Emergent gravity and higher spin on covariant quantum spaces

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Quantum Structure of Spacetime

Motivation

requirements for fundamental theory

- simple, constructive
- finite dof (per "volume"), pre-geometric
- gauge theory

string theory: far-reaching, but issues:

- compactification (why? landscape? testable?)
- definition

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Matrix Models as fundamental theories of space-time & matter

- string theory (IKKT model (this talk!), BFSS model)
- NC gauge theory

outline:

- quantum (NC) spaces from matrix models
- *4D covariant quantum spaces*: fuzzy S^4
- fluctuations $\rightarrow$ higher spin theory in M.M.
- metric, vielbein; towards gravity
- cosmological space-times

HS, arXiv:1606.00769
M. Sperling, HS arXiv:1704.02863
M. Sperling, HS arXiv:1707.00885
HS, arXiv:1710.00xxx

The IKKT model

IKKT or IIB model Ishibashi, Kawai, Kitazawa, Tsuchiya 1996

$$S[X,\Psi] = -Tr\left([X^a,X^b][X^{a'},X^{b'}]\eta_{aa'}\eta_{bb'} \;+\bar\Psi\gamma_a[X^a,\Psi]\right)$$

$$X^a = X^{a\dagger} \in Mat(N,\mathbb{C})\,, \qquad a=0,...,9, \qquad N \text{ large}$$

gauge symmetry $X^a \to UX^aU^{-1}$, $SO(9,1)$, SUSY

proposed as non-perturbative definition of IIB string theory

origins:

- quantized Schild action for IIB superstring
- reduction of $10D$ SYM to point, N large
- $\mathcal{N}=4$ SYM on noncommutative $\mathbb{R}^4_\theta$

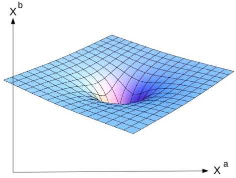
leads to "matrix geometry": (≈ NC geometry)

- $S_E \sim Tr[X^a, X^b]^2 \Rightarrow$ config's with small $[X^a, X^b] \neq 0$ dominate

  i.e. "almost-commutative" configurations, geometry
- $\exists$ quasi-coherent states $|x\rangle$, minimize $\sum_a \langle x|\Delta X_a^2|x\rangle$

$X^a \approx$ diag., spectrum $=: \mathcal{M} \subset \mathbb{R}^{10}$

$$\langle x|X^a|x'\rangle \approx \delta(x - x')x^a, \qquad x \in \mathcal{M}$$



NC branes embedded in target space $\mathbb{R}^{10}$

$$X^a \sim x^a : \qquad \mathcal{M} \hookrightarrow \mathbb{R}^{10}$$

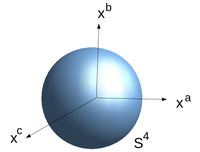
how to preserve Lorentz / $SO(4)$ covariance in 4D ?

- obstacle: NC spaces: $0 \neq [X^\mu, X^\nu] =: i\theta^{\mu\nu}$
  $\theta^{\mu\nu}$ breaks Lorentz invariance (in $D > 2$)
- $\exists$ fully covariant fuzzy four-sphere S^4_N
  Grosse-Klimcik-Presnajder 1996; Castelino-Lee-Taylor; Ramgoolam; Kimura; Hasebe; Medina-O'Connor; Karabail-Nair; Zhang-Hu 2001 (QHE!) ...
  price to pay: "internal structure" $\rightarrow$ higher spin theory
- here:
  work out higher spin modes on S^4_N
  higher-spin gauge theory from matrix models

covariant fuzzy four-spheres

5 hermitian matrices $X_a, \;\; a = 1, ..., 5$ acting on $\mathcal{H}_N$

$$\sum_a X_a^2 = R^2$$



covariance: $X_a \in End(\mathcal{H}_N)$ transform as vectors of $SO(5)$

$$\begin{aligned} [\mathcal{M}_{ab}, X_c] &= i(\delta_{ac} X_b - \delta_{bc} X_a), \\ [\mathcal{M}_{ab}, \mathcal{M}_{cd}] &= i(\delta_{ac}\mathcal{M}_{bd} - \delta_{ad}\mathcal{M}_{bc} - \delta_{bc}\mathcal{M}_{ad} + \delta_{bd}\mathcal{M}_{ac}) \,. \end{aligned}$$

$\mathcal{M}_{ab}$ ... $so(5)$ generators acting on $\mathcal{H}_N$

denote

$$[X^a, X^b] =: i\Theta^{ab}$$

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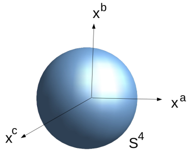

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oscillator construction: Grosse-Klimcik-Presnajder 1996; ...

$$X_a = \psi^\dagger_\alpha (\Gamma_a)^\alpha_\beta \psi^\beta, \qquad [\psi^\beta, \psi^\dagger_\alpha] = \delta^\beta_\alpha$$

acting on

$$\mathcal{H}_N = \psi^\dagger_{\alpha_1}...\psi^\dagger_{\alpha_N}|0\rangle \ \cong\ (\mathbb{C}^4)^{\otimes_S N} \ \cong\ (0, N)_{so(5)}$$

relations:

$$
\begin{aligned}
\Theta^{ab} &= r^2\,\mathcal{M}^{ab} \\
X_a X_a &= R^2 \sim \tfrac{1}{4} r^2 N^2 \\
\epsilon^{abcde} X_a X_b X_c X_d X_e &= (N+2) R^2 r^3 \qquad \text{(volume quantiz.)}
\end{aligned}
$$

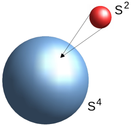
geometry from coherent states $|p\rangle$:

$$
\boxed{\{p_a = \langle p|X_a|p\rangle\} = S^4}
$$

closer inspection:

degeneracy of coherent states at $\to$ “internal” fuzzy S^2_{N+1} fiber semi-classical picture: hidden bundle structure

$$\begin{array}{ccc} \mathbb{C}P^3 & \ni & \psi \\ \downarrow & & \downarrow \\ S^4 & \ni & x^a = \psi^+ \Gamma^a \psi \end{array}$$



Ho-Ramgoolam, Medina-O'Connor, Abe, ...

fuzzy case: $[\Psi, \Psi^\dagger] = \delta$

$$X^a = \Psi^\dagger \Gamma^a \Psi$$

$$M^{ab} = \Psi^\dagger \Sigma^{ab} \Psi$$

...functions on fuzzy $\mathbb{C}P^3_N$

fuzzy S^4_N is really fuzzy $\mathbb{C}P^3_N$, hidden extra dimensions S^2 !

Poisson tensor

$$\theta^{\mu\nu}(x,\xi) \sim -i[X^\mu, X^\nu]$$

rotates along fiber $\xi \in S^2$!

is averaged $[\theta^{\mu\nu}(x,\xi)]_0 = 0$ over fiber $\to$ local $SO(4)$ preserved,

4D "covariant" quantum space

fields and harmonics on S^4_N

"functions" on S^4_N:

$$\phi \in End(\mathcal{H}_N) \cong \bigoplus_{s\leq n\leq N} (n-s, 2s) \quad = \bigoplus \ydiagram{5,2}$$

$(n, 0)$ modes = scalar functions on S^4:

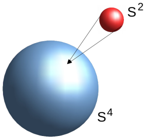
$$\phi(X) = \phi_{a_1\ldots a_n} X^{a_1} \ldots X^{a_n} \quad = \ydiagram{4}$$

$(n, 2)$ modes = selfdual 2-forms on S^4

$$\phi_{bc}(X)\mathcal{M}^{bc} = \phi_{a_1\ldots a_n b;c} X^{a_1} \ldots X^{a_n} \mathcal{M}^{bc} \quad = \ydiagram{4,1}$$

etc.

tower of higher spin modes, $s = 0, 1, 2, ..., N$
from "twisted" would-be KK modes on S^2



(local $SO(4)$ acts non-trivially on S^2 fiber) relation with spin s fields: isomorphism

$$(*, 2s) \quad \cong \quad T^{*\otimes_S s} S^4$$

$$\phi^{(s)} = \phi^{(s)}_{b_1\ldots b_s;c_1\ldots c_s}(x)\,\theta^{b_1c_1}\ldots\theta^{b_sc_s} \quad \mapsto \quad \phi^{(s)}_{c_1\ldots c_s}(x) = \phi^{(s)}_{b_1\ldots b_s;c_1\ldots c_s} x^{b_1}\ldots x^{b_s}$$

... "symbol" of $\phi \in \mathcal{C}^s$

M. Sperling & HS, arXiv:1707.00885

$(*, 2s)$ = symm., traceless, tang., div.-free rank s tensor field on S^4

$$\begin{aligned} \phi_{c_1\ldots c_s}(x) x^{c_i} &= 0\,, \\ \phi_{c_1\ldots c_s}(x) g^{c_1c_2} &= 0\,, \\ \partial^{c_i}\phi_{c_1\ldots c_s}(x) &= 0\,. \end{aligned}$$

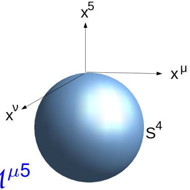
functions on $S^4_N \quad \cong \quad \mathfrak{hs}$ - valued functions on S^4

cf. Vasiliev theory!

local description: pick north pole $p \in S^4$

$\rightarrow$ tangential & radial generators

$$X^a = \begin{pmatrix} X^\mu \\ X^5 \end{pmatrix}, \qquad \mu = 1, ..., 4 \text{...tangential coords at } p$$



separate $SO(5)$ into $SO(4)$ & translations

$$\mathcal{M}^{ab} = \begin{pmatrix} \mathcal{M}^{\mu\nu} & \mathcal{P}^\mu \\ -\mathcal{P}^\mu & 0 \end{pmatrix} \qquad \text{where} \quad \mathcal{P}^\mu = \mathcal{M}^{\mu 5}$$

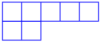
rescale

$$P_\mu = \frac{1}{R} g_{\mu\nu} \mathcal{P}^\nu \qquad \text{(cf. Wigner contraction)}$$

Poisson algebra

$$\begin{aligned} \{P_\mu, X^\nu\} &= \delta^\nu_\mu, \\ \{P_\mu, P_\nu\} &= \frac{1}{R^2} \mathcal{M}^{\mu\nu} \qquad \rightarrow 0 \\ \{X^\mu, X^\nu\} &=: \theta^{\mu\nu} = i r^2 \mathcal{M}^{\mu\nu} \qquad \approx 0 \qquad \text{(cf. Snyder space)} \end{aligned}$$

local form of $s = 2$ modes:

$$\phi^{(2)} = \phi_{a_1 \ldots a_n bc;de} x^{a_1} \ldots x^{a_n} \theta^{bd} \theta^{ce} \in$$ 

$$=: h_{\mu\nu}(x) P^\mu P^\nu + \omega_{\mu:\alpha\beta}(x) P^\mu \mathcal{M}^{\alpha\beta} + \Omega_{\alpha\beta;\mu\nu}(x) \mathcal{M}^{\alpha\beta} \mathcal{M}^{\mu\nu}$$

where

$$\omega_{\mu;\alpha\beta} = -\frac{n+1}{(n+2)(n+3)} (\partial_\alpha h_{\mu\beta} - \partial_\beta h_{\mu\alpha})$$

$$\Omega_{\alpha\beta;\mu\nu} = -\frac{1}{(n+2)(n+3)} \mathcal{R}_{\alpha\beta\mu\nu}[h]$$

... (lin.) spin connection and curvature determined by $h_{\mu\nu}$

(irrep of $\mathfrak{so}(5)$)

---

action for higher spin gauge theory on S^4_N ? $\rightarrow$ matrix models !

IKKT model

$$S = Tr(-[Y^a, Y^b][Y_a, Y_b] + \mu^2 Y^a Y_a)$$

eom: $(\Box + \frac{\mu^2}{2})X^a = 0, \quad \Box = [X^a, [X_a, .]]$

fact: S^4_N is solution (for $\mu^2 < 0$) (cf. HS arXiv:1510.05779)

add fluctuations $Y^a = X^a + \mathcal{A}^a$

expand action to second oder in $\mathcal{A}^a$

$$S[Y] = S[X] + \frac{2}{g^2}\mathrm{Tr}\mathcal{A}_a\Big((\Box + \mu^2)\delta^a_b + 2[[X^a, X^b], .] - [X^a, [X^b, .]]\Big)\mathcal{A}_b$$

fluctuations $\mathcal{A}$ describe

- gauge theory (NCFT) on $\mathcal{M}$ ("open strings“ ending on $\mathcal{M}$)
  effective metric $G^{\mu\nu}(x) \sim \theta^{\mu\mu'}\theta^{\nu\nu'} g_{\mu'\nu'}$
  (review: H.S. arXiv:1003.4134)
- matrix model provides off-shell formulation of $\mathfrak{hs}$ gauge theory

tangential fluctuations at $p \in S^4$:

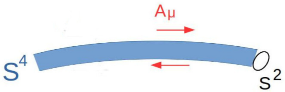
$$\boxed{\mathcal{A}^\mu = \theta^{\mu\nu} \mathbf{A}_\nu}$$

where

$$\mathbf{A}_\nu(x) = A_\nu(x) + \underbrace{A_{\nu\rho}(x) P^\rho + A_{\nu\rho\sigma}(x) \mathcal{M}^{\rho\sigma}}_{A_{\nu ab}\mathcal{M}^{ab} \ \ldots SO(5)} + \ldots$$

connection

... $\mathfrak{hs}$ - valued gauge field



rank 2 tensor field

$$A_{\nu\rho}(x) = \frac{1}{2}(h_{\nu\rho} + a_{\nu\rho}) \qquad h_{\nu\rho} = h_{\rho\nu} \quad \text{... metric fluctuation}$$

rank 3 tensor field

$$A_{\nu\rho\sigma}(x) \mathcal{M}^{\rho\sigma} \quad \ldots \ \mathfrak{so}(4) \text{ connection}$$

rank 1 field $A_\nu(x)$ ... $U(1)$ gauge field gauge transformations:

$Y^a \to UY^aU^{-1} = U(X^a + \mathcal{A}^a)U^{-1}$ leads to $(U = e^{i\Lambda})$

$$\delta\mathcal{A}^a = i[\Lambda, X^a] + i[\Lambda, \mathcal{A}^a]$$

expand

$$\Lambda = \Lambda_0 + \frac{1}{2}\Lambda_{ab}\mathcal{M}^{ab} + \ldots$$

... $U(1) \times SO(5) \times$ ... - valued gauge trafos

diffeos from $\delta_v := i[v_\rho P^\rho, .]$

$$\begin{aligned}
\delta h_{\mu\nu} &= (\partial_\mu v_\nu + \partial_\nu v_\mu) - v^\rho\partial_\rho h_{\mu\nu} + (\Lambda \cdot h)_{\mu\nu} \\
\delta A_{\mu\rho\sigma} &= \tfrac{1}{2}\partial_\mu\Lambda_{\sigma\rho}(x) - v^\rho\partial_\rho A_{\mu\rho\sigma} + (\Lambda \cdot A)_{\mu\rho\sigma}
\end{aligned}$$

etc.

metric and vielbein

consider scalar field $\phi = \phi(X)$ (= transversal fluctuation $\mathcal{A}^a(X)$)

kinetic term

$$-[X^\alpha, \phi][X_\alpha, \phi] \sim e^\alpha \phi e_\alpha \phi = \gamma^{\mu\nu} \partial_\mu \phi \partial_\nu \phi,$$

vielbein

$$\begin{aligned} e^\alpha &:= \{X^\alpha, .\} = e^{\alpha\mu} \partial_\mu \\ e^{\alpha\mu} &= \theta^{\alpha\mu} \end{aligned}$$

Poisson structure $\rightarrow$ frame bundle!

metric

$$\gamma^{\mu\nu} = g_{\alpha\beta} e^{\alpha\mu} e^{\beta\nu} = \frac{1}{4} \Delta^4 g^{\mu\nu}$$

averaging over internal S^2:

$$[e^{\alpha\nu}]_0 = 0, \qquad [\gamma^{\mu\nu}]_0 = \frac{\Delta^4}{4} g^{\mu\nu}$$ ... SO(5) invariant !

perturbed vielbein: $Y^a = X^a + \mathcal{A}^a$

$$e^a \quad := \{Y^a, .\} \sim e^{a\mu}[\mathcal{A}]\partial_\mu \qquad \text{... vielbein}$$

$$e^{\alpha\mu}[\mathcal{A}] \quad \sim \theta^{\alpha\beta}(\delta^\mu_\beta + A_{\beta\rho}g^{\rho\mu}) + \frac{1}{r^2}\theta^{\alpha\nu}\theta^{\rho\sigma}\{A_{\nu\rho\sigma}, x^\mu\}$$

using $\{P^\rho, X^\mu\} \sim g^{\rho\mu}$ (!)

linearize & average over fiber $\to$

$$\gamma^{\mu\nu} \sim e^{\alpha\mu}[\mathcal{A}]e_\alpha^{\ \nu}[\mathcal{A}] = \bar{\gamma}^{\mu\nu} + [\delta\gamma^{\mu\nu}]_0$$

complication:

graviton is combination

$$h_{\mu\nu} := [\delta\gamma_{\mu\nu}]_0 = \frac{\Delta^4}{4}(A_{\mu\nu} + \partial^\rho A_{\mu\rho\nu} + \partial^\rho A_{\nu\rho\mu})$$

- basic S^4_N : $\partial^\rho A_{\mu\rho\nu} \approx \sqrt{\Box} A_{\mu\nu}$ dominates
- generalized S^4_Λ : modes $A_{\mu\nu}, A_{\mu\rho\nu}$ independent, issue should be resolved (?)

action for spin 2 modes:

expand IKKT action to second oder in $\mathcal{A}^a$

$$S[Y] = S[X] + \frac{2}{g^2}\mathrm{Tr}\mathcal{A}_a \underbrace{\Big((\Box + \frac{1}{2}\mu^2)\delta^a_b + 2[[X^a, X^b], .\,] - [X^a, [X^b, .\,]]\Big)}_{\mathcal{D}^2} \mathcal{A}_b$$

for spin 2 modes $\mathcal{A}^\mu \sim \theta^{\mu\nu} A_{\nu\rho} P^\rho + \ldots$

$$\int \mathcal{A}\mathcal{D}^2\mathcal{A} \sim \int h_{\mu\nu} h^{\mu\nu}$$

coupling to matter:

$$S[\mathrm{matter}] \sim \int_{\mathcal{M}} d^4x\, h^{\mu\nu} T_{\mu\nu}$$

$\rightarrow$ auxiliary field $h_{\mu\nu} \sim T_{\mu\nu}$!

"graviton" doesn't propagate, due to constraint $h_{\mu\nu} \leftrightarrow A_{\mu\rho\nu} \sim \partial h$

exact treatment of spin 2 modes on S^4_N:

Marcus Sperling & HS, arXiv:1707.00885

3 independent "graviton" modes

$$
\begin{aligned}
h_{\mu\nu}[A^B] &\sim -\, c\,\left(1 + \tfrac{2}{\Box}\right)\; T_{\mu\nu} \\
h_{\mu\nu}[A^C] &\sim -\, 3c\left(1 + \tfrac{1}{2}\tfrac{1}{\sqrt{\Box}}\right)\; T_{\mu\nu} \\
h_{\mu\nu}[A^D] &\sim -\, \tfrac{1}{3}c\left(1 - \tfrac{1}{2}\tfrac{1}{\sqrt{\Box}}\right)\; T_{\mu\nu}
\end{aligned}
$$

$$
c = \frac{4}{5L_{NC}^4}\frac{g^2\ vol(\mathcal{S}^4)}{\dim(\mathcal{H})}
$$

combined metric fluct:

$$
h_{\mu\nu} \sim (1 + \frac{1}{R|P|} + \frac{1}{R^2|P|^2})T_{\mu\nu}
$$

... unphysical gravity

possible ways out:

1. 1-loop $\to$ induced gravity action $\sim \int h_{\mu\nu} \Box h^{\mu\nu}$
   $\to$ (lin.) Einstein equations (fine-tuning ...)
2. generalized fuzzy sphere S^4_Λ
   - extra $A_{\mu\nu}$ modes, promising — HS, arXiv:1606.00769
   - fuzzy extra dims

   Aschieri Grammatikopoulos HS Zoupanos hep-th/0606021

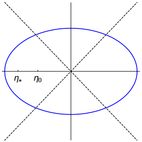
   HS, Zahn arXiv:1409.1440, Sperling, HS arXiv:1704.02863
3. cosmological space-times:
   H^4_n : $A_{\mu\rho\nu}$ no longer dominant (?)

Covariant cosmological space-times & BB

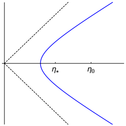
Lorentzian IKKT model

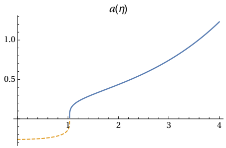
$$S = Tr(-[X^a, X^b][X^{a'}, X^{b'}]\eta_{aa'}\eta_{bb'} + m^2 X^i X^i - m_0^2 X^0 X^0)$$

- choose different masses $m_0^2 \neq m^2$
- $S^4_N \subset \mathbb{R}^{1,4}$ is solution for $m_0^2 < 0 < m^2$
  recollapsing universe



- $H^4_n \subset \mathbb{R}^{1,4}$ is solution for $m^2 < m_0^2 < 0$
  open universe





- fully $SO(4)$ covariant
- BB from signature change & 4-form flux

(H.S. arXiv:1710.00xxx)

summary

- $\exists$ 4D covariant quantum spaces, e.g. fuzzy S^4_N
  $\to$ regularized higher spin theory
- UV cutoff, finite d.o.f. per volume
  Poisson structure $\to$ frame bundle
- closely related to Vasiliev theory
- all ingredients for gravity,
  unrealistic for class. IKKT model on S^4_N
- gravity expected for
  - generalized S^4_Λ
  - cosmological H^4_n (?)
  - (modified action)
- Minkowski: cosmological space-times & BB (ongoing)

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- Minkowski: cosmological space-times & BB (ongoing)
- main problem: no position available ...