Implications of flavour anomalies for new physics

Nazila Mahmoudi
Lyon University & CERN


Thanks to T. Hurth, S. Neshatpour, D. Martinez Santos and V. Chobanova

Corfu Summer Institute: Workshop on the Standard Model and Beyond
Corfu, Greece, September 2-10, 2017
Indirect search for New Physics

I will focus on **indirect hints for new physics** from Flavour sector

Flavour physics is sensitive to new physics at $\Lambda_{NP} \gg E_{\text{experiments}}$

→ can discover new physics or probe it before it is directly observed in experiments
Indirect search for New Physics

I will focus on **indirect hints for new physics** from Flavour sector

Flavour physics is sensitive to new physics at \( \Lambda_{\text{NP}} \gg E_{\text{experiments}} \)

\( \rightarrow \) can discover new physics or probe it before it is directly observed in experiments

Rare decays in particular are very important as:

- They occur at loop level
  \( \rightarrow \) The SM contributions are very small and the NP contributions can have a comparable magnitude.

- The theory ingredients are known at a very good accuracy!
  \( \rightarrow \) In particular: QCD corrections are known with a good precision!

- The experimental situation is very promising
  \( \rightarrow \) Branching ratios can be measured precisely
Indirect search for New Physics

I will focus on **indirect hints for new physics** from Flavour sector

Flavour physics is sensitive to new physics at $\Lambda_{NP} \gg E_{\text{experiments}}$

$\rightarrow$ can discover new physics or probe it before it is directly observed in experiments

Rare decays in particular are very important as:

- They occur at loop level
  $\rightarrow$ The SM contributions are very small and the NP contributions can have a comparable magnitude.

- The theory ingredients are known at a very good accuracy!
  $\rightarrow$ In particular: QCD corrections are known with a good precision!

- The experimental situation is very promising
  $\rightarrow$ Branching ratios can be measured precisely

Many flavour observables under investigation!
I will focus on **indirect hints for new physics** from Flavour sector

Flavour physics is sensitive to new physics at $\Lambda_{NP} \gg E_{\text{experiments}}$

→ can discover new physics or probe it before it is directly observed in experiments

Rare decays in particular are very important as:

- They occur at loop level
  → The SM contributions are very small and the NP contributions can have a comparable magnitude.

- The theory ingredients are known at a very good accuracy!
  → In particular: QCD corrections are known with a good precision!

- The experimental situation is very promising
  → Branching ratios can be measured precisely

Many flavour observables under investigation!

There are currently some tensions (anomalies)
Confirmations are needed, but they are still among our best bets!
(LHCb) Observables and Anomalies
Impressive effort in studying exclusive $b \to s \ell \ell$ transitions at LHCb with the measurements of a large number of independent angular observables!

Deviations from the SM predictions in $B \to K^* \mu^+ \mu^-$, $B_s \to \phi \mu^+ \mu^-$ and $R_{K(*)}$: "anomalies"
Theoretical framework

**Effective field theory**

\[ \mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left( \sum_{i=1 \cdots 10, S, P} (C_i(\mu)\mathcal{O}_i(\mu) + C'_i(\mu)\mathcal{O}'_i(\mu)) \right) \]

Separation between short distance (Wilson coefficients) and long distance (local operators) effects

**Operator set for \( b \to s \) transitions:**

- **4-quark operators**
  - \( \mathcal{O}_{1 \cdots 6} \)
  - \( \mathcal{O}_{1,2} \propto (\bar{s}\Gamma_\mu c)(\bar{c}\Gamma^\mu b) \)
  - \( \mathcal{O}_{3,4} \propto (\bar{s}\Gamma_\mu b)\sum_q (\bar{q}\Gamma^\mu q) \)

- **Chromomagnetic dipole operator**
  - \( \mathcal{O}_8 \)
  - \( \mathcal{O}_8 \propto (\bar{s}\sigma^{\mu\nu} T^a P_R)G^{a}_{\mu\nu} \)

- **Electromagnetic dipole operator**
  - \( \mathcal{O}_7 \)
  - \( \mathcal{O}_7 \propto (\bar{s}\sigma^{\mu\nu} P_R)F^{a}_{\mu\nu} \)

- **Semileptonic operators**
  - \( \mathcal{O}_{9,10} \)
  - \( \mathcal{O}_9 \propto (\bar{s}\gamma^\mu b_L)(\bar{\ell}\gamma_\mu \ell) \)
  - \( \mathcal{O}_{10} \propto (\bar{s}\gamma^\mu b_L)(\bar{\ell}\gamma_\mu \gamma_5 \ell) \)

+ the chirality flipped counter-parts of the above operators, \( \mathcal{O}'_i \)
Wilson coefficients

The Wilson coefficients are calculated perturbatively

**Two main steps:**

- matching between the effective and full theories $\rightarrow$ extraction of the $C_i^{\text{eff}}(\mu)$ at scale $\mu \sim M_W$

$$C_i^{\text{eff}}(\mu) = C_i^{(0)\text{eff}}(\mu) + \frac{\alpha_s(\mu)}{4\pi} C_i^{(1)\text{eff}}(\mu) + \cdots$$

- Evolving the $C_i^{\text{eff}}(\mu)$ to the scale relevant for $B$ decays, $\mu \sim m_b$ using the RGE runnings.

The Wilson coefficients are process independent.

SM contributions to the Wilson coefficients known to NNLL:

(Bobeth, Misiak, Urban '99; Misiak, Steinhauser '04, Gorbahn, Haisch '04; Gorbahn, Haisch, Misiak '05; Czakon, Haisch, Misiak '06,...)

$$C_7 = -0.294 \quad C_9 = 4.20 \quad C_{10} = -4.01$$
The Wilson coefficients are calculated perturbatively

**Two main steps:**

- matching between the effective and full theories \( \rightarrow \) extraction of the \( C_i^{\text{eff}}(\mu) \) at scale \( \mu \sim M_W \)

\[
C_i^{\text{eff}}(\mu) = C_i^{(0)\text{eff}}(\mu) + \frac{\alpha_s(\mu)}{4\pi} C_i^{(1)\text{eff}}(\mu) + \cdots
\]

- Evolving the \( C_i^{\text{eff}}(\mu) \) to the scale relevant for \( B \) decays, \( \mu \sim m_b \) using the RGE runnings.

The Wilson coefficients are process independent.

**SM contributions to the Wilson coefficients known to NNLL:**
(Bobeth, Misiak, Urban '99; Misiak, Steinhauser '04, Gorbahn, Haisch '04; Gorbahn, Haisch, Misiak '05; Czakon, Haisch, Misiak '06,...)

\[
C_7 = -0.294 \quad C_9 = 4.20 \quad C_{10} = -4.01
\]
Hadronic quantities

To compute the amplitudes:
\[ \mathcal{A}(A \to B) = \langle B | H_{\text{eff}} | A \rangle = \frac{G_F}{\sqrt{2}} \sum \lambda_i C_i(\mu) \langle B | O_i | A \rangle(\mu) \]

\[ \langle B | O_i | A \rangle : \text{hadronic matrix element} \]

How to compute matrix elements?
- Model building, Lattice simulations, light/heavy flavour symmetries, ...
- Describe hadronic matrix elements in terms of hadronic quantities
  - Decay constants
  - Form factors

Main source of uncertainty!

- design observables where the hadronic uncertainties cancel (e.g. ratios,...)

Prime example: \( B \to K^* \mu^+ \mu^- \)
gives access to a variety of observables!

Nazila Mahmoudi
Corfu, September 5th 2017

7 / 28
To compute the amplitudes:

$$A(A \rightarrow B) = \langle B|\mathcal{H}_{\text{eff}}|A\rangle = \frac{G_F}{\sqrt{2}} \sum_i \lambda_i C_i(\mu) \langle B|O_i|A\rangle(\mu)$$

$$\langle B|O_i|A\rangle$$: hadronic matrix element

How to compute matrix elements?

→ Model building, Lattice simulations, light/heavy flavour symmetries, ...
→ Describe hadronic matrix elements in terms of hadronic quantities

Decay constants \hspace{2cm} Form factors

Main source of uncertainty!

→ design observables where the hadronic uncertainties cancel (e.g. ratios,...)

Prime example: $$B \rightarrow K^* \mu^+ \mu^-$$
gives access to a variety of observables!
Hadronic quantities

To compute the amplitudes:
\[ \mathcal{A}(A \rightarrow B) = \langle B | \mathcal{H}_\text{eff} | A \rangle = \frac{G_F}{\sqrt{2}} \sum_i \lambda_i C_i(\mu) \langle B | O_i | A \rangle(\mu) \]

\[ \langle B | O_i | A \rangle: \text{hadronic matrix element} \]

How to compute matrix elements?

→ Model building, Lattice simulations, light/heavy flavour symmetries, ...

→ Describe hadronic matrix elements in terms of **hadronic quantities**

\[ \text{Decay constants} \rightarrow \text{Form factors} \]

Main source of uncertainty!

→ **design observables where the hadronic uncertainties cancel** (e.g. ratios,...)

Prime example: \( B \rightarrow K^* \mu^+ \mu^- \)
gives access to a variety of observables!
The full angular distribution of the decay $\bar{B}^0 \rightarrow \bar{K}^* 0 \ell^+ \ell^-$ ($\bar{K}^* 0 \rightarrow K^- \pi^+$) is completely described by four independent kinematic variables: $q^2$ (dilepton invariant mass squared), $\theta_\ell$, $\theta_{K^*}$, $\phi$.

**Differential decay distribution:**

$$\frac{d^4 \Gamma}{dq^2 \ d\cos \theta_\ell \ d\cos \theta_{K^*} \ d\phi} = \frac{9}{32 \pi} J(q^2, \theta_\ell, \theta_{K^*}, \phi)$$

$$J(q^2, \theta_\ell, \theta_{K^*}, \phi) = \sum_i J_i(q^2) f_i(\theta_\ell, \theta_{K^*}, \phi)$$

\- angular coefficients $J_{1-9}$
\- functions of the transversity amplitudes $A_0$, $A_\parallel$, $A_\perp$, $A_t$, and $A_S$
\- or alternatively, helicity amplitudes $H_V$, $H_A$ and $H_S$

Transversity/helicity amplitudes: functions of Wilson coefficients and form factors
The full angular distribution of the decay $\bar{B}^0 \rightarrow \bar{K}^*\ell^+\ell^-$ ($\bar{K}^* \rightarrow K^-\pi^+$) is completely described by four independent kinematic variables: $q^2$ (dilepton invariant mass squared), $\theta_\ell$, $\theta_K^*$, $\phi$

Differential decay distribution:

$$\frac{d^4\Gamma}{dq^2 \, d\cos\theta_\ell \, d\cos\theta_K^* \, d\phi} = \frac{9}{32\pi} J(q^2, \theta_\ell, \theta_K^*, \phi)$$

$$J(q^2, \theta_\ell, \theta_K^*, \phi) = \sum_i J_i(q^2) \, f_i(\theta_\ell, \theta_K^*, \phi)$$

- angular coefficients $J_1-9$
- functions of the transversity amplitudes $A_0$, $A_\parallel$, $A_\perp$, $A_t$, and $A_S$
- or alternatively, helicity amplitudes $H_V$, $H_A$ and $H_S$

Transversity/helicity amplitudes: functions of Wilson coefficients and form factors
Optimised observables: form factor uncertainties cancel at leading order

\[ \langle P_1 \rangle_{\text{bin}} = \frac{1}{2} \int_{\text{bin}} dq^2 [J_3 + \bar{J}_3] \]

\[ \langle P_2 \rangle_{\text{bin}} = \frac{1}{8} \int_{\text{bin}} dq^2 [J_{6s} + \bar{J}_{6s}] \]

\[ \langle P_4' \rangle_{\text{bin}} = \frac{1}{N'_{\text{bin}}} \int_{\text{bin}} dq^2 [J_4 + \bar{J}_4] \]

\[ \langle P_5' \rangle_{\text{bin}} = \frac{1}{2N'_{\text{bin}}} \int_{\text{bin}} dq^2 [J_5 + \bar{J}_5] \]

\[ \langle P_6' \rangle_{\text{bin}} = \frac{-1}{2N'_{\text{bin}}} \int_{\text{bin}} dq^2 [J_7 + \bar{J}_7] \]

\[ \langle P_8' \rangle_{\text{bin}} = \frac{-1}{N'_{\text{bin}}} \int_{\text{bin}} dq^2 [J_8 + \bar{J}_8] \]

with

\[ N'_{\text{bin}} = \sqrt{-\int_{\text{bin}} dq^2 [J_{2s} + \bar{J}_{2s}] \int_{\text{bin}} dq^2 [J_{2c} + \bar{J}_{2c}]} \]

+ CP violating clean observables and other combinations

J. Matias et al., JHEP 1204 (2012) 104
S. Descotes-Genon et al., JHEP 1305 (2013) 137

Or alternatively:

\[ S_i = \frac{J_i(s,c) + \bar{J}_i(s,c)}{\frac{d\Gamma}{dq^2} + \frac{d\bar{\Gamma}}{dq^2}} \]

\[ P'_{4,5,8} = \frac{S_{4,5,8}}{\sqrt{F_L(1 - F_L)}} \]
The LHCb anomalies (1)

\[ B \rightarrow K^* \mu^+ \mu^- \] angular observables, in particular \( P_5'/S_5 \)

Long standing anomaly 2-3\( \sigma \):
- 2013 (1 fb\(^{-1}\)): disagreement with the SM for \( P_2 \) and \( P_5' \) \( (PRL 111, 191801 (2013)) \)
- March 2015 (3 fb\(^{-1}\)): confirmation of the deviations \( (LHCb-CONF-2015-002) \)
- Dec. 2015: 2 analysis methods, both show the deviations \( (JHEP 1602, 104 (2016)) \)

\[ LHCb, JHEP 02 (2016) 104; Belle, PRL 118 (2017); ATLAS, ATLAS-CONF-2017-023; CMS, CMS-PAS-BPH-15-008 \]

- Also measured by ATLAS, CMS and Belle
The LHCb anomalies (2)

$B_s \rightarrow \phi \mu^+ \mu^-$ branching fraction

- Same theoretical description as $B \rightarrow K^* \mu^+ \mu^-$
  - Replacement of $B \rightarrow K^*$ form factors with the $B_s \rightarrow \phi$ ones
  - Also consider the $B_s - \bar{B}_s$ oscillations

- June 2015 (3 fb$^{-1}$): the differential branching fraction is found to be $3.2\sigma$ below the SM predictions in the [1-6] GeV$^2$ bin

JHEP 1509 (2015) 179
The LHCb anomalies (3)

Lepton flavour universality in $B^+ \rightarrow K^+ \ell^+\ell^-$

- Theoretical description similar to $B \rightarrow K^* \mu^+\mu^-$, but different since $K$ scalar

SM prediction very accurate (leading corrections from QED, giving rise to large logarithms involving the ratio $m_B/m_{\mu,e}$)

$$R_K = \frac{BR(B^+ \rightarrow K^+ \mu^+\mu^-)}{BR(B^+ \rightarrow K^+ e^+e^-)}$$

$$R_K^{exp} = 0.745^{+0.090}_{-0.074}(stat) \pm 0.036(syst)$$

$$R_K^{SM} = 1.0006 \pm 0.0004$$

If confirmed this would be a groundbreaking discovery and a very spectacular fall of the SM

The updated analysis is eagerly awaited!
The LHCb anomalies (4)

Lepton flavour universality in $B^0 \rightarrow K^{*0} \ell^+ \ell^-$

- LHCb measurement (April 2017):
  \[ R_{K^*} = \frac{BR(B^0 \rightarrow K^{*0} \mu^+ \mu^-)}{BR(B^0 \rightarrow K^{*0} e^+ e^-)} \]
  \[ R_{K^*}^{\text{exp}, \text{bin}1} = 0.660^{+0.110}_{-0.070} \text{(stat)} \pm 0.024 \text{(syst)} \]
  \[ R_{K^*}^{\text{exp}, \text{bin}2} = 0.685^{+0.113}_{-0.069} \text{(stat)} \pm 0.047 \text{(syst)} \]

- Two $q^2$ regions: [0.045-1.1] and [1.1-6.0] GeV$^2$

BaBar, PRD 86 (2012) 032012; Belle, PRL 103 (2009) 171801
The LHCb anomalies (4)

Lepton flavour universality in $B^0 \rightarrow K^{*0} \ell^+ \ell^-$

- LHCb measurement (April 2017):
  \[ R_{K^*} = \frac{BR(B^0 \rightarrow K^{*0} \mu^+ \mu^-)}{BR(B^0 \rightarrow K^{*0} e^+ e^-)} \]

- Two $q^2$ regions: [0.045-1.1] and [1.1-6.0] GeV$^2$

![Graph showing $R_{K^*}$ versus $q^2$]

- LHCb Preliminary
- BaBar, PRD 86 (2012) 032012; Belle, PRL 103 (2009) 171801

\[
R^\text{exp,bin1}_{K^*} = 0.660^{+0.110}_{-0.070} \text{(stat)} \pm 0.024 \text{(syst)} \\
R^\text{exp,bin2}_{K^*} = 0.685^{+0.113}_{-0.069} \text{(stat)} \pm 0.047 \text{(syst)} \\
R^\text{SM,bin1}_{K^*} = 0.906 \pm 0.020_{\text{QED}} \pm 0.020_{\text{FF}} \\
R^\text{SM,bin2}_{K^*} = 1.000 \pm 0.010_{\text{QED}} \\
\]

Bordone, Isidori, Pattori, arXiv:1605.07633

2.2-2.5$\sigma$ tension with the SM predictions in each bin
Effective Hamiltonian for $b \to sll$ transitions

$$H_{\text{eff}} = H_{\text{eff}}^{\text{had}} + H_{\text{eff}}^{sll}$$

$$H_{\text{eff}}^{sll} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left[ \sum_{i=7,9,10} C_i^{(t)} O_i^{(t)} \right]$$

$$\langle \bar{K}^* | H_{\text{eff}}^{sll} | B \rangle: B \to K^* \text{ form factors } V, A_{0,1,2}, T_{1,2,3}$$

Transversity amplitudes:

$$A_{L,R}^{\perp} \approx N_\perp \left\{ (C_9^+ + C_{10}^+) \frac{V(q^2)}{m_B + m_{K^*}} + \frac{2m_b}{q^2} C_7^+ T_1(q^2) \right\}$$

$$A_{L,R}^{\parallel} \approx N_\parallel \left\{ (C_9^- + C_{10}^-) \frac{A_1(q^2)}{m_B - m_{K^*}} + \frac{2m_b}{q^2} C_7^- T_2(q^2) \right\}$$

$$A_{0,L,R} \approx N_0 \left\{ (C_9^- + C_{10}^-) \left[ (\cdots) A_1(q^2) + (\cdots) A_2(q^2) \right] + 2m_b C_7^- \left[ (\cdots) T_2(q^2) + (\cdots) T_3(q^2) \right] \right\}$$

$$A_S = N_S (C_S - C_S') A_0(q^2) \quad (C_i^\pm \equiv C_i \pm C_i')$$
Effective Hamiltonian for $b \rightarrow s\ell\ell$ transitions

$$\mathcal{H}_{\text{eff}} = \mathcal{H}_{\text{eff}}^{\text{had}} + \mathcal{H}_{\text{eff}}^{\text{sl}}$$

$$\mathcal{H}_{\text{eff}}^{\text{had}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left[ \sum_{i=1}^{6} C_i O_i + C_8 O_8 \right]$$

$$A^{(\text{had})}_\lambda = -i \frac{e^2}{q^2} \int d^4x e^{-iq \cdot x} \langle \ell^+ \ell^- | j_{\mu}^{\text{em,lept}}(x) | 0 \rangle$$

$$\times \int d^4y e^{iq \cdot y} \langle \bar{K}^* | T \{ j^{\text{em, had}, \mu}(y) \mathcal{H}_{\text{eff}}^{\text{had}}(0) \} | \bar{B} \rangle$$

$$\equiv e^2 \epsilon_{\mu} L_V^{\mu} \left[ \text{LO in } \mathcal{O}(\frac{\Lambda}{m_b}, \frac{\Lambda}{E_K^*}) + h_\lambda(q^2) \right]$$

Non-Fact., QCDf

Beneke et al.; 106067; 0412400
Effective Hamiltonian for $b \rightarrow s\ell\ell$ transitions

$$H_{\text{eff}} = H_{\text{eff}}^{\text{had}} + H_{\text{eff}}^{\text{sl}}$$

$$H_{\text{eff}}^{\text{had}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left[ \sum_{i=1\ldots6} C_i O_i + C_8 O_8 \right]$$

$$A_\lambda^{(\text{had})} = -i \frac{e^2}{q^2} \int d^4 x e^{-iq \cdot x} \langle \ell^+ \ell^- | j_{\mu}^{\text{em,lept}}(x) | 0 \rangle$$

$$\times \int d^4 y e^{iq \cdot y} \langle \bar{K}^* | T \{ j_{\mu}^{\text{em,had}}(y) H_{\text{eff}}^{\text{had}}(0) \} | \bar{B} \rangle$$

$$\equiv \frac{e^2}{q^2} \epsilon_\mu L_\nu^\mu \left[ \text{LO in } O(\frac{\Lambda}{m_b}, \frac{\Lambda}{E_K^*}) + \text{power corrections} \right]$$

Beneke et al.: 106067; 0412400

Non-Fact., QCDf

$\rightarrow \text{unknown}$

partial calculation: Khodjamirian et al., 1006.4945
Effective Hamiltonian for $b \rightarrow s \ell\ell$ transitions

$$H_{\text{eff}} = H_{\text{eff}}^{\text{had}} + H_{\text{eff}}^{\text{sl}}$$

$$H_{\text{eff}}^{\text{had}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left[ \sum_{i=1 \ldots 6} C_i O_i + C_8 O_8 \right]$$

$$A_{\lambda}^{(\text{had})} = -i \frac{e^2}{q^2} \int d^4 x e^{-i q \cdot x} \langle \ell^+ \ell^- | j_{\mu}^{\text{em,lept}}(x) | 0 \rangle$$

$$\times \int d^4 y e^{i q \cdot y} \langle \bar{K}^*_{\lambda} | T \{ j_{\mu}^{\text{em,had},\mu}(y) H_{\text{eff}}^{\text{had}}(0) \} | \bar{B} \rangle$$

$$\equiv \frac{e^2}{q^2} \epsilon_{\mu} L_{\nu}^{\mu} \left[ \begin{array}{c}
\text{LO in } \mathcal{O}\left( \frac{\Lambda}{m_b}, \frac{\Lambda}{E_K^*} \right) + h_\lambda(q^2) \\
\text{Non-Fact., QCDf}
\end{array} \right]$$

The significance of the anomalies depends on the assumptions made for the unknown power corrections!

This does not affect $R_K$ and $R_K^*$ of course, but does affect the combined fits!
Implications
Global fits

Many observables → Global fits

NP manifests itself in shifts of individual coefficients with respect to SM values:

\[ C_i(\mu) = C_i^{\text{SM}}(\mu) + \delta C_i \]

→ Scans over the values of \( \delta C_i \)
→ Calculation of flavour observables

Theoretical uncertainties and correlations

- Monte Carlo analysis
- variation of the "standard" input parameters: masses, scales, CKM, ...
- decay constants taken from the latest lattice results
- \( B \to K(\ast) \) and \( B_s \to \phi \) form factors are obtained from the lattice+LCSR combinations (1411.3161, 1503.05534), including all the correlations
- Parameterisation of uncertainties from power corrections:

\[ A_k \to A_k \left(1 + a_k \exp(i\phi_k) + \frac{q^2}{6 \text{ GeV}^2} b_k \exp(i\theta_k)\right) \]

\( |a_k| \) between 10 to 60%, \( b_k \sim 2.5a_k \)

Low recoil: \( b_k = 0 \)

⇒ Computation of a (theory + exp) correlation matrix
Global fits

Global fits of the observables obtained by minimisation of

$$\chi^2 = (\vec{O}^{th} - \vec{O}^{exp}) \cdot (\Sigma_{th} + \Sigma_{exp})^{-1} \cdot (\vec{O}^{th} - \vec{O}^{exp})$$

$$(\Sigma_{th} + \Sigma_{exp})^{-1}$$ is the inverse covariance matrix.

More than 100 observables relevant for leptonic and semileptonic decays:

- $BR(B \rightarrow X_s \gamma)$
- $BR(B \rightarrow X_d \gamma)$
- $\Delta_0(B \rightarrow K^* \gamma)$
- $BR^{low}(B \rightarrow X_s \mu^+ \mu^-)$
- $BR^{high}(B \rightarrow X_s \mu^+ \mu^-)$
- $BR^{low}(B \rightarrow X_s e^+ e^-)$
- $BR^{high}(B \rightarrow X_s e^+ e^-)$
- $BR(B_s \rightarrow \mu^+ \mu^-)$
- $BR(B_d \rightarrow \mu^+ \mu^-)$
- $BR(B \rightarrow K^0 \mu^+ \mu^-)$
- $BR(B \rightarrow K^{*+} \mu^+ \mu^-)$
- $BR(B \rightarrow K^+ \mu^+ \mu^-)$
- $BR(B \rightarrow K^* e^+ e^-)$
- $R_K, R_{K^*}$
- $B \rightarrow K^{*0} \mu^+ \mu^-$: $BR, F_L, A_{FB}, S_3, S_4, S_5, S_7, S_8, S_9$
  in 8 low $q^2$ and 4 high $q^2$ bins
- $B_s \rightarrow \phi \mu^+ \mu^-$: $BR, F_L, S_3, S_4, S_7$
  in 3 low $q^2$ and 2 high $q^2$ bins

Computations performed using SuperIso public program
New physics or hadronic effects?

Description in terms of helicity amplitudes:

\[ H_V(\lambda) = -i N' \left\{ C_9 \tilde{V}_{L\lambda}(q^2) + C'_9 \tilde{V}_{R\lambda}(q^2) + \frac{m_B^2}{q^2} \left[ \frac{2 \hat{m}_b}{m_B} \left( C_7 \tilde{T}_{L\lambda}(q^2) + C'_7 \tilde{T}_{R\lambda}(q^2) \right) - 16\pi^2 N_\lambda(q^2) \right] \right\} \]

\[ H_A(\lambda) = -i N' \left( C_{10} \tilde{V}_{L\lambda}(q^2) + C'_{10} \tilde{V}_{R\lambda}(q^2) \right), \quad N_\lambda(q^2) = \text{leading nonfact.} + h_\lambda \]

\[ H_S = i N' \hat{m}_b \frac{m_W}{m_W} (C_S - C'_S) \tilde{S}(q^2) \]

Helicity FFs \( \tilde{V}_{L/R}, \tilde{T}_{L/R}, \tilde{S} \) are combinations of the standard FFs \( V, A_{0,1,2}, T_{1,2,3} \)

A possible parametrisation of the non-factorisable power corrections \( h_\lambda(=+,–,0)(q^2) \):

\[ h_\lambda(q^2) = h_\lambda^{(0)} + \frac{q^2}{1\text{GeV}^2} h_\lambda^{(1)} + \frac{q^4}{1\text{GeV}^4} h_\lambda^{(2)} \]


M. Ciuchini et al., JHEP 1606 (2016) 116

It seems

\[ h_\lambda^{(0)} \rightarrow C_{7NP}^N, \quad h_\lambda^{(1)} \rightarrow C_{9NP}^N \]

and \( h_\lambda^{(2)} \) terms cannot be mimicked by \( C_7 \) and \( C_9 \)

M. Ciuchini et al., JHEP 1606 (2016) 116

However, \( \tilde{V}_{L(R)\lambda} \) and \( \tilde{T}_{L(R)\lambda} \) both have a \( q^2 \) dependence!
New physics or hadronic effects?

Description in terms of helicity amplitudes:

\[ H_V(\lambda) = -i N' \left\{ C_9 \tilde{V}_{L\lambda}(q^2) + C'_9 \tilde{V}_{R\lambda}(q^2) + \frac{m_B^2}{q^2} \left[ \frac{2 \hat{m}_b}{m_B} (C_7 \tilde{T}_{L\lambda}(q^2) + C'_7 \tilde{T}_{R\lambda}(q^2)) - 16\pi^2 N_\lambda(q^2) \right] \right\} \]

\[ H_A(\lambda) = -i N' (C_{10} \tilde{V}_{L\lambda}(q^2) + C'_{10} \tilde{V}_{R\lambda}(q^2)), \]

\[ N_\lambda(q^2) = \text{leading nonfact.} + h_\lambda \]

\[ H_S = i N' \frac{\hat{m}_b}{m_W} (C_S - C'_S) \tilde{S}(q^2) \]

\[ \left( N' = -\frac{4G_F m_B}{\sqrt{2}} \frac{e^2}{16\pi^2} V_{tb} V_{ts}^* \right) \]

Helicity FFs \( \tilde{V}_{L/R}, \tilde{T}_{L/R}, \tilde{S} \) are combinations of the standard FFs \( V, A_{0,1,2}, T_{1,2,3} \)

A possible parametrisation of the non-factorisable power corrections \( h_\lambda(=+,-,0)(q^2) \):

\[ h_\lambda(q^2) = h_\lambda^{(0)} + \frac{q^2}{1\text{GeV}^2} h_\lambda^{(1)} + \frac{q^4}{1\text{GeV}^4} h_\lambda^{(2)} \]


M. Ciuchini et al., JHEP 1606 (2016) 116

It seems

\[ h_\lambda^{(0)} \rightarrow C_{7NP}^{NP}, \quad h_\lambda^{(1)} \rightarrow C_{9NP}^{NP} \]

and \( h_\lambda^{(2)} \) terms cannot be mimicked by \( C_7 \) and \( C_9 \)

M. Ciuchini et al., JHEP 1606 (2016) 116

However, \( \tilde{V}_{L(R)\lambda} \) and \( \tilde{T}_{L(R)\lambda} \) both have a \( q^2 \) dependence!
New physics or hadronic effects?

\[ q^4 \text{ terms can rise due to terms which multiply Wilson coefficients} \]

\[ \implies C_7^{NP} \text{ and } C_9^{NP} \text{ can each cause effects similar to } h^{(0,1,2)}_\lambda \]
New physics or hadronic effects?

Hadronic power correction effect:

\[ \delta H^\text{p.c.}_V(\lambda) = iN' m_B^2 \frac{16\pi^2}{q^2} h_\lambda(q^2) = iN' m_B^2 \frac{16\pi^2}{q^2} \left( h^{(0)}_\lambda + q^2 h^{(1)}_\lambda + q^4 h^{(2)}_\lambda \right) \]

New Physics effect:

\[ \delta H^\text{NP}_V (\lambda) = -iN' \tilde{V}_L(q^2) C^\text{NP}_9 = iN' m_B^2 \frac{16\pi^2}{q^2} \left( a_\lambda C^\text{NP}_9 + q^2 b_\lambda C^\text{NP}_9 + q^4 c_\lambda C^\text{NP}_9 \right) \]

and similarly for \( C_7 \)

\[ \Rightarrow \text{NP effects can be embedded in the hadronic effects.} \]

We can do a fit for both (hadronic quantities \( h^{(0,1,2)}_{+,-,0} \) (18 parameters) and Wilson coefficients \( C^\text{NP}_i \) (2 or 4 parameters))

Due to this embedding the two fits can be compared with the Wilk's test
Wilk’s test

SM vs 2 parameters and 4 parameters p-values were independently computed through 2D profile likelihood integration, and they give similar results

\[ q^2 \text{ up to } 8 \text{ GeV}^2 \]

<table>
<thead>
<tr>
<th></th>
<th>2 ((\delta C_9))</th>
<th>4 ((\delta C_7, \delta C_9))</th>
<th>18 ((h_{+,-,0}^{(0,1,2)}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3.7 \times 10^{-5} (4.1(\sigma))</td>
<td>6.3 \times 10^{-5} (4.0(\sigma))</td>
<td>6.1 \times 10^{-3} (2.7(\sigma))</td>
</tr>
<tr>
<td>2</td>
<td>(\cdot)</td>
<td>0.13 (1.5(\sigma))</td>
<td>0.45 (0.76(\sigma))</td>
</tr>
<tr>
<td>4</td>
<td>(\cdot)</td>
<td>(\cdot)</td>
<td>0.61 (0.52(\sigma))</td>
</tr>
</tbody>
</table>

→ Adding \(\delta C_9\) improves over the SM hypothesis by 4.1\(\sigma\)
→ Including in addition \(\delta C_7\) or hadronic parameters improves the situation only mildly
→ One cannot rule out the hadronic option

Adding 16 more parameters does not really improve the fits

The situation is still inconclusive
NP Fit results: single operator

Best fit values considering all observables besides \( R_K \) and \( R_K^* \)
(under the assumption of 10% non-factorisable power corrections)

<table>
<thead>
<tr>
<th></th>
<th>b.f. value</th>
<th>( \chi^2_{\text{min}} )</th>
<th>Pull_{SM}</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta C_9 )</td>
<td>-0.24</td>
<td>70.5</td>
<td>4.1\sigma</td>
</tr>
<tr>
<td>( \Delta C_9' )</td>
<td>-0.02</td>
<td>87.4</td>
<td>0.3\sigma</td>
</tr>
<tr>
<td>( \Delta C_{10} )</td>
<td>-0.02</td>
<td>87.3</td>
<td>0.4\sigma</td>
</tr>
<tr>
<td>( \Delta C_{10}' )</td>
<td>+0.03</td>
<td>87.0</td>
<td>0.7\sigma</td>
</tr>
<tr>
<td>( \Delta C_9^\mu )</td>
<td>-0.25</td>
<td>68.2</td>
<td>4.4\sigma</td>
</tr>
<tr>
<td>( \Delta C_9^e )</td>
<td>+0.18</td>
<td>86.2</td>
<td>1.2\sigma</td>
</tr>
<tr>
<td>( \Delta C_{10}^\mu )</td>
<td>-0.05</td>
<td>86.8</td>
<td>0.8\sigma</td>
</tr>
<tr>
<td>( \Delta C_{10}^e )</td>
<td>-2.14</td>
<td>86.3</td>
<td>1.1\sigma</td>
</tr>
<tr>
<td></td>
<td>+0.14</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( \rightarrow \) \( C_9 \) and \( C_9^\mu \) solutions are favoured with SM pulls of 4.1 and 4.4\sigma
\( \rightarrow \) Primed operators have a very small SM pull
\( \rightarrow \) \( C_{10} \)-like solutions do not play a role
NP Fit results: single operator

Best fit values considering all observables besides $R_K$ and $R_{K^*}$
(under the assumption of 10% non-factorisable power corrections)

<table>
<thead>
<tr>
<th></th>
<th>b.f. value</th>
<th>$\chi^2_{\text{min}}$</th>
<th>$\text{Pull}_{\text{SM}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta C_9$</td>
<td>−0.24</td>
<td>70.5</td>
<td>4.1$\sigma$</td>
</tr>
<tr>
<td>$\Delta C_9'$</td>
<td>−0.02</td>
<td>87.4</td>
<td>0.3$\sigma$</td>
</tr>
<tr>
<td>$\Delta C_{10}$</td>
<td>−0.02</td>
<td>87.3</td>
<td>0.4$\sigma$</td>
</tr>
<tr>
<td>$\Delta C_{10}'$</td>
<td>+0.03</td>
<td>87.0</td>
<td>0.7$\sigma$</td>
</tr>
<tr>
<td>$\Delta C_9''$</td>
<td>−0.25</td>
<td>68.2</td>
<td>4.4$\sigma$</td>
</tr>
<tr>
<td>$\Delta C_9'^e$</td>
<td>+0.18</td>
<td>86.2</td>
<td>1.2$\sigma$</td>
</tr>
<tr>
<td>$\Delta C_{10}''$</td>
<td>−0.05</td>
<td>86.8</td>
<td>0.8$\sigma$</td>
</tr>
<tr>
<td>$\Delta C_{10}''^e$</td>
<td>−2.14</td>
<td>86.3</td>
<td>1.1$\sigma$</td>
</tr>
</tbody>
</table>

→ $C_9$ and $C_9''$ solutions are favoured with SM pulls of 4.1 and 4.4$\sigma$
→ Primed operators have a very small SM pull
→ $C_{10}$-like solutions do not play a role

Best fit values in the one operator fit considering only $R_K$ and $R_{K^*}$

<table>
<thead>
<tr>
<th></th>
<th>b.f. value</th>
<th>$\chi^2_{\text{min}}$</th>
<th>$\text{Pull}_{\text{SM}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta C_9$</td>
<td>−0.48</td>
<td>18.3</td>
<td>0.3$\sigma$</td>
</tr>
<tr>
<td>$\Delta C_9'$</td>
<td>+0.78</td>
<td>18.1</td>
<td>0.6$\sigma$</td>
</tr>
<tr>
<td>$\Delta C_{10}$</td>
<td>−1.02</td>
<td>18.2</td>
<td>0.5$\sigma$</td>
</tr>
<tr>
<td>$\Delta C_{10}'$</td>
<td>+1.18</td>
<td>17.9</td>
<td>0.7$\sigma$</td>
</tr>
<tr>
<td>$\Delta C_9''$</td>
<td>−0.35</td>
<td>5.1</td>
<td>3.6$\sigma$</td>
</tr>
<tr>
<td>$\Delta C_9''^e$</td>
<td>+0.37</td>
<td>3.5</td>
<td>3.9$\sigma$</td>
</tr>
<tr>
<td>$\Delta C_{10}''$</td>
<td>−1.66</td>
<td>2.7</td>
<td>4.0$\sigma$</td>
</tr>
<tr>
<td>$\Delta C_{10}''^e$</td>
<td>−2.36</td>
<td>2.2</td>
<td>4.0$\sigma$</td>
</tr>
</tbody>
</table>

→ NP in $C_9^e, C_9'', C_{10}^e,$ or $C_{10}''$ are favoured by the $R_K(\ast)$ ratios (significance: 3.6 – 4.0$\sigma$)
→ NP contributions in primed operators do not play a role.
Fit results for two operators

\((C_9^\mu - C_9^e)\)

\((C_9^\mu - C_{10}^\mu)\)

The two sets are compatible at least at the 2\(\sigma\) level.
1) **Unknown power corrections**

- Significance of the anomalies depends on the assumptions on the power corrections
- Towards a calculation...
- Problem: they are not calculable in QCD factorisation
- Alternative approaches exist based on light cone sum rule techniques

  Khodjamirian et al. JHEP 1009 (2010) 089  
  A more recent approach based on the analyticity structure: Bobeth et al. arXiv:1707.07305

2) **Cross-check with inclusive modes**

Inclusive decays are theoretically cleaner  
(see e.g. T. Huber, T. Hurth, E. Lunghi, JHEP 1506 (2015) 176)

→ Belle-II will check the NP interpretation with theoretically clean modes

  T. Hurth, FM, JHEP 1404 (2014) 097  
How to resolve the issue?

3) Cross-check with other $R_{\mu/e}$ ratios

- $R_K$ and $R_{K^*}$ ratios are theoretically very clean
- The tensions cannot be explained by hadronic uncertainties

Cross-checks needed with other ratios:

<table>
<thead>
<tr>
<th>Obs.</th>
<th>$C_9^\mu$</th>
<th>$C_9^e$</th>
<th>$C_{10}^\mu$</th>
<th>$C_{10}^e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{FL}^{1.1,6.0}$</td>
<td>[0.785, 0.913]</td>
<td>[0.909, 0.933]</td>
<td>[1.005, 1.042]</td>
<td>[1.001, 1.018]</td>
</tr>
<tr>
<td>$R_{AFB}^{1.1,6.0}$</td>
<td>[6.048, 14.819]</td>
<td>[−0.288, −0.153]</td>
<td>[0.816, 0.928]</td>
<td>[0.974, 1.061]</td>
</tr>
<tr>
<td>$R_9^{1.1,6.0}$</td>
<td>[−0.787, 0.394]</td>
<td>[0.603, 0.697]</td>
<td>[0.881, 1.002]</td>
<td>[1.053, 1.146]</td>
</tr>
<tr>
<td>$R_{FL}^{15,19}$</td>
<td>[0.999, 0.999]</td>
<td>[0.998, 0.998]</td>
<td>[0.997, 0.998]</td>
<td>[0.998, 0.998]</td>
</tr>
<tr>
<td>$R_{AFB}^{15,19}$</td>
<td>[0.616, 0.927]</td>
<td>[1.002, 1.061]</td>
<td>[0.860, 0.994]</td>
<td>[1.046, 1.131]</td>
</tr>
<tr>
<td>$R_9^{5}$</td>
<td>[0.615, 0.927]</td>
<td>[1.002, 1.061]</td>
<td>[0.860, 0.994]</td>
<td>[1.046, 1.131]</td>
</tr>
<tr>
<td>$R_{K^*}^{15,19}$</td>
<td>[0.621, 0.803]</td>
<td>[0.577, 0.771]</td>
<td>[0.589, 0.778]</td>
<td>[0.586, 0.770]</td>
</tr>
<tr>
<td>$R_{K}^{15,19}$</td>
<td>[0.597, 0.802]</td>
<td>[0.590, 0.778]</td>
<td>[0.659, 0.818]</td>
<td>[0.632, 0.805]</td>
</tr>
<tr>
<td>$R_{\phi}^{1.1,6.0}$</td>
<td>[0.748, 0.852]</td>
<td>[0.620, 0.805]</td>
<td>[0.578, 0.770]</td>
<td>[0.578, 0.764]</td>
</tr>
<tr>
<td>$R_{\phi}^{15,19}$</td>
<td>[0.623, 0.803]</td>
<td>[0.577, 0.771]</td>
<td>[0.586, 0.776]</td>
<td>[0.583, 0.769]</td>
</tr>
</tbody>
</table>

A confirmation of the deviations in the ratios would indirectly confirm the NP interpretation of the anomalies in the angular observables!
How to resolve the issue?

4) Future LHCb upgrade

Global fits using the angular observables only (NO theoretically clean $R$ ratios)

Considering several luminosities, assuming the current central values

LHCb will be able to establish new physics within the angular observables even in the pessimistic case that there will be no theoretical progress on non-factorisable power corrections!
How to resolve the issue?

Pull\textsubscript{SM} for the fit to $\Delta C^\mu_9$ based on the ratios $R_K$ and $R_{K^*}$ for the LHCb upgrade

Assuming current central values remain.

<table>
<thead>
<tr>
<th>$\Delta C^\mu_9$</th>
<th>Syst. Pull\textsubscript{SM}</th>
<th>Syst./2 Pull\textsubscript{SM}</th>
<th>Syst./3 Pull\textsubscript{SM}</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 fb\textsuperscript{-1}</td>
<td>6.1\textsigma (4.3\textsigma)</td>
<td>7.2\textsigma (5.2\textsigma)</td>
<td>7.4\textsigma (5.5\textsigma)</td>
</tr>
<tr>
<td>50 fb\textsuperscript{-1}</td>
<td>8.2\textsigma (5.7\textsigma)</td>
<td>11.6\textsigma (8.7\textsigma)</td>
<td>12.9\textsigma (9.9\textsigma)</td>
</tr>
<tr>
<td>300 fb\textsuperscript{-1}</td>
<td>9.4\textsigma (6.5\textsigma)</td>
<td>15.6\textsigma (12.3\textsigma)</td>
<td>19.5\textsigma (16.1\textsigma)</td>
</tr>
</tbody>
</table>

(): assuming 50% correlation between each of the $R_K$ and $R_{K^*}$ measurements

Only a small part of the 50 fb\textsuperscript{-1} is needed to establish NP in the $R_{K(*)}$ ratios even in the pessimistic case that the systematic errors are not reduced by then at all.

This is independent of the hadronic uncertainties!
The full LHCb Run 1 results still show some tensions with the SM predictions.

Significance of the anomalies depends on the assumptions on the power corrections.

Model independent fits point to about 25% reduction in $C_9$, and new physics in muonic $C_9^\mu$ is preferred.

Comparing the fits for NP and hadronic parameters through the Wilk’s test shows that at the moment adding the hadronic parameters does not improve the fit compared to the new physics fit, but the situation is inconclusive.

The recent measurement of $R_{K^*}$ supports the NP hypothesis, but the experimental errors are still large and the update of $R_K$ is eagerly awaited!

The LHCb upgrade will have enough precision to distinguish between NP and hadronic effects.
Backup
Global fit results

Fit with 2 parameters (complex $C_9$)

low $q^2$ bins (up to 8 GeV$^2$)

About $3\sigma$ tension for $\text{Re}(\delta C_9)$
Global fit results

Fit with 2 parameters (complex $C_9$)

Fit with 4 parameters (complex $C_7$ and $C_9$)

About $3\sigma$ tension for $Re(\delta C_9)$
Fits with different assumptions for the form factor uncertainties:
- correlations ignored (solid line)
- normal form factor errors (filled areas)
- $2 \times$ form factor errors (dashed line)
- $4 \times$ form factor errors (dotted line)

\[
(C_9 - C_{10}) \quad (C_9 - C'_9) \quad (C^e_9 - C^\mu_9)
\]
Fits with different assumptions for the form factor uncertainties:

- correlations ignored (solid line)
- normal form factor errors (filled areas)
- $2 \times$ form factor errors (dashed line)
- $4 \times$ form factor errors (dotted line)

$$\left( C_9 - C_{10} \right)$$

$$\left( C_9 - C'_9 \right)$$

$$\left( C^e_9 - C^\mu_9 \right)$$
Fit results for two operators: form factor dependence

Fits with different assumptions for the form factor uncertainties:

- correlations ignored (solid line)
- normal form factor errors (filled areas)
- $2 \times$ form factor errors (dashed line)
- $4 \times$ form factor errors (dotted line)

The size of the form factor errors has a crucial role in constraining the allowed region!
Fit results for four operators: \( \{ C^\mu_9, C^e_9, C^\mu_{10}, C^e_{10} \} \)

No reason that only 2 Wilson coefficients receive contributions from new physics

Larger ranges are allowed for the Wilson coefficients

Considering 4 operator fits considerably relaxes the constraints on the Wilson coefficients leaving room for more diverse new physics contributions which are otherwise overlooked.
Fit results for four operators: \( \{ C_9^\mu, C_9'^\mu, C_9^e, C_9'^e \} \)

No reason that only 2 Wilson coefficients receive contributions from new physics

Larger ranges are allowed for the Wilson coefficients
Fit results for four operators: \( \{ C_9, C_9', C_{10}, C_{10}' \} \)

No reason that only 2 Wilson coefficients receive contributions from new physics

Larger ranges are allowed for the Wilson coefficients