Do Unimodular Gravity and General Relativity have the same S matrix?

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Details in

* JHEP 1508 (2015) 078 by E. Álvarez, S. González-Martín, M. Herrero-Valea & C.P.M.
* Eur.Phys.J. C76 (2016) no.10, 554 by E. Álvarez, S. González-Martín, M.& C.P.M.
*JCAP 1707 (2017) no.07, 019 by C.P.M.
*PLB 773(2017)585 by S. González-Martín, M.& C.P.M.
*forthcoming paper by S. González-Martín, M.& C.P.M.

PLAN

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What's UG?

- What's Unimodular Gravity (UG)?

  It's a gravity theory with action

$$S_{UG} \equiv -M_P^{n-2} \int d^n x \sqrt{-\hat{g}} \left(R[\hat{g}_{\mu\nu}] + L_{\text{matt}}[\psi_i, \hat{g}_{\mu\nu}] \right)$$

  BUT with the space of metrics restricted by the condition

$$\hat{g} = -1$$

  ie, $\det \hat{g}_{\mu\nu}$ is not a dynamical variable!!!!

  Hence, additions of the type

$$\Lambda_0 \int d^n x \sqrt{-\hat{g}}$$

  are physically irrelevant.

UG Eq. of Motion

- EM = Trace-free equations (TFE)(see book by A. Zee)

$$R_{\mu\nu} - \frac{1}{n} R \hat{g}_{\mu\nu} = M_P^{2-n}(T_{\mu\nu} - \frac{1}{n} T \hat{g}_{\mu\nu})$$

obtained by variations of S_{UG}, with momentarily unconstrained $g_{\mu\nu}, \hat{g}_{\mu\nu} \to g_{\mu\nu}$, under traceless variations

$$\delta g_{\mu\nu} = \text{unconstrained variation}$$
$$\delta^t g_{\mu\nu} = \delta g_{\mu\nu} - \tfrac{1}{n} \delta g^{\mu}_{\mu}\, g_{\mu\nu} \implies \delta^t g = 0.$$

- Now, the 2nd Bianchi identity $\nabla_\mu R^{\mu\nu} = \frac{1}{2}\nabla^\nu R$ and TFE imply

$$\nabla_\mu((n-2)R + 2M_P^{2-n} T) = 0 \implies (n-2)R + 2M_P^{2-n} T = -2nC$$

- TFE and the previous consistency condition imply

$$R_{\mu\nu} - \frac{1}{2} R \hat{g}_{\mu\nu} - C\hat{g}_{\mu\nu} = M_P^{2-n} T_{\mu\nu}$$

ie, Einstein equations with a cosmological constant term but with $\hat{g}_{\mu\nu}/\ \hat{g} = -1$.

My motivation: Why Unimodular Gravity?

- 1) Solves in a Wilsonian way the huge disparity between the QFT "'prediction" for the vacuum energy and the experimentally observed cosmological constant: Vacuum energy is not seen by gravity. See
  i) S. Weinberg, Rev. Mod. Phys. 61 (1989) 1
  ii) G.F.R. Ellis, H. Van Elst, J. Murugan and J.P. Uzan, Class. Quantum Grav. 28(2011)22 5007
  iii) G.F.R Ellis, Gen. Relativ. Gravit. (2014) 46, 1619.
  In ii) there is a paragraph that runs thus

  "What about experiments? The experimental predictions for the two theories [General Relativity and UG] are the same, so no experiment can tell the difference between them, except for one fundamental feature: the EFE [Einstein's field equations](confirmed in the solar system and by binary pulsar measurements to high accuracy) together with QFT prediction for the vacuum energy density (confirmed by Casimir force measurements) give the wrong answer by many orders of magnitud; the TFE [UG] does not suffer this problem. In this respect, the TFE [UG] are strongly preferred by experiment."

  in i), one can read

  "In my view, the key question in deciding whether this [UG] is a plausible classical theory of gravitation is whether it can be obtained as the classical limit of any physically satisfactory quantum theory of gravitation.

- 2) When ordinary differential geometry is look at from the noncommutative geometry point of view some kind of quantisation of the volume form seems (at least to me!) to occur. See
  i) A. Chamseddine, A. Connes & V. Mukhanov, PRL 114 (2015) no.9, 091302
  ii) J.M. Gracia-Bondia, "Notes on quantum gravity and noncommutative geometry", Lect.Notes Phys. 807 (2010) 3-58

Quantum Unimodular Gravity as a theory of gravitons: free theory

- Redundancies (ie, Gauge symmetries) of UG: Not Full Diff. but Transverse Diff, since $g = -1$:

$$\delta^{trans}\hat{g}_{\mu\nu} = \nabla_\mu \varepsilon_\nu + \nabla_\nu \varepsilon_\mu \quad \text{with} \quad \nabla_\mu \varepsilon^\mu = 0$$

- Redundancies enough to go from 9 mathematical d.o.f to 2 physical d.o.f:

$$+9 \leftarrow e_{\mu\nu}(k) \quad \text{polarizations with} \quad e_{\mu\nu}(k) = e_{\nu\mu}(k), \quad e^\mu_\mu(k) = 0$$
$$-4 \leftarrow k^\mu e_{\mu\nu}(k) = 0(transversalityconditions)$$
$$\underline{-3} \leftarrow e_{\mu\nu}(k) \equiv e_{\mu\nu}(k) + k_\mu \varepsilon_\nu(k) + k_\nu \varepsilon_\mu(k), \quad k_\mu \varepsilon^\mu(k) = 0$$
$$+2 \leftarrow 2 \text{ helicity states}$$

- Actually, J.J. Van der Bij, H. Van Dam and Y.J. NG (Physica 116A (1982)307 showed that the UG free propagator in Minkowski space yields the propagation of gravitons (massless helicity 2 particles) between two sources and that the amplitude of this process is the same as in GR.

Quantum Unimodular Gravity as a theory of gravitons: free theory

- Actually, as shown by E. Alvarez, D. Blas, J. Garriga & E. Verdaguer NPB 756 (2006) 148, if one asks which quadratic actions on Minkowski of the general type

$$
\begin{aligned}
&s_{quad} \equiv \sum_{i=1}^{4} C_i \; \mathscr{O}^{(i)} \\
&\mathscr{O}^{(1)} \equiv \tfrac{1}{4}\partial_\mu h_{\rho\sigma}\partial^\mu h^{\rho\sigma}, \; \mathscr{O}^{(2)} \equiv -\tfrac{1}{2}\partial^\rho h_{\rho\sigma}\partial_\mu h^{\mu\sigma} \\
&\mathscr{O}^{(3)} \equiv \tfrac{1}{2}\partial_\mu h \partial_\lambda h^{\mu\lambda}, \; \mathscr{O}^{(4)} \equiv -\tfrac{1}{4}\partial_\mu h \partial^\mu h
\end{aligned}
$$

  are invariant under linear transverse diff

$$\delta^{trans} h_{\mu\nu} = \partial_\mu \varepsilon_\nu + \partial_\nu \varepsilon_\mu \quad \text{with} \quad \partial_\mu \varepsilon^\mu = 0$$

  one ends up with only two choices, namely

  - 1) Fierz-Pauli (corresponding to LDiff) and
  - 2) Linear Unimodular Gravity (corresponding to TDiff)

- Generalizes to curved space: C. Barcelo, R. Carballo-Rubio & L.J.Garay, PRD 89 (2014) 124019.

UG at one-loop: The 't Hooft and Veltman computation counterpart

- A classic paper in quantum GR (one-loop): 't Hooft & Veltman Ann.Inst.H.Poincare Phys.Theor. A20 (1974) 69-94
- Another classic paper (two-loop) in quantum GR: Goroff & Sagnotti Nucl.Phys. B266 (1986) 709-736
- Carry out the analogous computations for UG and compare with GR.
- The 't Hooft and Veltman computation counterpart is technically involved enough

Quantum Unimodular Gravity: Interacting theory

- Quantum Interacting theory defined by a path integral, a la BRST, over configuration space of
  - metric: $\hat{g}_{\mu\nu}$ with $\hat{g} = -1$, ghosts: c^T_μ with $\nabla^\mu c^T_\mu = 0$, etc....
- But, this is a constrained space which is not linear: Functional integration needs definition.
- Our approach: solve the constraints in terms of unconstrained fields as follows:
  - $\hat{g}_{\mu\nu} = (-g)^{-\frac{1}{n}} g_{\mu\nu}$, with $g_{\mu\nu}$ unconstrained
  - $c^T_\mu = (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu - R_{\mu\nu})c^\nu$, c_μ unconstrained
- This way 2 new redundancies (gauge symmetries) are introduced, namely
  - Weyl: $g_{\mu\nu} \to e^{2\omega} g_{\mu\nu}$ and
  - a gauge symmetry for the ghost: $c_\mu \to \nabla_\mu \phi$
- We have ended with –what is called in the Batalin-Vilkovisky formalism parlance– first-stage reducible gauge transformations:
  - a whole cascade of ghosts and antighosts are to be introduced and integrated over. SEE NEXT FRAME

Quantum Unimodular Gravity: Batalin-Vilkovisky field content

Notation: $(field)^{(n,m)}$, n = Grassmann no., m = Ghost number no.

- Unimodular Background metric $\tilde{g}^{(0,0)}_{\mu\nu} = (-\bar{g})^{-\frac{1}{n}}\,\bar{g}_{\mu\nu}$ and quantum gravitational field $h^{(0,0)}_{\mu\nu}$: $g_{\mu\nu} = \bar{g}_{\mu\nu} + (-\bar{g})^{\frac{1}{n}}\,h_{\mu\nu}$
- Transverse Diffeomorphisms
  - Ghost fields: $c^{(1,1)}_{\mu}$ (from TDiff of $h^{(0,0)}_{\mu\nu}$), $\phi^{(0,2)}$ (from transversality of c^{T}_{μ})
  - Antighosts and auxiliary fields couples
    - $(b^{(1,-1)}_{\mu}, f^{(0,0)}_{\mu})$ to gauge-fix the symmetry parametrized by $c^{(1,1)}_{\mu}$
    - $(\bar{c}^{(0,2)}, \pi^{(1,-1)})$ and $(c'^{(0,0)}, \pi'^{(1,1)})$ to gauge-fix the symmetry parametrized by $\phi^{(0,2)}$
- Weyl transformations
  - Ghost fields: $c^{(1,1)}$ (from Weyl of $h^{(0,0)}_{\mu\nu}$)
  - Antighosts and auxiliary fields couple
    - $(b^{(1,-1)}, f^{(0,0)})$ to gauge-fix Weyl

Background BRST transformations

- The FULL BRST operator

$$s = s_D + s_W$$

acting of the fields is introduced, so that
  - $s_D^2 = 0$, $s_W^2 = 0$ and $\{s_D, s_W\} = 0$
  - s, s_D and s_W all have Grassmann no. = 1 and ghost no = 1.
- s_D, coming form the TDiff, and s_W coming from Weyl transformations, act on the fields as follows:

Background BRST transformations definition

field	s_D	s_W
$\tilde{g}_{\mu\nu}$	0	0
$h_{\mu\nu}$	$\nabla_\mu c^T_\nu + \nabla_\nu c^T_\mu + c^{\rho\, T} \nabla_\rho h_{\mu\nu} + \nabla_\mu c^{\rho\, T} h_{\rho\nu} + \nabla_\nu c^{\rho\, T} h_{\rho\mu}$	$2c^{(1,1)} \left(\tilde{g}_{\mu\nu} + h_{\mu\nu}\right)$
$c^{(1,1)\mu}$	$\left(Q^{-1}\right)^{\mu}_{\nu} \left(c^{\rho\, T} \nabla_\rho c^{T\nu}\right) + \nabla^\mu \phi^{(0,2)}$	0
$\phi^{(0,2)}$	0	0
$b^{(1,-1)}_\mu$	$f^{(0,0)}_\mu$	0
$f^{(0,0)}_\mu$	0	0
$\bar{c}^{(0,-2)}$	$\pi^{(1,-1)}$	0
$\pi^{(1,-1)}$	0	0
$c'^{\,(0,0)}$	$\pi'^{\,(1,1)}$	0
$\pi'^{\,(1,1)}$	0	0
$c^{(1,1)}$	$c^{T\rho} \nabla_\rho c^{(1,1)}$	0
$b^{(1,-1)}$	$c^{T\rho} \nabla_\rho b^{(1,-1)}$	$f^{(0,0)}$
$f^{(0,0)}$	$c^{T\rho} \nabla_\rho f^{(0,0)}$	0

where $\left(Q^{-1}\right)^{\mu}_{\nu}$ denotes the inverse of the operator $Q_{\mu\nu} = \tilde{g}_{\mu\nu}\Box - R_{\mu\nu}$,
∇_μ and $R_{\mu\nu}$ defined with respect to $\tilde{g}_{\mu\nu}$

Quantum UG: The DeWitt effective action

- The DeWitt effective action $W[\tilde{g}_{\mu\nu}]$

$$e^{iW[\tilde{g}_{\mu\nu}]} = \int \mathscr{D}h_{\mu\nu}\mathscr{D}c_\mu\mathscr{D}b_\mu\mathscr{D}f_\mu\mathscr{D}\bar{c}\mathscr{D}\pi\mathscr{D}c'\mathscr{D}\pi'\mathscr{D}c\mathscr{D}b\mathscr{D}f \quad e^{iS_{UG}[\tilde{g}_{\mu\nu}+h_{\mu\nu}]+iS_{gf}}$$

$$S_{UG}[g_{\mu\nu}] = -M_P^{n-2}\int d^n x\, R[(-g)^{-\frac{1}{n}}g_{\mu\nu}]$$
$$S_{gf} = \int d^n x\, s(X_{TD}+X_W)\,,$$

- X_{TD} and X_W are to be chosen so that the term quadratic in the quantum fields is the closest to a minimal –the large energy behaviour is of Laplacian (to some power) type– differential operator: See JHEP 1508 (2015) 078.
- Recall that $W[\tilde{g}_{\mu\nu}]$ is gauge-invariant when $\tilde{g}_{\mu\nu}$ satisfies the classical equations of motion (ie, it's on-shell)

Quantum UG: Nonminimal Operator

The operator involving $h_{\mu\nu}$, f and c' is non-minimal. We need to use the Barvinsky & Vilkovisky technique (A. O. Barvinsky and G. A. Vilkovisky, Phys. Rept. **119**, 1 (1985)) to compute it.
The non minimal piece can be written

$$S = \int d^n x \, \Psi^A F_{AB} \Psi^B$$

$$\Psi^A = \begin{pmatrix} h^{\mu\nu} \\ f \\ c' \end{pmatrix}$$

Barvinsky & Vilkovisky method

The main idea is to introduce a parameter λ in the non-minimal part of the operator

$$F_{AB}(\nabla|\lambda) = \gamma_{AB}\Box + \lambda J^{\alpha\beta}_{AB}\nabla_\alpha\nabla_\beta + M_{AB} = D_{AB}(\nabla|\lambda) + M_{AB} \qquad 0 \leq \lambda \leq 1$$

so the effective action can be defined as

$$W(1) = W(0) - \frac{1}{2}\int_0^1 d\lambda' \, Tr\left[\frac{d\hat{F}(\lambda)}{d\lambda'}\hat{G}(\lambda')\right]$$

And if we find the inverse of $\hat{F}$ in the sense

$$\hat{F}(\nabla)\hat{K}(\nabla) = \Box^m + \hat{M}(\nabla)$$

we can expand the Green function as a power series in $\hat{M}$

$$\hat{G} = -\hat{K}\sum_{p=0}^{4}(-1)^p\,\hat{M}_p\frac{\mathbb{I}}{\Box^{m(p+1)}} + \ldots$$

so the trace can be computed with some effort and help from Mathematica's xAct. Indeed,$\rightarrow$

Quantum UG: An involved trace

$$
\begin{aligned}
\nabla_\mu \nabla_\nu \nabla_\alpha \nabla_\beta \frac{\mathbb{I}}{\Box^2} \quad = \quad & \frac{\sqrt{g}}{8(n-4)\pi^2} \Big\{ \Big[\frac{1}{36} \left(R_{\mu\nu} R_{\alpha\beta} + R_{\mu\alpha} R_{\nu\beta} + R_{\mu\beta} R_{\nu\alpha} \right) + \frac{1}{180} \left(R^\lambda_\mu \left(11 R_{\nu\alpha\beta\lambda} - R_{\beta\alpha\nu\lambda} \right) + R^\lambda_\nu \left(11 R_{\mu\alpha\beta\lambda} - R_{\beta\alpha\mu\lambda} \right) + R^\lambda_\alpha \left(11 R_{\mu\nu\beta\lambda} - R_{\beta\nu\mu\lambda} \right) + R^\lambda_\beta \left(11 R_{\mu\nu\alpha\lambda} - R_{\alpha\nu\mu\lambda} \right) \right) \\
& + \frac{1}{90} \left(R_\mu{}^\lambda{}_\nu{}^\sigma \left(R_{\lambda\alpha\sigma\beta} + R_{\lambda\beta\sigma\alpha} \right) + R_\mu{}^\lambda{}_\alpha{}^\sigma \left(R_{\lambda\nu\sigma\beta} + R_{\lambda\beta\sigma\nu} \right) + R_\mu{}^\lambda{}_\beta{}^\sigma \left(R_{\lambda\nu\sigma\alpha} + R_{\lambda\alpha\sigma\nu} \right) \right) + \\
& + \frac{1}{20} \left(\nabla_\mu \nabla_\nu R_{\alpha\beta} + \nabla_\mu \nabla_\alpha R_{\nu\beta} + \nabla_\mu \nabla_\beta R_{\nu\alpha} + \nabla_n \nabla_a R_{\mu\beta} + \nabla_\nu \nabla_\beta R_{\mu\alpha} + \nabla_a \nabla_\beta R_{\mu\nu} \right) \Big] \mathbb{I} + \\
& + \frac{1}{12} \left[R_{\mu\nu} \hat{\mathscr{R}}_{\alpha\beta} + R_{\mu\alpha} \hat{\mathscr{R}}_{\nu\beta} + R_{\mu\beta} \hat{\mathscr{R}}_{\nu\alpha} + R_{\nu\alpha} \hat{\mathscr{R}}_{\mu\beta} + R_{\nu\beta} \hat{\mathscr{R}}_{\mu\alpha} + R_{\alpha\beta} \hat{\mathscr{R}}_{\mu\nu} \right] + \\
& + \frac{1}{2} \left[\nabla_\mu \nabla_\nu \hat{\mathscr{R}}_{\alpha\beta} + \nabla_\mu \nabla_\alpha \hat{\mathscr{R}}_{\nu,b} + \nabla_\nu \nabla_\alpha \hat{\mathscr{R}}_{\mu\beta} \right] + \frac{1}{8} \Big[\hat{\mathscr{R}}_{\mu\nu} \hat{\mathscr{R}}_{\alpha\beta} + \hat{\mathscr{R}}_{\alpha\beta} \hat{\mathscr{R}}_{\mu\nu} + \hat{\mathscr{R}}_{\mu\alpha} \hat{\mathscr{R}}_{\nu\beta} + \\
& + \hat{\mathscr{R}}_{\nu\beta} \hat{\mathscr{R}}_{\mu\alpha} + \hat{\mathscr{R}}_{\mu\beta} \hat{\mathscr{R}}_{\nu\alpha} + \hat{\mathscr{R}}_{\nu\alpha} \hat{\mathscr{R}}_{\mu\beta} \Big] - \frac{1}{12} \Big[\hat{\mathscr{R}}_{\mu\lambda} \left(R^\lambda_{\alpha\nu\beta} + R^\lambda_{\beta\nu\alpha} \right) + \hat{\mathscr{R}}_{\nu\lambda} \left(R^\lambda_{\alpha\mu\beta} + R^\lambda_{\beta\mu\alpha} \right) + \\
& + \hat{\mathscr{R}}_{\alpha\lambda} \left(R^\lambda_{\nu\mu\beta} + R^\lambda_{\beta\mu\nu} \right) + \hat{\mathscr{R}}_{\beta\lambda} \left(R^\lambda_{\mu\nu\alpha} + R^\lambda_{\alpha\nu\mu} \right) \Big] - \frac{1}{2} \left[-\frac{1}{9} \left(R_{\alpha\mu\beta\nu} + R_{\beta\mu\alpha\nu} \right) R \mathbb{I} + \right. \\
& + g_{\mu\nu} \Big[\Big(\frac{1}{36} R_{\alpha\beta} R + \frac{1}{90} R^{\lambda\sigma} R_{\lambda\alpha\sigma\beta} + \frac{1}{90} R_{\rho\sigma\lambda\alpha} R^{\rho\sigma\lambda}{}_\beta - \frac{1}{45} R_{\alpha\lambda} R^\lambda_\beta + \frac{1}{60} \Box R_{\alpha\beta} + \frac{1}{20} \nabla_\alpha \nabla_\beta R \Big) \mathbb{I} + \\
& + \frac{1}{12} \left(\hat{\mathscr{R}}_{\alpha\lambda} \hat{\mathscr{R}}^\lambda_\beta + \hat{\mathscr{R}}_{\beta\lambda} \hat{\mathscr{R}}^\lambda_\alpha \right) - \frac{1}{12} \left(\nabla_\alpha \nabla^\lambda \hat{\mathscr{R}}_{\lambda\beta} + \nabla_\beta \nabla^\lambda \hat{\mathscr{R}}_{\lambda\alpha} \right) + \frac{1}{12} R \hat{\mathscr{R}}_{\alpha\beta} \Big] + \\
& + g_{\mu\alpha} \Big[\Big(\frac{1}{36} R_{\nu\beta} R + \frac{1}{90} R^{\lambda\sigma} R_{\lambda\nu\sigma\beta} + \frac{1}{90} R_{\rho\sigma\lambda\nu} R^{\rho\sigma\lambda}{}_\beta - \frac{1}{45} R_{\nu\lambda} R^\lambda_\beta + \frac{1}{60} \Box R_{\nu\beta} + \frac{1}{20} \nabla_\nu \nabla_\beta R \Big) \mathbb{I} + \\
& + \frac{1}{12} \left(\hat{\mathscr{R}}_{\nu\lambda} \hat{\mathscr{R}}^\lambda_\beta + \hat{\mathscr{R}}_{\beta\lambda} \hat{\mathscr{R}}^\lambda_\nu \right) - \frac{1}{12} \left(\nabla_\nu \nabla^\lambda \hat{\mathscr{R}}_{\lambda\beta} + \nabla_\beta \nabla^\lambda \hat{\mathscr{R}}_{\lambda\nu} \right) + \frac{1}{12} R \hat{\mathscr{R}}_{\nu\beta} \Big] + \\
& + g_{\mu\beta} \Big[\Big(\frac{1}{36} R_{\nu\alpha} R + \frac{1}{90} R^{\lambda\sigma} R_{\lambda\nu\sigma\alpha} + \frac{1}{90} R_{\rho\sigma\lambda\nu} R^{\rho\sigma\lambda}{}_\alpha - \frac{1}{45} R_{\nu\lambda} R^\lambda_\alpha + \frac{1}{60} \Box R_{\nu\alpha} + \frac{1}{20} \nabla_\nu \nabla_\alpha R \Big) \mathbb{I} + \\
& + \frac{1}{12} \left(\hat{\mathscr{R}}_{\nu\lambda} \hat{\mathscr{R}}^\lambda_\alpha + \hat{\mathscr{R}}_{\alpha\lambda} \hat{\mathscr{R}}^\lambda_\nu \right) - \frac{1}{12} \left(\nabla_\nu \nabla^\lambda \hat{\mathscr{R}}_{\lambda\alpha} + \nabla_\alpha \nabla^\lambda \hat{\mathscr{R}}_{\lambda\nu} \right) + \frac{1}{12} R \hat{\mathscr{R}}_{\nu\alpha} \Big] + \\
& + g_{\nu\alpha} \Big[\Big(\frac{1}{36} R_{\mu\beta} R + \frac{1}{90} R^{\lambda\sigma} R_{\lambda\mu\sigma\beta} + \frac{1}{90} R_{\rho\sigma\lambda\mu} R^{\rho\sigma\lambda}{}_\beta - \frac{1}{45} R_{\mu\lambda} R^\lambda_\beta + \frac{1}{60} \Box R_{\mu\beta} + \frac{1}{20} \nabla_\mu \nabla_\beta R \Big) \mathbb{I} + \\
& + \frac{1}{12} \left(\hat{\mathscr{R}}_{\mu\lambda} \hat{\mathscr{R}}^\lambda_\beta + \hat{\mathscr{R}}_{\beta\lambda} \hat{\mathscr{R}}^\lambda_\mu \right) - \frac{1}{12} \left(\nabla_\mu \nabla^\lambda \hat{\mathscr{R}}_{\lambda\beta} + \nabla_\beta \nabla^\lambda \hat{\mathscr{R}}_{\lambda\mu} \right) + \frac{1}{12} R \hat{\mathscr{R}}_{\mu\beta} \Big] + \\
& + g_{\nu\beta} \Big[\Big(\frac{1}{36} R_{\mu\alpha} R + \frac{1}{90} R^{\lambda\sigma} R_{\lambda\mu\sigma\alpha} + \frac{1}{90} R_{\rho\sigma\lambda\mu} R^{\rho\sigma\lambda}{}_\alpha - \frac{1}{45} R_{\mu\lambda} R^\lambda_\alpha + \frac{1}{60} \Box R_{\mu\alpha} + \frac{1}{20} \nabla_\mu \nabla_\alpha R \Big) \mathbb{I} + \\
& + \frac{1}{12} \left(\hat{\mathscr{R}}_{\mu\lambda} \hat{\mathscr{R}}^\lambda_\alpha + \hat{\mathscr{R}}_{\alpha\lambda} \hat{\mathscr{R}}^\lambda_\mu \right) - \frac{1}{12} \left(\nabla_\mu \nabla^\lambda \hat{\mathscr{R}}_{\lambda\alpha} + \nabla_\alpha \nabla^\lambda \hat{\mathscr{R}}_{\lambda\mu} \right) + \frac{1}{12} R \hat{\mathscr{R}}_{\mu\alpha} \Big] + \\
& + g_{\alpha\beta} \Big[\Big(\frac{1}{36} R_{\mu\nu} R + \frac{1}{90} R^{\lambda\sigma} R_{\lambda\mu\sigma\nu} + \frac{1}{90} R_{\rho\sigma\lambda\mu} R^{\rho\sigma\lambda}{}_\nu - \frac{1}{45} R_{\mu\lambda} R^\lambda_\nu + \frac{1}{60} \Box R_{\mu\nu} + \frac{1}{20} \nabla_\mu \nabla_\nu R \Big) \mathbb{I} + \\
& + \frac{1}{12} \left(\hat{\mathscr{R}}_{\mu\lambda} \hat{\mathscr{R}}^\lambda_\nu + \hat{\mathscr{R}}_{\nu\lambda} \hat{\mathscr{R}}^\lambda_\mu \right) - \frac{1}{12} \left(\nabla_\mu \nabla^\lambda \hat{\mathscr{R}}_{\lambda\nu} + \nabla_\nu \nabla^\lambda \hat{\mathscr{R}}_{\lambda\mu} \right) + \frac{1}{12} R \hat{\mathscr{R}}_{\mu\nu} \Big] \Big] + \\
& + \frac{1}{4} \left(g_{\mu\nu} g_{\alpha\beta} + g_{\mu\alpha} g_{\nu\beta} + g_{\mu\beta} g_{\nu\alpha} \right) \Big[\Big[\frac{1}{180} \left(R_{\lambda\sigma\rho\gamma} R^{\lambda\sigma\rho\gamma} - R_{\lambda\delta} R^{\lambda\sigma} \right) + \frac{1}{30} \Box R - \frac{1}{72} R^2 \Big] \mathbb{I} + \frac{1}{12} \hat{\mathscr{R}}_{\lambda\sigma} \hat{\mathscr{R}}^{\lambda\sigma} \Big] \Big\}
\end{aligned}
\tag{1}
$$

Quantum UG: logarithmic UV divergencies

By doing this we find (the UV divergent part of) the off-shell effective action

$$W_\infty = \frac{1}{16\pi^2}\frac{1}{n-4}\int d^4x\left(\frac{119}{90}R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} + \left(\frac{1}{6\alpha^2} - \frac{359}{90}\right)R_{\mu\nu}R^{\mu\nu} + \frac{1}{72}\left(22 - \frac{3}{\alpha^2}\right)R^2\right)$$

We can get the on-shell result using the equations of motion of the background field

$$R_{\mu\nu} - \frac{1}{4}Rg_{\mu\nu} = 0$$

$$R_{\mu\nu}R^{\mu\nu} = \frac{1}{4}R^2$$

$$R = \text{constant}$$

and

$$E_4 \equiv R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4R_{\mu\nu}R^{\mu\nu} + R^2$$

Quantum UG: logaritmic On-shell UV divergencies

The one loop on-shell DeWitt effective action reads

$$W_\infty^{\text{on-shell}} = \frac{1}{16\pi^2}\frac{1}{n-4}\int d^4x \left(\frac{119}{90}R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - \frac{83}{120}R^2\right) =$$
$$= \frac{1}{16\pi^2}\frac{1}{n-4}\int d^4x \left(\frac{119}{90}E_4 - \frac{83}{120}R^2\right)$$

This is physically irrelevant, in contrast with GR with a Cosmological Constant term (Christensen-Duff, NPB 170[FSI] (1980) 480)

$$W_\infty^{GR} \equiv \frac{1}{16\pi^2(n-4)}\int \sqrt{|g|}\, d^4x \left(\frac{53}{45}\,E_4 - \frac{522}{45}\Lambda^2\right)$$

Tree level Scattering Amplitudes I

Tree level scattering amplitudes, in UG, are constructed from the unimodular gravity propagator

$$P^{\rm UG}_{\mu\nu,\rho\sigma}=\frac{i}{2k^2}\left(\eta_{\mu\sigma}\eta_{\nu\rho}+\eta_{\mu\rho}\eta_{\nu\sigma}\right)-\frac{i}{k^2}\frac{\alpha^2n^2-n+2}{\alpha^2n^2(n-2)}\eta_{\mu\nu}\eta_{\rho\sigma}+\frac{2i}{n-2}\left(\frac{k_\rho k_\sigma\eta_{\mu\nu}}{k^4}+\frac{k_\mu k_\nu\eta_{\rho\sigma}}{k^4}\right)-\frac{2in}{n-2}\frac{k_\mu k_\nu k_\rho k_\sigma}{k^6}$$

and 3-point, 4-point etc vertices,

$$V^{\mu\nu,\rho\sigma,\alpha\beta}_{(p1,p2,p3)},\qquad V^{\mu\nu,\rho\sigma,\alpha\beta,\eta\lambda}_{(p1,p2,p3,p4)},\qquad \ldots.$$

which are daunting

Tree level Scattering Amplitudes II

$$
\begin{aligned}
V^{\mu\nu,\rho\sigma,\alpha\beta}_{(p1,p2,p3)} = i\kappa \mathscr{S}\Bigg\{ &-\frac{\left(2+n\right)(p_1.p_2)\eta^{\alpha\rho}\eta^{\beta\sigma}\eta^{\mu\nu}}{n^2} - \frac{(p_1.p_2)\eta^{\alpha\beta}\eta^{\mu\rho}\eta^{\nu\sigma}}{2n} + \frac{\left(2+n\right)(p_1.p_2)\eta^{\alpha\beta}\eta^{\mu\nu}\eta^{\rho\sigma}}{2n^3} \\
&+ \frac{2\eta^{\beta\nu}\eta^{\rho\sigma}p_1^m p_2^\alpha}{n} + \frac{1}{2}\eta^{mr}\eta^{\nu\sigma}p_1^\alpha p_2^\beta - \frac{\left(2+n\right)\eta^{\mu\nu}\eta^{\rho\sigma}p_1^\alpha p_2^\beta}{2n^2} - 2\eta^{\beta\sigma}\eta^{\nu\rho}p_1^\alpha p_2^\mu - \eta^{\alpha\nu}\eta^{\beta\sigma}p_1^\rho p_2^\mu \\
&+ \frac{\eta^{\alpha\beta}\eta^{\nu\sigma}p_1^\rho p_2^\mu}{n} + \frac{2\eta^{\beta\mu}\eta^{\rho\sigma}p_1^\alpha p_2^\nu}{n} - \frac{2\eta^{\alpha\beta}\eta^{\rho\sigma}p_1^\mu p_2^\nu}{n^2} + \frac{2\eta^{\alpha\mu}\eta^{\beta\nu}p_1^\sigma p_2^\rho}{\nu} + (p_1.p_2)\eta^{\alpha\nu}\eta^{\beta\sigma}\eta^{\mu\rho} \Bigg\}
\end{aligned}
$$

$\mathscr{S}$: Symmetrization over $(p_1,\mu\nu)$; $(p_2,\rho\sigma)$; $(p_3,\alpha\beta)$; $p_4,\eta\lambda)$ and $\mu\nu$, $\rho\sigma$, $\alpha\beta$, $\eta\lambda$

$$
\begin{aligned}
V^{\mu\nu,\rho\sigma,\alpha\beta,\eta\lambda}_{(p1,p2,p3,p4)} = i\kappa^2\, \mathscr{S}\Bigg\{ &\frac{\left(2+n\right)(p_3.p_4)g^{\mu\nu}g^{\rho\sigma}g^{\alpha\beta}g^{\eta\lambda}}{4n^4} - \frac{\left(2+n\right)(p_3.p_4)g^{\mu\rho}g^{\alpha\beta}g^{\eta\lambda}g^{\nu\sigma}}{4n^3} \\
&+ \frac{\left(2+n\right)(p_3.p_4)g^{\mu\eta}g^{\rho\alpha}g^{\nu\lambda}g^{\sigma\beta}}{2n^2} - \frac{\left(2+n\right)(p_3.p_4)g^{\mu\nu}g^{\rho\eta}g^{\alpha\beta}g^{\sigma\lambda}}{n^3} + \frac{\left(2+n\right)(p_3.p_4)g^{\mu\rho}g^{\alpha\beta}g^{\eta\sigma}g^{\nu\lambda}}{n^2} \\
&- \frac{(p_3.p_4)g^{\mu\nu}g^{\rho\sigma}g^{\alpha\eta}g^{\beta\lambda}}{4n^2} + \frac{(p_3.p_4)g^{\mu\nu}g^{\rho\eta}g^{\alpha\sigma}g^{\beta\lambda}}{n} + g^{\mu\eta}g^{\alpha\sigma}g^{\beta\lambda}p_3^\nu p_4^\rho \\
&+ \frac{\left(2+n\right)g^{\mu\rho}g^{\alpha\beta}g^{\eta\lambda}p_3^\sigma p_4^\nu}{2n^2} - \frac{1}{2}g^{\mu\rho}g^{\alpha\eta}g^{\beta\lambda}p_3^\sigma p_4^\nu + \frac{\left(2+n\right)g^{\mu\alpha}g^{\eta\lambda}g^{\nu\beta}p_3^\rho p_4^\sigma}{n^2} \\
&+ \frac{g^{\mu\nu}g^{\alpha\eta}g^{\beta\lambda}p_3^\rho p_4^\sigma}{2n} - g^{\mu\alpha}g^{\eta\nu}g^{\beta\lambda}p_3^\rho p_4^\sigma - 2\frac{g^{\mu\alpha}g^{\rho\beta}g^{\eta\lambda}p_3^\nu p_4^\sigma}{n} - \frac{\left(2+n\right)g^{\mu\nu}g^{\alpha\beta}g^{\eta\lambda}p_3^\rho p_4^\sigma}{2n^3} \\
&-2\frac{g^{\mu\alpha}g^{\eta\sigma}g^{\nu\beta}p_3^\rho p_4^\lambda}{n} + 2\frac{g^{\mu\nu}g^{\rho\alpha}g^{\eta\lambda}p_3^\sigma p_4^\beta}{n^2} - 2\frac{g^{\mu\rho}g^{\alpha\sigma}g^{\eta\lambda}p_3^\nu p_4^\beta}{n} \\
&+2g^{\mu\eta}g^{\rho\lambda}g^{\alpha\nu}p_3^\sigma p_4^\beta - 2\frac{g^{\mu\nu}g^{\rho\eta}g^{\alpha\lambda}p_3^\sigma p_4^\beta}{n} - 2\frac{g^{\mu\eta}g^{\rho\alpha}g^{\nu\lambda}p_3^\sigma p_4^\beta}{n} + 2g^{\mu\rho}g^{\alpha\lambda}g^{\eta\sigma}p_3^\nu p_4^\beta - 2\frac{g^{\mu\rho}g^{\alpha\beta}g^{\eta\sigma}p_3^\nu p_4^\lambda}{n} \\
&+2\frac{g^{\mu\nu}g^{\rho\eta}g^{\alpha\beta}p_3^\sigma p_4^\lambda}{n^2} + \frac{g^{\mu\nu}g^{\rho\sigma}g^{\alpha\eta}p_3^\lambda p_4^\beta}{2n^2} - \frac{g^{\mu\nu}g^{\rho\eta}g^{\alpha\sigma}p_3^\lambda p_4^\beta}{n} + g^{\mu\rho}g^{\alpha\nu}g^{\eta\sigma}p_3^\lambda p_4^\beta - \frac{g^{\mu\rho}g^{\alpha\eta}g^{\nu\sigma}p_3^\lambda p_4^\beta}{2n} \\
&- \frac{g^{\mu\nu}g^{\rho\sigma}g^{\alpha\beta}p_3^\eta p_4^\lambda}{n^3} + \frac{g^{\mu\rho}g^{\alpha\beta}g^{\nu\sigma}p_3^\eta p_4^\lambda}{n^2} - 2\frac{g^{\mu\rho}g^{\alpha\sigma}g^{\nu\beta}p_3^\eta p_4^\lambda}{n} + 2\frac{g^{\mu\nu}g^{\rho\alpha}g^{\sigma\beta}p_3^\eta p_4^\lambda}{n^2} \\
&- \frac{1}{2}(p_3.p_4)g^{\mu\eta}g^{\rho\lambda}g^{\alpha\nu}g^{\sigma\beta} - (p_3.p_4)g^{\mu\rho}g^{\alpha\nu}g^{\eta\sigma}g^{\beta\lambda} + \frac{(p_3.p_4)g^{\mu\rho}g^{\alpha\eta}g^{\nu\sigma}g^{\beta\lambda}}{4n} \Bigg\}
\end{aligned}
$$

Tree level Scattering Amplitudes III

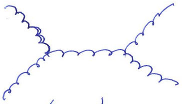
- Can we use modern on-shell techniques to compute tree level scattering amplitudes?
  Not with our UG propagator, since it has higher order poles in the momentum. Then
- Is there a gauge (Lorentz covariant) in which the propagator has a simple pole?
  No, if we are to reproduce Newton's law in the non relativistic limit (See Eur.Phys.J. C76 (2016) no.10, 554)–> Dead End
- To carry out a non trivial check we computed (with the help of Mathematica and FORM) the first nontrivial MHV amplitudes

$$\mathscr{A}(1^-,2^-;3^+,4^+), \qquad \mathscr{A}(1^-2^-;3^+,4^+,5^+)$$

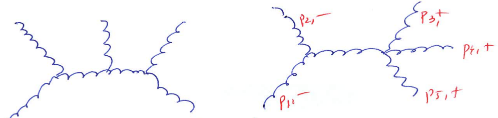
  and verified that they agree the corresponding amplitudes in GENERAL RELATIVITY.

Tree level Scattering Amplitudes IV

To give you a flavour of the lengthy computations one has to carry out, let us list the type diagrams to be worked out



4-point



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$\mathcal{A}(1^-, 2^-, 3^+, 4^+, 5^+)$

Tree level Scattering Amplitudes V

$$
\begin{aligned}
\mathscr{D}(1^-,2^-;3^+;4^+,5^+) = &-\frac{i\kappa^3(\varepsilon_1.p_2)^2(\varepsilon_2.\varepsilon_4)^2(\varepsilon_3.p_2)^2(\varepsilon_5.p_2)^2}{(p_1+p_2)^2(p_4+p_5)^2} \\
&-\frac{i\kappa^3(\varepsilon_1.p_2)^2(\varepsilon_2.\varepsilon_3)^2(\varepsilon_4.p_2)^2(\varepsilon_5.p_3)^2}{(p_1+p_2)^2(p_4+p_5)^2} - \frac{i\kappa^3(\varepsilon_1.p_2)^2(\varepsilon_2.\varepsilon_3)^2(\varepsilon_4.p_3)^2(\varepsilon_5.p_2)^2}{(p_1+p_2)^2(p_4+p_5)^2} \\
&-\frac{i\kappa^3(\varepsilon_1.p_2)^2(\varepsilon_2.\varepsilon_4)^2(\varepsilon_3.p_2)^2(\varepsilon_5.p_3)^2}{(p_1+p_2)^2(p_4+p_5)^2} - \frac{2i\kappa^3(\varepsilon_1.p_2)^2(\varepsilon_2.\varepsilon_4)^2(\varepsilon_3.p_2)^2(\varepsilon_5.p_2)(\varepsilon_5.p_3)}{(p_1+p_2)^2(p_4+p_5)^2} \\
&+\frac{2i\kappa^3(\varepsilon_1.p_2)^2(\varepsilon_2.\varepsilon_3)(\varepsilon_2.\varepsilon_4)(\varepsilon_3.p_2)(\varepsilon_4.p_2)(\varepsilon_5.p_2)(\varepsilon_5.p_3)}{(p_1+p_2)^2(p_4+p_5)^2} - \frac{2i\kappa^3(\varepsilon_1.p_2)^2(\varepsilon_2.\varepsilon_3)(\varepsilon_2.\varepsilon_4)(\varepsilon_3.p_2)(\varepsilon_4.p_3)(\varepsilon_5.p_2)(\varepsilon_5.p_3)}{(p_1+p_2)^2(p_4+p_5)^2} \\
&+\frac{2i\kappa^3(\varepsilon_1.p_2)^2(\varepsilon_2.\varepsilon_3)^2(\varepsilon_4.p_2)(\varepsilon_4.p_3)(\varepsilon_5.p_2)(\varepsilon_5.p_3)}{(p_1+p_2)^2(p_4+p_5)^2} + \frac{2i\kappa^3(\varepsilon_1.p_2)^2(\varepsilon_2.\varepsilon_3)(\varepsilon_2.\varepsilon_4)(\varepsilon_3.p_2)(\varepsilon_4.p_2)(\varepsilon_5.p_3)^2}{(p_1+p_2)^2(p_4+p_5)^2} \\
&-\frac{2i\kappa^3(\varepsilon_1.p_2)^2(\varepsilon_2.\varepsilon_3)(\varepsilon_2.\varepsilon_4)(\varepsilon_3.p_2)(\varepsilon_4.p_3)(\varepsilon_5.p_2)^2}{(p_1+p_2)^2(p_4+p_5)^2}
\end{aligned}
$$

Lepton anomalous magnetic moment I

- Another classic computation in GR+Matter is Gravitational contribution to the (g-2) factor of leptons's: Berends & Gastmans PLB 55 (1975) 311
- The relevant part of QED coupled to UG reads

$$S_{QED} = -\frac{1}{4}\int d^n x\, F_{\mu\nu}F^{\mu\nu} + \int d^n x\, \bar{\psi}(i\partial\!\!\!/ - eA\!\!\!/ - m)\psi - \frac{\kappa}{2}\int d^n x\, T^{\mu\nu}\hat{h}_{\mu\nu} + O(\kappa^2).$$

$$\hat{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{n}h^\rho_\rho\eta_{\mu\nu}$$

$$T^{\mu\nu} = \frac{i}{4}\bar{\psi}(\gamma^\mu\overrightarrow{\partial}^\nu + \gamma^\nu\overrightarrow{\partial}^\mu)\psi - \frac{i}{4}\bar{\psi}(\gamma^\mu\overleftarrow{\partial}^\nu + \gamma^\nu\overleftarrow{\partial}^\mu)\psi + F^\mu{}_\rho F^{\rho\nu} + \frac{1}{4}F_{\sigma\rho}F^{\sigma\rho}\eta^{\mu\nu}$$
$$-\frac{e}{2}\bar{\psi}(\gamma^\mu A^\nu + \gamma^\nu A^\mu)\psi$$

$h_{\mu\nu}$ is the graviton field

- FOR UG, unlike for GR, $T^{\mu\nu}$ couples to $\hat{h}_{\mu\nu}$, NOT to $h_{\mu\nu}$!!!
- We need

$$\langle\hat{h}_{\mu\nu}(k)\hat{h}_{\rho\sigma}(-k)\rangle = \Delta^{(GR)}_{\mu\nu,\rho\sigma}(k) + \Delta_{\mu\nu,\rho\sigma}(k)$$
$$\Delta^{(GR)}_{\mu\nu,\rho\sigma}(k) = \frac{i}{2k^2}\left(\eta_{\mu\sigma}\eta_{\nu\rho} + \eta_{\mu\rho}\eta_{\nu\sigma} - \frac{2}{n-2}\eta_{\mu\nu}\eta_{\rho\sigma}\right),$$
$$\Delta_{\mu\nu,\rho\sigma}(k) = \frac{2i}{n-2}\frac{k_\mu k_\nu\eta_{\rho\sigma} + k_\rho k_\sigma\eta_{\mu\nu}}{(k^2)^2} - \frac{2in}{n-2}\frac{k_\mu k_\nu k_\rho k_\sigma}{(k^2)^3}.$$

Lepton anomalous magnetic moment II

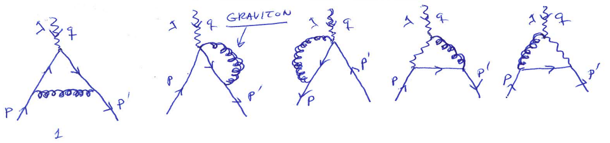
- The UG contributions to the $(g-2)_l$ factor are given by

$$(g-2)_l^{UG} = 2\,m^2\kappa^2 \lim_{q^2/m^2\to 0^-} F_2\Big(\frac{q^2}{m^2}\Big),$$

where $F_2\Big(\frac{q^2}{m^2}\Big)$ is obtained from the structures of the form

$$-i\,e\,m^2\kappa^2\,F_2\Big(\frac{q^2}{m^2}\Big)\,\bar{u}(p')\,\frac{i\sigma^{\lambda\rho}q_\rho}{2m}\,u(p), \quad \sigma^{\lambda\rho} = \frac{i}{2}[\gamma^\lambda,\gamma^\rho],$$

which occur in



Lepton anomalous magnetic moment III

- The difference between the GR and UG contributions to the $(g-2)_l$ coming from the previous diagrams read
  Contribution from the diagram in fig. 1:

$$-ie\frac{m^2\kappa^2}{8}\frac{1}{n-2}\frac{1}{16\pi^2}\left[2\left(\frac{1}{\varepsilon}-\gamma-\ln\left(\frac{m^2}{4\pi\mu^2}\right)+3\right)\right]\bar{u}(p')\frac{i\sigma^{\lambda\rho}q_\rho}{2m}u(p)$$

  Contribution from the diagrams in fig. 2:

$$-ie\frac{m^2\kappa^2}{8}\frac{2}{n-2}\frac{1}{16\pi^2}\left[2\left(\frac{1}{\varepsilon}-\gamma-\ln\left(\frac{m^2}{4\pi\mu^2}\right)+3\right)\right]\bar{u}(p')\frac{i\sigma^{\lambda\rho}q_\rho}{2m}u(p)$$

  Contribution from the diagrams in fig. 3:

$$-ie\frac{m^2\kappa^2}{8}\frac{2}{n-2}\frac{1}{16\pi^2}\left[-4\left(\frac{1}{\varepsilon}-\gamma-\ln\left(\frac{m^2}{4\pi\mu^2}\right)+3\right)\right]\bar{u}(p')\frac{i\sigma^{\lambda\rho}q_\rho}{2m}u(p)$$

- EACH CONTRIBUTION IS UV DIVERGENT, BUT THE SUM OF ALL OF THEM VANISHES!!!

Lepton anomalous magnetic moment IV

- CONCLUSION:
- GR and UG YIELD the SAME CONTRIBUTION to the $(g-2)_l$ FACTOR(at least in DIM REG and DIM RED)
- This result is surprising since the $(g-2)_l$ gravitational contribution is not an OBSERVABLE: the counterterm

$$C\,\frac{m^2}{M_{pl}^2}\int d^4x\;\bar{\psi}(x)\,\frac{i\sigma^{\mu\nu}}{2m}\,\psi(x)\,F_{\mu\nu}(x),$$

can be added to the bare action –we are dealing with a nonrenormalizable theory.

The quartic and Yukawa beta functions I

- A FACT: In peturbatively renormalizable field theories the coupling constant beta functions have invaluable physical information.
- In Phys.Rev.Lett. 104 (2010) 081301, the GR corrections to the beta functions of the quartic, λ and Yukawa, g couplings was computed in the MS scheme in the de Donder gauge:

$$\beta_\lambda^{\mathrm{GR}} = -\frac{1}{4\pi^2}\kappa^2 m_\phi^2\,\lambda, \quad \beta_g^{\mathrm{GR}} = \frac{1}{16\pi^2}\kappa^2\left\{m_\phi^2\left[\frac{1}{2}\right] + m_\Psi^2\left[-1\right]\right\}$$

$$m_\phi = \text{mass of the Scalar}, \quad m_\Psi = \text{mass of the fermion}$$

- Interesting results due to the NEGATIVE value both of them in the SM case, or so it was thought!!!
- So, we decided to carry out the very same type of computations in the UG case and found

$$\beta_\lambda^{\mathrm{UG}} = 0, \quad \beta_g^{\mathrm{UG}} = \frac{1}{16\pi^2}\kappa^2 m_\Psi^2\,\frac{3}{16}$$

- HENCE, ONE IS TEMPTED TO CONCLUDE THAT GR $\neq$ UG AT THE QUANTUM LEVEL
    - WRONG CONCLUSION!!!!

The quartic and Yukawa beta functions II

- INDEED: The beta functions defined as in Phys.Rev.Lett. 104 (2010) 081301(ie, by a textbook's standard MULTIPLICATIVE renormalization) lack intrinsic physical meaning, for they turn out to be gauge dependent
- We have obtained –for GR– that for a generalised de Donder gauge

$$\int d^n x\, \alpha \left(\partial^\mu h_{\mu\nu} - \frac{1}{2}\partial_\nu h\right)^2,$$

one has

$$\beta_\lambda^{GR} = -\frac{1}{4\pi^2}\kappa^2 m_\phi^2 \left(\frac{3}{2}+\alpha\right)\lambda$$
$$\beta_g^{GR} = \frac{1}{16\pi^2}\kappa^2 \left\{ m_\phi^2 \left[\frac{1}{2} - \left(\frac{1}{2}+\alpha\right)\right] + m_\Psi^2 \left[-1 - \left(\frac{1}{2}+\alpha\right)\frac{85}{16}\right]\right\}$$

The quartic and Yukawa beta functions III

- ...WHAT'S MORE
- By introducing a NONMULTIPLICATIVE (but local) MS WAVE FUNCTION RENORMALIZATION (as did, in the YM case, J.Ellis & N. Mavromatos Phys.Lett. B711 (2012) 139)

$$
\begin{aligned}
&g_0 = \mu^{-\varepsilon} Z_g Z_\psi^{-1} Z_\phi^{-1/2} g, Z_\Psi, = 1 + \delta Z_\Psi, Z_\phi = 1 + \delta Z_\phi,\\
&\phi_0 = \phi + \frac{1}{2}\delta Z_\phi \phi,\\
&\Psi_0 = \Psi + \frac{1}{2}\delta Z_\Psi \Psi + \frac{1}{2} a_1 \kappa^2 m_\Psi^2 \phi \Psi + \frac{1}{2} b_1 \kappa^2 m_\phi^2 \phi \Psi, m_{\Psi_0} = (1 + \delta Z_{m_\Psi}) m_\Psi,\\
&\bar{\Psi}_0 = \bar{\Psi} + \frac{1}{2}\delta Z_\Psi \bar{\Psi} + \frac{1}{2} a_1 \kappa^2 m_\Psi^2 \bar{\Psi} \phi + \frac{1}{2} b_1 \kappa^2 m_\phi^2 \bar{\Psi} \phi,,\\
&m_{\phi_0} = (1 + \delta Z_{m_\phi}) m_\phi.
\end{aligned}
$$

one obtains that

$$\beta_g^{GR} = 0 = \beta_g^{UG}$$

- SO THAT β_g^{GR} and β_g^{UG} have no intrinsic physical meaning.
- Analogous analysis for β_λ^{GR} and β_λ^{UF}. See PLB 773 (2017) 585.
- In this regard there is no disagreement between GR and UG (ie, both contributions to β_g and β_λ can be set to zero by nonmultiplicative MS field renormalizations), but this does not settle the question of the physical equivalence between GR and UG coupled to the $\lambda\phi^4$ and Yukawa theories.
- Similar to what happens in GR w.r.t the gauge couplings: J. Ellis & N. Mavromatos

The UV behaviour of S-matrix elements I

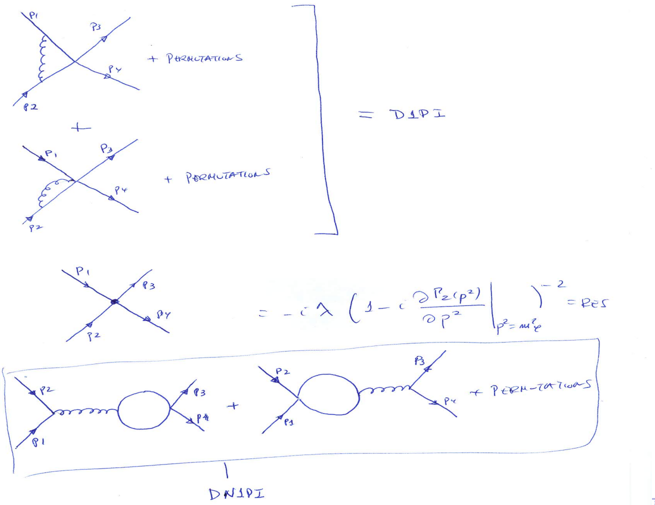
- To check whether UV divergent behaviour of the GR contributions to the S-matrix elements of the $\lambda\phi^4$ and Yukawa theory agree with those of UG, we decided to compute such behaviour for the scattering processes

$$\phi+\phi\rightarrow\phi+\phi \qquad \& \qquad \Psi+\Psi\rightarrow\Psi+\Psi,$$

  at one-loop
- After a lengthy computation we have shown that the GR and UG contributions agree, although this agreement is achieved after summing over all Feynman diagrams.

The UV behaviour of S-matrix elements II

- The one-loop diagrams of order κ^2 are



The UV behaviour of S-matrix elements III

- FOR GR, the divergences read (we are onshell)

$$\begin{aligned}
\mathrm{D1PI} &= -\frac{i}{16\pi^2\varepsilon}\kappa^2 m_\phi^2\lambda\left(1+[\tfrac{1}{2}+\alpha]\right)(-2)\\
\mathrm{Res} &= -\frac{i}{16\pi^2\varepsilon}\kappa^2 m_\phi^2\lambda\left(1+[\tfrac{1}{2}+\alpha]\right)(-2)\\
\mathrm{DN1PI} &= -\frac{i}{16\pi^2\varepsilon}\kappa^2 m_\phi^2\lambda(-5/6)\\
\text{FINAL RESULT} &= -\frac{i}{16\pi^2\varepsilon}\kappa^2 m_\phi^2\lambda(-5/6)
\end{aligned}$$

- FOR UG, the divergences read (we are onshell)

$$\begin{aligned}
\mathrm{D1PI} &= 0\\
\mathrm{Res} &= 0\\
\mathrm{DN1PI} &= -\frac{i}{16\pi^2\varepsilon}\kappa^2 m_\phi^2\lambda(-5/6)\\
\text{FINAL RESULT} &= -\frac{i}{16\pi^2\varepsilon}\kappa^2 m_\phi^2\lambda(-5/6)
\end{aligned}$$

- THERE IS COMPLETE AGREEMENT BETWEEN GR AND UG!!!!!!
- The same conclusion for $\Psi+\Psi\to\Psi+\Psi$ (details in S. Gonzalez-Martin and CPM, forthcoming paper)

CONCLUSION

- As far as we can tell there is no difference between quantum GR and quantum UG when the Cosmological Constant vanishes
- Plenty of work still to be done:
- eg, does UG come from String Theory ? Recall the evidence that UG and GR have the same S-matrix.
- Goroff and Sagnotti computation