Critical Fluctuations in QCD Matter

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1. The QCD critical endpoint; universality class; the order parameter

2. The Ising-QCD effective action

3. Finite-size scaling; the origin of critical fluctuations

4. A route to observation of critical fluctuations

5. Concluding remarks
The QCD critical point

- Critical Point (CP) separates first order line ($\mu > \mu_c$) from analytical crossover ($\mu < \mu_c$)

- $T_c \approx 150 \text{ MeV}$, Lattice QCD (chiral susceptibility)

- $\mu_c \approx 1.8 \cdot T_c$ (sign problem)

Search for critical fluctuations at BNL-RHIC (BES) and CERN-SPS (NA61) ⇒ Universality and scaling

from H-T. Ding et al,

Universality class; order parameter

- Second order phase transition at QCD-CP (if exists) in the 3d Ising universality class

- Critical exponents: $\alpha \approx 0$, $\beta \approx \frac{1}{3}$, $\gamma \approx \frac{4}{3}$, $\nu = \frac{2}{3}$ and $\delta \approx 5$

- Order parameter $\Rightarrow$ mixing between chiral condensate ($\sigma$-field) and baryon-number density $n_b$

- Baryon-number density $= \text{slow component of the order parameter}$

\[\downarrow\]

appropriate thermodynamic quantity for description of critical fluctuations in QCD matter
3d Ising effective action

Partition function for 3d-Ising system in an external field $h$:

$$Z(\text{Ising}) = \sum_{\{\phi_i\}} \exp \left[ \beta \sum_{i,j} \phi_i \phi_j + h \sum_i \phi_i \right] ; \quad \phi_i = \pm 1$$

$$\Downarrow$$

Effective action for 3d scalar field theory from Monte-Carlo simulations:

$$S_{\text{eff}}(\text{scalar}) = \int d^3x \left[ \frac{1}{2} |\nabla \phi|^2 + \frac{1}{2} m^2 \phi^2 + g_4 m \phi^4 + g_6 \phi^{\delta+1} - h\phi \right]$$


- $S_{\text{eff}}$ is **universal** ($g_4 \approx 1$, $g_6 \approx 2$) describing any system in 3d Ising universality class close to the CP

- $\delta \approx 5$ (isothermal critical exponent)
Ising-QCD effective action

QCD critical point

\[ S_{\text{eff}}(\text{Ising-QCD}) = S_{\text{eff}}(\text{scalar}) \text{ with:} \]

\[ \phi = \beta^3 n_b \]

\[ h = (\mu - \mu_c) \beta_c \]

Length scale: \( \beta_c = \frac{1}{k_B T_c} \)

Mass parameter (dimensionless) \( m = \beta_c \zeta^{-1} \), \( \zeta = \text{correlation length} \)

\[ m = m_+ |t|^\nu \text{ with } \frac{m_+}{m_-} = \frac{1}{2} \text{ and } t = \frac{T - T_c}{T_c} \quad (\nu = \text{critical exponent}) \]
The Ising-QCD partition function $Z_{QCD}^{Ising}$ is determined by the effective action $S_{eff}(\text{Ising} - \text{QCD})$:

$$Z_{QCD}^{Ising} = \sum_\{\phi\} \exp (- S_{eff}(\text{Ising} - \text{QCD}))$$

Summing over the zero modes (constant $\phi$-configurations) we obtain:

$$Z_{QCD}^{Ising} = \sum_{N_b} \zeta^{N_b} \exp \left[ - \frac{1}{2} m_\pm^2 |t|^{2\nu} \frac{N_b^2}{V} - m_\pm g_4 |t|^{\nu} \frac{N_b^4}{V^3} - g_6 \frac{N_b^{\delta+1}}{V^\delta} \right]$$

with $V$: volume in units $\beta_c^3$, $\zeta = \exp \left( \frac{\mu - \mu_c}{T_c} \right)$, $N_b$ the total baryon-number multiplicity and

$$m_+ = 1 \quad \text{for} \quad T \gtrsim T_c \quad ; \quad m_- = 2 \quad \text{for} \quad T \lesssim T_c$$
The partition function $Z_{Ising}^{QCD}$:

- Describes dynamical fluctuations near the QCD CP on the basis of scaling theory and universality.
- Involves two fundamental indices ($\nu, q = \frac{d_F}{d}$).
- These two determine the complete set of critical exponents:

  $$\alpha = 2 - \nu d \ ; \ \beta = \nu d (1 - q) \ ; \ \gamma = \nu d (2q - 1) \ ; \ \delta = \frac{q}{1 - q}$$

  occurring in the associated scaling laws.
- The relevant scaling laws are linked to Ising-QCD universality class if:

  $$\nu \approx \frac{2}{3} \ ; \ q \approx \frac{5}{6}$$
Scaling behaviour of QCD matter

General finite-size scaling theory (FSST) predictions for QCD matter at the critical point \( (t = 0, \zeta = 1) \):

- **Baryon-number density** → \( n_b(L) \propto L^{-\beta/\nu} \)
- **Baryon-number susceptibility** → \( \chi_b(L) \propto L^{\gamma/\nu} \)

Ising-QCD predictions (compatible with FSST):

- \( Z^{Ising}_{QCD} = \sum_{N_b} \exp \left( -g_6 \frac{N_b^{\delta+1}}{V^{\delta}} \right) \rightarrow \langle n_b \rangle \propto L^{d_F-1} \); \( d_F = 1 - \frac{\beta}{\nu} \)
- \( \chi_b(L) \sim \frac{1}{V} \left[ \langle N_b^2 \rangle - \langle N_b \rangle^2 \right] \rightarrow \chi_b(L) \propto L^{d(2q-1)} \); \( 2q - 1 = \frac{\gamma}{\nu d} \)

with \( L = V^{1/d} \) the linear size
Scaling behaviour of QCD matter (continued)

The results of Ising-QCD reveal the nature of global critical fluctuations in the form of a monofractal structure:

\[
\langle N^k \rangle \propto L^{\kappa d_F} \quad ; \quad d_F = 1 - \frac{\beta}{\nu}, \quad \kappa = 1, 2, \ldots
\]

Away from CP (\( t \neq 0, \zeta \neq 1 \)):

\[
\langle N \rangle \sim V^{\tilde{q}}
\]

Critical region: \( \frac{3}{4} < \tilde{q} < 1 \)

\( \tilde{q} = \frac{3}{4} \Rightarrow \phi^4\)-dominance (mean field) over \( \phi^6 \) (3d Ising)

Critical region narrow along \( \mu_b \):

for \( t = 0 \) is \( \frac{\delta \mu_b}{T_c} \approx 0.03 \)

N.G. Antoniou et al, hep-ph:1705.09124v1
In the finite-size scaling regime the fractal structure of critical fluctuations

\[ \langle N_b \rangle \propto L^{d_F} \]

implies correlations at scales close to the correlation length \( \xi > L \):

\[ \langle n_b(x)n_b(x') \rangle \sim |x - x'|^{-(3-d_F)} \]

\[ d_F = \frac{5}{2} \]

\( \downarrow \)

Scaling is transferred to momentum space for small momentum differences (Fourier transform):

\[ \lim_{k \to k'} \langle n_b(k)n_b(k') \rangle \propto |k - k'|^{-d_F} \]

A fractal structure in momentum space with $\hat{d}_F = d - d_F$ is locally formed!

At midrapidity region the momentum space fractal becomes a cartesian product ($d = 3$):

Transverse momentum $\otimes$ Longitudinal momentum

leading to the **transverse momentum** scaling law:

$$\lim_{k_\perp \to k'_\perp} \langle n_b(k_\perp)n_b(k'_\perp) \rangle \propto |k_\perp - k'_\perp|^{-\frac{2d_F}{3}}$$

$\Downarrow$

2$d$-fractal in transverse momentum space with $\hat{d}_{F,\perp} = 2 - \frac{2}{3}d_F$

$\Downarrow$

Local, power-law distributed, fluctuations in transverse momentum space!
Measuring $\tilde{q}$ - Intermittency

Experimental observation of local, power-law distributed fluctuations

\[ \Downarrow \]

Intermittency in transverse momentum space (net protons at mid-rapidity)

(Critical opalescence in ion collisions)

- Transverse momentum space is partitioned into $M^2$ cells
- Calculate second factorial moments $F_2(M)$ as a function of cell size $\Leftrightarrow$ number of cells $M$:

\[
F_2(M) \equiv \frac{\sum_m \langle n_m(n_m - 1) \rangle}{\sum_m \langle n_m \rangle^2},
\]

where $\langle \ldots \rangle$ denotes averaging over events.

$n_m$: number of particles in $m_{th}$ bin

$m_{th}$ bin
For local power-law fluctuations:

\[ F_2(M) \propto (M^2)^{\phi_2} \quad \text{for} \quad M^2 \gg 1 \]

with \( \phi_2 = \frac{1}{2}(2 - \hat{d}_{F,\perp}) \) → Intermittency index

↓

**Critical fluctuations** linked to the QCD critical point imply:

\[ \phi_2 = q = \frac{d_F}{3} = \frac{5}{6} \quad \text{; since} \quad d_F = \frac{5}{2} \quad \text{for 3d Ising} \]

**Critical Intermittency**

For \( t \neq 0 \) and/or \( \zeta \neq 1 \) the intermittency index is \( \phi_2 = \tilde{q} \)

↓

**Measurement of** \( \phi_2 \equiv \text{Measurement of} \ \tilde{q}! \)
Roadmap towards critical fluctuations

- Measurement of $\tilde{q}$ (transverse momentum intermittency analysis)
- For $\frac{3}{4} < \tilde{q} < 1 \rightarrow$ indication for critical fluctuations (CP: $\tilde{q} = q = \frac{5}{6}$)
- Measurement of chemical freeze-out states $\Rightarrow$ location in the critical region
- Pilot measurement in Si+Si at SPS (NA49 experiment): $\tilde{q} \approx 0.96$ (compatible with $T_c \approx 150$ MeV, $\mu_c \approx 256$ MeV)

The proposed method is directly applicable to current experiments at CERN and BNL (SPS-NA61, RHIC-BES)

(I) Pb + Pb, Au + Au: The crucial energy range for these processes at CERN (Pb + Pb) and BNL (Au + Au) is:

- Pb + Pb at $12.3 \text{ GeV} < \sqrt{s_{NN}} < 17.2 \text{ GeV}$ (SPS-NA61, beyond 2020)
- Au + Au at $14.5 \text{ GeV} < \sqrt{s_{NN}} < 19.6 \text{ GeV}$ (RHIC-BES)

(II) Be + Be, Ar + Sc, Xe + La at $\sqrt{s_{NN}} = 17.2 \text{ GeV}$ (SPS-NA61, highest energy)
Summary and conclusions

- A construction of Ising-QCD effective action was proposed, for studying critical fluctuations in QCD matter.

- A description of the QCD critical point in terms of baryon-number density, as the appropriate order parameter, was presented, compatible with universality and scaling theory.

- In particular, the phenomenon of finite-size scaling was revealed, linked to the fundamental critical exponents $D_h = qd$, $D_t = \frac{1}{\nu}$, which are incorporated in the Ising-QCD effective action.

- It was emphasized that finite-size scaling leads to global and local critical fluctuations in QCD matter, accessible to measurements (critical intermittency in proton transverse momenta).
Summary and conclusions (continued)

- A critical region in the phase diagram \((\mu_b, T)\), extremely narrow in the \(\mu_b\)-direction was found \(\left(\frac{\delta \mu_b}{T_c} \approx 0.03\right)\) providing an efficient instrument for the location of the QCD critical point.

- Finally, a route to observation was proposed, based on crucial intermittency measurements and chemical freeze-out studies, linked to current experiments at CERN (SPS-NA61) and BNL (RHIC-BES).
More about the QCD critical point in CPOD-CORFU 2018

THANK YOU!