

Multiquark Hadrons - Current Status and Future Directions

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DESY, Hamburg

Sept. 4, 2017

Corfu Summer Institute: Workshop on the SM and Beyond

- Experimental Evidence for Multiquark states X , Y , Z and P_c
- Models for X, Y, Z Mesons
- The Diquark model of Tetraquarks
- Mass Spectrum of the low-lying S and P Wave Tetraquark States
- A New Look at the excited Ω_c and the Y States in the Diquark Model
- The Pentaquarks $\mathbb{P}^\pm(4380)$ and $\mathbb{P}^\pm(4450)$ in the Diquark Model
- Summary

X(3872) - the poster Child of the X, Y, Z Mesons

VOLUME 91, NUMBER 26

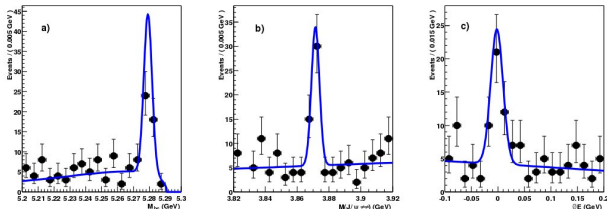
PHYSICAL REVIEW LETTERS

week ending
31 DECEMBER 2003

Observation of a Narrow Charmoniumlike State in Exclusive $B^{\pm} \rightarrow K^{\pm} \pi^+ \pi^- J/\psi$ Decays

S.-K. Choi,⁵ S.L. Olsen,⁶ K. Abe,⁷ T. Abe,⁷ I. Adachi,⁷ Byoung Sup Ahn,¹⁴ H. Aihara,⁴³ K. Akai,⁷ M. Akatsu,²⁰ M. Akemoto,⁷ Y. Asano,⁴⁸ T. Aso,⁴⁷ V. Aulchenko,¹ T. Aushev,¹¹ A. M. Bakich,³³ Y. Ban,³¹ S. Banerjee,³⁹ A. Bondar,¹ A. Bozek,²⁵ M. Bračko,^{18,12} J. Brodzicka,²⁵ T. E. Browder,⁶ P. Chang,²⁴ Y. Chao,²⁴ K.-F. Chen,²⁴ B. G. Cheon,³⁷ R. Chistov,¹¹ Y. Choi,³⁷ Y. K. Choi,³⁷ M. Danilov,¹¹ L. Y. Dong,⁹ A. Drutskoy,¹¹ S. Eidelman,¹ V. Eiges,¹¹ J. Flanagan,⁷ C. Fukunaga,⁴⁸ K. Furukawa,⁷ N. Gabyshev,⁷ T. Gershon,⁷ B. Golob,^{17,12} H. Guler,⁶ R. Guo,²² C. Hagner,²⁰ F. Handa,⁴² T. Hara,²⁹ N. C. Hastings,⁷ H. Hayashii,²¹ M. Hazumi,⁷ L. Hinz,¹⁶ Y. Hoshi,⁴¹ W.-S. Hou,²⁴ Y. B. Hsiung,^{24,28} H.-C. Huang,²⁴ T. Iijima,²⁰ K. Inami,²⁰ A. Ishikawa,²⁰ R. Itoh,⁷ M. Iwasaki,⁴³ Y. Iwasaki,⁷ J. H. Kang,⁵² S. U. Kataoka,²¹ N. Katayama,⁷ H. Kawai,² T. Kawasaki,²⁷ H. Kichimi,⁷ E. Kikutani,⁷ H. J. Kim,⁵² Hyunwoo Kim,¹⁴ J. H. Kim,²⁷ S. K. Kim,³⁶ K. Kinoshita,³ H. Koiso,⁷ P. Koppenburg,⁷ S. Korpar,^{18,12} P. Križan,^{17,12} P. Krokovny,¹ S. Kumar,³⁰ A. Kuzmin,⁴ J. S. Lange,^{4,33} G. Leder,¹⁰ S. H. Lee,¹⁶ T. Lesiak,²⁵ S.-W. Lin,²⁴ D. Liventsev,¹¹ J. MacNaughton,¹⁰ G. Majumder,³⁶ F. Mandl,¹⁰ D. Marlow,³² T. Matsumoto,⁴⁵ S. Michizono,⁷ T. Mimashi,⁷ W. Mitaroff,¹⁰ K. Miyabayashi,²¹ H. Miyake,²⁹ D. Mohapatra,¹⁹ T. Nagamine,⁴² Y. Nagasaka,⁸ T. Nakadaira,⁴³ T. T. Nakamura,⁷ M. Nakao,⁷ Z. Natkaniec,²⁵ S. Nishida,⁷ O. Nitoh,⁴⁶ T. Nozaki,⁷ S. Ogawa,⁴⁰ Y. Ogawa,⁷ K. Ohmi,⁷ Y. Ohnishi,⁷ T. Ohshima,²⁰ N. Ohuchi,⁷ K. Oide,⁷ T. Okabe,³⁰ S. Okuno,¹³ W. Ostrowicz,²⁵ H. Ozaki,⁷ H. Palka,²⁵ H. Park,¹⁵ N. Parslow,³⁸ L. E. Pilonen,⁵⁰ H. Sagawa,⁷ S. Saitoh,⁷ Y. Sakai,⁷ T. R. Sarangi,⁴⁹ M. Satpathy,⁴⁹ A. Satpathy,^{7,3} O. Schneider,¹⁶ A. J. Schwartz,³ S. Semenov,¹¹ K. Senyo,²⁰ R. Seuster,⁶ M. E. Sevior,¹⁹ H. Shibuya,⁴⁰ T. Shidara,⁷ B. Shwartz,¹ V. Sidorov,¹ N. Soni,³⁰ S. Stanić,^{48,8} M. Starić,¹² A. Sugiyama,⁴⁴ T. Sumiyoshi,⁴⁵ S. Suzuki,⁵¹ F. Takasaki,⁷ K. Tamai,⁷ N. Tamura,²⁷ M. Tanaka,⁷ M. Tawada,⁷ G. N. Taylor,¹⁹ Y. Teramoto,²⁸ T. Tomura,⁴³ K. Trabelsi,⁶ T. Tsukamoto,⁷ S. Uehara,⁷ K. Ueno,²⁴ Y. Unno,² S. Uno,⁷ G. Varner,⁶ K. E. Varvell,³⁸ C. C. Wang,²⁴ C. H. Wang,²³ J. G. Wang,⁵⁰ Y. Watanabe,⁴⁴ E. Wron,¹⁴ B. D. Yabsley,⁵⁰ Y. Yamada,⁷ A. Yamaguchi,⁴² Y. Yamashita,²⁶ H. Yanai,²⁷ Heyoung Yang,³⁰ J. Ying,³¹ M. Yoshida,⁷ C. C. Zhang,⁹ Z. P. Zhang,³⁵ and D. Žontar^{17,12}

(Belle Collaboration)



Ahmed Ali (DESY, Hamburg)

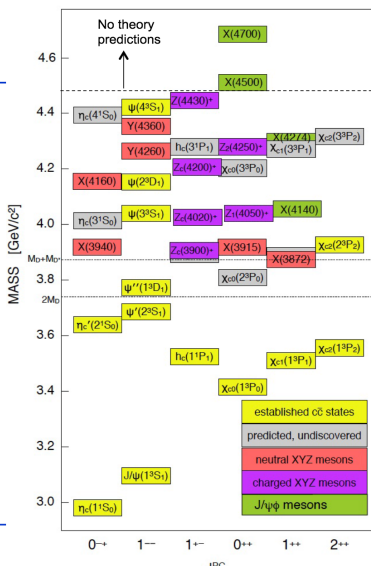
- Discovery Mode : $B \rightarrow J/\psi \pi^+ \pi^- K$
- $M = 3872.0 \pm 0.6 \pm 0.5 \text{ MeV}$
- $\Gamma < 2.3 \text{ MeV}$
- $J^{PC} = 1^{++}$ [LHCb] [PRL110, 22201(2013)]



Tetraquark summary

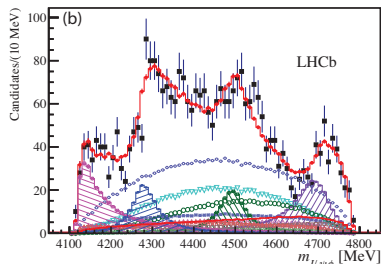
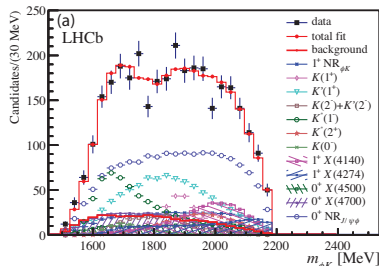
- Predicted neutral charmonium states compared with found $c\bar{c}$ states, & both neutral & charged exotic candidates
- Based on Olsen [[arXiv:1511.01589](https://arxiv.org/abs/1511.01589)]
- Added 4 new $J/\psi\phi$ states

Exotica, Islamabad, Feb. 8, 2017



The four $J/\psi\phi$ states

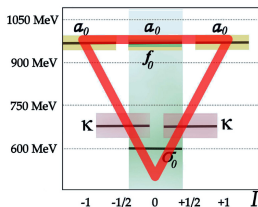
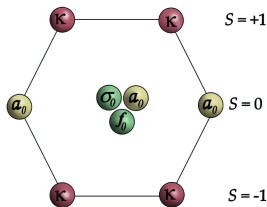
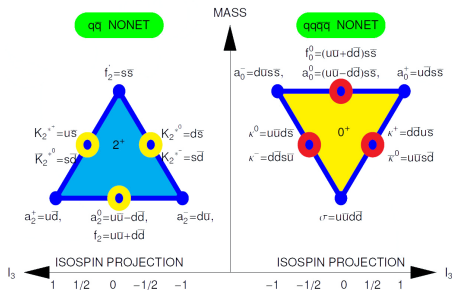
[R. Aaij *et al.*, Phys. Rev. D95 (2017) 012002]



Particle	J^P	σ	Mass (MeV)	Width (MeV)	Fit Fraction(%)
X(4140)	1^+	8.4	$4146 \pm 4.5^{+4.6}_{-2.8}$	$83 \pm 21^{+21}_{-14}$	$13.0 \pm 3.2^{+4.8}_{-2.0}$
X(4274)	1^+	6.0	$4273.3 \pm 8.3^{+17.2}_{-3.6}$	$56 \pm 11^{+8}_{-11}$	$7.1 \pm 2.5^{+3.5}_{-2.4}$
X(4500)	0^+	5.6	$4506 \pm 11^{+12}_{-15}$	$92 \pm 21^{+21}_{-14}$	$6.6 \pm 2.4^{+3.5}_{-2.3}$
X(4700)	0^+	5.6	$4704 \pm 10^{+14}_{-24}$	$120 \pm 31^{+42}_{-33}$	$12 \pm 5^{+9}_{-5}$
NR	0^+	6.4			$46 \pm 11^{+11}_{-21}$

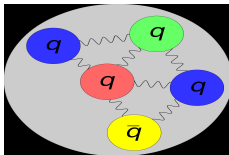
Constituent Quark Model and Light States

- Masses for light resonances in constituent model
 - Flavor nonets are arranged as triangles



- Tetraquark interpretation in agreement with experiment [’t Hooft, Isidori, Maiani, Polosa, Riquer, PLB (2008)]

Pentaquarks




- Pentaquarks remained cursed under the shadow of the botched discoveries of $\Theta(1540)$, $\Phi(1860)$, $\Theta_c(3100)$!

- Review on Pentaquarks [C.G. Wohl in PDG (2014)]:

There are two or three recent experiments that find weak evidence for signals near the nominal masses, but there is simply no point in tabulating them in view of the overwhelming evidence that the claimed pentaquarks do not exist. The whole story — is a curious episode in the history of science.



Observation of $J/\psi p$ resonances consistent with pentaquark states in $\Lambda_b^0 \rightarrow J/\psi K^- p$ decays

The LHCb collaboration 

Abstract

Observations of exotic structures in the $J/\psi p$ channel, which we refer to as charmonium-pentaquark states, in $\Lambda_b^0 \rightarrow J/\psi K^- p$ decays are presented. The data sample corresponds to an integrated luminosity of 3 fb^{-1} acquired with the LHCb detector from 7 and 8 TeV pp collisions. An amplitude analysis of the three-body final-state reproduces the two-body mass and angular distributions. To obtain a satisfactory fit of the structures seen in the $J/\psi p$ mass spectrum, it is necessary to include two Breit-Wigner amplitudes that each describe a resonant state. The significance of each of these resonances is more than 9 standard deviations. One has a mass of $4380 \pm 8 \pm 29 \text{ MeV}$ and a width of $205 \pm 18 \pm 86 \text{ MeV}$, while the second is narrower, with a mass of $4449.8 \pm 1.7 \pm 2.5 \text{ MeV}$ and a width of $39 \pm 5 \pm 19 \text{ MeV}$. The preferred J^P assignments are of opposite parity, with one state having spin $3/2$ and the other $5/2$.

The Pentaquarks P_c^+ (4380) and P_c^+ (4450) as resonant $J/\psi p$ states

- Discovery Channel (LHC; $\sqrt{s} = 7$ & 8 TeV; $\int Ldt = 3 \text{ fb}^{-1}$)

$$pp \rightarrow b\bar{b} \rightarrow \Lambda_b X; \quad \Lambda_b \rightarrow K^- J/\psi p$$

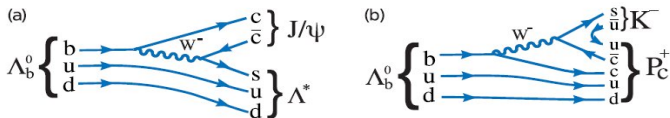


Figure 1: Feynman diagrams for (a) $\Lambda_b^0 \rightarrow J/\psi \Lambda^*$ and (b) $\Lambda_b^0 \rightarrow P_c^+ K^-$ decay.

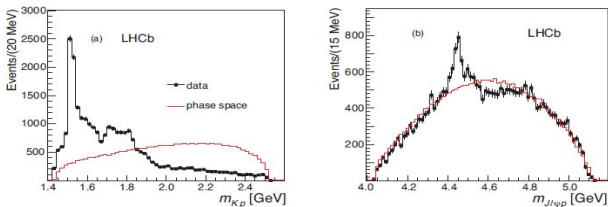
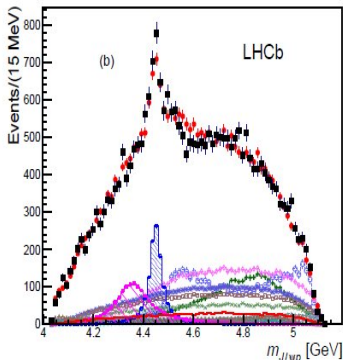
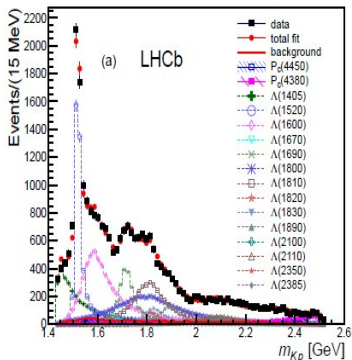


Figure 2: Invariant mass of (a) $K^- p$ and (b) $J/\psi p$ combinations from $\Lambda_b^0 \rightarrow J/\psi K^- p$ decays. The solid (red) curve is the expectation from phase space. The background has been subtracted.

Model fits with two [$P_c^+(4380)$ and $P_c^+(4450)$] states

- Fits with two P_c^+ states. Acceptable fits found for several J^P combinations
- The best fit yields $J^P = (3/2^-, 5/2^+)$ for [$P_c^+(4380), P_c^+(4450)$] states. Both the m_{Kp} and $m_{J/\psi p}$ projections are well described



Summary of the LHCb Pentaquark Measurements

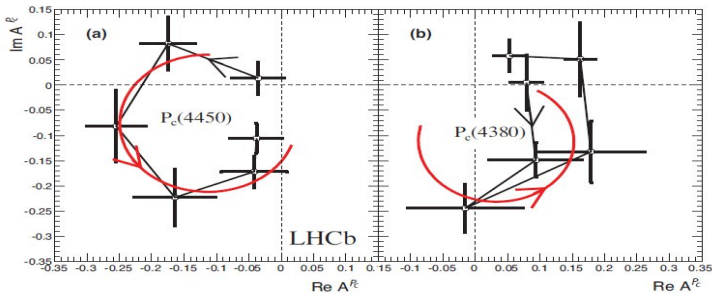
- Higher mass state (statistical significance 12σ)

$$M = 4449.8 \pm 1.7 \pm 2.5 \text{ MeV}; \Gamma = 39 \pm 5 \pm 19 \text{ MeV}$$

- Lower mass state (statistical significance 9σ)

$$M = 4380 \pm 8 \pm 29 \text{ MeV}; \Gamma = 205 \pm 18 \pm 86 \text{ MeV}$$

- Fitted Values of the real and imaginary parts of the amplitudes



- For $P_c^+(4450)$, fit shows a phase change in amplitudes consistent with a resonance

Summary of the LHCb Pentaquark Measurements (Contd.)

Possible J^P assignments and the energies of the nearby thresholds

	$P_c(4380)^+$	$P_c(4450)^+$
Mass	$4380 \pm 8 \pm 29$	$4449.8 \pm 1.7 \pm 2.5$
Width	$205 \pm 18 \pm 86$	$35 \pm 5 \pm 19$
Assignment 1	$3/2^-$	$5/2^+$
Assignment 2	$3/2^+$	$5/2^-$
Assignment 3	$5/2^+$	$3/2^-$
$\Sigma_c^{*+} \bar{D}^0$	4382.3 ± 2.4	
$\chi_{c1} p$		4448.93 ± 0.07
$\Lambda_c^{*+} \bar{D}^0$		4457.09 ± 0.35
$\Sigma_c^+ \bar{D}^{*0}$		4459.9 ± 0.5
$\Sigma_c^+ \bar{D}^0 \pi^0$		4452.7 ± 0.5

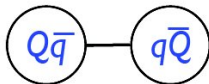
Models for XYZ Mesons

Quarkonium Tetraquarks

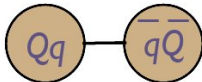
- compact tetraquark



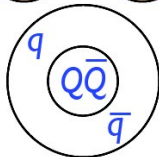
- meson molecule



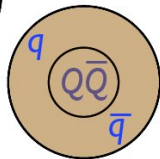
- diquark-onium



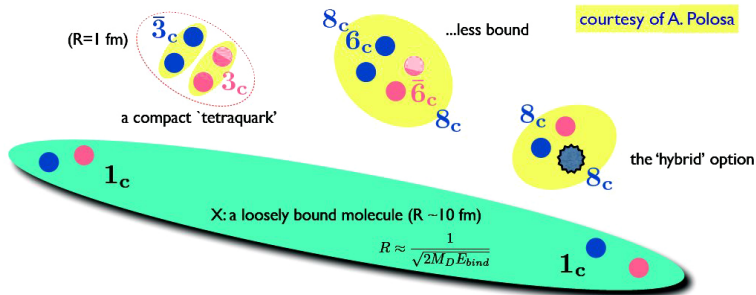
- hadro-quarkonium



- quarkonium adjoint meson

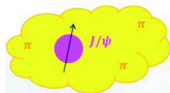


X, Y, Z Exotics



Hadro-charmonium

Voloshin arXiv:1304.0380



A $c\bar{c}$ state surrounded by light matter

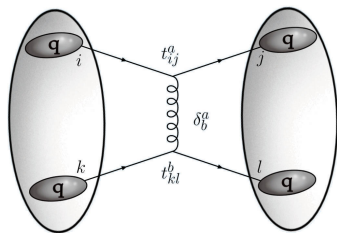
Decay into $\eta_c \rho$ forbidden by HQSS

-  quark (heavy or light)
-  antiquark
-  gluon

Diquarks: Color Representation

- One gluon exchange model [Jaffe, Phys.Rept.(2005)]

↪ Color factor determines binding:
Negative sign → Attractive



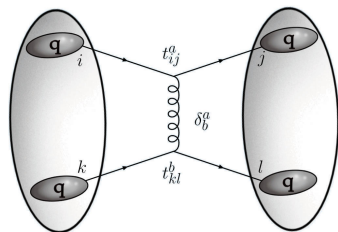
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- Quarks in diquark transform as:

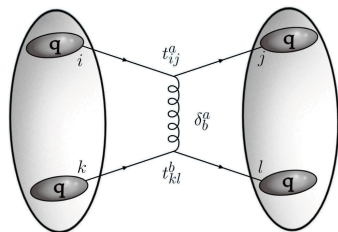
$$\mathbf{3} \otimes \mathbf{3} = \bar{\mathbf{3}} \oplus \mathbf{6}$$



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$$\mathbf{3} \otimes \mathbf{3} = \bar{\mathbf{3}} \oplus \mathbf{6}$$

- qq bound state color factor:

$$t_{ij}^a t_{kl}^a = -\frac{2}{3} \underbrace{(\delta_{ij}\delta_{kl} - \delta_{il}\delta_{kj})/2}_{\text{antisymmetric: projects } \bar{\mathbf{3}}} + \frac{1}{3} \underbrace{(\delta_{ij}\delta_{kl} + \delta_{il}\delta_{kj})/2}_{\text{symmetric: projects } \mathbf{6}}$$

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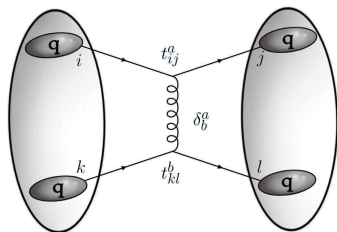
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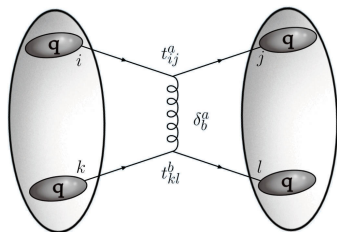
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✓ ~~✗~~

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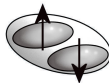
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Diquarks: Spin representation

$s=1/2$



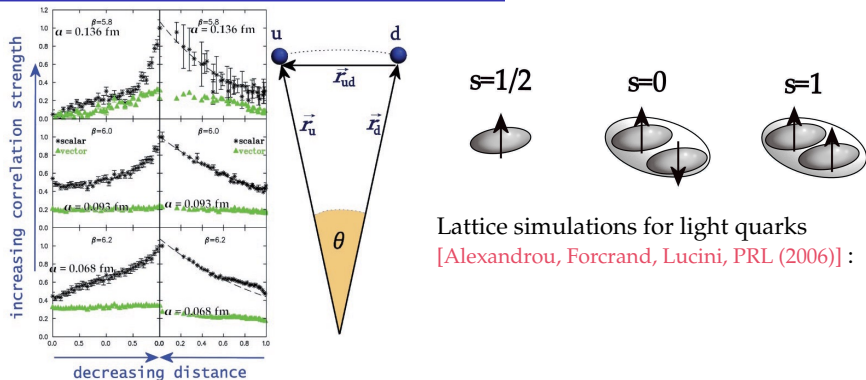
$s=0$



$s=1$



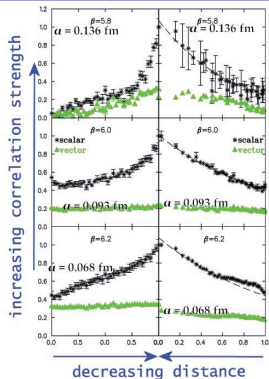
Diquarks: Spin representation



Lattice simulations for light quarks
 [Alexandrou, Forcrand, Lucini, PRL (2006)] :

- Calculation of 2 quark correlation strength
- Decreasing distance
- Increasing strength for "good" diquarks
- Diquark size $\mathcal{O}(1\text{fm})$

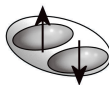
Diquarks: Spin representation



$s=1/2$



$s=0$



$s=1$



Lattice simulations for light quarks

[Alexandrou, Forcrand, Lucini, PRL (2006)] :

- Binding for “good” spin 0 diquarks
- No binding for “bad” spin 1 diquarks

■ Calculation of 2 quark correlation strength

■ Decreasing distance

↪ Increasing strength for “good” diquarks

■ Diquark size $\mathcal{O}(1\text{fm})$

Spin decoupling in HQ-Limit

↪ “Bad” diquarks in b -sector might bind

Diquark Model of Tetra- and Pentaquarks

Diquarks and Anti-diquarks are the building blocks of Tetraquarks

Color representation: $3 \otimes 3 = \bar{3} \oplus 6$; only $\bar{3}$ is attractive; $C_{\bar{3}} = 1/2 C_3$

Interpolating diquark operators for the two spin-states of diquarks

$$\begin{aligned} \text{Scalar: } 0^+ \quad \mathcal{Q}_{i\alpha} &= \epsilon_{\alpha\beta\gamma} (\bar{c}_c^\beta \gamma_5 q_i^\gamma - \bar{q}_{i_c}^\beta \gamma_5 c^\gamma) \\ \text{Axial-Vector: } 1^+ \quad \vec{\mathcal{Q}}_{i\alpha} &= \epsilon_{\alpha\beta\gamma} (\bar{c}_c^\beta \vec{\gamma} q_i^\gamma + \bar{q}_{i_c}^\beta \vec{\gamma} c^\gamma) \end{aligned} \quad \alpha, \beta, \gamma: SU(3)_C \text{ indices}$$

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$$\text{Axial-Vector: } 1^+ \quad \vec{Q}_{i\alpha} = \epsilon_{\alpha\beta\gamma} (\bar{c}_c^\beta \vec{\gamma} q_i^\gamma + \bar{q}_{i_c}^\beta \vec{\gamma} c^\gamma)$$

NR limit: States parametrized by Pauli matrices :

$$\text{Scalar: } 0^+ \quad \Gamma^0 = \frac{\sigma_2}{\sqrt{2}}$$

$$\text{Axial-Vector: } 1^+ \quad \vec{\Gamma} = \frac{\sigma_2 \vec{\sigma}}{\sqrt{2}}$$

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Diquark spin $s_Q \rightarrow$ tetraquark total angular momentum J :

$$|Y_{[bq]}\rangle = |s_Q, s_{\bar{Q}}; J\rangle$$

$$\hookrightarrow \text{Tetraquarks: } |0_Q, 0_{\bar{Q}}; 0_J\rangle = \Gamma^0 \otimes \Gamma^0$$

$$|1_Q, 1_{\bar{Q}}; 0_J\rangle = \frac{1}{\sqrt{3}} \Gamma^i \otimes \Gamma_i \dots$$

$$|0_Q, 1_{\bar{Q}}; 1_J\rangle = \Gamma^0 \otimes \Gamma^i$$

NR Hamiltonian for Tetraquarks with hidden charm

Involves constituent diquark mass, spin-spin, spin-orbit, and tensor forces

$$H = 2m_Q + H_{SS}^{(qq)} + H_{SS}^{(q\bar{q})} + H_{SL} + H_{LL} + H_T$$

- In the following, assume $\kappa_{q\bar{q}'} \simeq 0$

$$H_{\text{eff}}(X, Y, Z) = 2m_Q + \frac{B_Q}{2} L^2 + 2A_Q(L \cdot S) + 2\kappa_{qQ} [s_q \cdot s_Q + s_{\bar{q}} \cdot s_{\bar{Q}}] + b_Y \frac{S_{12}}{4}$$

NR Hamiltonian for Tetraquarks with hidden charm

Involves constituent diquark mass, spin-spin, spin-orbit, and tensor forces

$$H = 2m_Q + H_{SS}^{(qq)} + H_{SS}^{(q\bar{q})} + H_{SL} + H_{LL} + H_T$$

with

← constituent mass

$$= b_Y [3(\mathbf{S}_Q \cdot \mathbf{n})(\mathbf{S}_{\bar{Q}} \cdot \mathbf{n}) - (\mathbf{S}_Q \cdot \mathbf{S}_{\bar{Q}})]; \quad (\mathbf{n} = \text{unit vector})$$

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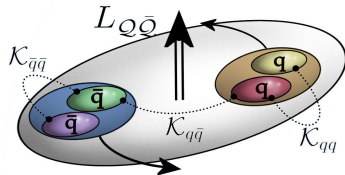
with

qq spin coupling

$q\bar{q}$ spin coupling

$$H_{SS}^{(qq)} = 2(\mathcal{K}_{cq})_{\bar{3}} [(\mathbf{S}_c \cdot \mathbf{S}_q) + (\mathbf{S}_{\bar{c}} \cdot \mathbf{S}_{\bar{q}})]$$

$$H_{SS}^{(q\bar{q})} = 2(\mathcal{K}_{c\bar{q}})(\mathbf{S}_c \cdot \mathbf{S}_{\bar{q}} + \mathbf{S}_{\bar{c}} \cdot \mathbf{S}_q) + 2\mathcal{K}_{c\bar{c}}(\mathbf{S}_c \cdot \mathbf{S}_{\bar{c}}) + 2\mathcal{K}_{q\bar{q}}(\mathbf{S}_q \cdot \mathbf{S}_{\bar{q}})$$



$$= b_Y [3(\mathbf{S}_Q \cdot \mathbf{n})(\mathbf{S}_{\bar{Q}} \cdot \mathbf{n}) - (\mathbf{S}_Q \cdot \mathbf{S}_{\bar{Q}})]; \quad (\mathbf{n} = \text{unit vector})$$

- In the following, assume $\kappa_{q\bar{q}'} \simeq 0$

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NR Hamiltonian for Tetraquarks with hidden charm

Involves constituent diquark mass, spin-spin, spin-orbit, and tensor forces

$$H = 2m_Q + H_{SS}^{(qq)} + H_{SS}^{(q\bar{q})} + H_{SL} + H_{LL} + H_T$$

with

LS coupling LL coupling

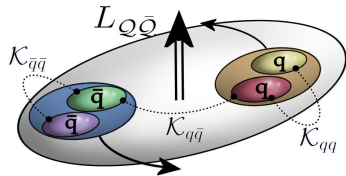
$$H_{SS}^{(qq)} = 2(\mathcal{K}_{cq})_{\bar{3}} [(\mathbf{S}_c \cdot \mathbf{S}_q) + (\mathbf{S}_{\bar{c}} \cdot \mathbf{S}_{\bar{q}})]$$

$$H_{SS}^{(q\bar{q})} = 2(\mathcal{K}_{c\bar{q}})(\mathbf{S}_c \cdot \mathbf{S}_{\bar{q}} + \mathbf{S}_{\bar{c}} \cdot \mathbf{S}_q) + 2\mathcal{K}_{c\bar{c}}(\mathbf{S}_c \cdot \mathbf{S}_{\bar{c}}) + 2\mathcal{K}_{q\bar{q}}(\mathbf{S}_q \cdot \mathbf{S}_{\bar{q}})$$

$$H_{SL} = 2A_Q(\mathbf{S}_Q \cdot \mathbf{L} + \mathbf{S}_{\bar{Q}} \cdot \mathbf{L})$$

$$H_{LL} = B_Q \frac{L_{Q\bar{Q}}(L_{Q\bar{Q}} + 1)}{2}$$

$$= b_Y [3(\mathbf{S}_Q \cdot \mathbf{n})(\mathbf{S}_{\bar{Q}} \cdot \mathbf{n}) - (\mathbf{S}_Q \cdot \mathbf{S}_{\bar{Q}})]; \quad (\mathbf{n} = \text{unit vector})$$



- In the following, assume $\kappa_{q\bar{q}'} \simeq 0$

$$H_{\text{eff}}(X, Y, Z) = 2m_Q + \frac{B_Q}{2}L^2 + 2A_Q(L \cdot S) + 2\kappa_{qQ}[s_q \cdot s_Q + s_{\bar{q}} \cdot s_{\bar{Q}}] + b_Y \frac{S_{12}}{4}$$

NR Hamiltonian for Tetraquarks with hidden charm

Involves constituent diquark mass, spin-spin, spin-orbit, and tensor forces

$$H = 2m_Q + H_{SS}^{(qq)} + H_{SS}^{(q\bar{q})} + H_{SL} + H_{LL} + H_T$$

with

$$H_{SS}^{(qq)} = 2(\mathcal{K}_{cq})_{\bar{3}} [(\mathbf{S}_c \cdot \mathbf{S}_q) + (\mathbf{S}_{\bar{c}} \cdot \mathbf{S}_{\bar{q}})]$$

$$H_{SS}^{(q\bar{q})} = 2(\mathcal{K}_{c\bar{q}})(\mathbf{S}_c \cdot \mathbf{S}_{\bar{q}} + \mathbf{S}_{\bar{c}} \cdot \mathbf{S}_q) + 2\mathcal{K}_{c\bar{c}}(\mathbf{S}_c \cdot \mathbf{S}_{\bar{c}}) + 2\mathcal{K}_{q\bar{q}}(\mathbf{S}_q \cdot \mathbf{S}_{\bar{q}})$$

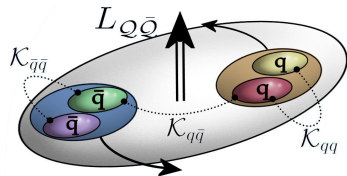
$$H_{SL} = 2A_Q(\mathbf{S}_Q \cdot \mathbf{L} + \mathbf{S}_{\bar{Q}} \cdot \mathbf{L})$$

$$H_{LL} = B_Q \frac{L_{Q\bar{Q}}(L_{Q\bar{Q}} + 1)}{2}$$

$$H_T = b_Y \frac{S_{12}}{4} = b_Y [3(\mathbf{S}_Q \cdot \mathbf{n})(\mathbf{S}_{\bar{Q}} \cdot \mathbf{n}) - (\mathbf{S}_Q \cdot \mathbf{S}_{\bar{Q}})]; \quad (\mathbf{n} = \text{unit vector})$$

- In the following, assume $\kappa_{q\bar{q}'} \simeq 0$

$$H_{\text{eff}}(X, Y, Z) = 2m_Q + \frac{B_Q}{2} L^2 + 2A_Q(L \cdot S) + 2\kappa_{qQ} [s_q \cdot s_Q + s_{\bar{q}} \cdot s_{\bar{Q}}] + b_Y \frac{S_{12}}{4}$$



Low-lying S-Wave Tetraquark States

- In the $|s_{qQ}, s_{\bar{q}\bar{Q}}; S, L\rangle_J$ and $|s_{q\bar{q}}, s_{Q\bar{Q}}; S', L'\rangle_J$ bases, the positive parity S-wave tetraquarks are listed below; $M_{00} = 2m_Q$

Label	J^{PC}	$ s_{qQ}, s_{\bar{q}\bar{Q}}; S, L\rangle_J$	$ s_{q\bar{q}}, s_{Q\bar{Q}}; S', L'\rangle_J$	Mass
X_0	0^{++}	$ 0, 0; 0, 0\rangle_0$	$(0, 0; 0, 0\rangle_0 + \sqrt{3} 1, 1; 0, 0\rangle_0) / 2$	$M_{00} - 3\kappa_{qQ}$
X'_0	0^{++}	$ 1, 1; 0, 0\rangle_0$	$(\sqrt{3} 0, 0; 0, 0\rangle_0 - 1, 1; 0, 0\rangle_0) / 2$	$M_{00} + \kappa_{qQ}$
X_1	1^{++}	$(1, 0; 1, 0\rangle_1 + 0, 1; 1, 0\rangle_1) / \sqrt{2}$	$ 1, 1; 1, 0\rangle_1$	$M_{00} - \kappa_{qQ}$
Z	1^{+-}	$(1, 0; 1, 0\rangle_1 - 0, 1; 1, 0\rangle_1) / \sqrt{2}$	$(1, 0; 1, 0\rangle_1 - 0, 1; 1, 0\rangle_1) / \sqrt{2}$	$M_{00} - \kappa_{qQ}$
Z'	1^{+-}	$ 1, 1; 1, 0\rangle_1$	$(1, 0; 1, 0\rangle_1 + 0, 1; 1, 0\rangle_1) / \sqrt{2}$	$M_{00} + \kappa_{qQ}$
X_2	2^{++}	$ 1, 1; 2, 0\rangle_2$	$ 1, 1; 2, 0\rangle_2$	$M_{00} + \kappa_{qQ}$

- The spectrum of these states depends on just two parameters, $M_{00}(Q)$ and κ_{qQ} , $Q = c, b$, hence very predictive
- Some of the states still missing and are being searched for at the LHC

Charmonium-like and Bottomonium-like Tetraquark Spectrum

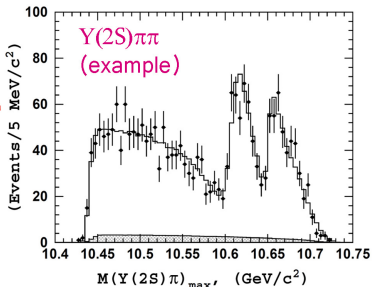
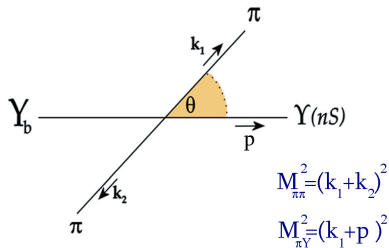
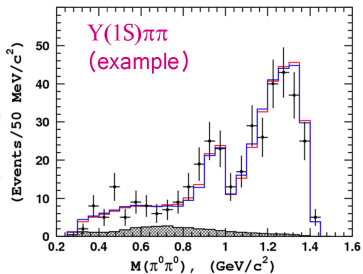
Parameters in the Mass Formula

	charmonium-like	bottomonium-like
M_{00} [MeV]	3957	10630
κ_{qQ} [MeV]	67	22.5

Label	J^{PC}	charmonium-like		bottomonium-like	
		State	Mass [MeV]	State	Mass [MeV]
X_0	0^{++}	—	3756	—	10562
X'_0	0^{++}	—	4024	—	10652
X_1	1^{++}	$X(3872)$	3890	—	10607
Z	1^{+-}	$Z_c^+(3900)$	3890	$Z_b^{+,0}(10610)$	10607
Z'	1^{+-}	$Z_c^+(4020)$	4024	$Z_b^+(10650)$	10652
X_2	2^{++}	—	4024	—	10652

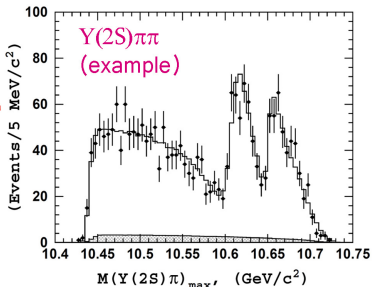
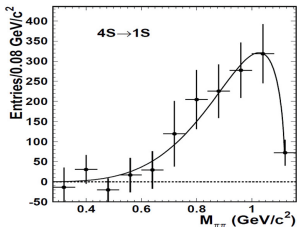
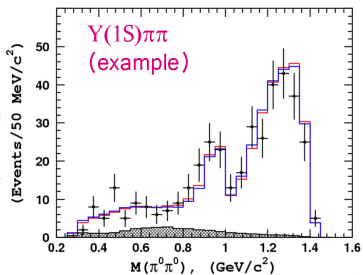
Dipion mass distributions in $Y(5S) \rightarrow Y(nS)\pi\pi$ decays?

[Belle Collaboration (2012)]



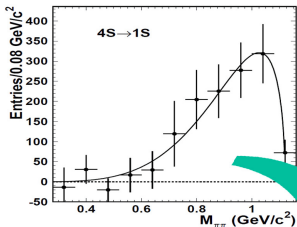
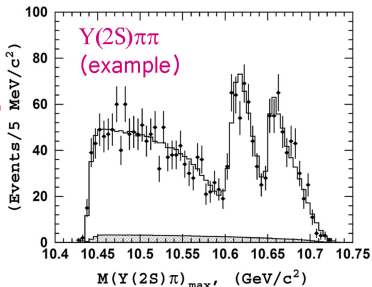
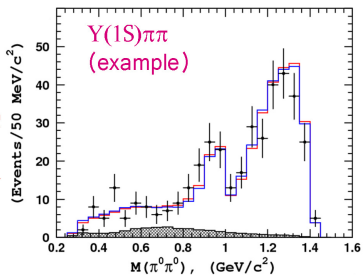
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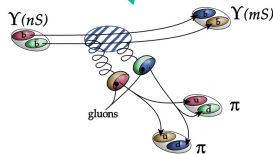
Dipion mass distributions in $Y(5S) \rightarrow Y(nS)\pi\pi$ decays?

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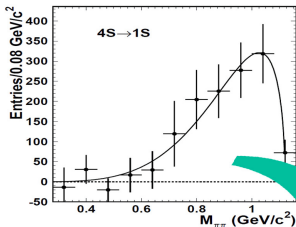
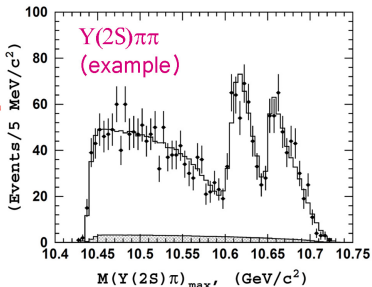
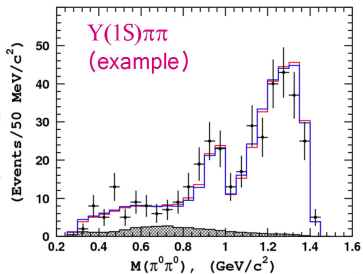
theory works
well (multipole exp.)
[Brown, Cahn PRL 75]
[Voloshin, JETP 75]

Process:

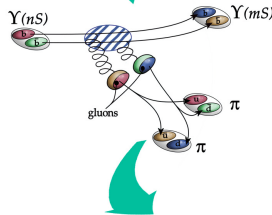


Dipion mass distributions in $Y(5S) \rightarrow Y(nS)\pi\pi$ decays?

[Belle Collaboration (2012)]



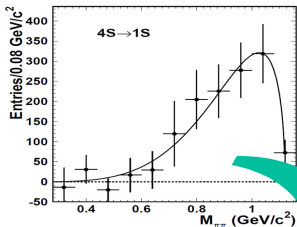
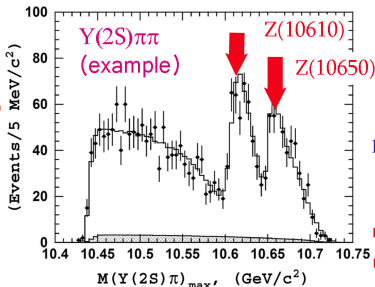
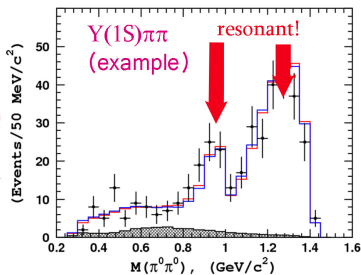
theory works well (multipole exp.)
[Brown, Cahn PRL 75]
[Voloshin, JETP 75]
Process:



- NO resonant structure
- Zweig forbidden

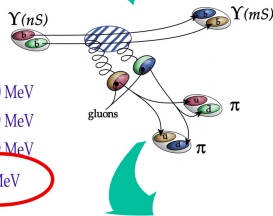
Dipion mass distributions in $Y(nS) \rightarrow Y(mS)\pi\pi$ decays?

[Belle Collaboration (2012)]



theory works
well (multipole exp.)
[Brown, Cahn PRL 75]
[Voloshin, JETP 75]

Process:



$$\Gamma(Y(2S) \rightarrow Y(1S)\pi\pi) \approx 0.0060 \text{ MeV}$$

$$\Gamma(Y(3S) \rightarrow Y(1S)\pi\pi) \approx 0.0009 \text{ MeV}$$

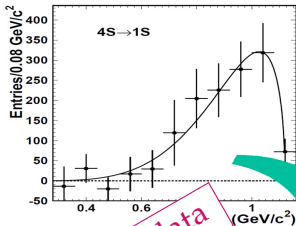
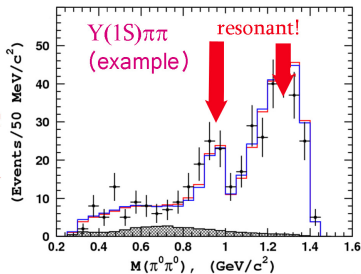
$$\Gamma(Y(4S) \rightarrow Y(1S)\pi\pi) \approx 0.0019 \text{ MeV}$$

$$\Gamma(Y(5S) \rightarrow Y(1S)\pi^+\pi^-) \approx 0.59 \text{ MeV}$$

- distinct resonant structure
- differs by two orders of Magnitude!
- NO resonant structure
- Zweig forbidden

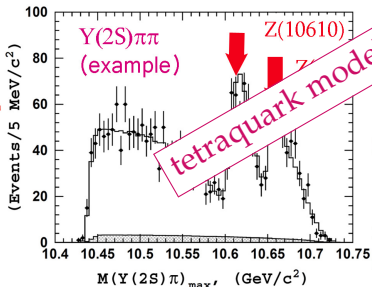
Dipion mass distributions in $Y(5S) \rightarrow Y(nS)\pi\pi$ decays?

[Belle Collaboration (2012)]



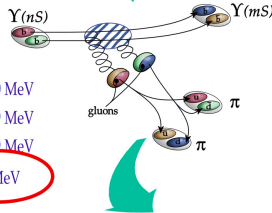
theory works
well (multipole exp.)
[Brown, Cahn PRL 75]
[Voloshin, JETP 75]

Process:



tetraquark model can explain data

- $\Gamma(Y(2S) \rightarrow Y(1S)\pi\pi) \approx 0.0060 \text{ MeV}$
- $\Gamma(Y(3S) \rightarrow Y(1S)\pi\pi) \approx 0.0009 \text{ MeV}$
- $\Gamma(Y(4S) \rightarrow Y(1S)\pi\pi) \approx 0.0019 \text{ MeV}$
- $\Gamma(Y(5S) \rightarrow Y(1S)\pi^+\pi^-) \approx 0.59 \text{ MeV}$



- distinct resonant structure
- differs by two orders of Magnitude!
- NO resonant structure
- Zweig forbidden

Heavy-Quark-Spin Flip in $Y(10890) \rightarrow Z_b/Z'_b + \pi \rightarrow h_b(1P, 2P)\pi\pi$

A.A., L. Maiani, A.D. Polosa, V. Riquer; PR D91, 017502 (2015)

Relative normalizations and phases for $s_{b\bar{b}}$: $1 \rightarrow 1$ and $1 \rightarrow 0$ transitions

Final State	$Y(1S)\pi^+\pi^-$	$Y(2S)\pi^+\pi^-$	$Y(3S)\pi^+\pi^-$	$h_b(1P)\pi^+\pi^-$	$h_b(2P)\pi^+\pi^-$
Rel. Norm.	$0.57 \pm 0.21^{+0.19}_{-0.04}$	$0.86 \pm 0.11^{+0.04}_{-0.10}$	$0.96 \pm 0.14^{+0.08}_{-0.05}$	$1.39 \pm 0.37^{+0.05}_{-0.15}$	$1.6^{+0.6+0.4}_{-0.4-0.6}$
Rel. Phase	$58 \pm 43^{+4}_{-9}$	$-13 \pm 13^{+17}_{-8}$	$-9 \pm 19^{+11}_{-26}$	187^{+44+3}_{-57-12}	$181^{+65+74}_{-105-109}$

- In $Y(10890)$, $S_{b\bar{b}} = 1$. In $h_b(nP)$, $S_{b\bar{b}} = 0$, transitions above involve heavy-quark spin-flip, yet rates not suppressed, violating heavy-quark-spin conservation
- This contradiction is only apparent. Expressing the states Z_b and Z'_b in the basis of definite $b\bar{b}$ and light quark $q\bar{q}$ spins

$$|Z_b\rangle = \frac{\alpha|1_{q\bar{q}}, 0_{b\bar{b}}\rangle - \beta|0_{q\bar{q}}, 1_{b\bar{b}}\rangle}{\sqrt{2}}, \quad |Z'_b\rangle = \frac{\beta|1_{q\bar{q}}, 0_{b\bar{b}}\rangle + \alpha|0_{q\bar{q}}, 1_{b\bar{b}}\rangle}{\sqrt{2}}$$

- and defining (g are the effective couplings at the vertices $Y Z_b \pi$ and $Z_b h_b \pi$)

$$g_Z \equiv g(Y \rightarrow Z_b \pi)g(Z_b \rightarrow h_b \pi) \propto -\alpha\beta\langle h_b | Z_b \rangle \langle Z_b | Y \rangle$$

$$g_{Z'} \equiv g(Y \rightarrow Z'_b \pi)g(Z'_b \rightarrow h_b \pi) \propto \alpha\beta\langle h_b | Z'_b \rangle \langle Z'_b | Y \rangle$$

Determination of α/β from $Y(10890) \rightarrow Z_b/Z'_b + \pi \rightarrow Y(nS)\pi\pi$ ($n = 1, 2, 3$)

- A comprehensive analysis of the Belle data including the direct and resonant components is required to test the underlying dynamics, yet to be carried out
- Parametrizing the amplitudes in terms of two Breit-Wigners, one can determine the ratio α/β

$s_{b\bar{b}} : 1 \rightarrow 1$ transition :

$$\overline{\text{Rel.Norm.}} = 0.85 \pm 0.08 = |\alpha|^2 / |\beta|^2$$

$$\overline{\text{Rel.Phase}} = (-8 \pm 10)^\circ$$

$s_{b\bar{b}} : 1 \rightarrow 0$ transition :

$$\overline{\text{Rel.Norm.}} = 1.4 \pm 0.3$$

$$\overline{\text{Rel.Phase}} = (185 \pm 42)^\circ$$

- Within errors, the tetraquark assignment with $\alpha = \beta = 1$ is supported, i.e.,

$$|Z_b\rangle = \frac{|1_{bq}, 0_{\bar{b}\bar{q}}\rangle - |0_{bq}, 1_{\bar{b}\bar{q}}\rangle}{\sqrt{2}}, \quad |Z'_b\rangle = |1_{bq}, 1_{\bar{b}\bar{q}}\rangle_{J=1}$$

$$|Z_b\rangle = \frac{|1_{q\bar{q}}, 0_{b\bar{b}}\rangle - |0_{q\bar{q}}, 1_{b\bar{b}}\rangle}{\sqrt{2}}, \quad |Z'_b\rangle = \frac{|1_{q\bar{q}}, 0_{b\bar{b}}\rangle + |0_{q\bar{q}}, 1_{b\bar{b}}\rangle}{\sqrt{2}}$$

A new look at the Υ tetraquarks and the excited Ω_c states in the Diquark model

- Observation of 5 narrow excited Ω_c baryons in $\Omega_c \rightarrow \Xi_c^+ K^-$ [LHCb, PRL 118, 182001 (2017)]
- Measured masses (in MeV) [LHCb] and plausible J^P quantum numbers, assuming diquark model $\Omega_c(=css) = c[ss]$ [M. Karliner, J.L. Rosner, PR D95, 114012 (2017)]

$$M(\Omega_c(3000)) = 3000.4 \pm 0.2 \pm 0.1; J^P = 1/2^-$$

$$M(\Omega_c(3050)) = 3050.2 \pm 0.1 \pm 0.1; J^P = 1/2^-$$

$$M(\Omega_c(3066)) = 3065.6 \pm 0.1 \pm 0.3; J^P = 3/2^-$$

$$M(\Omega_c(3090)) = 3090.2 \pm 0.3 \pm 0.5; J^P = 3/2^-$$

$$M(\Omega_c(3119)) = 3119.1 \pm 0.3 \pm 0.9; J^P = 5/2^-$$

- For the P states, important to take into account the tensor couplings

$$H_{\text{eff}} = m_c + m_{[ss]} + \kappa_{ss} S_s \cdot S_s + \frac{B_Q}{2} L^2 + V_{SD},$$

$$V_{SD} = a_1 L \cdot S_{[ss]} + a_2 L \cdot S_c + b \frac{\langle S_{12} \rangle}{4} + c S_{[ss]} \cdot S_c$$

Analysis of the excited Ω_c states in the Diquark-Quark model

$$\frac{\langle S_{12} \rangle}{2} = \langle 2Q(S_{[ss]}, S_c) \rangle = \langle Q(S, S) - Q(S_c, S_c) - Q(S_{[ss]}, S_{[ss]}) \rangle$$

$$\langle Q(S_X, S_X) \rangle = -\frac{3}{5} \langle [2(L \cdot S_X)^2 + (L \cdot S_X) - \frac{4}{3}(S_X \cdot S_X)] \rangle$$

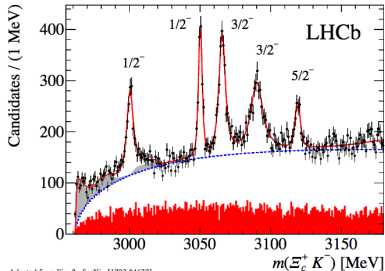
$$J = 1/2: \quad \frac{1}{4} \langle S_{12} \rangle = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -1 \end{pmatrix}$$

$$J = 3/2: \quad \frac{1}{4} \langle S_{12} \rangle = \begin{pmatrix} 0 & -\frac{1}{2\sqrt{5}} \\ -\frac{1}{2\sqrt{5}} & \frac{4}{5} \end{pmatrix}$$

$$J = 5/2: \quad \frac{1}{4} \langle S_{12} \rangle = -\frac{1}{5}$$

- Coeffs. determined from the masses of the J^P states. $M_0 \equiv m_c + m_{[ss]} + 2\kappa_{ss} + B_Q$

a_1	a_2	b	c	M_0
26.95	25.75	13.52	4.07	3079.94



Adapted from Fig. 2 of arXiv:1703.04639

Analysis of the tetraquark Y states in the diquark model

$$\begin{aligned}
 H_{\text{eff}} &= 2m_Q + \frac{B_Q}{2}L^2 - 3\kappa_{cq} + 2a_Y L \cdot S + b_Y \frac{\langle S_{12} \rangle}{4} \\
 &+ \kappa_{cq} [2(S_q \cdot S_c + S_{\bar{q}} \cdot S_{\bar{c}}) + 3] \\
 \frac{1}{4} \langle S_{12} \rangle &= \begin{pmatrix} 0 & 2/\sqrt{5} \\ 2/\sqrt{5} & -7/5 \end{pmatrix}
 \end{aligned}$$

- There are four $L = 1$ and one $L = 3$ tetraquark states with $J^{PC} = 1^{--}$
- Tensor couplings non-vanishing only for the states with $S_Q = S_{\bar{Q}} = 1$

P-wave ($J^{PC} = 1^{--}$) states

Label	J^{PC}	$ s_{qQ}, s_{\bar{q}\bar{Q}}; S, L\rangle_J$	$ s_{q\bar{q}}, s_{Q\bar{Q}}; S', L'\rangle_J$	Mass
Y_1	1^{--}	$ 0, 0; 0, 1\rangle_1$	$(0, 0; 0, 1\rangle_1 + \sqrt{3} 1, 1; 0, 1\rangle_1) / 2$	$M_{00} - 3\kappa_{qQ} + B_Q \equiv \tilde{M}_{00}$
Y_2	1^{--}	$(1, 0; 1, 1\rangle_1 + 0, 1; 1, 1\rangle_1) / \sqrt{2}$	$ 1, 1; 1, L'\rangle_1$	$\tilde{M}_{00} + 2\kappa_{qQ} - 2A_Q$
Y_3	1^{--}	$ 1, 1; 0, 1\rangle_1$	$(\sqrt{3} 0, 0; 0, 1\rangle_1 - 1, 1; 0, 1\rangle_1) / 2$	$\tilde{M}_{00} + 4\kappa_{qQ} + E_+$
Y_4	1^{--}	$ 1, 1; 2, 1\rangle_1$	$ 1, 1; 2, L'\rangle_1$	$\tilde{M}_{00} + 4\kappa_{qQ} + E_-$
Y_5	1^{--}	$ 1, 1; 2, 3\rangle_1$	$ 1, 1; 2, L'\rangle_1$	$M_{Y_2} + 2\kappa_{qQ} - 14A_Q + 5B_Q - 8/5b_Y$

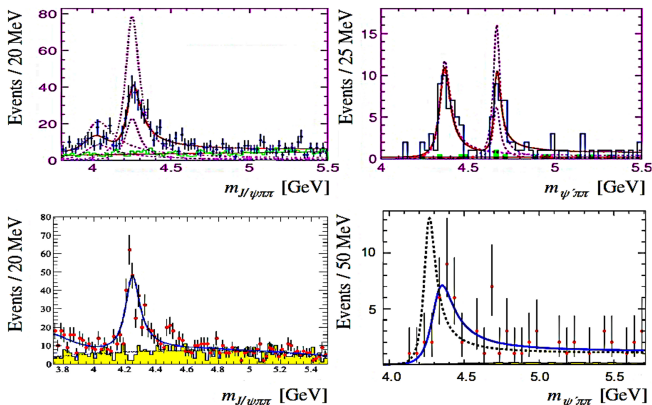
$$E_{\pm} = \frac{1}{10} (-30A_Q - 7b_Y \mp \sqrt{3} \sqrt{300A_Q^2 + 140A_Q b_Y + 43b_Y^2})$$

Experimental situation with the tetraquark Y states rather confusing

- Summary of the Y states observed in Initial State Radiation (ISR) processes in e^+e^- annihilation [BaBar, Belle, CLEO]

$$e^+e^- \rightarrow \gamma_{\text{ISR}} J/\psi \pi^+ \pi^-; \gamma_{\text{ISR}} \psi' \pi^+ \pi^-$$

$$\Rightarrow Y(4008), Y(4260), Y(4360), Y(4660)$$

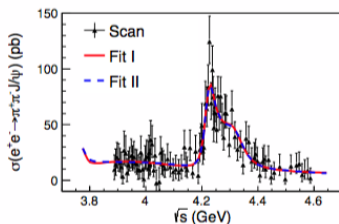
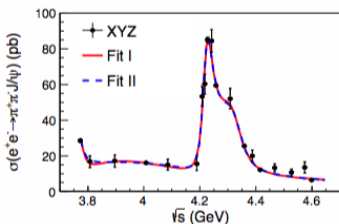


$e^+e^- \rightarrow J/\psi\pi^+\pi^-$ cross section at $\sqrt{s} = (3.77 - 4.60)$ GeV

(BESIII, PRL 118, 092001 (2017)

- Y(4008) is not confirmed; Y(4260) is split into 2 resonances: Y(4220) and Y(4320), with the Y(4220) probably the same as Y(4260)

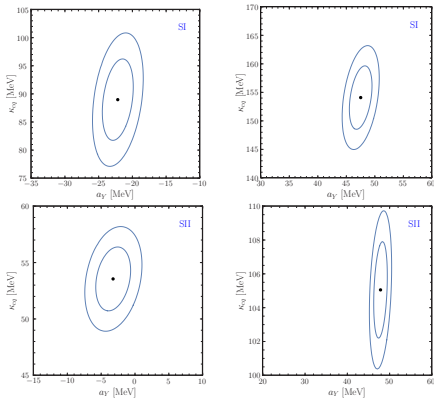
Parameters	Fit result
$M(R_1)$	$3812.6^{+61.9}_{-96.6} (\dots)$
$\Gamma_{\text{tot}}(R_1)$	$476.9^{+78.4}_{-64.8} (\dots)$
$M(R_2)$	4222.0 ± 3.1 (4220.9 ± 2.9)
$\Gamma_{\text{tot}}(R_2)$	44.1 ± 4.3 (44.1 ± 3.8)
$M(R_3)$	4320.0 ± 10.4 (4326.8 ± 10.0)
$\Gamma_{\text{tot}}(R_3)$	$101.4^{+25.3}_{-19.7}$ ($98.2^{+25.4}_{-19.6}$)



Two Experimental Scenarios for the Y States

[AA, L. Maiani, A. Borisov, I. Ahmed, A. Rehman, M.J. Aslam, A. Parkhomenko, A.D. Polosa, arxiv:1708.04650]

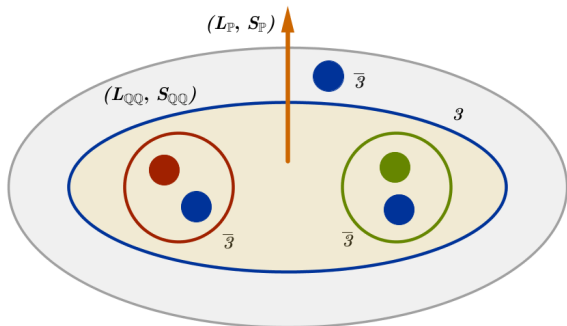
- SI (Based on CLEO, BaBaR, Belle): $Y(4008)$, $Y(4260)$, $Y(4360)$, $Y(4660)$
- SII (BESIII, PRL 118, 092001 (2017): $Y(4220)$, $Y(4320)$, $Y(4390)$, $Y(4660)$)



- SII $\implies \kappa_{cq}$ and a_Y for Y states similar to the ones in (X, Z) and Ω_c

Effective Hamiltonian for Pentaquarks

[Ahmed,Rehman,Aslam,AA, arxiv:1607.00987]



Diquark – Diquark – Antiquark Model of Pentaquarks

$$H_{\text{eff}}(\mathbb{P}) = H_{\text{eff}}([QQ]) + m_{\bar{c}} + \kappa_{\bar{c}[QQ]}(s_{\bar{c}} \cdot S_{[QQ]}) - 2a_{\mathbb{P}}(L_{\mathbb{P}} \cdot S_{\mathbb{P}}) + \frac{B_{\mathbb{P}}}{2} \langle L_{\mathbb{P}}^2 \rangle$$

- $S_{[QQ]}$ is the spin of the tetraquark; $s_{\bar{c}}$ is the spin of the \bar{c}
 $L_{\mathbb{P}}$ and $S_{\mathbb{P}}$ are the orbital angular momentum and spin of the pentaquark,
 respectively

Pentaquarks in the diquark model [Maiani et al., arxiv:1507.04980]

- $\Lambda_b(bud) \rightarrow \mathbb{P}^+ K^-$ decaying according to $\mathbb{P}^+ \rightarrow J/\Psi + p$
- \mathbb{P}^+ carry a unit of baryonic number and have the valence quarks

$$\mathbb{P}^+ = \bar{c}cuud$$

- Assume the assignments

$$\mathbb{P}^+(3/2^-) = \{\bar{c} [cq]_{s=1} [q'q'']_{s=1}, L = 0\}$$

$$\mathbb{P}^+(5/2^+) = \{\bar{c} [cq]_{s=1} [q'q'']_{s=0}, L = 1\}$$

- Mass difference:

- Level spacing for $\Delta L = 1$ in light baryons; $\Lambda(1405) - \Lambda(1116) \sim 290$ MeV

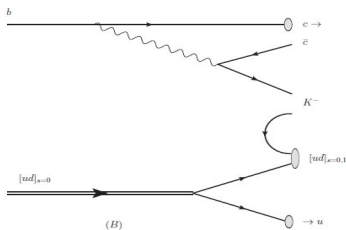
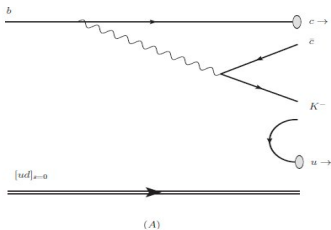
- Light-light diquark mass difference for $\Delta S = 1$:

$$[qq']_{s=1} - [qq']_{s=0} = \Sigma_c(2455) - \Lambda_c(2286) \simeq 170 \text{ MeV}$$

- Orbital gap $\mathbb{P}^+(3/2^-) - \mathbb{P}^+(5/2^+)$ is thereby reduced to 120 MeV, more or less in agreement with data, 70 MeV

Pentaquark production mechanisms in $\Lambda_b^0 \rightarrow K^- J/\psi p$

- Two possible mechanisms are proposed by Maiani et al.
 - In the first, b -quark spin is shared between the K^- , and the \bar{c} and $[cu]$ components, the final $[ud]$ diquark has spin-0, Fig. A
 - In the second, the $[ud]$ diquark is formed from the original d quark, and the u quark from the vacuum $u\bar{u}$; angular momentum is shared among all components, and the diquark $[ud]$ may have both spins, $s = 0, 1$, Fig. B
- Which of the two diagrams dominate is a dynamical question; semileptonic decays of Λ_b hint that the mechanism in Fig. B is dynamically suppressed



Flavor $SU(3)$ structure of Pentaquarks

- Pentaquarks are of two types:

$$\mathbb{P}_u = \epsilon^{\alpha\beta\gamma} \bar{c}_\alpha [cu]_{\beta,s=0,1} [ud]_{\gamma,s=0,1}$$

$$\mathbb{P}_d = \epsilon^{\alpha\beta\gamma} \bar{c}_\alpha [cd]_{\beta,s=0,1} [uu]_{\gamma,s=1}$$

- This leads to two distinct $SU(3)$ series of Pentaquarks

$$\mathbb{P}_A = \epsilon^{\alpha\beta\gamma} \left\{ \bar{c}_\alpha [cq]_{\beta,s=0,1} [q'q'']_{\gamma,s=0, L} \right\} = \mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}$$

$$\mathbb{P}_S = \epsilon^{\alpha\beta\gamma} \left\{ \bar{c}_\alpha [cq]_{\beta,s=0,1} [q'q'']_{\gamma,s=1, L} \right\} = \mathbf{3} \otimes \mathbf{6} = \mathbf{8} \oplus \mathbf{10}$$

- For S waves, the first and the second series have the angular momenta (multiplicity)

$$\mathbb{P}_A(L=0) : J = 1/2(2), 3/2(1)$$

$$\mathbb{P}_S(L=0) : J = 1/2(3), 3/2(3), 5/2(1)$$

- Maiani et al. propose to assign $\mathbb{P}(3/2^-)$ to the \mathbb{P}_A and $\mathbb{P}(5/2^+)$ to the \mathbb{P}_S series of Pentaquarks

Heavy quark symmetry and observed pentaquarks

[Ahmed,Rehman,Aslam,AA, arxiv:1607.00987]

Selection rules from the the data on $b \rightarrow c$ baryonic decays and HQS

$$P_c^+(4450) = \{\bar{c}[cu]_{s=1}[ud]_{s=0}; L_{\mathcal{P}} = 1, J^P = \frac{5}{2}^+\} \quad \text{Favored}$$

$$P_c^+(4380) = \{\bar{c}[cu]_{s=1}[ud]_{s=1}; L_{\mathcal{P}} = 0, J^P = \frac{3}{2}^-\} \quad \text{Disfavored}$$

$\implies \frac{3}{2}^-$ state may require a different interpretation.

$m[\Lambda_c^+(2625); J^P = \frac{3}{2}^-] - m[\Lambda_c^+(2286); J^P = \frac{1}{2}^+] \simeq 341 \text{ MeV} \implies$ the mass of $J^P = 3/2^-$ state to be about 4110 MeV.

In **diquark-diquark-antiquark** spectrum, $\frac{3}{2}^-$ state is **favored** by HQS,

$$\{\bar{c}[cu]_{s=1}[ud]_{s=0}; L_{\mathcal{P}} = 0, J^P = \frac{3}{2}^-\},$$

Third state anticipated in 4110-4130 MeV range. A renewed fit of the LHCb data by allowing a third resonance is called for.

Weak decays of the b -baryons into pentaquark states

$$\mathcal{A} = \langle \mathcal{P} \mathcal{M} | H_{\text{eff}}^W | \mathcal{B} \rangle, \text{ with } H_{\text{eff}}^W = \frac{4G_F}{\sqrt{2}} \left[V_{cb} V_{cq}^* (c_1 O_1^{(q)} + c_2 O_2^{(q)}) \right]$$

H_{eff}^W inducing the Cabibbo-allowed $\Delta I = 0, \Delta S = -1$ transition $b \rightarrow c\bar{c}s$, and the Cabibbo-suppressed $\Delta S = 0$ transition $b \rightarrow c\bar{c}d$.

$$O_1^{(q)} = (\bar{q}_\alpha c_\beta)_{V-A} (\bar{c}_\alpha b_\beta)_{V-A} \text{ and } O_2^{(q)} = (\bar{q}_\alpha c_\alpha)_{V-A} (\bar{c}_\beta b_\beta)_{V-A}$$

$$\mathcal{B}_{ij}(\mathbf{3}) = \Lambda_b^0(udb), \Xi_b^0(usb), \Xi_b^-(dsb), \quad \mathcal{C}_{ij}(\mathbf{6}) = \Sigma_b^-(ddb), \Sigma_b^0(udb), \Sigma_b^+(uub), \Xi_b'(dsb), \Xi_b^0(usb), \Omega_b^-(ssb)$$

$$\mathcal{M}_i^j = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta_8}{\sqrt{6}} \end{pmatrix}, \quad \mathcal{P}_i^j(J^P) = \begin{pmatrix} \frac{P_{\Sigma^0}}{\sqrt{2}} + \frac{P_\Lambda}{\sqrt{6}} & P_{\Sigma^+} & P_p \\ P_{\Sigma^-} & -\frac{P_{\Sigma^0}}{\sqrt{2}} + \frac{P_\Lambda}{\sqrt{6}} & P_n \\ P_{\Xi^-} & P_{\Xi^0} & -\frac{P_\Lambda}{\sqrt{6}} \end{pmatrix}.$$

A decuplet \mathcal{P}_{ijk} : $\mathcal{P}_{111} = P_{\Delta^{++}}, \mathcal{P}_{112} = P_{\Delta^{+}}/\sqrt{3}, \mathcal{P}_{122} = P_{\Delta^{0}}/\sqrt{3}, \mathcal{P}_{222} = P_{\Delta^{-}}, \mathcal{P}_{113} = P_{\Sigma^{+}}/\sqrt{3}, \mathcal{P}_{123} = P_{\Sigma^{0}}/\sqrt{6}, \mathcal{P}_{223} = P_{\Sigma^{-}}/\sqrt{3}, \mathcal{P}_{133} = P_{\Xi^{0}}/\sqrt{3}, \mathcal{P}_{233} = P_{\Xi^{-}}/\sqrt{3}$ and $\mathcal{P}_{333} = P_{\Omega^{-}}$.

◇ Calculating the decay amplitudes is a formidable challenge.

◇ $SU(3)_F$ symmetry relations provided useful guide for pentaquark searches, [Li et al.](#)

[\[arXiv:1507.08252\]](#)

$SU(3)$ based analysis of $\Lambda_b \rightarrow \mathbb{P}^+ K^- \rightarrow (J/\psi p) K^-$

- With respect to flavor $SU(3)$, $\Lambda_b(bud) \sim \bar{3}$, and is isosinglet $I = 0$
- The weak non-leptonic Hamiltonian for $b \rightarrow c\bar{c}s$ decays is:

$$H_W^{(3)}(\Delta I = 0, \Delta S = -1)$$

- With M a nonet of $SU(3)$ light mesons, $\langle \mathbb{P}, M | H_W | \Lambda_b \rangle$ requires $\mathbb{P} + M$ to be in $8 \oplus 1$ representation
- Recalling the $SU(3)$ group multiplication rule

$$8 \otimes 8 = 1 \oplus 8 \oplus 8 \oplus 10 \oplus \bar{10} \oplus 27$$

$$8 \otimes 10 = 8 \oplus 10 \oplus 27 \oplus 35$$

the decay $\langle \mathbb{P}, M | H_W | \Lambda_b \rangle$ can be realized with \mathbb{P} in either an octet (8) or a decuplet (10)

- The discovery channel $\Lambda_b \rightarrow \mathbb{P}^+ K^- \rightarrow J/\psi p K^-$ corresponds to \mathbb{P} in an octet (8)

Weak decays with \mathbb{P} in Decuplet representation

- Decays involving the decuplet (10) pentaquarks may also occur, if the light diquark pair having spin-0 $[ud]_{s=0}$ in Λ_b gets broken to produce a spin-1 light diquark $[ud]_{s=1}$

$$\Lambda_b \rightarrow \pi \mathbb{P}_{10}^{(S=-1)} \rightarrow \pi(J/\psi \Sigma(1385))$$

$$\Lambda_b \rightarrow K^+ \mathbb{P}_{10}^{(S=-2)} \rightarrow K^+(J/\psi \Xi^-(1530))$$

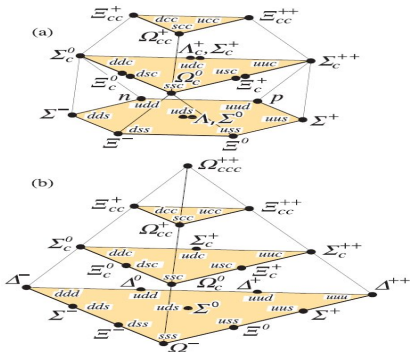
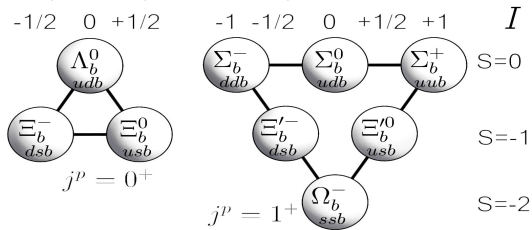


Figure 15.4: SU(4) multiplets of baryons made of u , d , s , and c quarks. (a) The 20-plet with an SU(3) octet. (b) The 20-plet with an SU(3) decuplet.

Weak decays with \mathbb{P} in Decuplet representation - Contd.

- Apart from $\Lambda_b(bud)$, several b -baryons, such as $\Xi_b^0(usb)$, $\Xi_b^-(dsb)$ and $\Omega_b^-(ssb)$ undergo weak decays



- Examples of bottom-strange b -baryon in various charge combinations, respecting $\Delta I = 0$, $\Delta S = -1$ are:

$$\Xi_b^0(5794) \rightarrow K(J/\psi \Sigma(1385))$$

which corresponds to the formation of the pentaquarks with the spin configuration $(q, q' = u, d)$

$$\mathbb{P}_{10}(\bar{c} [cq]_{s=0,1} [q's]_{s=0,1})$$

Weak decays with \mathbb{P} in Decuplet representation - Contd.

- The $s\bar{s}$ pair in Ω_b is in the symmetric (6) representation of flavor $SU(3)$ with spin 1; expected to produce decuplet Pentaquarks in association with a ϕ or a Kaon

$$\Omega_b(6049) \rightarrow \phi(J/\psi \Omega^-(1672))$$

$$\Omega_b(6049) \rightarrow K(J/\psi \Xi(1387))$$

- These correspond, respectively, to the formation of the following pentaquarks ($q = u, d$)

$$\mathbb{P}_{10}^-(\bar{c} [cs]_{s=0,1} [ss]_{s=1})$$

$$\mathbb{P}_{10}(\bar{c} [cq]_{s=0,1} [ss]_{s=1})$$

- These transitions are on firmer theoretical footings, as the initial $[ss]$ diquark in Ω_b is left unbroken; more transitions can be found relaxing this condition

Estimates of the ratio of decay widths for $J^P = \frac{5}{2}^+$

[Ahmed,Rehman,Aslam,AA, arxiv:1607.00987]

Decay Process	$\Gamma/\Gamma(\Lambda_b^0 \rightarrow P_p^{5/2} K^-)$	Decay Process	$\Gamma/\Gamma(\Lambda_b^0 \rightarrow P_p^{5/2} K^-)$
$\Lambda_b \rightarrow P_p^{\{Y_2\}c_1} K^-$	1	$\Xi_b^- \rightarrow P_{\Sigma^-}^{\{Y_2\}c_2} \bar{K}^0$	2.07
$\Lambda_b \rightarrow P_n^{\{Y_2\}c_1} \bar{K}^0$	1	$\Xi_b^0 \rightarrow P_{\Sigma^+}^{\{Y_2\}c_2} K^-$	2.07
$\Lambda_b \rightarrow P_{\Delta^0}^{\{Y_2\}c_3} \eta'$	0.03	$\Lambda_b \rightarrow P_{\Delta^0}^{\{Y_2\}c_3} \eta$	0.19
$\Xi_b^- \rightarrow P_{\Sigma^0}^{\{Y_2\}c_2} K^-$	1.04	$\Xi_b^- \rightarrow P_{\Sigma^0}^{\{Y_2\}c_2} K^-$	0.34
$\Omega_b^- \rightarrow P_{\Xi_{10}^-}^{\{Y_3\}c_5} \bar{K}^0$	0.14	$\Omega_b^- \rightarrow P_{\Xi_{10}^-}^{\{Y_3\}c_5} K^-$	0.14

Decay Process	$\Gamma/\Gamma(\Lambda_b^0 \rightarrow P_p^{5/2} K^-)$	Decay Process	$\Gamma/\Gamma(\Lambda_b^0 \rightarrow P_p^{5/2} K^-)$
$\Lambda_b \rightarrow P_p^{\{Y_2\}c_1} \pi^-$	0.08	$\Lambda_b \rightarrow P_n^{\{Y_2\}c_1} \pi^0$	0.04
$\Lambda_b \rightarrow P_n^{\{Y_2\}c_1} \eta$	0.01	$\Lambda_b \rightarrow P_n^{\{Y_2\}c_1} \eta'$	0
$\Xi_b^- \rightarrow P_{\Xi^-}^{\{Y_2\}c_4} K^0$	0.02	$\Xi_b^- \rightarrow P_{\Sigma^0}^{\{Y_2\}c_2} \pi^-$	0.08
$\Xi_b^- \rightarrow P_{\Sigma^-}^{\{Y_2\}c_2} \eta$	0.02	$\Xi_b^- \rightarrow P_{\Sigma^-}^{\{Y_2\}c_2} \eta'$	0.01
$\Xi_b^- \rightarrow P_{\Sigma^-}^{\{Y_2\}c_2} \pi^0$	0.08	$\Xi_b^0 \rightarrow P_{\Sigma^0}^{\{Y_2\}c_2} \pi^0$	0.04
$\Xi_b^0 \rightarrow P_{\Delta^0}^{\{X_2(Y_2)\}c_2} \eta$	0.01	$\Xi_b^0 \rightarrow P_{\Sigma^0}^{\{Y_2\}c_2} \eta'$	0.01
$\Xi_b^0 \rightarrow P_{\Delta^0}^{\{Y_2\}c_2} \pi^0$	0.01	$\Omega_b^- \rightarrow P_{\Xi_{10}^-}^{\{Y_3\}c_5} \pi^0$	0.01
$\Omega_b^- \rightarrow P_{\Xi_{10}^-}^{\{Y_3\}c_5} \pi^-$	0.02		

■ We have used the pentaquark masses estimated in this work.

■ $\Delta S = 0$ are suppressed by $|V_{cd}^*/V_{cs}^*|^2$ compared to $\Delta S = 1$.

Estimates of the ratio of decay widths for $J^P = \frac{3}{2}^-$

[Ahmed,Rehman,Aslam,AA, arxiv:1607.00987]

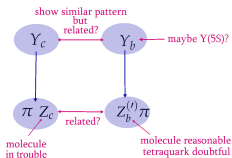
Decay Process	$\Gamma/\Gamma(\Lambda_b^0 \rightarrow P_p^{\{X_2\}_{c_1} K^-})$	Decay Process	$\Gamma/\Gamma(\Lambda_b^0 \rightarrow P_p^{\{X_2\}_{c_1} K^-})$
$\Lambda_b \rightarrow P_p^{\{X_2\}_{c_1} K^-}$	1	$\Xi_b^- \rightarrow P_{\Sigma^-}^{\{X_2\}_{c_2} \bar{K}^0}$	1.38
$\Lambda_b \rightarrow P_n^{\{X_2\}_{c_1} \bar{K}^0}$	1	$\Xi_b^0 \rightarrow P_{\Sigma^+}^{\{X_2\}_{c_2} K^-}$	1.38
$\Lambda_b \rightarrow P_{\Lambda^0}^{\{X_2\}_{c_3} \eta'}$	0.17	$\Lambda_b \rightarrow P_{\Lambda^0}^{\{X_2\}_{c_3} \eta}$	0.22
$\Xi_b^- \rightarrow P_{\Sigma_0^-}^{\{X_2\}_{c_2} K^-}$	0.69	$\Xi_b^- \rightarrow P_{\Lambda_0^-}^{\{X_2\}_{c_2} K^-}$	0.23
$\Omega_b^- \rightarrow P_{\Xi_{10}^-}^{\{X_3\}_{c_5} \bar{K}^0}$	0.24	$\Omega_b^- \rightarrow P_{\Xi_{10}^-}^{\{X_3\}_{c_5} K^-}$	0.24

Decay Process	$\Gamma/\Gamma(\Lambda_b^0 \rightarrow P_p^{\{X_2\}_{c_1} K^-})$	Decay Process	$\Gamma/\Gamma(\Lambda_b^0 \rightarrow P_p^{\{X_2\}_{c_1} K^-})$
$\Lambda_b \rightarrow P_p^{\{X_2\}_{c_1} \pi^-}$	0.06	$\Lambda_b \rightarrow P_n^{\{X_2\}_{c_1} \pi^0}$	0.03
$\Lambda_b \rightarrow P_n^{\{X_2\}_{c_1} \eta}$	0.01	$\Lambda_b \rightarrow P_n^{\{X_2\}_{c_1} \eta'}$	0.01
$\Xi_b^- \rightarrow P_{\Xi^-}^{\{X_2\}_{c_4} \bar{K}^0}$	0.02	$\Xi_b^- \rightarrow P_{\Sigma_0^-}^{\{X_2\}_{c_2} \pi^-}$	0.03
$\Xi_b^- \rightarrow P_{\Sigma^-}^{\{X_2\}_{c_2} \eta}$	0.02	$\Xi_b^- \rightarrow P_{\Sigma^-}^{\{X_2\}_{c_2} \eta'}$	0.01
$\Xi_b^- \rightarrow P_{\Sigma^-}^{\{X_2\}_{c_2} \pi^0}$	0.04	$\Xi_b^0 \rightarrow P_{\Sigma^0}^{\{X_2\}_{c_2} \pi^0}$	0.02
$\Xi_b^0 \rightarrow P_{\Lambda_0}^{\{X_2\}_{c_2} \eta}$	0	$\Xi_b^0 \rightarrow P_{\Lambda_0}^{\{X_2\}_{c_2} \eta'}$	0
$\Xi_b^0 \rightarrow P_{\Lambda_0}^{\{X_2\}_{c_2} \pi^0}$	0.01	$\Omega_b^- \rightarrow P_{\Xi_{10}^-}^{\{X_3\}_{c_5} \pi^0}$	0.01
$\Omega_b^- \rightarrow P_{\Xi_{10}^-}^{\{X_3\}_{c_5} \pi^-}$	0.02		

- Neutron, η and η' , and possibly π^0 : only decays with K^- , or \bar{K}^0 , or π^- .

Summary

- A new facet of QCD is opened by the discovery of the exotic X, Y, Z , and the pentaquark states $\mathbb{P}(4380)$ and $\mathbb{P}(4450)$
- Using the heavy quark symmetry, we predict a lower-mass $J^P = 3/2^-$ state at 4110 MeV!
- A very rich spectrum of tetraquark and pentaquark states is anticipated, including tetraquarks with a single c , or a single b quark
- Important puzzles remain in the complex:



- What is the nature of $Y_c(4260)$? A tetraquark? or a $c\bar{c}g$ hybrid?
- What exactly is $Y(10888)$? Is it just $Y(5S)$? Does $Y_b(10890)$ still exist?
- Hadroproduction and Drell-Yan mechanism are potential sources of multi-quark states
- We look forward to decisive experimental results from Belle-II, LHC, in particular, from the LHCb

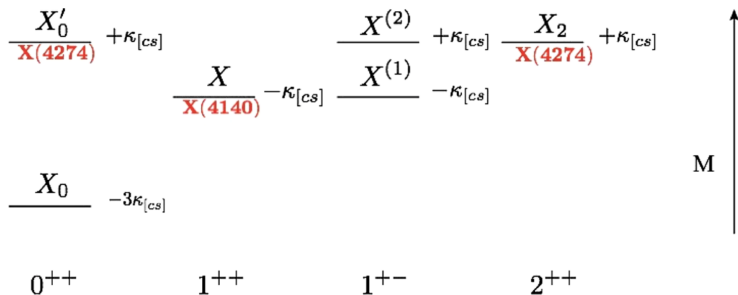
Backup: Recent Reviews on X, Y, Z, P_c Exotics

- Exotics: Heavy Pentaquarks and Tetraquarks, A.A., J.S. Lange, S. Stone, arXiv: 1706.00610 (2017).
- Hadronic Molecules, F.K. Guo, C. Hanhart, U.G. Meissner, Q. Wang, Q. Zhao, B.-S. Zou, arXiv: 1705.00141 (2017).
- Heavy-Quark QCD Exotica, R.F. Lebed, R.E. Mitchell, E.S. Swanson, Prog. Part. Nucl. Phys. 93 (2017) 143, arXiv: 1610.04528 (2016).
- Multiquark Resonances, A. Esposito, A. Pilloni, A.D. Polosa, Physics Reports 668 (2016)1, arXiv: 1611.07920 (2016).
- Hidden-Charm Pentaquark and Tetraquark States, H.X. Chen, W. Chen, X. Liu, S.-L. Zhu, Physics Reports 639 (2016)1, arXiv: 1601.02092 (2016).
- **Multiquark states also receiving increasing attention on the Lattice**
 - Hadron Spectroscopy and Interactions from Lattice QCD, A. Prelovsek, EPJ Web Conf. (2016) 00018.
 - Hadronic Interactions, T. Yamazaki, PoS Lattice2014 (2015) 009.
 - Few-body Physics, R.A. Bricero, PoS Lattice2014 (2015) 008.

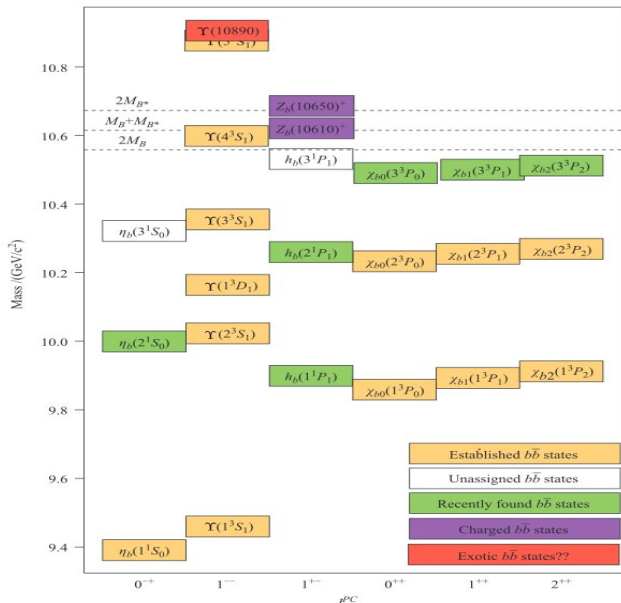
Tetraquark interpretation of the axial $J/\psi\phi$ states

[L. Maiani, A.D. Polosa, V. Riquer, Phys. Rev. D94 (2016) 054026]

- Interpret the four $J/\psi\phi$ states as $[cs][\bar{c}\bar{s}]$ tetraquarks, with $X(4140)$, $X(4274)$ as 1S states, and $X(4500)$, $X(4700)$ as 2S states
- $X(4274)$ having $J^{PC} = 1^{++}$ is not compatible with the tetraquark interpretation
- Prefer two almost degenerate $J^{PC} = 0^{++}$ & 2^{++} states - this remains to be seen



Bottomonia and Bottomonium-like Hadrons (Olsen, 1411.7738)



Production of $J^{PC} = 1^{--}$ hadrons $\phi(2170)$, $Y(4260)$, and " $Y_b(10890)$ " via Drell-Yan mechanism

[A.A., Wei Wang; PRL 106 (2011), 192001]

$$pp(\bar{p}) \rightarrow \gamma^* \rightarrow V + \dots; \quad V = \phi(2170), \quad Y(4260), \quad "Y_b(10890)"$$

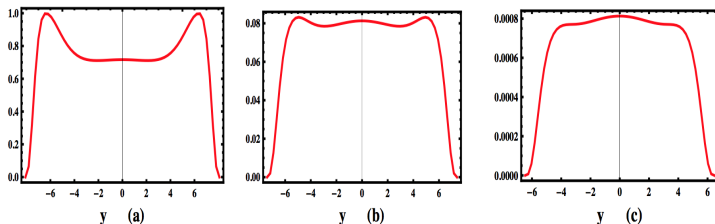
- Decay Modes: $\phi(2170) \rightarrow \phi(1020)f_0(980) \rightarrow K^+K^-\pi^+\pi^-$
 $Y(4260) \rightarrow J/\psi\pi^+\pi^- \rightarrow \ell^+\ell^-\pi^+\pi^-$
 $"Y_b(10890)" \rightarrow Y(nS)\pi^+\pi^- \rightarrow \ell^+\ell^-\pi^+\pi^-; \quad (nS = 1S, 2S, 3S)$

	m_V (MeV)	Γ (MeV)	$\Gamma_{ee}\mathcal{B}$ (eV)
$\phi(2170)$	2175 ± 15	61 ± 18	2.5 ± 0.9 ^a
$X(4260)$	4263^{+8}_{-9}	108 ± 21 [4]	$6.0^{+4.9}_{-1.3}$ ^b [4]
$Y_b(10890)$	$10888.4^{+3.0}_{-2.9}$ [12]	$30.7^{+8.9}_{-7.7}$ [12]	$0.69^{+0.23}_{-0.20}$ ^c [12]
$\mathcal{B}_{\phi \rightarrow K^+K^-}$	$(48.9 \pm 0.5)\%$	$\mathcal{B}_{f_0(980) \rightarrow \pi^+\pi^-}$	$(50^{+7}_{-9})\%$ [23]
$\mathcal{B}_{J/\psi \rightarrow \mu^+\mu^-}$	$(5.93 \pm 0.06)\%$	$\mathcal{B}_{Y(1S) \rightarrow \mu^+\mu^-}$	$(2.48 \pm 0.05)\%$
$\mathcal{B}_{Y(2S) \rightarrow \mu^+\mu^-}$	$(1.93 \pm 0.17)\%$	$\mathcal{B}_{Y(3S) \rightarrow \mu^+\mu^-}$	$(2.18 \pm 0.21)\%$

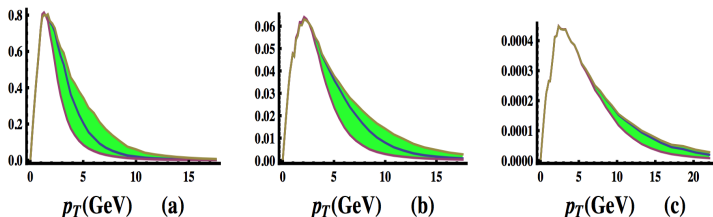
Rapidity & p_T -distributions at the LHC

[A.A., Wei Wang; PRL 106 (2011), 192001]

- Rapidity-distributions for a): $\phi(2170)$, b): $\Upsilon(4260)$, c): " $\Upsilon_b(10890)$ "



- p_T -distributions for a): $\phi(2170)$, b): $\Upsilon(4260)$, c): " $\Upsilon_b(10890)$ "



Cross sections at the Tevatron and LHC (in pb)

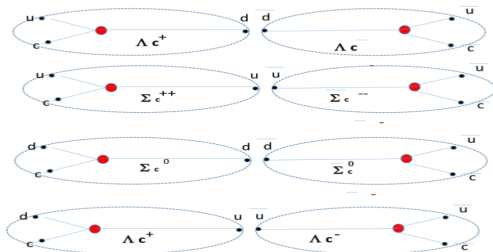
- $\phi(2170) : pp(\bar{p}) \rightarrow \gamma^* \rightarrow \phi(2170) \rightarrow \phi(1020)f_0(980) \rightarrow K^+K^-\pi^+\pi^- + X$
- $X(4260) : pp(\bar{p}) \rightarrow \gamma^* \rightarrow Y(4260) \rightarrow J/\psi\pi^+\pi^- \rightarrow \ell^+\ell^-\pi^+\pi^- + X$
- $Y_b(10890) : pp(\bar{p}) \rightarrow \gamma^* \rightarrow Y_b(10890) \rightarrow Y(nS)\pi^+\pi^- \rightarrow \ell^+\ell^-\pi^+\pi^- + X$

	$\phi(2170)$	$X(4260)$	$Y_b(10890)$
Tevatron ($ y < 2.5$)	$2.3^{+0.9}_{-0.9}$	$0.23^{+0.19}_{-0.05}$	$0.0020^{+0.0006}_{-0.0005}$
LHC 7TeV ($ y < 2.5$)	$3.6^{+1.4}_{-1.4}$	$0.40^{+0.32}_{-0.09}$	$0.0040^{+0.0013}_{-0.0011}$
LHCb 7TeV ($1.9 < y < 4.9$)	$2.2^{+1.2}_{-1.1}$	$0.24^{+0.20}_{-0.07}$	$0.0023^{+0.0007}_{-0.0006}$
LHC 14TeV ($ y < 2.5$)	$4.5^{+1.9}_{-1.9}$	$0.54^{+0.44}_{-0.12}$	$0.0060^{+0.0019}_{-0.0016}$
LHCb 14TeV ($1.9 < y < 4.9$)	$2.7^{+1.9}_{-1.6}$	$0.31^{+0.27}_{-0.11}$	$0.0033^{+0.0011}_{-0.0010}$

- Drell-Yan mechanism is a potential source of $J^{PC} = 1^{--}$ exotica at the LHC!

Stringy picture of tetraquarks

- $\Upsilon(4630)$ is a likely candidate for a tetraquark state, whose most natural decay, is into a charm baryon-antibaryon ($\Lambda_c^+ \Lambda_c^-$), [L. Maiani]
- This is the general pattern of tetraquark decays, anticipated in stringy formalism [G. Rossi, G. Veneziano, JHEP 06 (2016) 041] and in the holography inspired perspective [J. Sonnenschein, D. Weissman, Nucl. Phys. B920 (2017) 319].
- Tetraquarks $[cu][\bar{c}\bar{u}]$ and $[cu][\bar{c}\bar{d}]$ and their decay products [J. Sonnenschein, D. Weissman, Nucl. Phys. B920 (2017) 319].



Regge Trajectory of the $Y(4630)$ and other tetraquarks with $b\bar{b}$, $c\bar{c}$ and $s\bar{s}$

- Tetraquarks, like other mesons, lie on a Regge trajectory. This is a fundamental difference to all other multiquark scenarios (cusps, hadron molecule etc.)
- Regge Formula from rotating string ($\alpha' = 1/2\pi T$, T is string tension, β is the velocity of the string end-point) [J. Sonnenschein, D. Weissman]

$$E = 2m \left(\frac{\beta \arcsin(\beta) + \sqrt{1 - \beta^2}}{1 - \beta^2} \right)$$

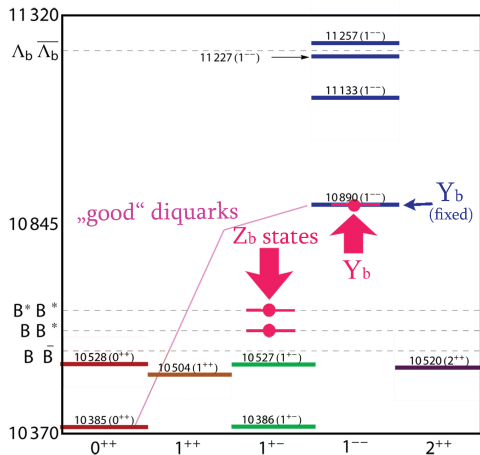
$$J + n - a = 2\pi\alpha' m^2 \frac{\beta^2}{(1 - \beta^2)^2} \left(\arcsin(\beta) + \beta\sqrt{1 - \beta^2} \right)$$

- Predictions for tetraquarks and their first few excited states

Tetraquarks containing $c\bar{c}$ (decaying to $\Lambda_c \bar{\Lambda}_c$)				Tetraquarks containing $b\bar{b}$ (decaying to $\Lambda_b \bar{\Lambda}_b$)			Tetraquarks containing $s\bar{s}$ (decaying to $\Lambda \bar{\Lambda}$)		
n	J^{PC}	M	Γ	n	J^{PC}	M	n	J^{PC}	M
0	1^{--}	4634^{+9}_{-11}	92^{+41}_{-32}	0	1^{--}	11280 ± 40	0	1^{--}	2270 ± 40
0	2^{++}	4800 ± 40	$150 - 250$	0	2^{++}	11410 ± 40	0	2^{++}	2510 ± 40
1	1^{--}	4870 ± 50	$150 - 250$	1	1^{--}	11460 ± 40	1	1^{--}	2540 ± 40
0	3^{--}	4960 ± 40	$180 - 280$	0	3^{--}	11550 ± 40	0	3^{--}	2730 ± 40
2	1^{--}	5100 ± 60	$200 - 300$	2	1^{--}	11640 ± 40	2	1^{--}	2780 ± 40

$[bq][\bar{b}\bar{q}]$ Constituent Model Spectrum

[A.A., Hambrock, Ahmed, Aslam, PLB684 (2010)]



- $Y_b(10890)$ probably not an independent state, but is the same as $Y(5S)$ [Belle 2016]
- Decays of " $Y(5S)$ " saturated by exotic final states!
- The entire exotic bottomonium sector remains to be tested
- A great opportunity for the LHCb to make decisive progress!!

Enigmatic $Y(5S)$ Decays!

PRL 100, 112001 (2008)

21.7 fb⁻¹ at 10.580 GeV



$$\Upsilon(5S) \rightarrow \Upsilon(1S)\pi^+\pi^- \quad 0.59 \pm 0.04 \pm 0.09$$

$$\Upsilon(5S) \rightarrow \Upsilon(2S)\pi^+\pi^- \quad 0.85 \pm 0.07 \pm 0.16$$

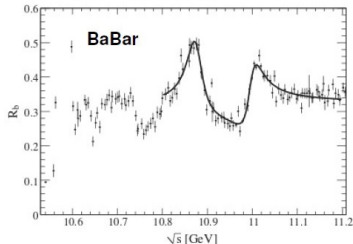
$$\Upsilon(5S) \rightarrow \Upsilon(3S)\pi^+\pi^- \quad 0.52^{+0.20}_{-0.17} \pm 0.10$$

$$\Upsilon(2S) \rightarrow \Upsilon(1S)\pi^+\pi^- \quad 0.0060$$

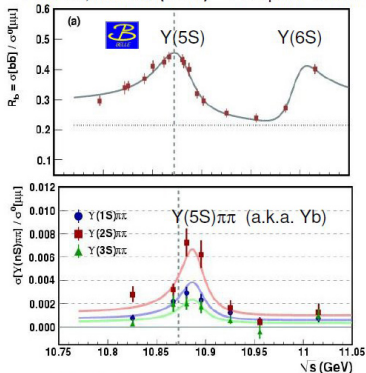
$$\Upsilon(3S) \rightarrow \Upsilon(1S)\pi^+\pi^- \quad 0.0009$$

$$\Upsilon(4S) \rightarrow \Upsilon(1S)\pi^+\pi^- \quad 0.0019$$

PRL 102, 012001 (2009)



PRD 89, 091106 (2010) ~1 fb⁻¹/point SCAN



Belle 2010

$$M(5S)b\bar{b} = 10869 \pm 2 \text{ MeV}$$

$$M(5S)\pi\pi = 10888.4 \pm 2.7 \pm 1.2 \text{ MeV}$$

$$M(5S) - M(5S)\pi\pi = -9 \pm 4 \text{ MeV}$$

- Is there a $Y_b(10890)$ close to $Y(5S)$? If yes, what is it??

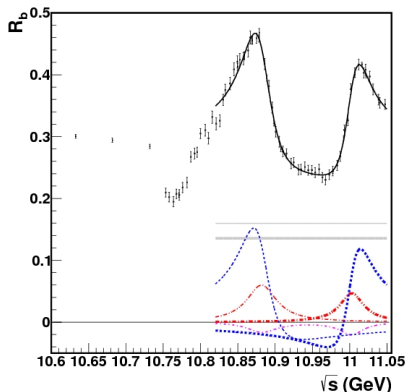
[AA. Hambrock. Ishtiaq Ahmed. Jamil Aslam. PLB 684 (2010) 28]

Ahmed Ali (DESY, Hamburg)

$\sigma(e^+e^- \rightarrow b\bar{b})$ in the $Y(10860)$ and $Y(11020)$ resonance region [Belle]

R'_b data and fit

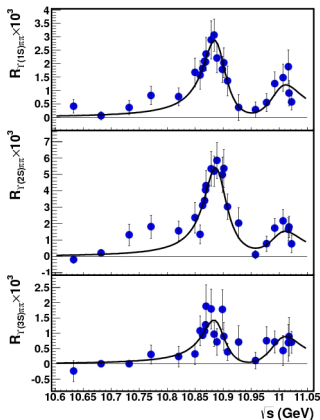
- $F_{b\bar{b}} = |A_{nr}|^2 + |A_r + A_{5S}e^{i\phi_{5S}}f_{5S} + A_{6S}e^{i\phi_{6S}}f_{6S}|^2$
- $f_{nS} = M_{nS}\Gamma_{nS} / [(s - M_{nS}^2) + iM_{nS}\Gamma_{nS}]$ [BW]; A_r and A_{nr} [Continuum]
- No peaking structure seen at 10.9 GeV, hinted by the BaBar data;
 $\Gamma(e^+e^-) < 9$ eV (@ 90% C.L.)



$\sigma(e^+e^- \rightarrow Y(nS)\pi^+\pi^-)$ in the $Y(10860)$ and $Y(11020)$ resonance region

[D. Santel et al. (Belle), arxiv:1501.01137 (2015)]

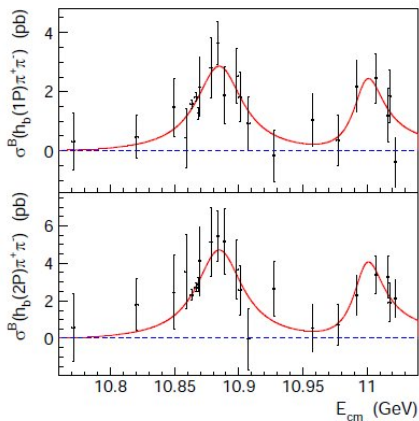
- Fit Values (MeV): $M_{10860} = 10891.1 \pm 3.2^{+0.6}_{-1.5}$; $\Gamma_{10860} = 53.7^{+7.1}_{-5.6} \text{ }^{+0.9}_{-5.4}$
- $M_{5S}(Y(nS)\pi\pi) - M_{5S}(b\bar{b}) = 9.2 \pm 3.4 \pm 1.9$ MeV ?
- Fit Values (MeV): $M_{11020} = 10987.5^{+6.4}_{-2.5} \text{ }^{+9.0}_{-2.1}$; $\Gamma_{11020} = 61^{+9}_{-19} \text{ }^{+2}_{-20}$



$\sigma(e^+e^- \rightarrow h_b(1P,2P)\pi^+\pi^-)$ in the $Y(10860)$ and $Y(11020)$ resonance region

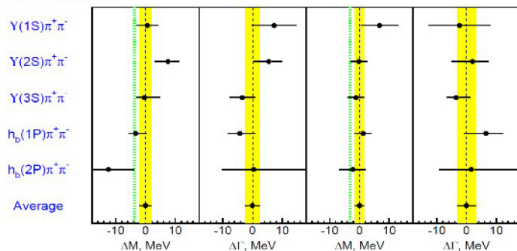
[A. Abdesselam et al. (Belle), arxiv:1508.06562 (2015)]

- Fit Values (MeV): $M_{10860} = 10884.7^{+3.2}_{-2.9} +8.6_{-0.6}$; $\Gamma_{10860} = 44.2^{+1.9}_{-7.8} +2.2_{-15.8}$
- Fit Values (MeV): $M_{11020} = 10998.6 \pm 6.1^{+16.1}_{-1.1}$; $\Gamma_{11020} = 29^{+20}_{-11} +2_{-7}$



Evidence for $Z_b(10610)^\pm$ and $Z_b(10650)^\pm$ (Belle)

PRL108,122001



Mass and Γ
measured in 5
different
final states agree

Angular analysis suggests $J^P = 1^+$

$Z_b(10610)$

$M = 10608 \text{ pm } 2.0 \text{ MeV}$

$\Gamma = 15.6 \text{ pm } 2.5 \text{ MeV}$

$Z_b(10650)$

$M = 10653 \text{ pm } 1.5 \text{ MeV}$

$\Gamma = 14.4 \text{ pm } 3.2 \text{ MeV}$

The Di Pion transitions from the $Y(5S)$ proceed via the intermediate charged state Z_b

The transition does not imply spin flip

Masses are close to B^*B and B^*B^* thresholds
Molecules?

The $Y(5S)$ is an unexpected source of h_b

Theoretical Interpretations of the LHCb Pentaquarks

Rescattering-induced kinematic effects

- Feng-Kun Guo, Ulf-G.Meißner, Wei Wang, Zhi Yang, arxiv:1507.04950
- Xiao-Hai Liu, Qian Wang, Qiang Zhao, arxiv:1507.05359
- M. Mikhasenko, arxiv:1507.06552
- Ulf-G.Meißner, Jose A. Oller, arxiv:1507.07478

Open-charm-baryon and -meson bound states

- Hua-Xing Chen, Wei Chen, Xiang Liu, T.G. Steele, Shi-Lin Zhu, arxiv:1507.03717
- Jun He, arxiv:1507.05200
- L. Roca, J. Nieves, E. Oset, arxiv:1507.04249
- Rui Chen, Xiang-Liu, arxiv:1507.03704
- C. W. Xiao and Ulf-G.Meißner, arxiv:1508.00924

Pentaquarks as Baryocharmonia

- Formation of hidden-charm pentaquarks in photon-nucleon collisions
V. Kubarovsky and M.B. Voloshin, arxiv:1508.00888

Theoretical Interpretations of the LHCb Pentaquarks (Contd.)

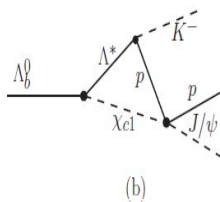
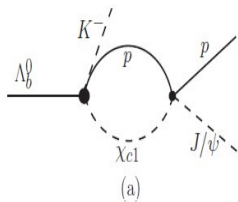
Compact Pentaquarks

- L. Maiani, A.D. Polosa, V. Riquer, arxiv: 1507.04980
- Richard F. Lebed, arxiv:1507.05867
- Guan-Nan Li, Xiao-Gang He, Min He, arxiv:1507.08252
- A. Mironov, A. Morozov, arxiv:1507.04694
- A.V. Anisovich et al., arxiv:1507.07652
- R. Ghosh, A. Bhattacharya, B. Chakrabarti, arxiv:1508.00356
- Zhi-Gang Wang, arxiv:1508.01468
- Zhi-Gang Wang, Tao Huang, arxiv:1508.04189
- H.Y. Cheng, C.K. Chua, arxiv:1509.03708
- G.N. Li, X.G. he, M. He, arxiv:1507.08252
- A. Ali, I. Ahmed, A. Rehman, M.J. Aslam, arxiv:1607.00987

Pentaquarks as rescattering-induced kinematic effects

[Feng-Kun Guo et al.; arxiv:1507.04950]

- Hypothesis: Kinematic effects can result in a narrow structure around the $\chi_{c1} p$ threshold in the $J/\psi p$ invariant mass of the decay $\Lambda_b^0 \rightarrow K^- J/\psi p$
 $M_{P_c(4450)} - M_{\chi_{c1}} - M_p = (0.9 \pm 3.1) \text{ MeV}$
- Two possible mechanisms:
 - a) 2-point loop with a 3-body production $\Lambda_b^0 \rightarrow K^- \chi_{c1} p$ followed by the rescattering process $\chi_{c1} p \rightarrow J/\psi p$
 - b) The $K^- p$ is produced from an intermediate Λ^* and the proton rescatters with the χ_{c1} into a $J/\psi p$



Pentaquarks as rescattering-induced kinematic effects (Contd.)

[Feng-Kun Guo et al.; arxiv:1507.04950]

- Amplitude for Fig. (a) ($\mu =$ reduced mass and $f_\Lambda(\vec{q}^2) = \exp(-2\vec{q}^2/\Lambda^2)$)

$$G_\Lambda(E) = \int \frac{d^3q}{(2\pi)^3} \frac{f_\Lambda(\vec{q}^2)}{E - m_p - m_{\chi_{c1}} - \vec{q}^2/(2\mu)}$$

- Analytic result

$$G_\Lambda(E) = \frac{\mu\Lambda}{(2\pi)^{3/2}} (k^2 + \Lambda^2/4) + \frac{\mu k^3}{2\pi} \exp^{-2k^2/\Lambda^2} \left[\operatorname{erfc}\left(\frac{\sqrt{2}k}{\Lambda}\right) - i \right]$$

where $k = \sqrt{2\mu(E - m_1 - m_2 + i\epsilon)}$. This function has a characteristic phase motion reflecting the error function (erfc), as shown below

- LHCb data is in better agreement with a Breit-Wigner phase motion

