Three dimensional higher spin holography

I. Lovrekovic, S. Fredenhagen

Universität Wien

Training school “Quantum spacetime and physics models”
CORFU 2017

Content

- Introduction and motivation for higher spin theory
- Holographic reconstruction
- Determination of coupling
- Outline

Introduction and motivation

- Higher spin (HS) theory, introduced by Vasiliev - missing link in the evolution from the field theories of lower spins to superstring theories

- It is a minimal theory with spectrum of HS fields

- Considered as a ultimate symmetry since it does not result from symmetry breaking

- It has no energy scale and can be thought as a toy model for fundamental theory beyond Planck scale

[Didenko, Skvortsov, 2015]

Introduction and motivation

- one can approach the question of what is string theory (e.g. M-theory) from the perspective of spectrum of elementary excitations
- massless modes of lower spins, e.g. graviton
- infinite tower of massive excitations of all spins
- HS theory had problems on flat space
- solution: consistent higher spin gravitational interactions in the cubic order at the action level, and to all orders in equations of motion on AdS
- strings that propagate in AdS spacetime are expected to be described by some theory of higher spins

Introduction and motivation

- Theories developed by Vasiliev: toy models of such a theory.
- The AdS/CFT dualities for higher spins allow further insight into a duality and this way also string theories
- In four bulk dimensions, Klebanov-Polyakov have conjectured that HS theory on AdS4 corresponds to large N limit of the 2+1 dimensional O(N) vector model: provide evidence towards the correct duality
- three dimensional analog is between Vasiliev theory in D=3 coupled to a pair of complex scalar fields and 't Hooft limit of the Wn minimal model CFT
- knowing this duality holds, one can investigate observables such as correlation functions

Introduction and motivation

- The CFT side can be represented with $\dfrac{SU(N)_k \oplus SU(N)_1}{SU(N)_{k+1}}$

$$N, k \to \infty, \qquad \lambda = \frac{N}{k+N}$$

- The correspondence of the three point functions between

  AdS_3 and CFT_2 has been determined

- We are interested to further analyse the cubic interaction

Holographic reconstruction

- For holographic reconstruction one has to analyse the correlation function from the CFT side using standard methods, and from the bulk side using the Witten diagrams
- From the CFT side, three point function is

$$\langle J_{s_1}(x_1|z_1)J_{s_2}(x_2|z_2)J_{s_3}(x_3|z_3)\rangle = N\prod_{i=1}^{3} c_{s_i} q_i^{\frac{1}{2}-\frac{\Delta}{4}}\Gamma\left(\frac{\Delta}{2}\right) J_{\frac{\Delta-2}{2}}(\sqrt{q_i})\frac{Y_1^{s_1}Y_2^{s_2}Y_3^{s_3}}{(x_{12}^2)^{\Delta/2}(x_{23}^2)^{\Delta/2}(x_{31}^2)^{\Delta/2}}$$

Holographic reconstruction

- For holographic reconstruction one has to analyse the correlation function from the CFT side using standard methods, and from the bulk side using the Witten diagrams
- From the CFT side, three point function is

$$\langle J_{s_1}(x_1|z_1)J_{s_2}(x_2|z_2)J_{s_3}(x_3|z_3)\rangle = N\prod{}_{i=1}^{3} c_{s_i} q_i^{\frac{1}{2}-\frac{\Delta}{4}}\Gamma\left(\frac{\Delta}{2}\right) J_{\frac{\Delta-2}{2}}(\sqrt{q_i})\frac{Y_1^{s_1}Y_2^{s_2}Y_3^{s_3}}{(x_{12}^2)^{\Delta/2}(x_{23}^2)^{\Delta/2}(x_{31}^2)^{\Delta/2}}$$

higher spin currents

Holographic reconstruction

- For holographic reconstruction one has to analyse the correlation function from the CFT side using standard methods, and from the bulk side using the Witten diagrams
- From the CFT side, three point function is

$$\langle J_{s_1}(x_1|z_1)J_{s_2}(x_2|z_2)J_{s_3}(x_3|z_3)\rangle = N\prod{}_{i=1}^{3} c_{s_i} q_i^{\frac{1}{2}-\frac{\Delta}{4}}\Gamma\left(\frac{\Delta}{2}\right) J_{\frac{\Delta-2}{2}}(\sqrt{q_i})\frac{Y_1^{s_1}Y_2^{s_2}Y_3^{s_3}}{(x_{12}^2)^{\Delta/2}(x_{23}^2)^{\Delta/2}(x_{31}^2)^{\Delta/2}}$$

contraction with auxiliary vector

Holographic reconstruction

- For holographic reconstruction one has to analyse the correlation function from the CFT side using standard methods, and from the bulk side using the Witten diagrams
- From the CFT side, three point function is

$$\langle J_{s_1}(x_1|z_1)J_{s_2}(x_2|z_2)J_{s_3}(x_3|z_3)\rangle = N\prod_{i=1}^{3} c_{s_i} q_i^{\frac{1}{2}-\frac{\Delta}{4}}\Gamma\left(\frac{\Delta}{2}\right)J_{\frac{\Delta-2}{2}}(\sqrt{q_i})\frac{Y_1^{s_1}Y_2^{s_2}Y_3^{s_3}}{(x_{12}^2)^{\Delta/2}(x_{23}^2)^{\Delta/2}(x_{31}^2)^{\Delta/2}}$$

coefficient determined from conventions

Holographic reconstruction

- For holographic reconstruction one has to analyse the correlation function from the CFT side using standard methods, and from the bulk side using the Witten diagrams
- From the CFT side, three point function is

$$\langle J_{s_1}(x_1|z_1)J_{s_2}(x_2|z_2)J_{s_3}(x_3|z_3)\rangle = N\prod_{i=1}^{3} c_{s_i} q_i^{\frac{1}{2}-\frac{\Delta}{4}}\Gamma\left(\frac{\Delta}{2}\right) J_{\frac{\Delta-2}{2}}(\sqrt{q_i})\frac{Y_1^{s_1}Y_2^{s_2}Y_3^{s_3}}{(x_{12}^2)^{\Delta/2}(x_{23}^2)^{\Delta/2}(x_{31}^2)^{\Delta/2}}$$

$$q_i = 2H_i\partial_{Y_{i+1}}\partial_{Y_{i+2}}$$

objects used for parametrisation of the most general conformal structure in CFT

Holographic reconstruction

- For holographic reconstruction one has to analyse the correlation function from the CFT side using standard methods, and from the bulk side using the Witten diagrams
- From the CFT side, three point function is

$$\langle J_{s_1}(x_1|z_1)J_{s_2}(x_2|z_2)J_{s_3}(x_3|z_3)\rangle = N\prod_{i=1}^{3} c_{s_i} q_i^{\frac{1}{2}-\frac{\Delta}{4}}\,\Gamma\left(\frac{\Delta}{2}\right) J_{\frac{\Delta-2}{2}}(\sqrt{q_i})\,\frac{Y_1^{s_1}Y_2^{s_2}Y_3^{s_3}}{(x_{12}^2)^{\Delta/2}(x_{23}^2)^{\Delta/2}(x_{31}^2)^{\Delta/2}}$$

scaling dimension d-2

Holographic reconstruction

- For holographic reconstruction one has to analyse the correlation function from the CFT side using standard methods, and from the bulk side using the Witten diagrams
- From the CFT side, three point function is

$$\langle J_{s_1}(x_1|z_1)J_{s_2}(x_2|z_2)J_{s_3}(x_3|z_3)\rangle = N\prod_{i=1}^{3} c_{s_i} q_i^{\frac{1}{2}-\frac{\Delta}{4}}\Gamma\left(\frac{\Delta}{2}\right)J_{\frac{\Delta-2}{2}}(\sqrt{q_i})\frac{Y_1^{s_1}Y_2^{s_2}Y_3^{s_3}}{(x_{12}^2)^{\Delta/2}(x_{23}^2)^{\Delta/2}(x_{31}^2)^{\Delta/2}}$$

standard Bessel function

Holographic reconstruction

- For holographic reconstruction one has to analyse the correlation function from the CFT side using standard methods, and from the bulk side using the Witten diagrams
- From the CFT side, three point function is

$$\langle J_{s_1}(x_1|z_1)J_{s_2}(x_2|z_2)J_{s_3}(x_3|z_3)\rangle = N\prod_{i=1}^{3} c_{s_i} q_i^{\frac{1}{2}-\frac{\Delta}{4}}\Gamma\left(\frac{\Delta}{2}\right) J_{\frac{\Delta-2}{2}}(\sqrt{q_i})\frac{Y_1^{s_1}Y_2^{s_2}Y_3^{s_3}}{(x_{12}^2)^{\Delta/2}(x_{23}^2)^{\Delta/2}(x_{31}^2)^{\Delta/2}}$$

$$x_{ij} = x_i - x_j$$

Holographic reconstruction

- normalising the currents to one we obtain

$$\langle O_0 O_0 J_s \rangle = \frac{\Gamma\left(\frac{d-2}{2}+s\right)}{\Gamma\left(\frac{d-2}{2}\right)} \sqrt{\frac{\Gamma\left(d+s-3\right)}{\Gamma\left(d+2s-3\right)}} \frac{2^{\frac{s+3}{2}}}{\sqrt{\Gamma\left(s+1\right)N}} \cdot f(x,z)$$

function that depends solely on the space-time coordinates and auxiliary vector

Holographic reconstruction

- From the bulk side we consider on-shell cubic interaction

in the ambient framework

$$\mathcal{V} = \sum_{s_1,s_2,s_3} g_{s_1,s_2,s_3} I^{0,0,0}_{s_1,s_2,s_3}(\Phi_i)$$

cubic vertex

Holographic reconstruction

- From the bulk side we consider on-shell cubic interaction

in the ambient framework

$$\mathcal{V} = \sum_{s_1,s_2,s_3} \underbrace{g_{s_1,s_2,s_3}}_{\text{coupling}} I^{0,0,0}_{s_1,s_2,s_3}(\Phi_i)$$

Holographic reconstruction

- From the bulk side we consider on-shell cubic interaction

in the ambient framework

$$\mathcal{V} = \sum_{s_1,s_2,s_3} g_{s_1,s_2,s_3} I^{0,0,0}_{s_1,s_2,s_3}(\Phi_i)$$

amplitude

- Computing a vertex of three scalars and an additional operator in this formalism

Holographic reconstruction

$$\langle O_0 O_0 J_s \rangle = g_{00s}\mathcal{A}_{00s} \Rightarrow$$

$$\boxed{\Rightarrow \frac{2^{\frac{3}{2}-\frac{s}{2}}\Gamma\left(\frac{d}{2}+s-1\right)}{\Gamma\left(\frac{d}{2}-1\right)\sqrt{\frac{N s!\Gamma(d+2s-3)}{\Gamma(d+s-3)}}}}$$

[Sleight, Taronna]

- We can see the agreement between the bulk and the CFT

side

Determination of coupling

- Three point function for arbitrary λ has been computed

$$\langle O_\pm(z_1)\overline{O}_\pm(z_2)J^{(s)}(z_3)\rangle = \frac{(-1)^{s-1}}{2\pi}\frac{\Gamma(s)^2}{\Gamma(2s-1)}\frac{\Gamma(s\pm\lambda)}{\Gamma(1\pm\lambda)}\left(\frac{z_{12}}{z_{13}z_{23}}\right)^s\langle O_\pm(z_1)\overline{O}_\pm(z_2)\rangle$$

[Ammon, Kraus, Perlmutter]

- taking into account normalisation of scalars
- comparing to 3pt function from holographic reconstruction

Determination of coupling

- we obtain the coupling

$$g_{00s} \to g_{00s}(\lambda, s)$$

- In this order we are interested to compare it to a computation from the Vasiliev's equations of motion from which we expect to obtain equal result

Determination of coupling

- From the most specific feature of higher spin theory, the fact that we do not have interacting action above the spin action for spin 3, but only equations of motion, we can verify the coupling

- The linearised equations of motion for the scalar field in the higher spin background field have the same coupling

Outline

- We have obtained the coupling of the 00s three point function for the higher spin theory in AdS_3/CFT_2 holography

- The result provides insight in the Vasiliev's higher spin theory, from which the result for the three point coupling is verifiable

- Analogous computation can be done for the higher point functions from both sides, however, in Vasiliev's theory equations quickly become more complicated, so computations of the couplings can for convenience be determined using the holographic reconstruction method and field theory computations

Thank you!