On non-supersymmetric string phenomenology

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• Introduction
• Non supersymmetric string models - Coordinate dependent compactifications
• A class of semi-realistic non supersymmetric string vacua
• One loop potential for the moduli fields - Cosmological constant
• Gauge coupling Thresholds - The decompactification problem
• Conclusions
The Standard Model

The Standard Model of particle interactions is a very successful theory.

However, it leaves a number of unanswered questions (Mass origin, flavor puzzle, charge quantization, a number of parameters, dark matter, hierarchy problem, gravity...)

Supersymmetry has been introduced to provide a solution to the gauge hierarchy problem and guarantee stability towards quantum corrections without fine-tuning. The introduction of SUSY at a few TeV leads also to coupling unification.

If SUSY were an exact symmetry of the nature every particle and its superpartner would have degenerate masses. However, this is not verified experimentally so SUSY must be broken.
Space-time supersymmetry is not required for consistency in string theory.

From the early days of the first string revolution it was known that heterotic strings comprise the SUSY $E_8 \times E_8$ and $SO(32)$ models as well as the non-supersymmetric tachyon free $SO(16) \times SO(16)$ theory.

However, non-supersymmetric model building has not received much attention.
Non-supersymmetric strings

Tachyons  Cosmological constant  ..............

Pandora’s box
Any scenario of supersymmetry breaking in the context of string theory has to address some important issues, as

- Resolve $M_W/M_P$ hierarchy
- Compatibility with gauge coupling evolution (unification) and weak string coupling constant
- Account for the smallness of the cosmological constant
- Resolve possible instabilities (tachyons)
- Moduli field stabilisation
A stringy Scherk–Schwartz mechanism involves an extra dimension $X^5$ and a conserved charge $Q$.

\[ \Phi (X^5 + 2\pi R) = e^{iQX^5} \Phi (X^5) \]

As a result we obtain a shifted tower of Kaluza–Klein states for charged fields, starting at $M_{KK} = \frac{|Q|}{2\pi R}$

\[ \Phi (X^5) = e^{iQX^5} \sum_{n \in \mathbb{Z}} e^{inX^5/R} \]

$Q = \text{Fermion number} \Rightarrow \text{leads to different masses for fermions-bosons (lying in the same supermultiplet) and thus to spontaneous breaking of supersymmetry.}$

The scale of SUSY breaking is related to the compactification radius $M \sim \frac{1}{R}$
Consider a big class of semirealistic $Z_2 \times Z_2$ heterotic string vacua for explicit realisations of the Scherk–Schwarz scenario. Study chirality, moduli potential and thresholds.

To this end we utilise both the free fermionic formulation and orbifold formulation. In the former we have full control of the spectrum in the latter we have explicit moduli dependence.
The class of models

We consider the class of four dimensional $N = 1$ heterotic models spontaneously broken to $N = 0$ via the Scherk–Schwarz mechanism.

The $E_8 \times E_8$ gauge symmetry is reduced to

$$SO(10) \times SO(8)^2 \times U(1)^2$$

We select models using the following criteria

- absence of tachyons
- $SO(10)$ chirality
- compatibility with Scherk–Schwarz of $N = 1$ SUSY
The free fermions in the light-cone gauge are:
left: $\psi^\mu, \chi^1,\ldots,^6, \ y^1,\ldots,^6, \omega^1,\ldots,^6$
right: $\bar{y}^1,\ldots,^6, \bar{\omega}^1,\ldots,^6, \bar{\eta}^1,^2,^3, \bar{\psi}^1,\ldots,^5, \bar{\phi}^1,\ldots,^8$

The class of vacua under consideration is defined by

$$
\beta_1 = 1 = \{\psi^\mu, \chi^1,\ldots,^6, \ y^1,\ldots,^6, \omega^1,\ldots,^6 | \bar{y}^1,\ldots,^6, \bar{\omega}^1,\ldots,^6, \bar{\eta}^1,^2,^3, \bar{\psi}^1,\ldots,^5, \bar{\phi}^1,\ldots,^8\} \\
\beta_2 = S = \{\psi^\mu, \chi^1,\ldots,^6\} \\
\beta_3 = T_1 = \{y^{12}, \omega^{12} | \bar{y}^{12}, \bar{\omega}^{12}\} \\
\beta_4 = T_2 = \{y^{34}, \omega^{34} | \bar{y}^{34}, \bar{\omega}^{34}\} \\
\beta_5 = T_3 = \{y^{56}, \omega^{56} | \bar{y}^{56}, \bar{\omega}^{56}\} \\
\beta_6 = b_1 = \{\chi^{34}, \chi^{56}, y^{34}, y^{56} | \bar{y}^{34}, \bar{y}^{56}, \bar{\psi}^1,\ldots,^5, \bar{\eta}^1\} \\
\beta_7 = b_2 = \{\chi^{12}, \chi^{56}, y^{12}, y^{56} | \bar{y}^{12}, \bar{y}^{56}, \bar{\psi}^1,\ldots,^5, \bar{\eta}^2\} \\
\beta_8 = z_1 = \{\bar{\phi}^1,\ldots,^4\} \\
\beta_9 = z_2 = \{\bar{\phi}^5,\ldots,^8\}
$$

and a variable set of $2^{9(9-1)/2} + 1 = 2^{36} + 1 \sim 10^{11}$ phases $c[\beta_i]$. 
Fermion generations, transforming as SO(10) spinorials, arise from $B^l_{pq} = S + b^l_{pq}$, $l = 1, 2, 3$ where $b^1_{pq} = b^1 + pT_2 + qT_3$, $b^2_{pq} = b^2 + pT_1 + qT_2$, $b^3_{pq} = x + b^1 + b^2 + pT_1 + qT_2$, with $p, q \in \{0, 1\}$, and $x = 1 + S + \sum_{i=1}^{3} T_i + \sum_{k=1}^{2} z_k$.

Number of generations \[ N = \sum_{l=1,2,3} \chi^l \] where

\[
\chi^1_{pq} = -4c \left[ B^1_{pq} \begin{bmatrix} B^1_{pq} \\ S + b_2 + (1 - q)T_3 \end{bmatrix} \right] P^1_{pq},
\]

\[
\chi^2_{pq} = -4c \left[ B^2_{pq} \begin{bmatrix} B^2_{pq} \\ S + b_1 + (1 - q)T_3 \end{bmatrix} \right] P^2_{pq},
\]

\[
\chi^3_{pq} = -4c \left[ B^3_{pq} \begin{bmatrix} B^3_{pq} \\ S + b_1 + (1 - q)T_1 \end{bmatrix} \right] P^3_{pq},
\]

and

\[
P^l_{pq} = \frac{1}{2^3} \left( 1 - c \left[ B^l_{pq} \begin{bmatrix} \end{bmatrix} T_l \right] \right) \left( 1 - c \left[ B^l_{pq} \begin{bmatrix} \end{bmatrix} z_1 \right] \right) \left( 1 - c \left[ B^l_{pq} \begin{bmatrix} \end{bmatrix} z_2 \right] \right)
\]
The one-loop partition function at the generic point reads

\[ Z = \frac{1}{\eta^{12}\eta^{24}} \frac{1}{2^3} \sum_{h_1, h_2, H} \frac{1}{2^3} \sum_{a, k, \rho} \frac{1}{2^3} \sum_{b, \ell, \sigma} (\eta h_1 - h_2) \]

\[ \times \vartheta[a][b] \vartheta[a+h_1][b+h_2] \vartheta[a-h_1-h_2][b-g_2] \]

\[ \times \Gamma^{(1)}_{2, 2} \left[ G_1 | h_1 \right] (T^{(1)}, U^{(1)}) \Gamma^{(2)}_{2, 2} \left[ G_2 | h_2 \right] (T^{(2)}, U^{(2)}) \Gamma^{(3)}_{2, 2} \left[ G_3 | \right] (T^{(3)}, U^{(3)}) \]

\[ \times \vartheta[k][\ell] \vartheta[k+h_1][\ell+g_2] \vartheta[k-h_1-h_2][\ell-g_1-g_2] \vartheta[\varphi] \vartheta[\varphi+H] \]

Where \( T^{(i)} = T_1^{(i)} + iT_2^{(i)} \), \( U^{(i)} = U_1^{(i)} + iU_2^{(i)} \) are the moduli of the three two tori and \( \eta(\tau) \) is the Dedekind eta function and \( \vartheta[\alpha][\beta](\tau) \) stand for the Jacobi theta functions.

**Connection with fermionic formulation**

Fermionic point \( T = \imath \) and \( U = (1 + \imath)/2 \)

Phase \( \Phi \left( c[\beta_i][\beta_j] \right) \)
Twisted/shifted lattices

\[ \Gamma_{2,2}^{[H_i |_G]}(T, U) = \begin{cases} \left| \frac{2\eta^3}{\sqrt{r^{[1-h]}}} \right|^2 , & (H_i, G_i) = (0, 0) \text{ or } (H_i, G_i) = (h, g) \\ \Gamma_{\text{shift}}^{[H_i]}(T, U), & h = g = 0 \\ 0, & \text{otherwise} \end{cases} \]

\[ \Gamma_{\text{shift}}^{[H_i]}(T, U) = \sum_{m_1, m_2} (-1)^{G(m_1 + n_2)} q_4^{1/4} |P_L|^2 \bar{q}_4^{1/4} |P_R|^2 , \]

with

\[ P_L = \frac{m_2 + \frac{H_i}{2} - U m_1 + T (n_1 + \frac{H_i}{2} + U n_2)}{\sqrt{T_2 U_2}} , \]

\[ P_R = \frac{m_2 + \frac{H_i}{2} - U m_1 + \bar{T} (n_1 + \frac{H_i}{2} + U n_2)}{\sqrt{T_2 U_2}} . \]
A preliminary scan shows that a number of approximately $7 \times 10^4$ models in the class under consideration satisfy all criteria.
Gravitino mass

At tree level the gravitino receives a mass

\[ m_{3/2} = \frac{|U^{(1)}|}{\sqrt{T_2^{(1)} U_2^{(1)}}} = \frac{1}{R_1} \]

for a square torus: \( T^{(1)} = \nu R_1 R_2, U^{(1)} = \nu R_2 / R_1 \)

The moduli \( T, U \) remain massless.

At \( R_1 \rightarrow \infty \) we have \( m_{3/2} = 0 \) and the supersymmetry is restored.
The effective potential at one loop as a function moduli $t_I$ is obtained by integrating the string partition function $Z(\tau_1, \tau_2; t_I)$ over the moduli space of the worldsheet torus $\Sigma_1$

$$V_{\text{one-loop}}(t_I) = -\frac{1}{2(2\pi)^4} \int_{\mathcal{F}} \frac{d^2 \tau}{\tau_2^3} Z(\tau, \bar{\tau}; t_I),$$

where $\tau = \tau_1 + i\tau_2$ is the complex structure on $\Sigma_1$ and $\mathcal{F} = \text{SL}(2; \mathbb{Z}) \backslash \mathbb{H}^+$ is a fundamental domain. This potential cannot be calculated analytically and it is also hard to calculate numerically (for general values of the moduli).
One loop moduli potentials

Typical one-loop potential versus the modulus $T_2$.

Undesirable features: SUSY breaking at the string scale, huge cosmological constant, region of tachyon instabilities
One loop potential: Analytic results

\[ Z = \frac{1}{2^8 \eta^{12} \bar{\eta}^{24}} \sum_{H_1, G_1=0,1} \Gamma_{2,2}^{\text{shift}} \left[ \begin{array}{c} H_1 \\ G_1 \end{array} \right] \left( T^{(1)}, U^{(1)} \right) \]

\[ \times \sum_{h_2, H=0,1} \sum_{k, \rho, \gamma_2, \gamma_3=0,1} \sum_{g_2, G=0,1} \sum_{\ell, \sigma, \delta_3, \delta_4=0,1} (-1)^{\hat{\Phi}'} \times \psi \left[ 1 + H_1 + h_2 \right]^2 \psi \left[ 1 + G_1 + g_2 \right] \]

\[ \times \bar{\psi} \left[ k \right]^6 \bar{\psi} \left[ k + h_2 \right]^2 \bar{\psi} \left[ \rho \right]^4 \bar{\psi} \left[ \rho + H \right]^4 \bar{\psi} \left[ \gamma_2 \right] \bar{\psi} \left[ \gamma_2 + h_2 \right] \]

\[ \times \bar{\psi} \left[ \gamma_2 \right] \bar{\psi} \left[ \gamma_2 + h_2 \right] \psi \left[ \gamma_3 \right] \psi \left[ \gamma_3 - h_3 \right] \]

The one loop integral can be unfolded in orbits

\[ -2(2\pi)^4 V_{\text{one-loop}}(T, U) = l_{0}^{0} + l_{\text{deg}}^{1} + l_{\text{nd}}^{0} + l_{\text{nd}}^{1} + l_{\text{nd}}^{1}. \]
One loop potential: Asymptotic limit

The asymptotic behaviour of the potential is dominated by the contribution of the orbit

\[ l_{\text{deg}}^{[0]} = \frac{2c^{[0]}(0, 0)}{\pi^3 T_2^2} \sum_{m_1, m_2 \in \mathbb{Z}} \frac{U_2^3}{|m_1 + \frac{1}{2} + Um_2|^6} + \frac{4\sqrt{2}}{\sqrt{T_2}} \sum_{N \geq 1} N^{3/2} c^{[0]}(N, 0) \]

\[ \times \sum_{m_1, m_2 \in \mathbb{Z}} \frac{U_2^{3/2}}{|m_1 + \frac{1}{2} + Um_2|^3} K_3 \left( 2\pi \sqrt{\frac{NT_2}{U_2}} \left| m_1 + \frac{1}{2} + Um_2 \right|^2 \right) \]

where \( K_s(z) \) is the modified Bessel function of the second kind.

\[ V_{\text{one-loop}}(R) = -\frac{(n_B - n_F)}{2^4 \pi^7 R^4} \sum_{m_1, m_2 \in \mathbb{Z}} \frac{U_2^3}{|m_1 + \frac{1}{2} + U m_2|^6} + e^{-\sqrt{2\pi R}} + \ldots \]

Super no scale models \( n_B = n_F \) at the generic point.

Cosmological constant is exponentially small.
The asymptotic form of the one-loop potential versus the modulus $T_2$ (dashed line) matched against the direct numerical evaluation of the integral (in dots).
We expand the partition function in powers of $q_r = e^{-2\pi \tau_2}$

$$Z = \sum_{n \in \mathbb{Z}/2} W_n q_r^n$$

The constant term at the fermionic point $W_0$ or the generic point $W_0^G$ is proportional to $n_B - n_F$.

<table>
<thead>
<tr>
<th></th>
<th>$W_0 &lt; 0$</th>
<th>$W_0 = 0$</th>
<th>$W_0 &gt; 0$</th>
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<td>$W_0^G &lt; 0$</td>
<td>3560</td>
<td>0</td>
<td>1856</td>
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<tr>
<td>$W_0^G = 0$</td>
<td>96</td>
<td>0</td>
<td>8848</td>
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<tr>
<td>$W_0^G &gt; 0$</td>
<td>0</td>
<td>0</td>
<td>62192</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>3656</td>
<td>0</td>
<td>72896</td>
</tr>
</tbody>
</table>

Table 1: Number of chiral models for the subclasses of models with $W_0^G$ positive/negative/zero and $W_0$ positive/negative.
One loop potentials: Super no scale models

7. $\# = 1392$

3. $\# = 32$

6. $\# = 288$

1. $\# = 96$

$T_2$
One loop potentials: Super no scale models
One loop potentials: Super no scale models
Gauge coupling Running - Thresholds

The gauge coupling running is calculable in the context of string theory. It turns out that they depend on the compactification moduli. At the one loop level

\[
\frac{16\pi^2}{g_i^2(\mu)} = k_a \frac{16\pi^2}{g_s^2} + b_a \log \frac{M_s^2}{\mu^2} + \Delta_a
\]

where \( M_s = g_s M_P \), \( M_P = 1/\sqrt{32G_N} \).

\( b_a \leftrightarrow \text{Massless modes} \quad \Delta_a \leftrightarrow \text{Massive modes} \)

\[
\Delta_a = \Delta_a(t_i) + \hat{\Delta}_a
\]
Decompactification problem

\[ \Delta'_a - \Delta'_b = \sum_i \left\{ -\alpha^i_{ab} \log \left[ T^i_2 U^i_2 \right. \right. \left| \eta(T^i) \eta(U^i) \right|^4 \] 
\[ \left. \left. -\beta^i_{ab} \log \left[ T^i_2 U^i_2 \right| \vartheta_4(T^i) \vartheta_2(U^i) \right|^4 \right] \right. \] 
\[ \left. -\gamma^i_{ab} \log \left[ \left| \hat{j}_2(T^i/2) - \hat{j}_2(U^i) \right|^4 \right| j_2(U^i) - 24 \right|^4 \right\} , \]

\( \alpha^i_{ab}, \beta^i_{ab}, \gamma^i_{ab} \) model dependent coefficients

The dominant growth at \( T^i_2 \gg 1 \)

\[ \Delta'_a = \alpha^i_a \left( \frac{\pi}{3} T^i_2 - \log T^i_2 \right) + \ldots , \]

Solutions?

\( a^i_a = 0, \ldots \)

C. Angelantonj, I. Florakis and M. Tsulaia (2014)

Florakis (2015)

Antoniadis (1990)
Computation of the thresholds

The dominant moduli dependent contribution is

\[ \Delta'_a = -\frac{k_a}{48} Y + \hat{\beta}_a \Delta, \]

where the universal part \( Y \) is defined as

\[ Y = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2} \Gamma_2,2(T, U) \left( \frac{\hat{E}_2 \hat{E}_4 \hat{E}_6 - \hat{E}_4^3}{\Delta} + 1008 \right), \]

and

\[ \Delta = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2} \Gamma_2,2(T, U) = -\log \left[ T_2 U_2 \ |\eta(T) \eta(U)|^4 \right]. \]

At the limit \( T_2 \gg 1 \)

\[ Y = 48\pi T_2 + \mathcal{O}(T_2^{-1}) \quad , \quad \Delta = \frac{\pi}{3} T_2 - \log T_2 + \mathcal{O}(e^{-2\pi T_2}) \]

and finally

\[ \Delta_a = \left( \frac{\hat{\beta}_a}{3} - k_a \right) \pi T_2 + \mathcal{O}(\log T_2). \]
Computation of the thresholds

A comprehensive scan over a class of $7 \times 10^4$ models with $SO(10) \times SO(8)^2 \times U(1)^2$ gauge symmetry yields for the non-abelian gauge couplings

\[ \hat{\beta}_a = 3k_a \]

<table>
<thead>
<tr>
<th>$\hat{b}_{10}$</th>
<th>$\hat{b}_8$</th>
<th>$\hat{b}_8'$</th>
<th># of models</th>
<th>%</th>
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<td>3</td>
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<td>3</td>
<td>29456</td>
<td>38.5</td>
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<tr>
<td>9</td>
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<td>-3</td>
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<td>9</td>
<td>14000</td>
<td>18.3</td>
</tr>
<tr>
<td>.</td>
<td>.</td>
<td>.</td>
<td>...</td>
<td>22.5</td>
</tr>
</tbody>
</table>

In a big class of vacua there is no decompactification problem for the gauge couplings.
Gauge coupling running

For models satisfying the decompactification condition \( \hat{\beta}_a = 3k_a \) the coupling running is

\[
\frac{16\pi^2}{g_a^2(\mu)} = k_a \frac{16\pi^2}{g_s^2} + \beta_a \log \frac{M_s^2}{\mu^2} + \beta'_a \log \left( \frac{2e^{1-\gamma} M_{KK}^2}{3\pi \sqrt{3} M_s^2} \right) + \ldots
\]

Here, \( \gamma \) is the Euler-Mascheroni constant, \( M_{KK} = 1/\sqrt{T_2} \) is the Kaluza-Klein scale. \( \beta_a = b_a^{(1)} + b_a^{(2)} + b_a^{(3)} \) and \( \beta'_a = b_a^{(1)} + b_a^{(2)} \) with \( b_a^{(1)} = \hat{\beta}_a \)
A Standard Model scenario

\[
\frac{k_2 + k_Y}{\alpha_s} = \frac{1}{\alpha_{em}} - \frac{\beta_2 + \beta_Y}{4\pi} \log \frac{M_s^2}{M_Z^2} - \frac{\beta'_2 + \beta'_Y}{4\pi} \log \left( \frac{2e^{1-\gamma}}{3\pi \sqrt{3}} \frac{M_{KK}^2}{M_s^2} \right)
\]

\[
\sin^2 \theta_W = \frac{k_2}{k_2 + k_Y} + \frac{\alpha_{em}}{4\pi} \left[ \frac{k_Y \beta_2 - k_2 \beta_Y}{k_2 + k_Y} \log \frac{M_s^2}{M_Z^2} + \frac{k_Y \beta'_2 - k_2 \beta'_Y}{k_2 + k_Y} \log \left( \frac{2e^{1-\gamma}}{3\pi \sqrt{3}} \frac{M_{KK}^2}{M_s^2} \right) \right]
\]

\[
\frac{1}{\alpha_3(M_Z)} = \frac{k_3}{\alpha_{em}(k_2 + k_Y)} + \frac{1}{4\pi} \left[ \left( \beta_3 - \frac{k_3 (\beta_2 + \beta_Y)}{k_2 + k_Y} \right) \log \frac{M_s^2}{M_Z^2} \right.

\left. + \left( \beta'_3 - \frac{k_3 (\beta'_2 + \beta'_Y)}{k_2 + k_Y} \right) \log \left( \frac{2e^{1-\gamma}}{3\pi \sqrt{3}} \frac{M_{KK}^2}{M_s^2} \right) \right]
\]
A Standard Model scenario

For \((\beta_Y, \beta_2, \beta_3) = (-7, -\frac{19}{6}, \frac{41}{6})\), \((k_Y, k_2, k_3) = (\frac{5}{3}, 1, 1)\) and 
\((\beta'_Y, \beta'_2, \beta'_3) = (-\frac{15}{2}, -\frac{43}{6}, -\frac{23}{3})\).
Conclusions

We have analysed a class of non supersymmetric heterotic vacua where SUSY is spontaneously broken via the Scherk–Schwartz mechanism. In this context we have constructed semi-realistic models with the following interesting characteristics

- Fermion chirality
- Dynamical determination of supersymmetry breaking scale $M_{\text{susy}} \ll M_{\text{Planck}}$
- Exponentially small cosmological constant
- Finite gauge coupling running (no decompactification problem)
- Examine more realistic vacua (e.g Pati–Salam)
- Could the decompactification condition be used as a vacuum selection criterion?
In the Free Fermionic Formulation of the heterotic string we can reduce the critical dimension and construct models in $D = 4$ by fermionizing the left movers and introducing non-linear supersymmetry among them.

$$f \rightarrow -e^{-i \pi \alpha(f)} f$$

A model is defined by a set of basis vectors $B = \{ \beta_1, \beta_2, \ldots, \beta_N \}$ and a set of $2^n(n-1)$ phases $c_{v_i v_j}, i > j$.

The basis vectors and phases are subject to constraints due to modular invariance, string amplitude factorization.