Clockwork and its continuum limit

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Workshop on the SM and Beyond

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KC, Sang Hui Im, Chang Sub Shin [arXiv:1709.0xxxx]

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Outline

- Introduction

  Clockwork mechanism

- Continuum clockwork (CCW)

  Extra dimensional setup for light particles protected by a localized symmetry

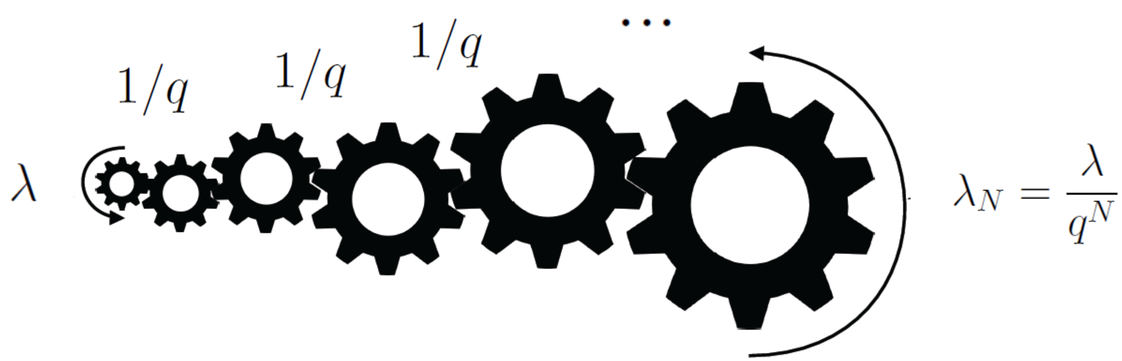
  * CCW for 5D periodic axion

  * Generalized linear dilaton model for CCW

- Conclusion

Clockwork (CW)

Generate exponentially suppressed effective couplings of light particles which are protected by a localized symmetry.



[Cartoon from Giudice, McCullough]

Clockwork axions [Choi, Kim, Yun 14; Choi, Im 15; Kaplan, Rattazzi 15]

Proposed to explain the hierarchical axion couplings (scales) required in various scenarios involving a rolling axion field:

* Natural inflation

$$f_{\rm eff} \sim \text{ axion field excursion}$$

$$V = \Lambda_{\rm inf}^4 \cos\left(\frac{\phi}{f_{\rm eff}}\right) + \Lambda_{\rm WG}^4 \cos\left(\frac{\phi}{f}\right) + \ldots \quad \Rightarrow \quad \frac{f_{\rm eff}}{f} \gtrsim S_{\rm ins}\sqrt{N_e} \gtrsim 10^2$$

Instanton effect suggested by the weak gravity conjecture

* Cosmological relaxation of the weak scale

$$V = \Lambda_{\rm Higgs}^4 \cos\left(\frac{\phi}{f_{\rm eff}}\right) + \mu^2 |H|^2 \cos\left(\frac{\phi}{f}\right) + \ldots \quad \Rightarrow \quad \frac{f_{\rm eff}}{f} \gtrsim \left(\frac{\Lambda_{\rm Higg}}{\rm TeV}\right)^4$$

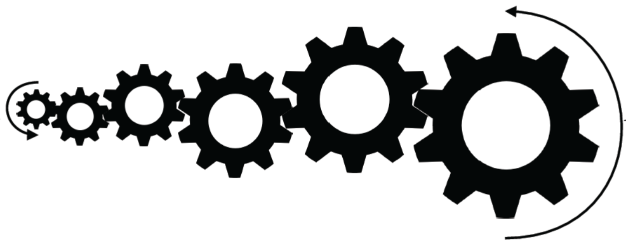
Higgs-dependent barrier potential to stop the relaxion

* Magnetogenesis driven by rolling ALP

$$\Lambda_{\rm inf}^4 \cos\left(\frac{\phi}{f_{\rm eff}}\right) + \frac{\alpha_{\rm em}}{4\pi}\frac{\phi}{f} F^{\mu\nu}\tilde{F}_{\mu\nu} \quad \Rightarrow \quad \frac{f_{\rm eff}}{f} \gtrsim 10\frac{4\pi}{\alpha_{\rm em}} \sim 10^4$$

ALP coupling for magnetogenesis

Clockwork axions:



Axions at each of $N+1$ sites with

the CW interactions between the nearest neighbor

$$\mathcal{L} = -\frac{1}{2} f^2 \left[\sum_{i=0}^{N} (\partial_\mu \theta_i)^2 - 2 \sum_{i=0}^{N-1} m^2 \cos\left(\theta_{i+1} - q\theta_i\right) \right]$$

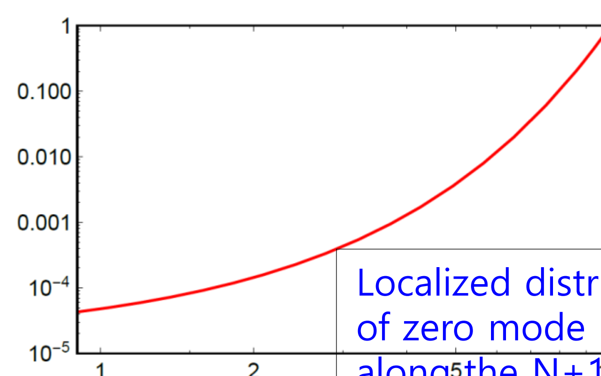
➔ Massless axion associated with unbroken $U(1)_{\rm CW}$ exponentially localized along the sites.

$$U(1)^{N+1} = \prod_i U(1)_i \quad \rightarrow \quad U(1)_{\rm CW}$$

$$\left[\, U(1)_i = e^{i\alpha_i Q_i} : \ \theta_i \rightarrow \theta_i + \alpha_i \,\right] \quad \left[Q_{\rm CW} \propto \sum_i q^i Q_i \right]$$

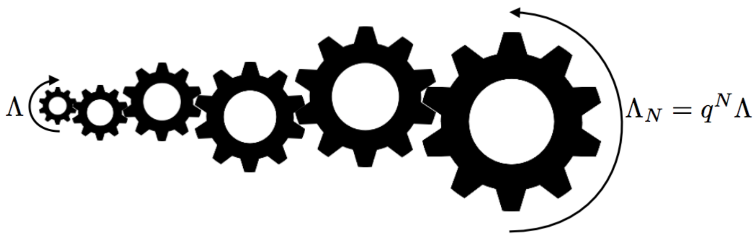
$$a^{(0)} = \text{canonically normalized massless mode}$$

$$\theta_i = q^i \frac{a^{(0)}}{f_{\rm eff}} + \text{massive modes}$$



Localized distribution of zero mode along the N+1 sites

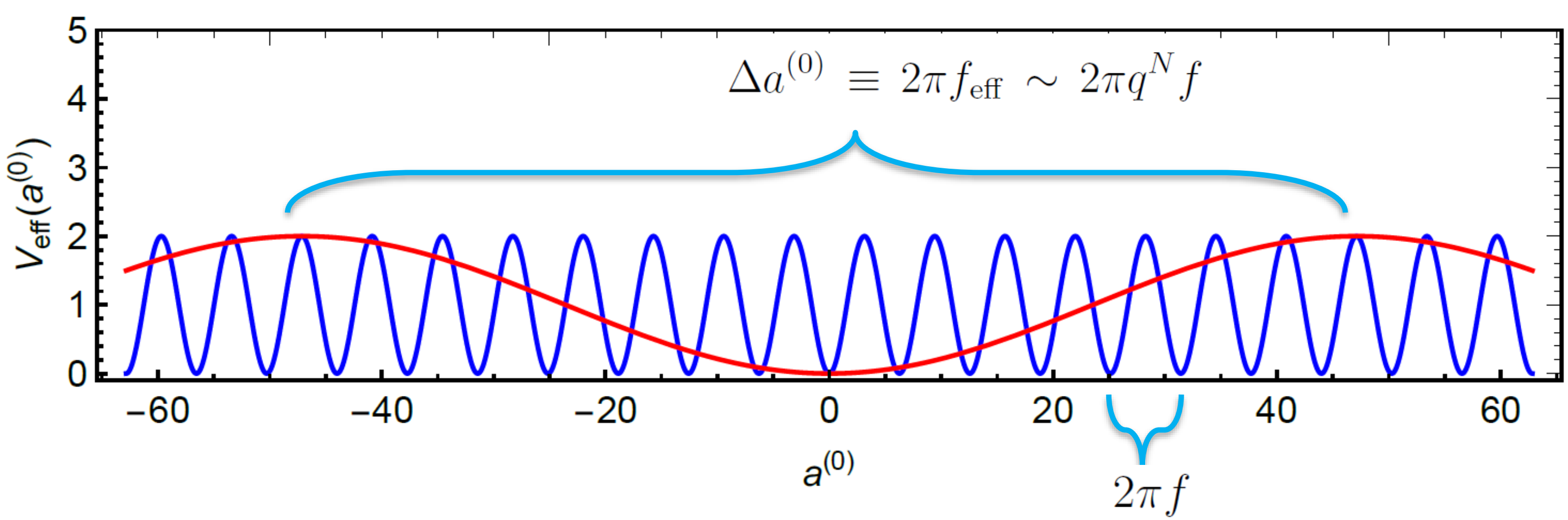
Extended field range and hierarchical couplings of zero mode axion



$$\Delta a^{(0)} \equiv 2\pi f_{\rm eff} \sim 2\pi q^N f$$

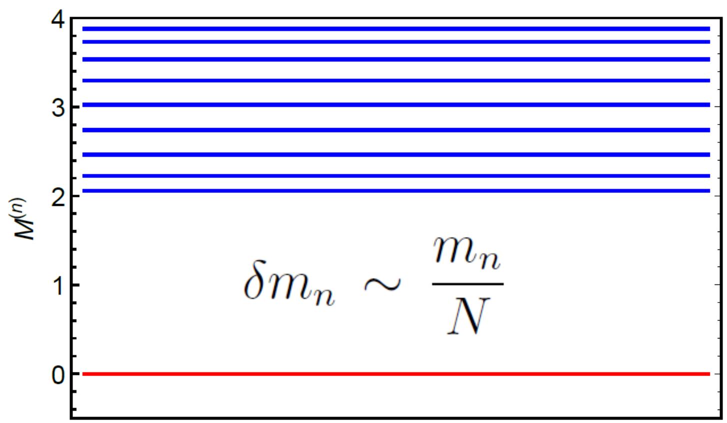
$$V_{\rm eff}(a^{(0)}) \;\; \text{induced by} \;\; \frac{1}{32\pi^2}\theta_0 G_0 \tilde{G}_0 = \frac{1}{32\pi^2}\frac{a^{(0)}}{f_{\rm eff}} G_0 \tilde{G}_0 + \ldots$$

$$\text{and} \;\; \frac{1}{32\pi^2}\theta_N G_N \tilde{G}_N = q^N \frac{1}{32\pi^2}\frac{a^{(0)}}{f_{\rm eff}} G_N \tilde{G}_N + \ldots$$

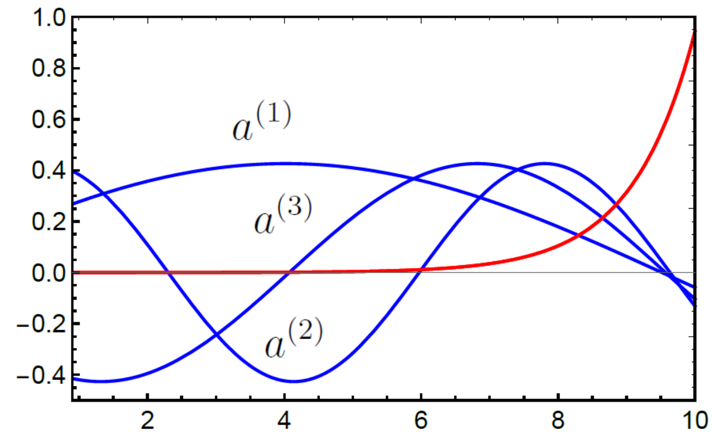


N massive axions with nearly degenerate masses
and oscillating distribution along the N+1 sites

[Giudice, McCullough 16]



Mass spectrum



Distribution of mass eigenstates along the N+1 sites

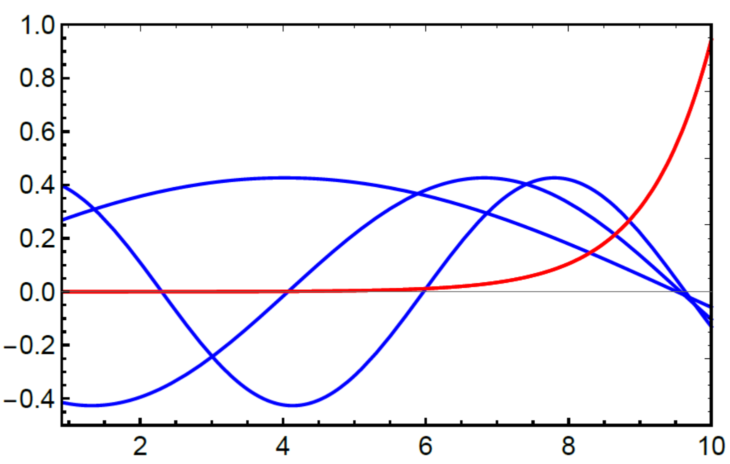
Summary

* Exponentially enlarged axion field range $f_{\rm eff} \sim q^N f$, which would avoid the *naive* WGC bound $f_{\rm eff} < M_{\rm Planck}/2\pi$.

* Exponential hierarchy ($\propto 1/q^N$) between the zero mode couplings at the 0-th site and the couplings at the N-th site
(Quantized couplings of the zero mode axion to instantons)

* Nearly degenerate N massive modes with $\delta m_n \sim m_n/N$.

* Exponential hierarchy ($\propto 1/q^N$) among the zero mode coupling and the massive mode couplings at the 0-th site



CW photons: *[Saraswat 16; Giudice, McCullough 16, H.M. Lee 17]*

(see H.M. Lee's talk)

$$\mathcal{L} = -\frac{1}{4g^2}\sum_{i=0}^{N} F_{\mu\nu i}F_i^{\mu\nu} - \frac{1}{2}m^2\sum_{i=0}^{N-1}\left[\partial_\mu\varphi_i - (A_{\mu i+1} - qA_{\mu i})\right]^2$$

➔ Exponentially localized zero mode photon associated with a localized unbroken $U(1)_{\rm CW}$ gauge symmetry

Mili-charged particle with exponentially small charge $Q_{\rm eff} = Q/q^N$ avoiding the WGC bound $Q_{\rm eff} > m/M_{\rm Planck}$.

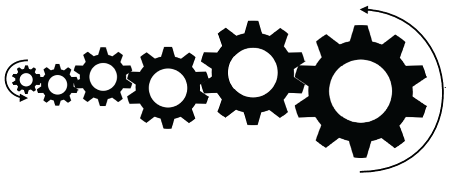
(Quantized gauge charges of the zero mode photon)

Other applications:

QCD axion, inflaton, flavor, neutrino, WIMP, (g-2)μ, ...

Higaki, Jeong, Kitajima, Takahashi 14,15
Giudice, McCullough 16
Kehagias, Riotto 16
Farina, Pappadopulo, Rompineve, Tesi 16
Hambye, Teresi, Tytgat 16
Ahmed, Dillon 16
Park, Shin 17
Hong, Kim, Shin 17

Continuum clockwork (CCW):



[Giudice, McCullough 16; Craig, Garcia, Sutherland 17]

$$\sum m^2 f^2 \cos\Big(\theta_{i+1} - q\theta_i\Big) = \sum m^2 f^2 \cos\Big((\theta_{i+1} - \theta_i) - (q-1)\theta_i\Big)$$

$$\Rightarrow \quad \int_0^{\pi R} dy\, \frac{f_5^3}{\epsilon^2} \cos\Big(\epsilon\,(\partial_y\theta - \mu\theta)\Big) \quad \Rightarrow \quad \int dy\, f_5^3\,(\partial_y\theta - \mu\theta)^2 = \int dy\,(\partial_y\Phi - \mu\Phi)^2$$

Continuum limit: $\epsilon \equiv \dfrac{\pi R}{N} = \text{lattice spacing} \;\rightarrow\; 0 \;\; \text{with} \;\; \ln q = \epsilon\mu = \dfrac{\mu\pi R}{N}$

$$\frac{1}{2} f^2 \sum_{i=0}^{N} \left(\partial_\mu \theta_i(x)\right)^2 + m^2 f^2 \sum_{i=0}^{N-1} \cos\left(\theta_{i+1} - q\theta_i\right)$$

$$\Rightarrow \qquad \frac{1}{2}\int dy \left[\left(\partial_\mu \Phi(x,y)\right)^2 - \left(\partial_y\Phi - \mu\Phi\right)^2\right]$$

The CW factor q can *not* be an integer anymore, so the periodicity of axions becomes obscure when the periodic 4D axions $f\theta_i(x)$ are promoted to a 5D field $\Phi(x,y)$, which limits the continuum CW to realize only partial features of the original discrete CW.

Two different approaches:

$$\frac{1}{2}\int dy\,\left[\left(\partial_\mu\Phi(x,y)\right)^2-\left(\partial_y\Phi-\mu\Phi\right)^2\right]$$ can be viewed as

[Giudice, McCullough 16]

$$\frac{1}{2}\int d^5x\,e^{2\mu y}\left((\partial_\mu\tilde{\Phi})^2-(\partial_y\tilde{\Phi})^2\right)=\int d^5x\sqrt{-G}\,G^{MN}\partial_M\tilde{\Phi}\partial_N\tilde{\Phi}$$

$\Big($ Linear dilaton model geometry $ds^2=e^{4\mu y/3}(\eta_{\mu\nu}dx^\mu dx^\nu+dy^2)$ for $\tilde{\Phi}=e^{-\mu y}\Phi\Big)$

or

[Craig, Garcia, Sutherland 17]

$$\frac{1}{2}\int d^5x\left[(\partial_\mu\Phi)^2-(\partial_y\Phi)^2-\mu^2\Phi^2-\mu\left(\delta(y)-\delta(y-\pi R)\right)\Phi^2\right]$$

(Bulk and boundary masses which are tuned appropriately to be invariant under the localized infinitesimal shift symmetry: $\delta\Phi=c_0e^{\mu y}$)

We are interested in 5D theory with a *periodic* $\Phi\equiv\Phi+2\pi f_5^{3/2}$, which clearly distinguishes Φ from $\tilde{\Phi}$.

Then these two approaches are not equivalent, and yield for instance different patterns of the zero mode axion couplings to the boundary fields.

CCW for 5D periodic axion

[KC, Im, Shin]

There are many good reasons to start with a periodic 5D axion:

$$\frac{1}{2}f_5^3\int d^5x\left[\left(\partial_\mu\theta\right)^2-\left(\partial_y\theta-\mu\theta\right)^2\right]$$

$$\Rightarrow\qquad \frac{1}{2}f_5^3\int d^5x\left[\left(\partial_\mu\theta\right)^2-\left(\partial_y\theta-\mu\sin\theta\right)^2\right]$$

$$=\frac{1}{2}f_5^3\int d^5x\left[\partial_M\theta\partial^M\theta+\frac{1}{2}\mu^2\cos 2\theta+2\mu\cos\theta\Big(\delta(y)-\delta(y-\pi R)\Big)\right]$$

The periodic μ-term for CW can not be attributed to the background geometry, but should originate from appropriately tuned bulk and boundary potentials.

The required form of the bulk and boundary potentials might be justified by the localized shift symmetry: $\delta\left(\tan\theta/2\right)=c_0e^{\mu y}$ and appropriate additional discrete symmetries.

While the physics of small fluctuations is same as the simple quadratic lagrangian, we can now study the behavior of the theory over the full range of the zero mode axion field, and compare to the case of discrete CW axions.

Canonically normalized zero mode axion:

$$\frac{a^{(0)}(x)}{f_4} = \left(e^{2\mu\pi R} - 1\right)^{1/2} \int^{\varphi^{(0)}} d\xi \left(1 + \frac{\xi^2}{4}\right)^{-1/2} \left(1 + \frac{\xi^2}{4} e^{2\mu\pi R}\right)^{-1/2}$$

$$\left(\tan\frac{\theta(x,y)}{2} = \frac{1}{2} e^{\mu y} \varphi^{(0)}(x), \quad f_4 = \sqrt{\frac{f_5^3}{2\mu}}\right)$$

Field range of $a^{(0)}$: $\quad \Delta a^{(0)} \simeq 2\pi f_4 \times \mu\pi R$

Couplings of $\delta a^{(0)}$ to the boundary instantons:

$$\int d^5x\, \theta(x,y) \left(\delta(y) \frac{\kappa_0}{32\pi^2} G_0^{\mu\nu} \tilde{G}_{0\mu\nu} + \delta(y-\pi R) \frac{\kappa_\pi}{32\pi^2} G_\pi^{\mu\nu} \tilde{G}_{\pi\mu\nu} \right)$$

$$\left(\kappa_{0,\pi} = \text{integers of order unity} \right)$$

$$\delta\theta(x,0) = e^{-\mu\pi R} \frac{\delta a^{(0)}}{f_4} + \text{massive fluctuations}$$

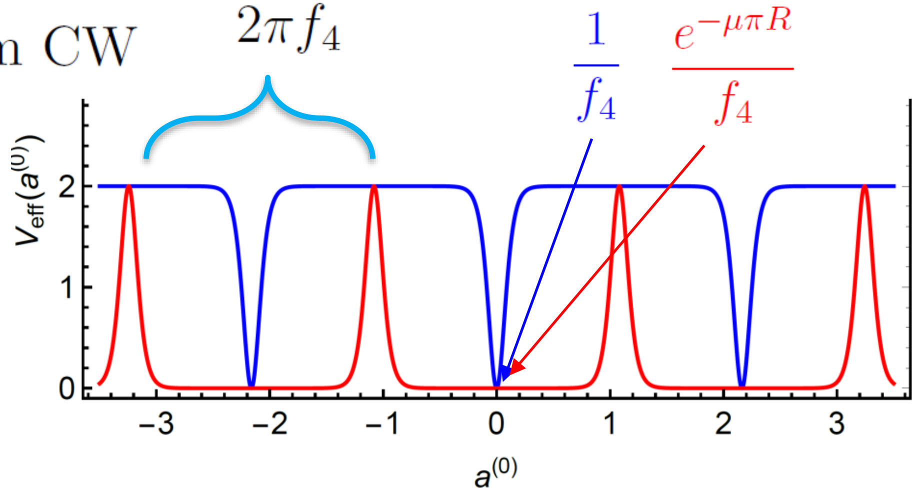
$$\delta\theta(x,\pi R) = \frac{\delta a^{(0)}}{f_4} + \text{massive fluctuations}$$

$$\Rightarrow \qquad \frac{e^{-\mu\pi R}\kappa_0}{32\pi^2} \frac{\delta a^{(0)}}{f_4} G_0^{\mu\nu} \tilde{G}_{0\mu\nu} + \frac{\kappa_\pi}{32\pi^2} \frac{\delta a^{(0)}}{f_4} G_\pi^{\mu\nu} \tilde{G}_{\pi\mu\nu}$$

There is no significant enhancement of the axion field range, which would satisfy the conventional WGC bound, although there is an exponential hierarchy among the couplings of the zero mode axion fluctuation.

Zero mode axion potential (over the full field range) induced by the boundary YM instantons $G_0\tilde{G}_0$ and $G_\pi\tilde{G}_\pi$:

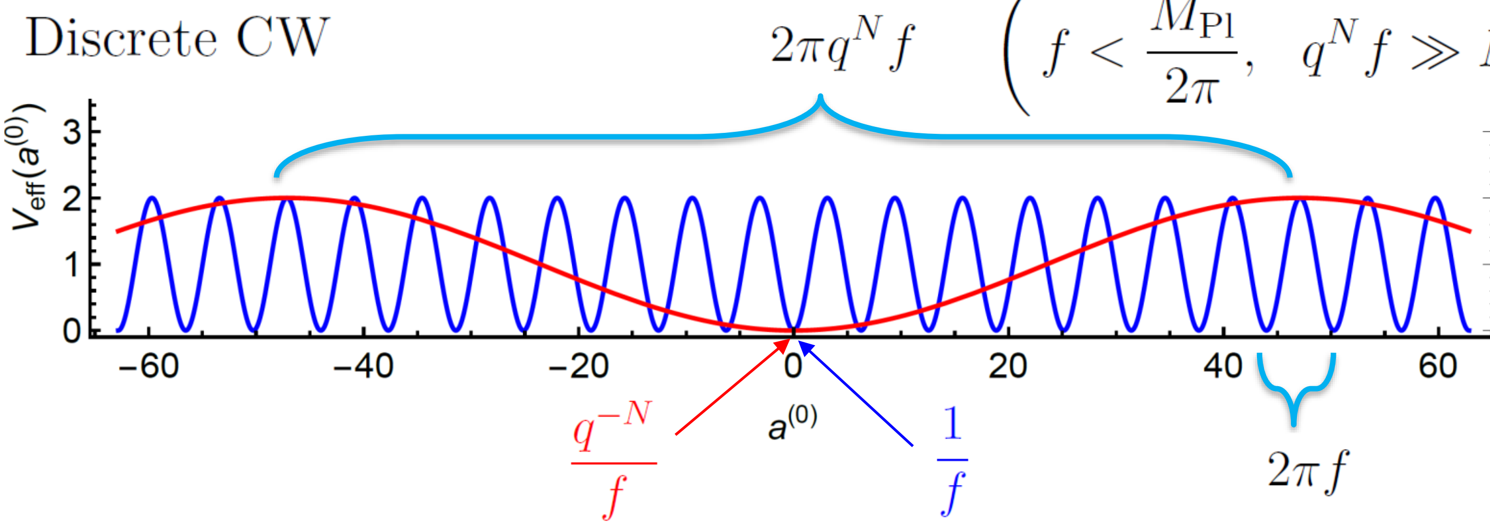
Continuum CW



$$\left(\frac{f_4}{M_{\rm Pl}} \sim \left(\frac{f_5}{M_5} \right)^{3/2} \right)$$

Discrete CW

$$\left(f < \frac{M_{\rm Pl}}{2\pi}, \quad q^N f \gg M_{\rm Pl} \right)$$



General continuum clockwork: [KC, Im, Shin]

$$\frac{1}{2}\int d^5x\,\left[\left(\partial_\mu\Phi\right)^2-\left(\partial_y\Phi-\mu\Phi\right)^2\right]$$

$$\Rightarrow\quad \frac{1}{2}\int d^5x\,\left[e^{2\mu_1 y}\left(\partial_\mu\Phi\right)^2-e^{2\mu_2 y}\left(\partial_y\Phi-\mu\Phi\right)^2\right]$$

$$\left(\text{Zero mode protected by a localized symmetry: } \delta\Phi=c_0e^{\mu y}\right)$$

$$\Leftarrow\quad \int d^5x\sqrt{-G}\left[G^{MN}\partial_M\Phi\partial_N\Phi-m_B^2(y)\Phi^2-m_b(y)\left(\frac{\delta(y)}{\sqrt{G_{55}}}-\frac{\delta(y-\pi R)}{\sqrt{G_{55}}}\right)\Phi^2\right]$$

$$\left(ds^2=G_{MN}dx^Mdx^N=e^{2k_1y}\eta_{\mu\nu}dx^\mu dx^\nu+e^{2k_2y}dy^2\right)$$

To get such extra-dimensional setup for light particles protected by localized symmetry from a 5D diffeomorphism and Lorentz invariant theory, we need a generalized version of the linear dilaton model yielding the necessary form of geometry and the y-dependent (dilaton-dependent) masses as a solution of the model.

Linear Dilaton model [Antoniadis, Arvanitaki, Dimopoulos, Giveon 11]

In Jordan frame

$$S_J = \frac{M_5^3}{2}\int d^5x\sqrt{-G}\,e^S\left[\mathcal{R} + G^{MN}\partial_M S\partial_N S + 4k^2 + \frac{8k}{\sqrt{G_{55}}}\left(\delta(y)-\delta(y-\pi R)\right)\right]$$

In Einstein frame

$$S_E = \frac{M_5^3}{2}\int d^5x\sqrt{-G}\left[\mathcal{R} + G^{MN}\partial_M S\partial_N S + 4\left(e^{-S/\sqrt{3}}k\right)^2 + \frac{8e^{-S/\sqrt{3}}k}{\sqrt{G_{55}}}\left(\delta(y)-\delta(y-\pi R)\right)\right]$$

Softly broken dilatonic shift symmetry: [Giudice, McCullough 16]

$$S \to S+\alpha, \quad k \to e^{\alpha/\sqrt{3}}k$$

Solution with same exponential warp factor and proper length factor:

$$ds^2 = e^{4ky/3}\left(\eta_{\mu\nu}dx^\mu dx^\nu + dy^2\right), \quad e^{-S/\sqrt{3}} = e^{-2ky/3}$$

Generalized Linear Dilaton model *[KC, Im, Shin]*

Take the dilatonic weight of the symmetry breaking spurion as a free parameter:

$$S \to S + \alpha, \quad k \to e^{c\alpha} k$$

$$S_E = \frac{M_5^3}{2} \int d^5x \sqrt{-G} \left[\mathcal{R} + G^{MN} \partial_M S \partial_N S + 4 \left(e^{-cS} k \right)^2 + \frac{8 e^{-cS} k}{\sqrt{G_{55}}} \left(\delta(y) - \delta(y - \pi R) \right) \right]$$

Generalized solution:

$$ds^2 = e^{2k_1 y} \eta_{\mu\nu} dx^\mu dx^\nu + e^{2k_2 y} dy^2, \quad e^{-cS} = e^{-k_2 y}$$

$$k_1 = \frac{2k}{\sqrt{12 - 9c^2}}, \quad k_2 = 3c^2 k_1$$

$c^2 \to 1/3$ (Linear dilaton limit), $c^2 \to 0$ (RS limit)

Generalized linear dilaton model with softly broken dilatonic shift symmetry provides a convenient framework for general CCW:

[KC, Im, Shin]

$$\frac{1}{2}\int d^5x\,\left[e^{2\mu_1 y}\left(\partial_\mu\Phi\right)^2 - e^{2\mu_2 y}\left(\partial_y\Phi - \mu\Phi\right)^2\right]$$

$$\left(\text{Zero mode protected by a localized symmetry: } \delta\Phi = c_0 e^{\mu y}\right)$$

$$\Leftarrow \quad \int d^5x\sqrt{-G}\left[G^{MN}\partial_M\Phi\partial_N\Phi - \left(m_B e^{-cS}\right)^2\Phi^2 - m_b e^{-cS}\left(\frac{\delta(y)}{\sqrt{G_{55}}} - \frac{\delta(y-\pi R)}{\sqrt{G_{55}}}\right)\Phi^2\right]$$

KK gravitons in generalized linear dilaton model

[KC, Im, Shin]

Graviton fluctuation around the solution:

$$ds^2 = \langle G_{MN} \rangle dx^M dx^N = e^{2k_1 y} \eta_{\mu\nu} dx^\mu dx^\nu + e^{2k_2 y} dy^2, \quad \langle e^{-cS} \rangle = e^{-k_2 y}$$

$$G_{\mu\nu} = e^{2k_1 y} \left(\eta_{\mu\nu} + 2h_{\mu\nu} \right)$$

$$\mathcal{L} = -M_5^3 \int_0^{\pi R} dy \, e^{(2k_1 + k_2)|y|} \left[\frac{1}{2} (\partial_\rho h_{\mu\nu})^2 + \frac{1}{2} e^{2(k_1 - k_2)|y|} (\partial_y h_{\mu\nu})^2 \right]$$

Large 4D Planck mass due to warping (k_1) + large volume (k_2)

$$M_{\rm Pl}^2 = \frac{M_5^3}{(2k_1 + k_2)} \left(e^{(2k_1 + k_2)\pi R} - 1 \right)$$

$$\boxed{\begin{aligned} k_{\rm eff} &\equiv k_1 + k_2/2 \\ \kappa &\equiv k_1 - k_2 \end{aligned}}$$

Detailed structure of the KK spectrum depends mostly on k_1-k_2

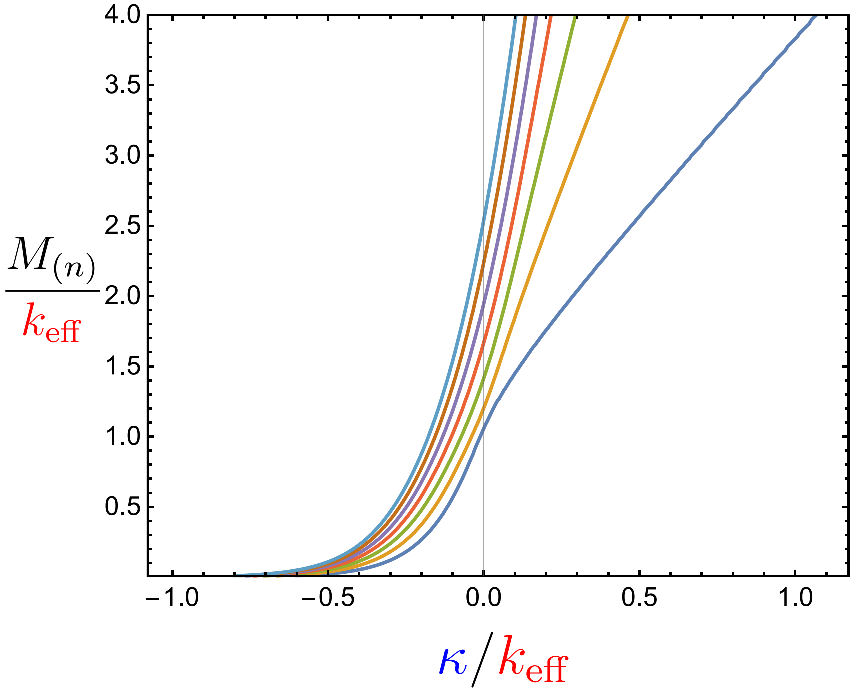
$$M_{(n)} \approx \left(n + \frac{1}{4} \right) \pi k_{\rm eff} \qquad \sqrt{k_{\rm eff}^2 + \frac{n^2}{R^2}} \qquad \frac{n}{L_5} \qquad \boxed{L_5 = \frac{1}{\pi} \int_0^{\pi R} dy \sqrt{G_{55}}}$$

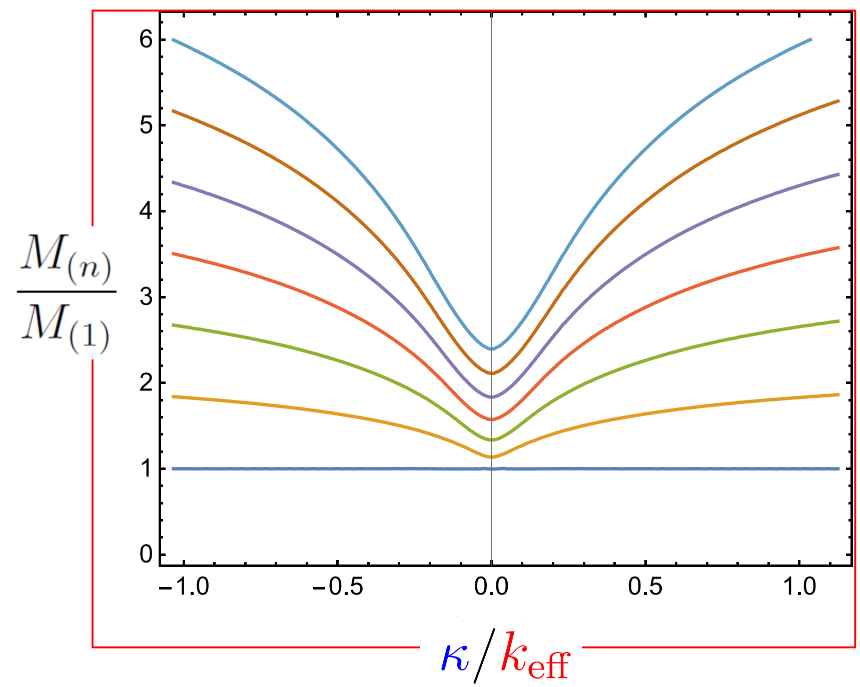
$$(\kappa = k_{\rm eff}, \ \text{RS}) \qquad (\kappa = 0, \ \text{CW}) \qquad (\kappa = -2k_{\rm eff}, \ \text{LED})$$

KK graviton spectrum

$$M_{(n)} = \begin{cases} (n - \frac{1}{4} + \frac{k_{\rm eff}}{2|\kappa|})\pi|\kappa|e^{-|\kappa|\pi R} & \text{for } \kappa \lesssim -k_{\rm eff} \\ \sqrt{k_{\rm eff}^2 + \frac{n^2}{R^2}} & \text{for } \kappa \simeq 0 \\ (n - \frac{1}{4} + \frac{k_{\rm eff}}{2\kappa})\pi\kappa & \text{for } \kappa \gtrsim k_{\rm eff} \end{cases}$$

$$k_{\rm eff} \equiv k_1 + k_2/2$$
$$\kappa \equiv k_1 - k_2$$

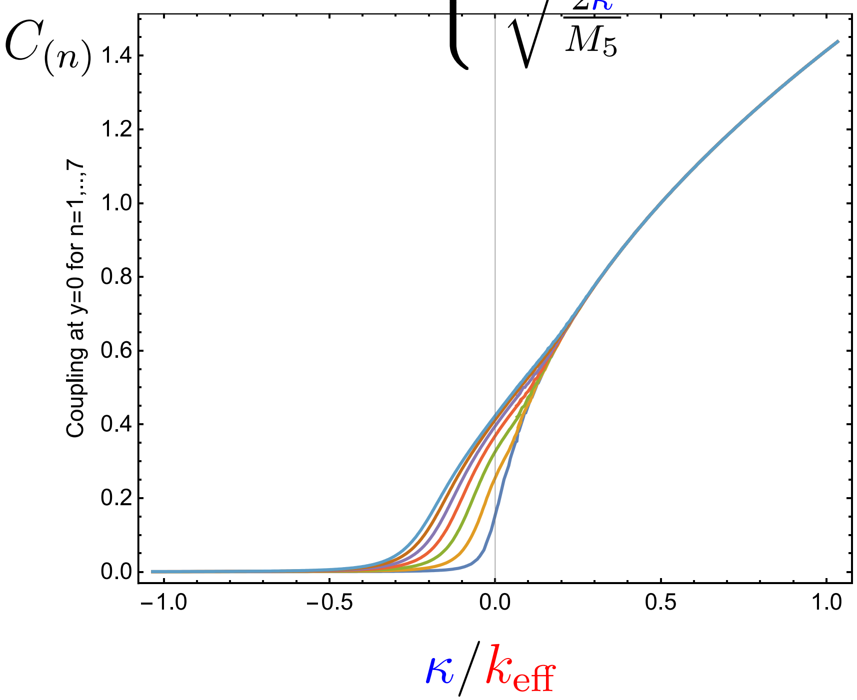




KK graviton couplings to the SM

$$\mathcal{L} = \sum_{n=1} \frac{C_{(n)}}{M_5}\, h^{(n)}_{\mu\nu}\, T^{\mu\nu}_{\rm SM}$$

$$C_{(n)} = \begin{cases} \sqrt{\frac{2|\kappa|\pi}{M_5}}\,\frac{e^{-k_{\rm eff}\pi R}}{\Gamma(k_{\rm eff}/|\kappa|)}\left[\frac{\pi}{2}\left(n-\frac{1}{4}+\frac{k_{\rm eff}}{2|\kappa|}\right)\right]^{\frac{k_{\rm eff}}{|\kappa|}-\frac{1}{2}} & \text{for } \kappa \lesssim -k_{\rm eff} \\ \sqrt{\frac{2}{M_5 \pi R}}\,\frac{n}{\sqrt{m^2R^2+n^2}} & \text{for } \kappa \simeq 0 \\ \sqrt{\frac{2\kappa}{M_5}} & \text{for } \kappa \gtrsim k_{\rm eff} \end{cases}$$



lighter KK gravitons : weaker couplings
CW limit : Compressed KK spectrum
RS limit : Universal (strong) couplings

Conclusion

- Clockwork (CW) is a mechanism to generate exponentially small couplings of light particle protected by a localized symmetry .

- CW axion and U(1) gauge boson indicate that the low energy implication of the Weak Gravity Conjecture crucially depend on the intermediate scale physics between the Quantum Gravity scale and the low energy scale.

- Continuum CW is an extra-dimensional realization of the clockwork mechanism, but has certain limitation (due to 5D locality, diffeomorphism, and Lorentz symmetries), so can reproduce only partial features of the discrete CW model.

- Yet the continuum CW is interesting as it can connect the CW mechanism with the known extra-dimensional solution of the hierarchy problem, which may have interesting phenomenological consequences.