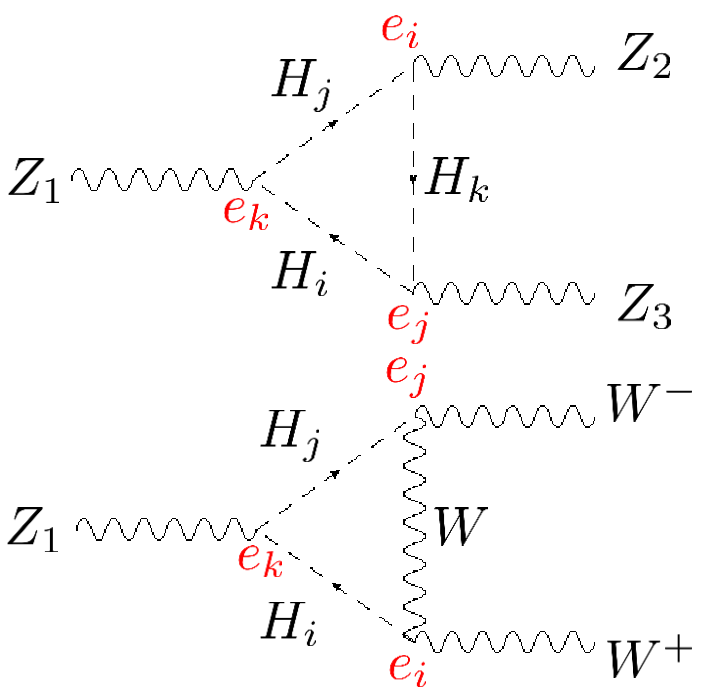
6 distinct cases of CP conservation:

- Case 1: M_1=M_2=M_3. Full mass degeneracy.
- Case 2: M_1=M_2 and $e_1q_2 = e_2q_1$
- Case 3: M_2=M_3 and $e_2q_3 = e_3q_2$
- Case 4: e_1=0 and q_1=0
- Case 5: e_2=0 and q_2=0
- Case 6: e_3=0 and q_3=0

If none of the above occur, then CP is broken!

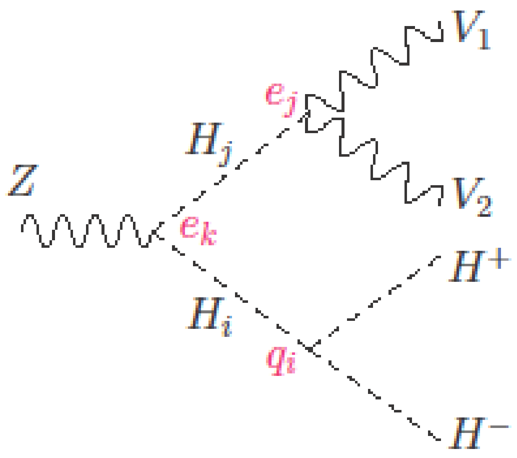
CP violating observables

- ZZZ and ZWW vertex both contain CP-violating form factors.
- Summing over all possible combinations of *i,j,k*, we find $\mathcal{M} \propto \mathrm{Im} J_2$

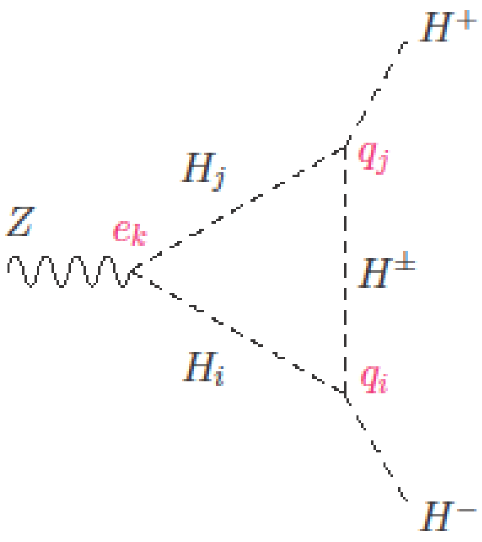


CP violating observables

- $Z \to VVH^+H^-$
- Summing over all possible combinations of *i,j,k*, we find $\mathcal{M}$ contains $\mathrm{Im}J_1$



- $Z \to H^+H^-$
- Summing over all possible combinations of *i,j,k*, we find $\mathcal{M}$ contains $\mathrm{Im}J_3$



From 2HDM to 3HDM

- CP violation in the 2HDM has been extensively studied and is well understood both in terms of invariants and masses/couplings.
- Not the case for 3HDM.
- Irremovable phases in 3HDM?

Want list:

- A set of invariants that guarantees CP conservation if all vanish, and CP violation when one is non-zero.
- Translation of this set into masses/couplings and a physical interpretation of the results.

Work in progress:

- Doable by working in the Higgs-basis as has been shown for 2HDM.
- Identify masses and couplings.
- Perform translation from potential parameters into masses/couplings.
- Systematic construct of invariants with imaginary part.
- Interpretation.

Very preliminary results for 3HDM

- Three invariants with imaginary parts that has been translated into masses/couplings.

$$
\begin{aligned}
\mathrm{Im}\, V_{ab} Y_{bc} Z_{cadd} &= \frac{1}{2v^3} \sum_{i,j} e_i M_i^2 \lambda_{ji} (q_{j11} + q_{j22}), \\
\mathrm{Im}\, V_{ab} Y_{bc} Z_{cdda} &= \frac{1}{2v^4} \sum_{i,j,k,l} e_i M_i^2 \mathrm{Im}\, \{ f_{ki}^* f_{lj} q_{jkl} \}, \\
\mathrm{Im}\, V_{ac} V_{bd} Y_{ce} Y_{df} Z_{eafb} &= -\frac{1}{2v^5} \sum_{i,j} e_i e_j M_i^2 M_j^4 \lambda_{ji}.
\end{aligned}
$$

- More to appear in an arXiv near you…
- Stay tuned!

$$
\begin{aligned}
e_i &: \quad H_i W^+ W^-, \\
\lambda_{ij} &: \quad H_i H_j Z, \\
f_{ij} &: \quad H_i H_j^+ W^-, \\
q_{ijk} &: \quad H_i H_j^+ H_k^-.
\end{aligned}
$$