Sudden Future Singularities in Quintessence and Scalar-Tensor Quintessence Models

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Introduction

- Theoretical challenges of ΛCDM model ⟶ alternative dark energy models which predict the existence of exotic cosmological singularities.

- These singularities can be either geodesically complete or incomplete

- Classification of Singularities

- Behaviour of the scale factor α(t) and/or its derivatives at the time t_s
- Energy density and pressure of the content of the universe at t_s.

- John D. Barrow ("Sudden future singularities" (2004), arXiv:gr-qc/0403084) [gr-qc])

Scale Factor: $$a(t) = \left(\frac{t}{t_s}\right)^m (a_s - 1) + 1 - \left(1 - \frac{t}{t_s}\right)^q$$ $1 < q < 2$

- Big-Bang singularity at $t = 0$
- New type of singularity at $t = t_s$ - a Sudden Future Singularity (SFS)

➢ Divergence of the scalar curvature $R = 6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right) \to \infty$

➢ Manifests as a singularity of pressure p (or $\ddot{a}$) only

➢ Leads to the dominant energy condition (DEC) $\rho \geq |p|$ violation only

➨ $a = finite$ ➨ $\dot{a} = finite$ ➨ $\ddot{a} \to -\infty$

➨ $\rho = finite$ ➨ $p \to \infty$ for $t = t_s$

Quintessence Models

- Action: $$\mathcal{S} = \int \left[\frac{1}{2}R + \frac{1}{2}g^{\mu\nu}\phi_{;\mu}\phi_{;\nu} - V(\phi)\right]\sqrt{-g}d^4x$$

- Potential: $$V(\phi) = A|\phi|^n, \qquad A > 0$$ $$0 < n < 1$$

- Dynamical equations: $$3H^2 = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$ $$\ddot{\phi} = -3H\dot{\phi} - An|\phi|^{n-1}\Theta(\phi)$$ $$2\dot{H} = -\dot{\phi}^2$$

  - $H, \dot{H}, \phi \to finite$
  - $\phi^{n-1} \to \infty, \ddot{\phi} \to \infty, \ddot{H} \to \infty$ as $t \to t_s\ (\phi \to 0)$

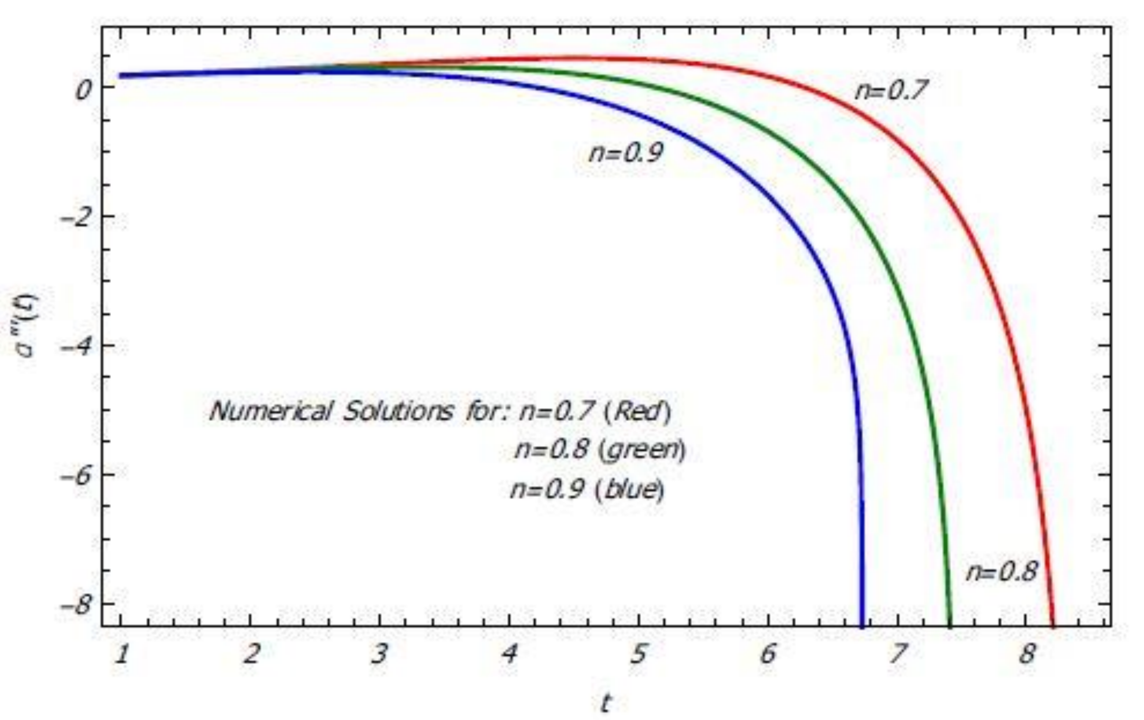
➢ $\dddot{a} \to \infty$ - a Generalized Sudden Future Singularity (GSFS)

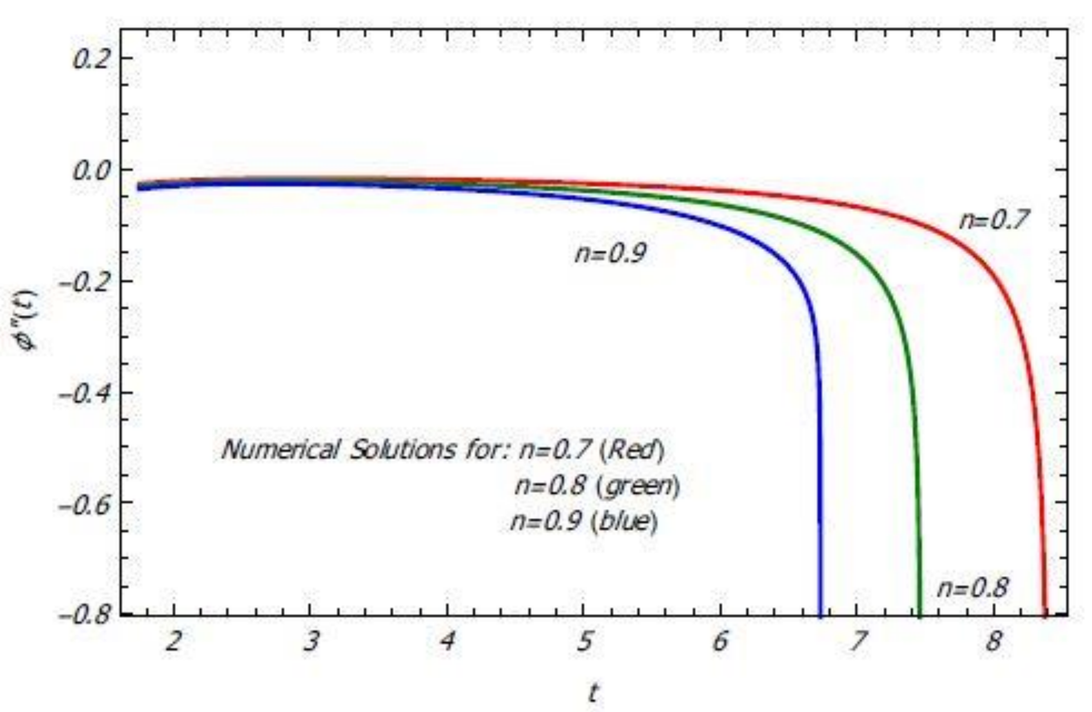
- Scale Factor: $a(t) = a_s + b(t_s - t) + c(t_s - t)^2 + d(t_s - t)^q$ $\quad 2 < q < 3$

As $t \to t_s$ ⟹ $a \to a_s$ and $\dddot{a} \to \infty$

- Scalar Field: $\phi(t) = f(t_s - t) + h(t_s - t)^r$ $\quad 1 < r < 2$

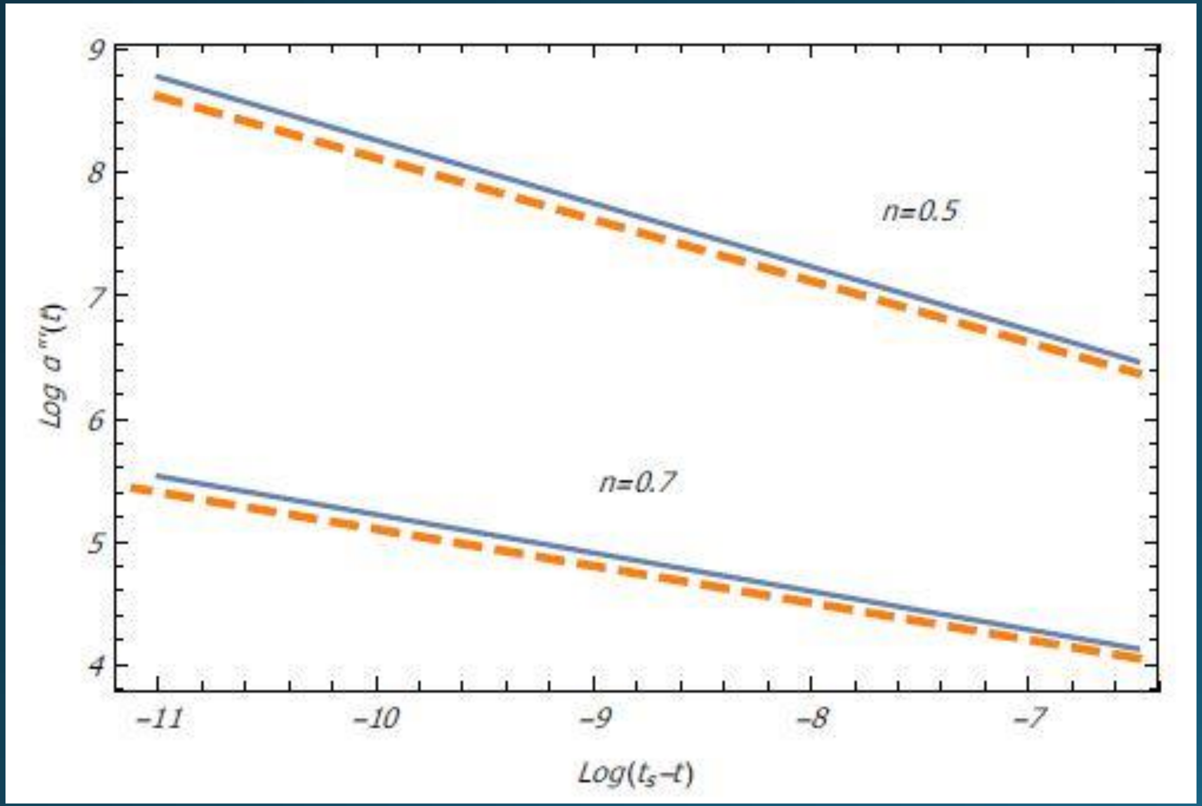
As $t \to t_s$ ⟹ $\phi \to 0$ and $\ddot{\phi} \to \infty$

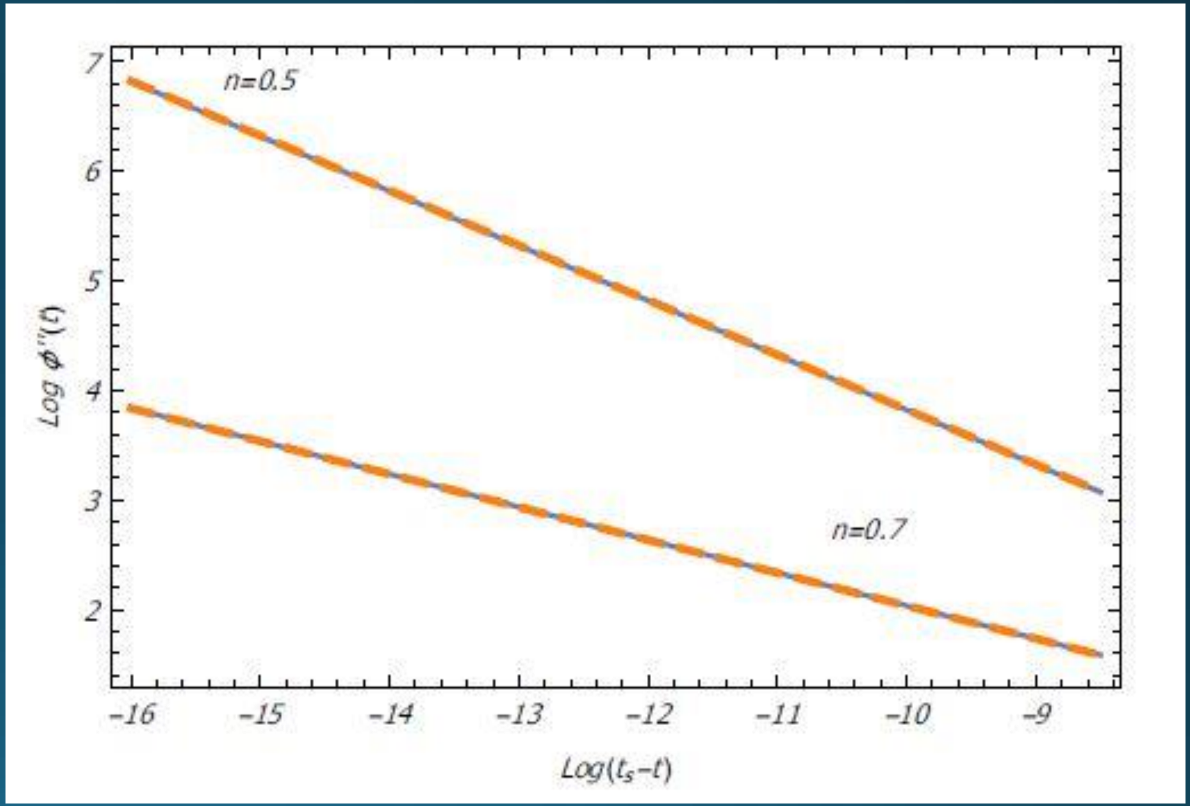




- Power Exponents: $r = n + 1$ $q = r + 1$ → $q = n + 2$

Consistent with the qualitatively expected range of $r, q,$ for $0 < n < 1$





- Observationally testable prediction of this class of models $H(z_s) = (1 + z_s)^3$

Scalar-Tensor Quintessence Models

- Action: $S = \int \left[\frac{1}{2}F(\phi)R + \frac{1}{2}g^{\mu\nu}\phi_{;\mu}\phi_{;\nu} - V(\phi)\right]\sqrt{-g}d^4x$ $\quad F = 1 - \lambda\phi$

- Potential: $V(\phi) = A|\phi|^n, \qquad A > 0$ $\quad 0 < n < 1$

- Dynamical equations: $3FH^2 = \frac{\dot{\phi}^2}{2} + V - 3H\dot{F}$

$$\ddot{\phi} + 3H\dot{\phi} - 3F_\phi\left(\frac{\ddot{a}}{a} + H^2\right) + An|\phi|^{(n-1)}\Theta(\phi) = 0$$

$$-2F\left(\frac{\ddot{a}}{a} - H^2\right) = \dot{\phi}^2 + \ddot{F} - H\dot{F}$$

- $H, \dot{\phi}, F, \dot{F} \rightarrow finite$
- $\phi^{n-1} \rightarrow \infty, \ddot{\phi} \rightarrow \infty, \ddot{F} \rightarrow \infty$ as $t \rightarrow t_s \; (\phi \rightarrow 0)$

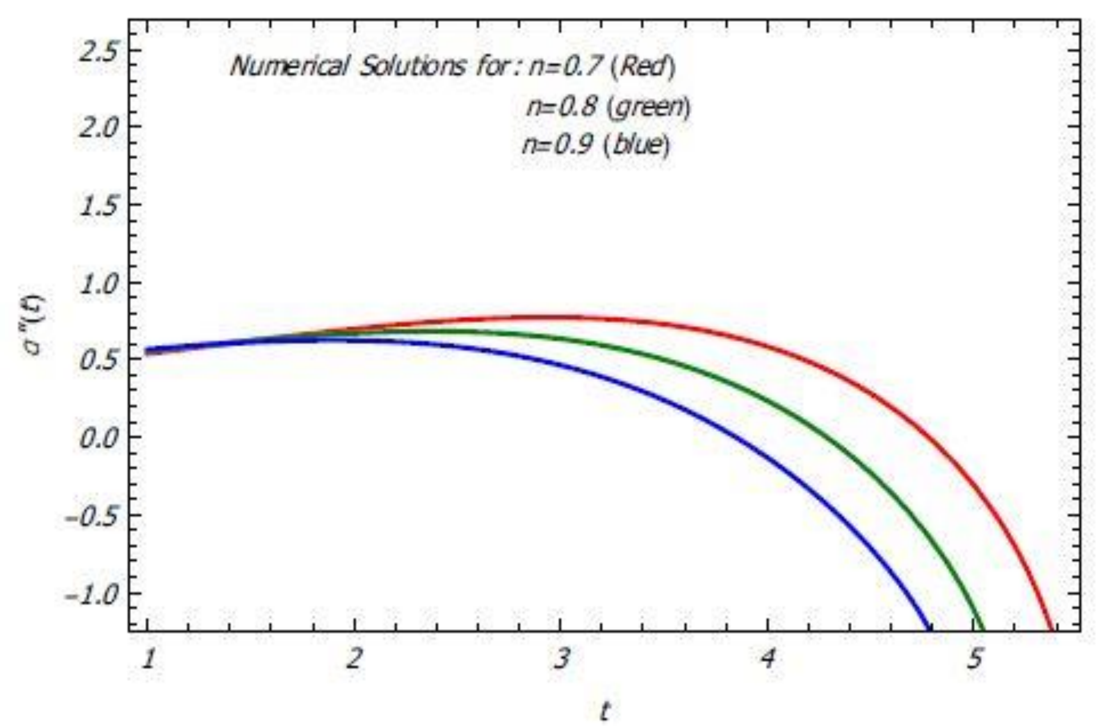
➢ $\ddot{a} \rightarrow \infty$ - a Sudden Future Singularity (SFS)

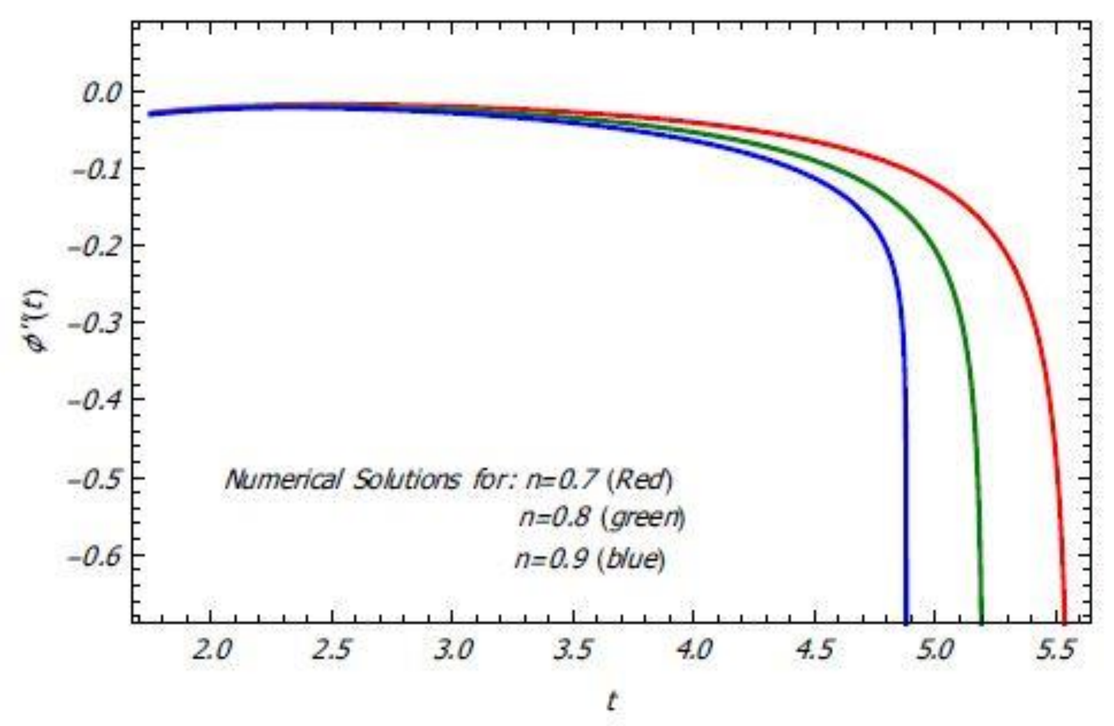
- Scale Factor: $a(t) = a_s + b(t_s - t) + c(t_s - t)^2 + d(t_s - t)^q$ $\quad 1 < q < 2$

As $t \to t_s$ ➡ $a \to a_s$ and $\ddot{a} \to \infty$

- Scalar Field: $\phi(t) = f(t_s - t) + h(t_s - t)^r$ $\quad 1 < r < 2$

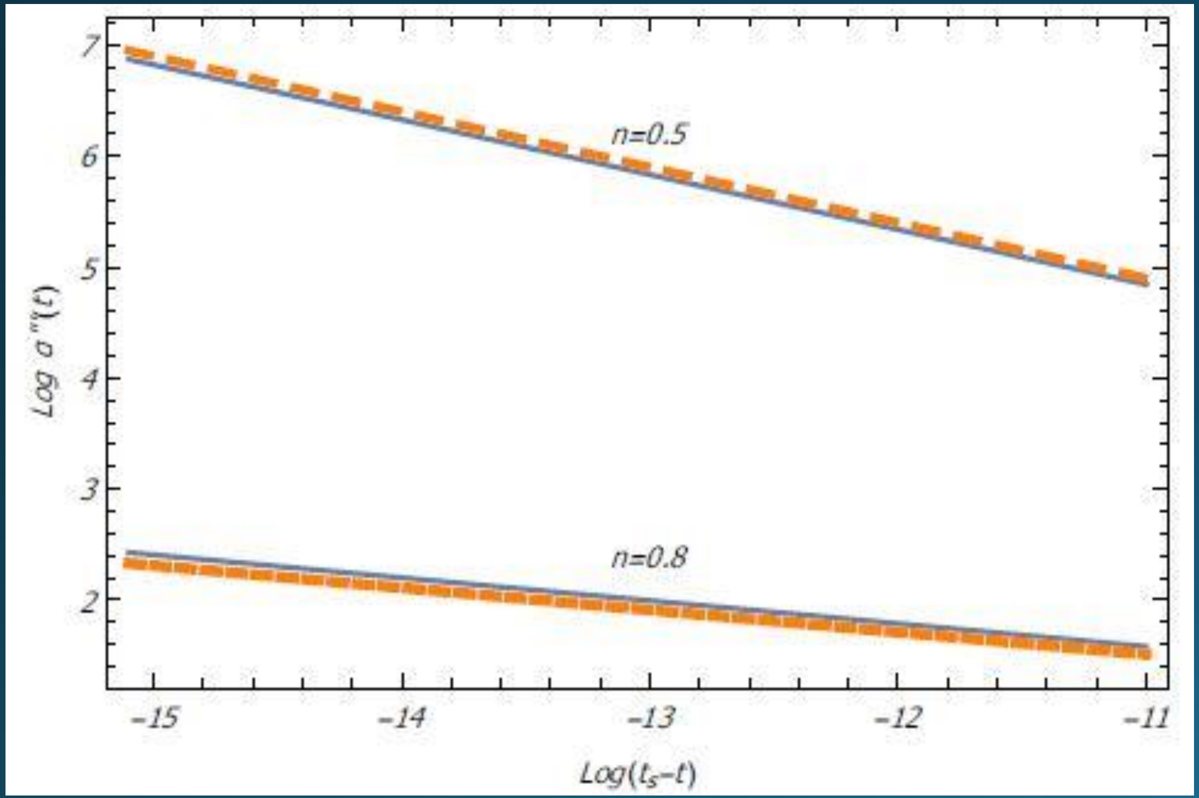
As $t \to t_s$ ➡ $\phi \to 0$ and $\ddot{\phi} \to \infty$

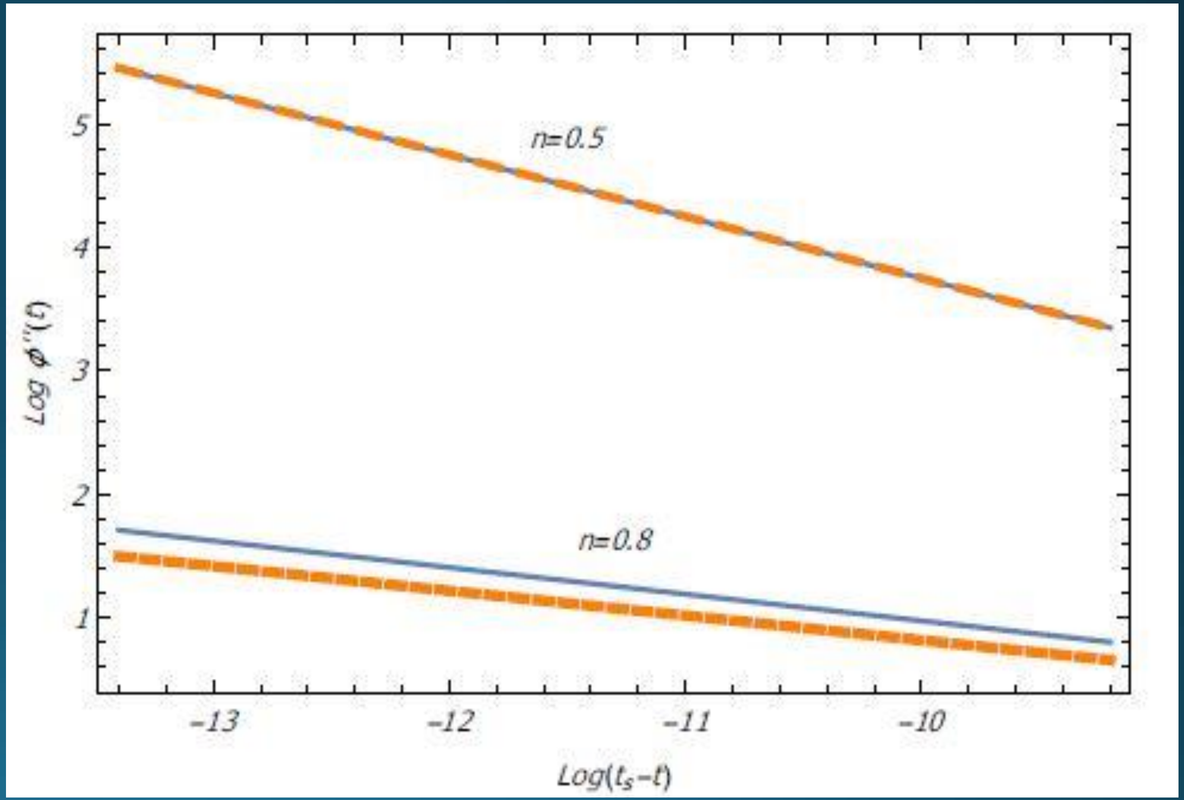




- Power Exponents: $q = r$ $r = n + 1$ → $q = n + 1$

Consistent with the qualitatively expected range of $r, q, \text{ for } 0 < n < 1$





Conclusions

- For quintessence models: $\dddot{a} \to \infty$ → GSFS singularity.
- Scalar-tensor quintessence models: $\ddot{a} \to \infty$ → a stronger singularity occurs - an SFS singularity (due to divergence of Ricci scalar).
- The additional linear and quadratic terms of $t_s - t$ in the form of the scale factor play an important role as $t \to t_s$. In the scalar-tensor case the quadratic term becomes subdominant close to the singularity.
- The relations of the Hubble parameter $H(z)$, may be used as observational signatures of such singularities in this class of models.

THANK YOU