SM effective action - vacuum stability and domain walls

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arXiv:1402.3826 (JHEP), arXiv:1505.05505, arXiv:1605.06713 (PRD),
arXiv:1411.6435 (PRD), arXiv:1606.07808 (JHEP), arXiv:1508.03297 (JHEP)
arXiv:1608.05719 (JCAP), update on DW/GW to appear soon

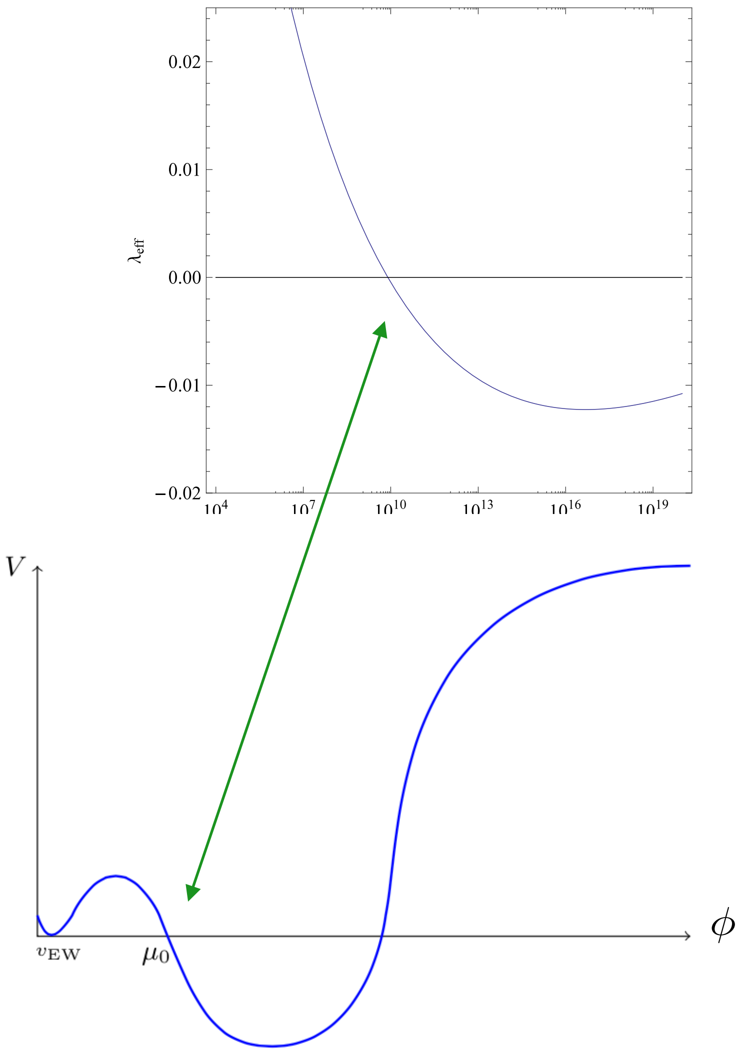
Corfu Summer Institute, 4th of September 2017

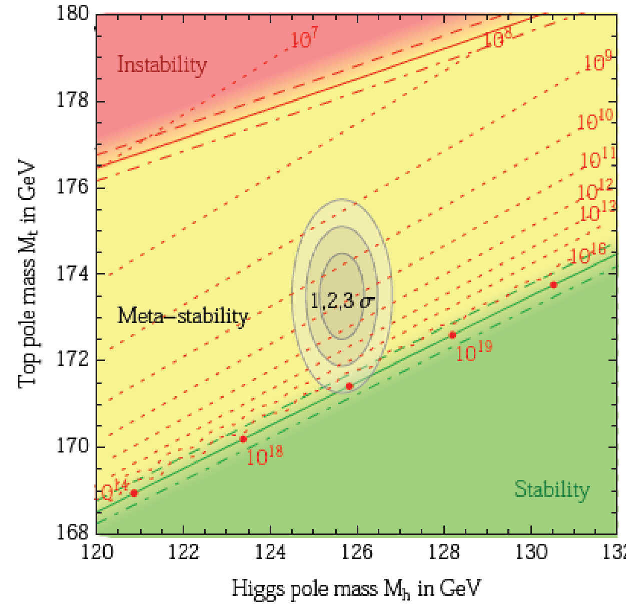
Outline:

- SM effective potential, tunneling and lifetime
- BSM physics via higher-order operators
- Modifications of the vacuum properties due to expanding background
- Coleman - De Luccia bounces
- Domain walls and gravitational waves
- Summary

SM Metastability

$$\lambda_{eff} < 0 \implies \text{Metastability}$$





D. Buttazzo, et al. [arXiv:1307.3536].
G. Degrassi, et al. [arXiv:1205.6497].

See lectures by G. Degrassi Corfu 2014

Tunneling

Standard semiclassical formalism

S. R. Coleman, Phys. Rev. D **15** (1977) 2929.
C. G. Callan, Jr. and S. R. Coleman, Phys. Rev. D **16** (1977) 1762.

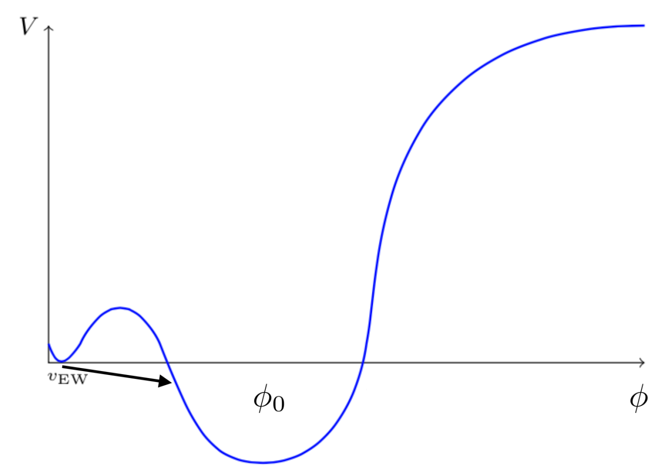
$O(4)$ symmetric solution to euclidean equation of motion

$$\ddot{\phi} + \frac{3}{s}\dot{\phi} = \frac{\partial V(\phi)}{\partial \phi},$$
$$s = \sqrt{\vec{x}^2 + x_4^2}.$$

with

- $\dot{\phi}(s = 0) = 0$ near the true vacuum
- $\phi(s = \infty) = \phi_{min}$ at the false vacuum

$$= v_{EW}$$



Tunneling

Action of the bounce solution

$$
\begin{aligned}
S_E &= \int d^4x \left\{ \frac{1}{2} \sum_{\alpha=1}^{4} \left(\frac{\partial \phi(\mathbf{x})}{\partial x^\alpha} \right)^2 + V(\phi(\mathbf{x})) \right\} \\
&= 2\pi^2 \int ds s^3 \left(\frac{1}{2} \dot{\phi}^2(s) + V(\phi(s)) \right),
\end{aligned}
$$

allows us to calculate decay probability dp of a volume d^3x

$$
dp = dt\, d^3x \frac{S_E^2}{4\pi^2} \left| \frac{\det'[-\partial^2 + V''(\phi)]}{\det[-\partial^2 + V''(\phi_0)]} \right|^{-1/2} e^{-S_E}.
$$

Simplifying

- normalisation factor replaced with width of the barrier $\propto \phi_0$
- size of the universe is $T_U = 10^{10}$yr

we can calculate the lifetime of the false vacuum $(p(\tau) = 1)$

$$
\boxed{\frac{\tau}{T_U} = \frac{1}{\phi_0^4 T_U^4} e^{S_E}.}
$$

Analytical solution

Analytical solutions for simple potentials

K. M. Lee and E. J. Weinberg, Nucl. Phys. B **267** (1986) 181.

Quartic potential:

$$V(\phi) = \frac{\lambda}{4}\phi^4 \qquad \Longrightarrow \qquad S_E = \frac{8\pi^2}{3|\lambda|}$$

for $\lambda < 0$.

Quartic and linear potential :

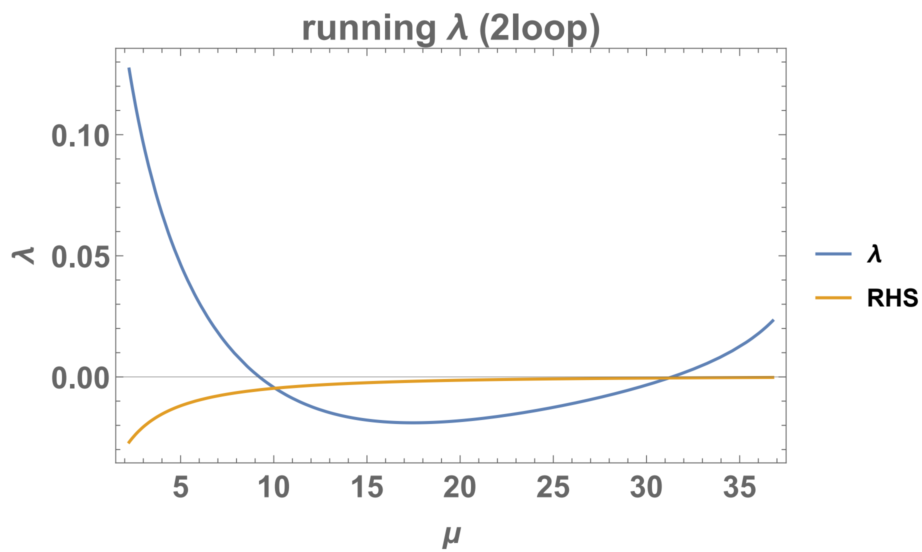
$$V_\eta(\phi) = \begin{cases} \frac{\lambda}{4}\phi^4 \, , & \phi \leqslant \eta \\ \frac{\lambda}{4}\eta^4 - K\left(\phi - \eta\right) , & \phi > \eta \end{cases} , \qquad \Longrightarrow \qquad \begin{array}{c} S_E = \frac{8\pi^2}{3|\lambda|}(1-(\gamma+1)^4) \\ \gamma = \frac{|\lambda|\eta^3}{K} \end{array}$$

for $\lambda < 0$ and $-1 < \gamma < 0$

New extrema created by quantum corrections
(Coleman-Weinberg mechanism)

condition for cancellation of corrections to the derivative of SM

$$\lambda = \frac{\hbar}{256\pi^2}\left[g_1^4 + 2g_1^2 g_2^2 + 3g_2^4 - 48h_t^4 - 3(g_1^2+g_2^2)^2 \log\frac{g_1^2+g_2^2}{4} - 6g_2^4\log\frac{g_2^2}{4} + 48y_t^4\log\frac{y_t^2}{2}\right]$$



Hence sensitivity to New Physics

We add new nonrenormalisable couplings
(similar to V. Branchina and E. Messina, [arXiv:1307.5193].)

$$V \approx \frac{\lambda_{eff}(\phi)}{4}\phi^4 + \frac{\lambda_6}{6!}\frac{\phi^6}{M_p^2} + \frac{\lambda_8}{8!}\frac{\phi^8}{M_p^4}.$$

New Physics at Planck scale

That modify the potential around the Planck scale:

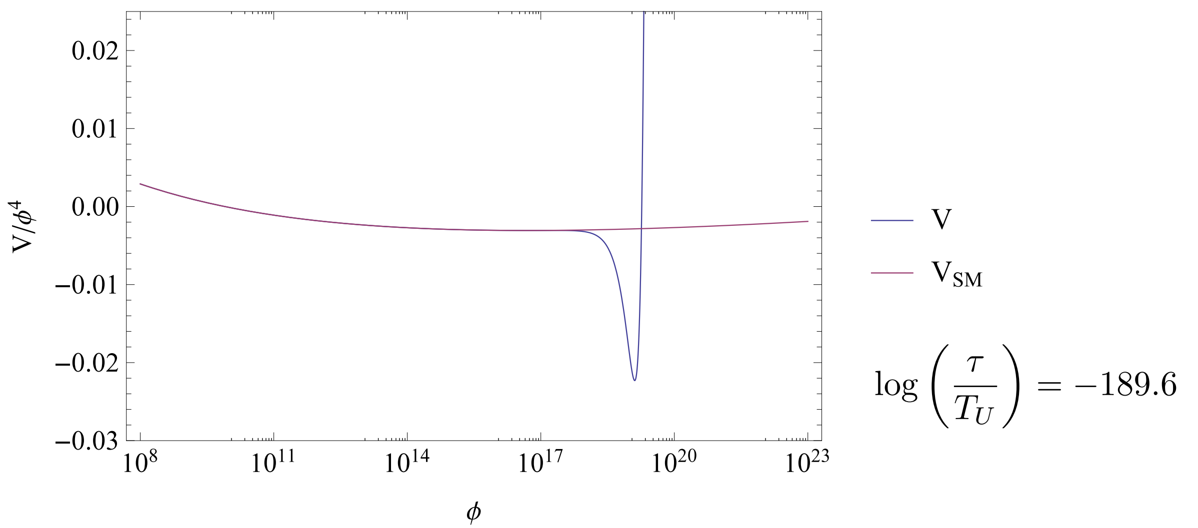


Figure: effective potential with $\lambda_6 = -1$ and $\lambda_8 = 1$.

Numerical vs Analytical again

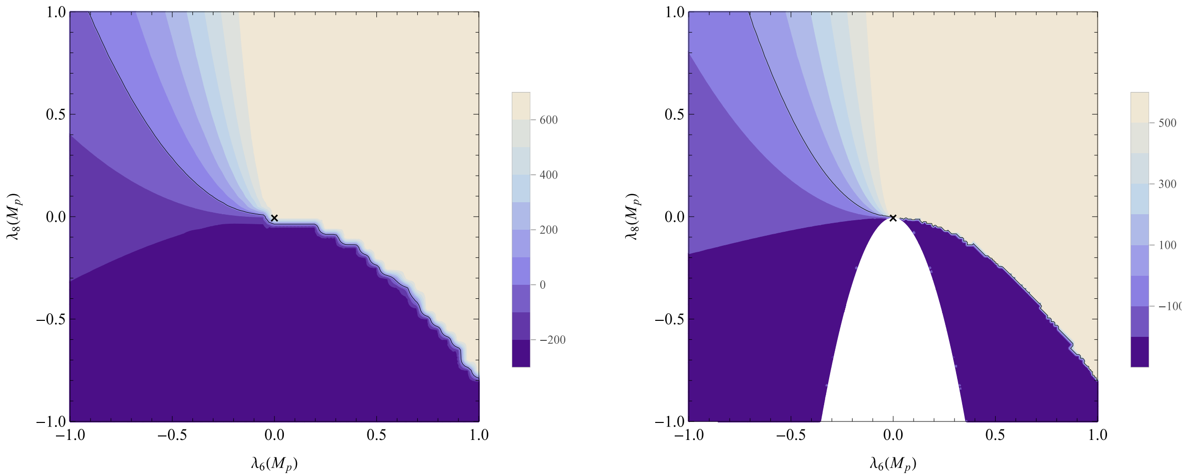


Figure: Decimal logatihm of lifetime of the universe in units of T_U as a function of the nonrenormalisable $\lambda_6(M_p)$ and $\lambda_8(M_p)$ couplings, calculated numerically (left panel) and analytically (right panel).

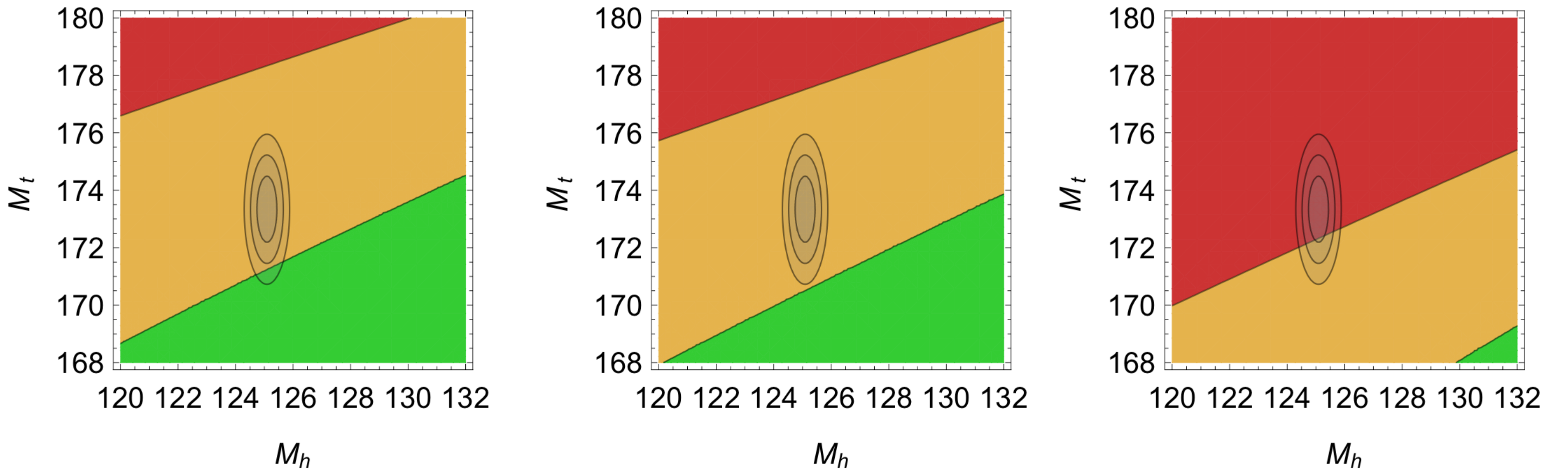


Figure 2: Standard Model phase diagram (left panel), the same diagram after including new operators $\lambda_6(M_p) = -1/2$ and $\lambda_8(M_p) = 1$ (middle panel) and $\lambda_6(M_p) = -1$ and $\lambda_8(M_p) = 1/2$ (right panel). The green region corresponds to absolute stability, the red region to instability, and the yellow region to metastability.

Magnitude of the suppression scale

Approximate lifetime:

$$\frac{\tau}{T_U} = \frac{1}{\mu^4(\lambda_{min})\, T_U^4} e^{\frac{8\pi^2}{3|\lambda_{min}|}}.$$

Positive λ_6 and λ_8 $\rightarrow$ stabilizing the potential

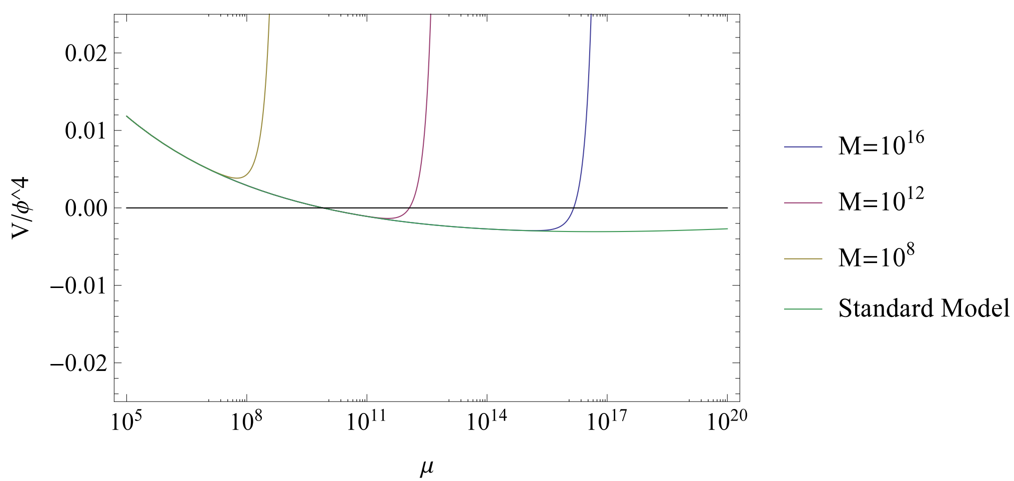


Figure: Scale dependence of $\frac{\lambda_{eff}}{4} = \frac{V}{\phi^4}$ with $\lambda_6 = \lambda_8 = 1$ for different values of suppression scale M. The lifetimes corresponding to suppression scales $M = 10^8, 10^{12}, 10^{16}$ are, respectively, $\log_{10}(\frac{\tau}{T_U}) = \infty, 1302, 581$ while for the Standard Model $\log_{10}(\frac{\tau}{T_U}) = 540$.

Magnitude of the suppression scale

Positive λ_8 and negative λ_6 $\rightarrow$ New Minimum

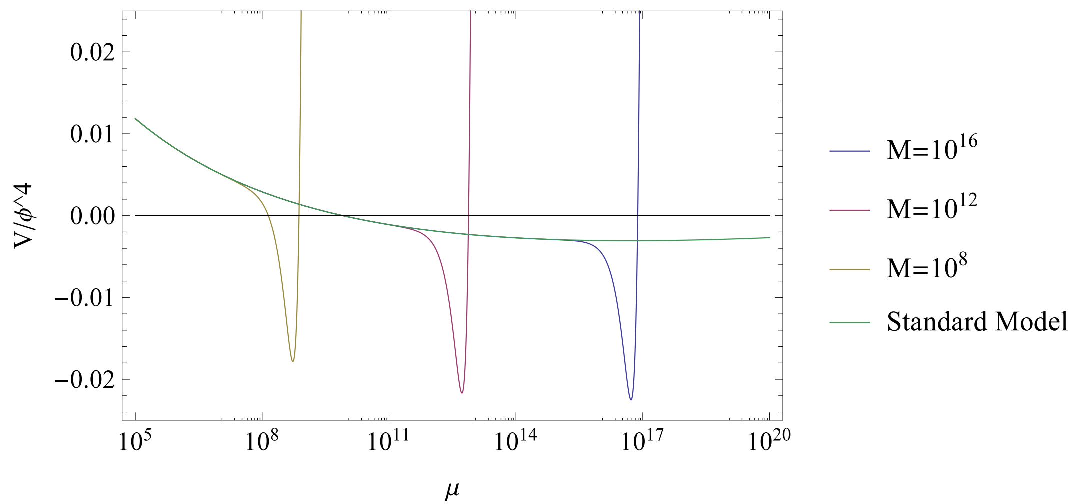


Figure: Scale dependence of $\frac{\lambda_{eff}}{4} = \frac{V}{\phi^4}$ with $\lambda_6 = -1$ and $\lambda_8 = 1$ for different values of suppression scale M. The lifetimes corresponding to suppression scales $M = 10^8, 10^{12}, 10^{16}$, are, respectively, $\log_{10}(\frac{\tau}{T_U}) = -45, -90, -110$ while for the Standard Model $\log_{10}(\frac{\tau}{T_U}) = 540$.

Gauge dependence of the tunneling rate

It is well known that the effective potential, and in general the effective action, are gauge-dependent objects

However, the statement about the spontaneous breaking of gauge symmetry is gauge invariant (N. K. Nielsen 1975)

The gauge invariant ”observables” are the values of the effective potential at the extrema, and the tunneling rate between different minima

When one computes the SM effective potential in a straightforward manner (say naively), nothing looks gauge independent - neither the value of the effective potential at the extrema (see L. Di Luzio and L. Mihaila 2014) nor the tunneling rate (ML,PO,ZL)

The leading gauge dependence comes from the gauge-dependent anomalous rescaling of the field

$$\mathcal{L}_{gauge\ fixing} = -\frac{1}{2\xi_W}(\partial^\mu W_\mu^{a2}) - \frac{1}{2\xi_B}(\partial^\mu B_\mu{}^2)$$

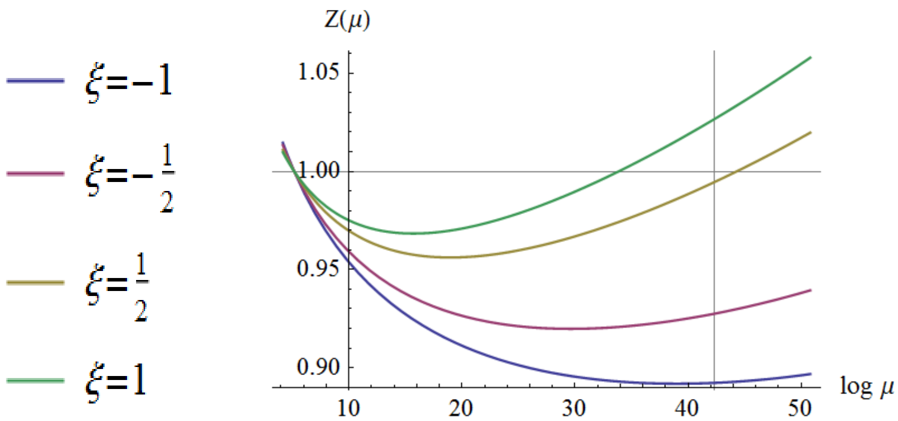
Contributes to:

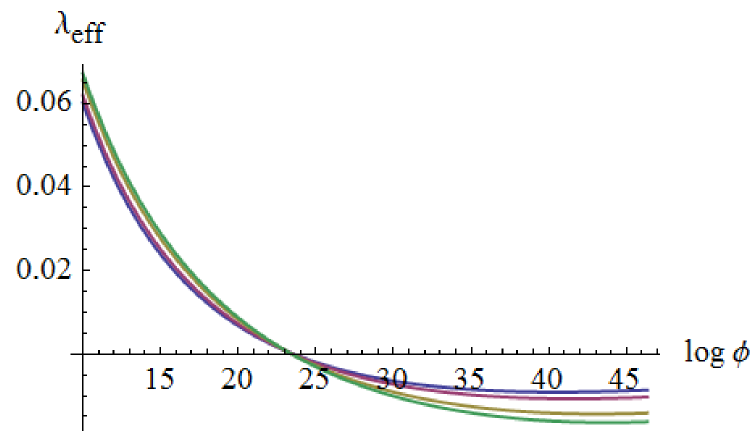
- 1-loop potential
- γ function of the scalar field

- More important.
- One needs to remember that kinetic contribution to the action is mulipied by Z.

$$\gamma = \frac{1}{16\pi^2}\left(\frac{9}{4}g_2^2 + \frac{9}{20}g_1^2 - 3y_t^2 - 3y_b^2 - y_\tau^2 + \frac{3}{20}\zeta_B g_1^2 + \frac{3}{4}\zeta_W g_2^2\right)$$

L. Di Luzio, L. Mihaila 1404.7450





$$\xi = \xi_W(m_t) = \xi_B(m_t)$$

This is due to the fact that the new extrema are created radiatively and already one loop effective potential, even in the RGE improved version, contains gauge-dependent terms

$$V^{1,\xi} = -\frac{1}{256\pi^2}\lambda h^4 \left[\xi_B g_1^2 \left(\log \frac{\lambda h^4(\xi_B g_1^2 + \xi_W g_2^2)}{4\mu^4} - 3\right) \right.$$
$$\left. + \xi_W g_2^2 \left(\log \frac{\lambda^3 h^{12}\xi_W^2 g_2^4(\xi_B^2 g_1^2 + \xi_W g_2^2)}{64\mu^{12}} - 9\right)\right]$$

As pointed out by A. Andreassen, W. Frost and M. Schwartz 2014, who followed E. Weinberg and D. Metaxas 1996 and S. Coleman and E. Weinberg 1973, the key to save in the calculations the gauge independence of the potantial at the extrema is to realize, that to create extrema radiatively, loop corrections have to cancel between themselves or the tree-level contributions

In CW model $\qquad \lambda \sim \dfrac{\hbar e^4}{16\pi^2}$

In the SM the equivalent condition is

$$\lambda = \frac{\hbar}{256\pi^2}\left[g_1^4 + 2g_1^2 g_2^2 + 3g_2^4 - 48h_t^4 - 3(g_1^2+g_2^2)^2\log\frac{g_1^2+g_2^2}{4} - 6g_2^4\log\frac{g_2^2}{4} + 48y_t^4\log\frac{y_t^2}{2}\right]$$

which holds at the extrema $h = \mu$

Hence λ is of the order $\hbar\, g^4$ and gives a higher order contribution

It has been shown that that taking this relation into account in counting radiative contributions in the SM makes the value of the potential at the extrema gauge independent at LO $(\hbar\, g^4)$ and NLO $(\hbar\, g^6)$

Gauge fixing independence

$$\text{EOM:} \quad \left.\frac{\delta\Gamma[\phi]}{\delta\phi}\right|_{\phi=\phi_{sol}} = 0\,,$$

$$\text{gauge fixing independence:} \quad \xi\frac{\partial}{\partial\xi}\Gamma[\phi_{sol}] = v\frac{\partial}{\partial v}\Gamma[\phi_{sol}] = 0$$

$$S_B = \Gamma_E[\varphi_B]\,, \qquad \left.\frac{\delta\Gamma_E[\phi]}{\delta\phi}\right|_{\phi=\varphi_B} = 0 \quad (+\text{ specific boundary conditions for } \phi_B)$$

desired property: $\xi\frac{\partial}{\partial\xi}S_B = v\frac{\partial}{\partial v}S_B = 0$

Nielsen identities:

$$\alpha\frac{\partial\Gamma[\phi]}{\partial\alpha} = \int C^\alpha[\phi]\frac{\delta\Gamma[\phi]}{\delta\phi}$$

$$\xi\frac{\partial K_{g^2}}{\partial\xi} = 2\frac{\partial C^\xi_{g^2}}{\partial\varphi_1}\,, \quad \xi\frac{\partial V_{g^6}}{\partial\xi} = C^\xi_{g^2}\frac{\partial V_{g^4}}{\partial\varphi_1} \qquad v\frac{\partial K_{g^2}}{\partial v} = 2\frac{\partial C^v_{g^2}}{\partial\varphi_1}\,, \quad v\frac{\partial V_{g^6}}{\partial v} = C^v_{g^2}\frac{\partial V_{g^4}}{\partial\varphi_1}$$

Gauge invariance of the action

$$S_B^0 = \int \mathrm{d}^4x\, \mathcal{L}^0(\varphi_B^0) \qquad \text{(which is explicitly gauge fixing independent)}$$

$$S_B^1 = \int \mathrm{d}^4x \left[\left. \frac{\delta \mathcal{L}^0(\varphi)}{\delta \varphi} \right|_{\varphi=\varphi_B^0} \cdot \varphi_B^1 \;+\; \mathcal{L}^1(\varphi_B^0) \right] = \int \mathrm{d}^4x\, \mathcal{L}^1(\varphi_B^0)\ .$$

Nielsen indentities

$$\alpha \frac{\partial}{\partial \alpha} S_B = \int \mathrm{d}^4x\, \alpha \frac{\partial}{\partial \alpha} \mathcal{L}^1(\varphi_B^0) =$$

$$= \int \mathrm{d}^4x \left. \left[\frac{1}{2} \alpha \frac{\partial}{\partial \alpha} K_{g^2}\, (\partial_\mu \varphi)^2 \;+\; \alpha \frac{\partial}{\partial \alpha} V_{g^6} \right] \right|_{\varphi=\varphi_B^0} = \int \mathrm{d}^4x \left. \left[\frac{\partial C^\alpha_{g^2}(\varphi)}{\partial \varphi}\, (\partial_\mu \varphi)^2 \;+\; C^\alpha \frac{\partial V_{g^4}}{\partial \varphi} \right] \right|_{\varphi=\varphi_B^0} =$$

$$= \int \mathrm{d}^4x \left[\underbrace{\left. \frac{\partial}{\partial\, x^\mu} \left(C^\alpha_{g^2}(\varphi)\, \partial_\mu \varphi \right) \right|_{\varphi=\varphi_B}}_{\text{boundary conditions for } \varphi_B^0} + C^\alpha \underbrace{\left. \left(-\partial_\mu^2 \varphi \;+\; \frac{\partial}{\partial \varphi} V_{g^4} \right) \right|_{\varphi=\varphi_B}}_{\text{EOM}} \right] \;=\; 0$$

Back to higher-order operators

$$\delta\mathcal{L}_{g^4} = \frac{\lambda_6}{6}\frac{(\varphi_i\varphi_i)^3}{\Lambda^2} + \frac{\lambda_8}{8}\frac{(\varphi_i\varphi_i)^4}{\Lambda^4} + \dots.$$

and new vertices: $\sim \varphi_1^4\varphi_2^2\,,\ \varphi_1^6\varphi_2^2\,,\ \dots$

$$\delta^{\mathrm{nonren}}\, V = \frac{\lambda_{60}}{6}\frac{\hat{\varphi}^6}{\Lambda^2} + \frac{\lambda_{80}}{8}\frac{\hat{\varphi}^8}{\Lambda^4} + \dots +$$

$$+ \frac{g_0^2}{32\pi^2}\left[v_0 - (2v_0 + \xi_0\hat{\varphi})\log\frac{-g_0^2 v_0\hat{\varphi}}{\bar{\mu}_0^2}\right]\cdot\left(\lambda_{60}\frac{\hat{\varphi}^4}{\Lambda^2} + \lambda_{80}\frac{\hat{\varphi}^6}{\Lambda^4} + \dots\right)$$

but Nielsen identity:

$$v\frac{\partial K_{g^2}}{\partial v} = 2\frac{\partial C^v_{g^2}}{\partial \varphi_1}\,, \qquad v\frac{\partial V_{g^6}}{\partial v} = C^v_{g^2}\frac{\partial V_{g^4}}{\partial \varphi_1}$$

Thus S_B for the bounce obtained in the presence of new operators is gauge fixing independent to the order g^6

Gravity Corrections in Curved Space

Effective action in curved background: gauge-less Higgs model

$$\Gamma = -\frac{1}{16\pi G}\int\sqrt{-g}d^4x(R+2\Lambda) + \int\sqrt{-g}d^4x\Big\{\bar{\chi}\left[i\gamma^\mu\nabla_\mu - \frac{1}{\sqrt{2}}yh\right]\chi +$$
$$+\frac{1}{2}\nabla_\mu h\nabla^\mu h - \frac{1}{2}\left(m_h^2 - \xi_h R\right)h^2 - \frac{\lambda_h}{4}h^4 - \frac{\lambda_{hX}}{4}h^2X^2 +$$
$$+\frac{1}{2}\nabla_\mu X\nabla^\mu X - \frac{1}{2}\left(m_X^2 - \xi_X R\right)X^2 - \frac{\lambda_X}{4}X^4 +$$
$$+\frac{\hbar}{64\pi^2}\Big[\frac{1}{2}y_t^2\bar{\chi}\left(i\gamma^\mu\nabla_\mu + 2\frac{1}{\sqrt{2}}y_t h\right)\chi - \frac{3}{2}y_t^2\nabla_\mu h\nabla^\mu h - 2y_t^2\ln\left(\frac{b}{\mu^2}\right)\nabla_\nu h\nabla^\nu h +$$
$$-\frac{1}{3}tr\left(\Box a\ln\left(\frac{a}{\mu^2}\right)\right) + \frac{8}{3}\Box b\ln\left(\frac{b}{\mu^2}\right) - tr\left(a^2\ln\left(\frac{a}{\mu^2}\right)\right) + \frac{3}{2}tra^2 + 8b^2\ln\left(\frac{b}{\mu^2}\right) - 12b^2 +$$
$$+\frac{1}{3}y_t^2h^2\ln\left(\frac{b}{\mu^2}\right)R - y_t^4h^4\ln\left(\frac{b}{\mu^2}\right) +$$
$$-\frac{4}{180}\left(-R_{\alpha\beta}R^{\alpha\beta} + R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}\right)\left(\ln\left(\frac{a_+}{\mu^2}\right) + \ln\left(\frac{a_-}{\mu^2}\right) - 2\ln\left(\frac{b}{\mu^2}\right)\right) +$$
$$-\frac{4}{3}R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}\ln\left(\frac{b}{\mu^2}\right)\Big]\Big\}, \tag{2.32}$$

$$\beta_{y_t} = \frac{\hbar}{(4\pi)^2}\frac{5}{2}y_t^3,$$

$$\beta_{\lambda_h} = \frac{\hbar}{(4\pi)^2}\left[18\lambda_h^2 - 2y_t^4 + 4y_t^2\lambda_h + \frac{1}{2}\lambda_{hX}^2\right],$$

$$\beta_{\lambda_X} = \frac{\hbar}{(4\pi)^2}\left[18\lambda_X^2 + \frac{1}{2}\lambda_{hX}^2\right],$$

$$\beta_{\lambda_{hX}} = \frac{\hbar}{(4\pi)^2}\left[4\lambda_{hX}^2 + 6\lambda_{hX}(\lambda_h + \lambda_X) + 2\lambda_{hX}y^2\right]$$

$$\beta_{m_h^2} = \frac{\hbar}{(4\pi)^2}\left[6\lambda_h m_h^2 + 2y_t^2 m_h^2 + \lambda_{hX} m_X^2\right],$$

$$\beta_{m_X^2} = \frac{\hbar}{(4\pi)^2}\left[6\lambda_x m_X^2 + \lambda_{hX} m_h^2\right],$$

$$\beta_{\xi_h} = \frac{\hbar}{(4\pi)^2}\left[6\lambda_h(\xi_h - \frac{1}{6}) + \lambda_{hX}(\xi_x - \frac{1}{6}) + 2y_t^2(\xi_h - \frac{1}{6})\right],$$

$$\beta_{\xi_X} = \frac{\hbar}{(4\pi)^2}\left[6\lambda_X(\xi_X - \frac{1}{6}) + \lambda_{hX}(\xi_h - \frac{1}{6})\right].$$

$$\gamma_h = \frac{\hbar}{(4\pi)^2}y_t^2,$$

$$\gamma_X = 0,$$

$$\gamma_\chi = \frac{\hbar}{(4\pi)^2}\frac{1}{4}y_t^2.$$

In Robertson-Walker background one may express curvature invariant through energy density and preassure

$$R = -3\bar{M}_P^{\,-2}\left[-p + \frac{1}{3}\rho\right]$$

RD

$$-R_{\alpha\beta}R^{\alpha\beta} + R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu} = -12H^2\frac{\ddot{a}}{a} = 2M_P^{-4}\rho\left(\frac{1}{3}\rho + p\right) = \frac{4}{3}\left(\bar{M}_P^{-2}\rho\right)^2$$

$$R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu} = 12\left[H^4 + \left(\frac{\ddot{a}}{a}\right)^2\right] = 12M_P^{-4}\left[\frac{1}{9}\rho^2 + \frac{1}{4}\left(\frac{1}{3}\rho + p\right)^2\right] = \frac{8}{3}\left(\bar{M}_P^{-2}\rho\right)^2$$

$$\rho = \sigma\nu^4 + \left(\frac{y_t}{\sqrt{2}}h\right)^4$$

$$\mu = \frac{y_t}{\sqrt{2}}h$$

Large field region

Stability in RD

$$V(h^4) = \frac{\lambda_{eff}(h)}{4} h^4 + V^{(1)}_{grav}$$

$$b = \frac{y_t^2 h_0^2}{2}$$

$$V(h^4) = \frac{\lambda_{eff}(h)}{4} h^4 + \frac{1}{64\pi^2}\frac{4}{3} R_{\alpha\beta\mu\nu} R^{\alpha\beta\mu\nu} \ln\left(\frac{b}{\mu^2}\right) =$$

$$= \frac{1}{4}\left[\lambda_{eff}(h) + \frac{4}{64\pi^2}\frac{4}{3}\frac{8}{3}\left(\bar{M}_P^{-2}\rho\right)^2 \frac{\tilde{c}}{h^4}\right]_{|h=h_0} h^4 = \frac{1}{4}\bar{\lambda}_{eff}(h) h^4,$$

$$\bar{\lambda}_{eff}(\rho, h_0) = 0 \ ?$$

For $h_0 = 10^{10}$ GeV and $\lambda_{off} = -0.02$ one obtain the scale $\nu \approx 10^{14}$ GeV

Stability in dS

$$V(h^4) = \frac{1}{4}\left[\lambda_{eff}(h) - 2\xi_h \frac{R}{h^2}\right]_{|h=h_0} h^4$$

$$\bar{\lambda}_{eff}(\rho, h_0) = 0 \ ? \quad \Longrightarrow \quad \nu \sim 7 \cdot 10^{13}\,\text{GeV}$$

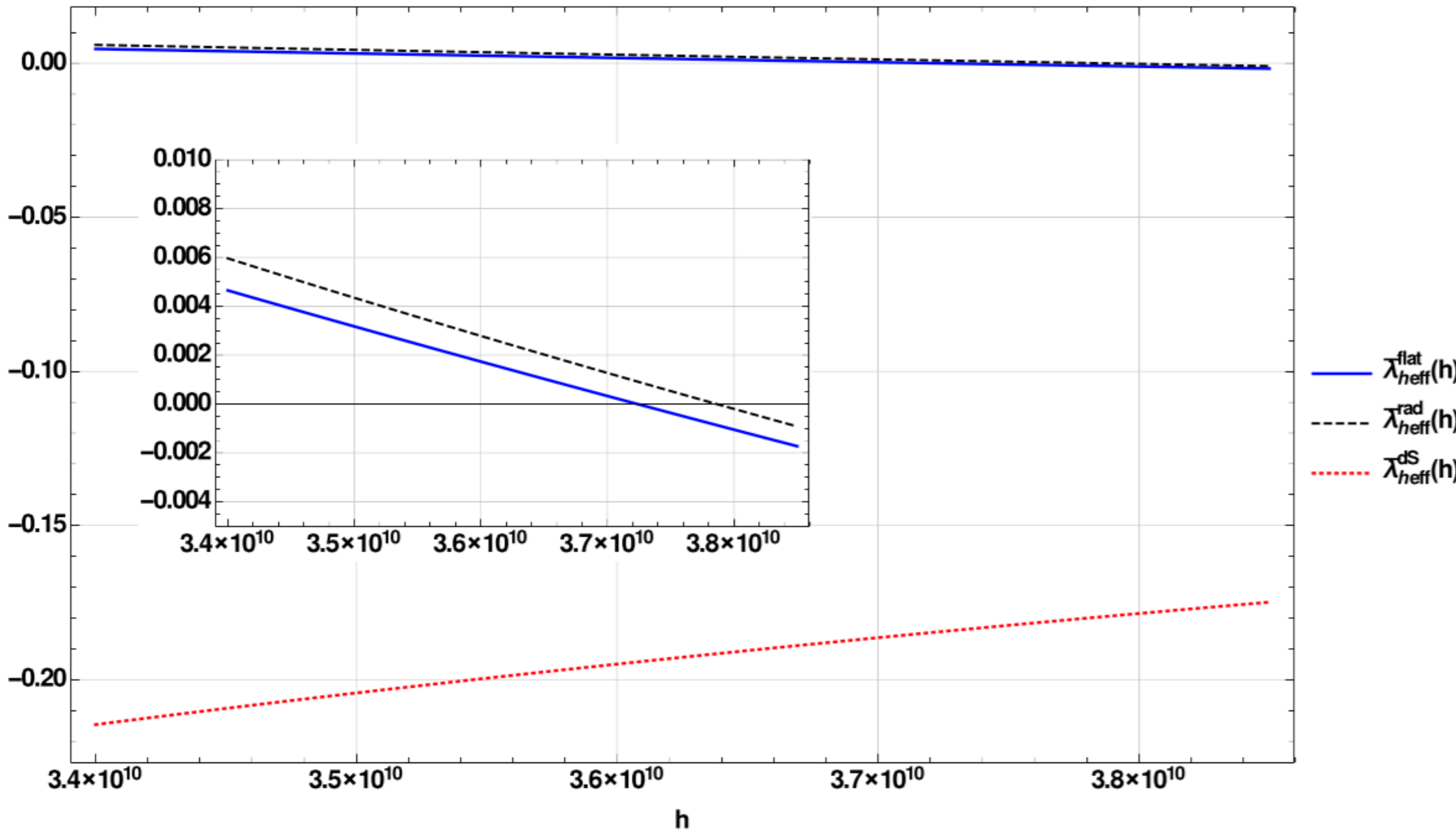


Figure 10: The effective quartic Higgs coupling, as defined by the relation $\bar{\lambda}_{heff}(h) \equiv \frac{4V^{(1)}(h)}{h^4}$, for various equations of state: $flat$ – flat spacetime result, rad – radiation dominance ($p = \frac{1}{3}\rho$), dS – de Sitter like ($p = -\rho$). The energy density was given by $\rho = \rho_{hc} + (\frac{y_t h}{\sqrt{2}})^4$, where ρ_{hc} was specified by the relation (4.36) and equal to $\rho_{hc} = (2.04 \cdot 10^{14} GeV)^4$. The X field was constant and set as equal to $X = v_X$. The non-minimal couplings were $\xi_h = \xi_X = 0$ at the $\mu = m_t$. The insert shows a close up of the difference for the flat spacetime and the radiation dominated era.

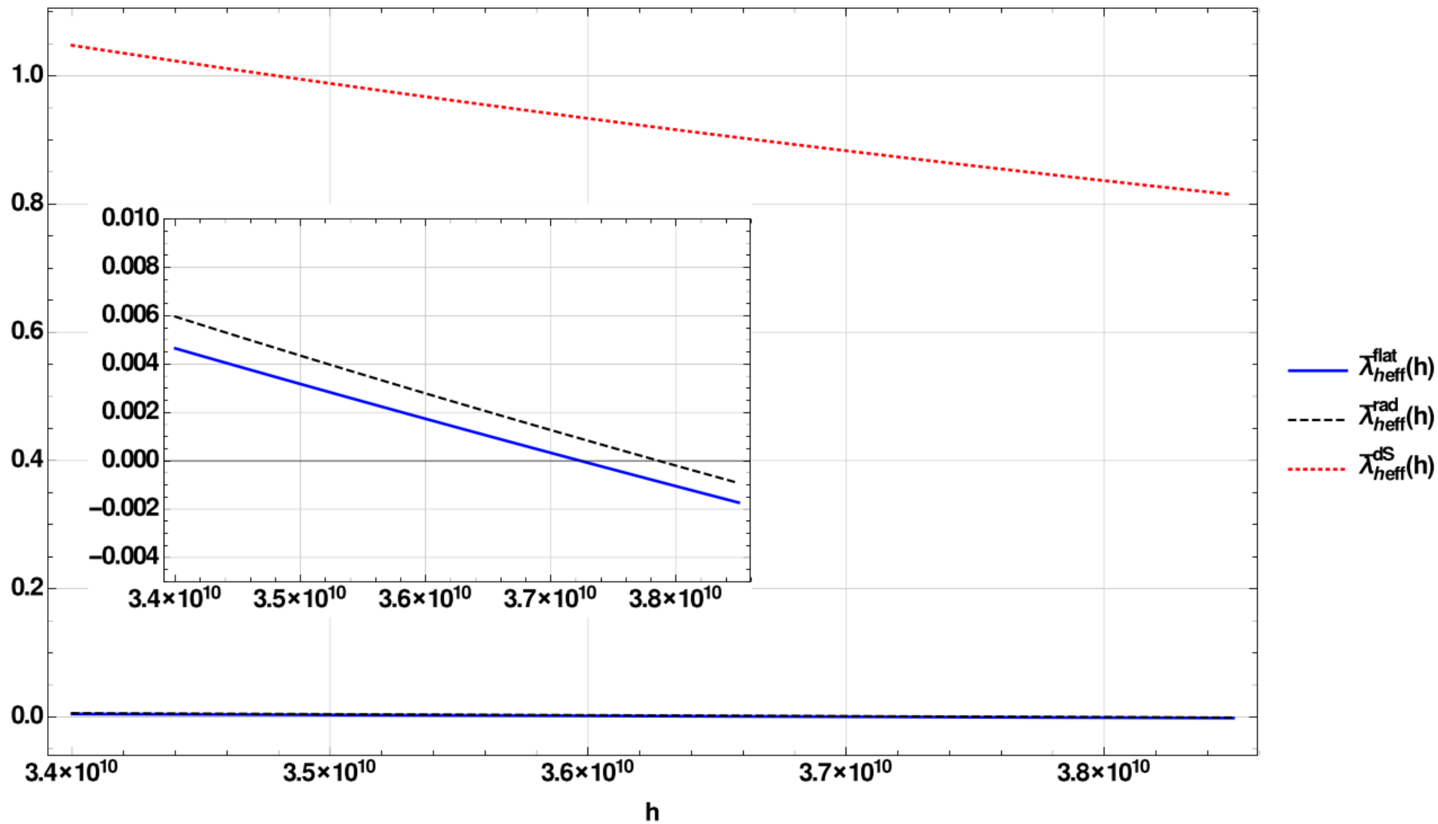
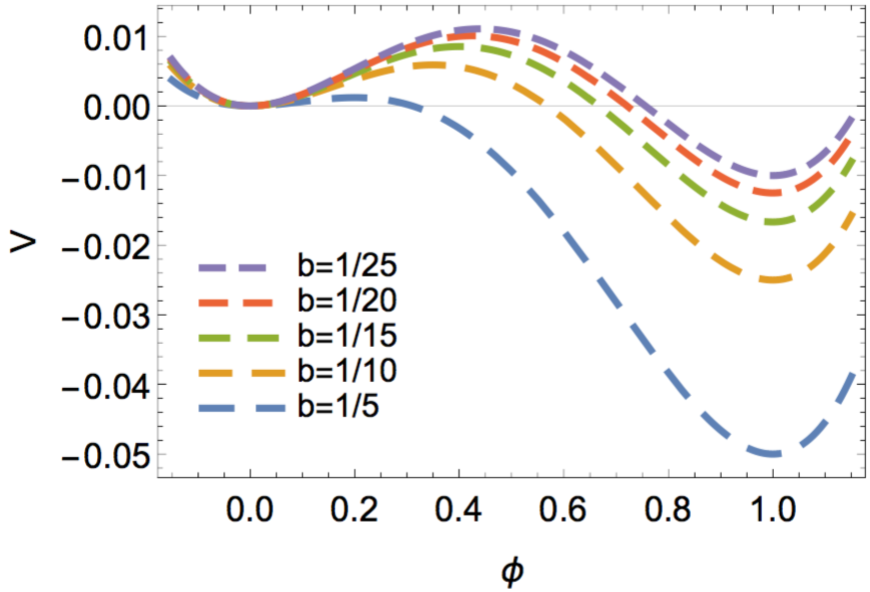


Figure 11: The effective quartic Higgs coupling, as defined by the relation $\bar{\lambda}_{heff}(h) \equiv \frac{4V^{(1)}(h)}{h^4}$, for various equations of state: *flat* – flat spacetime result, *rad* – radiation dominance ($p = \frac{1}{3}\rho$), *dS* – de Sitter like ($p = -\rho$). The energy density was given by $\rho = \rho_{hc} + (\frac{y_t h}{\sqrt{2}})^4$, where ρ_{hc} was specified by the relation (4.36) and equal to $\rho_{hc} = (2.04 \cdot 10^{14} GeV)^4$. The X field was constant and set as equal to $X = v_X$. The non-minimal couplings were $\xi_h = \xi_X = \frac{1}{3}$ at the $\mu = m_t$. The insert shows a close up of the difference for the flat spacetime and the radiation dominated era.

Coleman-De Luccia bounces

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - V, \quad V = -a^2(3b-1)\phi^2 + a(b-1)\phi^3 + \frac{1}{4}\phi^4 + C$$

two minima at $\phi = 0$ and $\phi = 2a$



$C = 0$

Figure 1. Our toy model potential for different values of b parameter. In this example vacuum energy vanishes $c = 0$. Different choices of vacuum energy, we will discuss, simply mean adding a constant to the potential.

Transition probability

$$\Gamma = Ae^{-S}$$

$$S_{\text{CDL}} = S[\phi_{\text{CDL}}] - S[\phi_{\text{fv}}]$$

$$S[\phi_{\text{fv}}] = -\frac{24\pi^2}{V_{\min}}, \quad \text{(for dS)}$$
$$S[\phi_{\text{fv}}] = 0, \quad \text{(for Minkowski)}$$

$$ds^2 = d\tau^2 + \rho(\tau)^2 (d\Omega)^2$$

$$S_E = 2\pi^2 \int d\tau \rho^3 \left(\frac{1}{2}\dot{\phi}^2 + V + \frac{1}{2}R \right)$$

$$R = 6\left(\frac{\ddot{\rho}}{\rho} + \left(\frac{\dot{\rho}}{\rho}\right)^2 - \frac{1}{\rho^2} \right) \text{ and } \dot{\phi} = \frac{d\phi}{d\tau}$$

Equations of motion

$$\ddot{\phi} + 3\frac{\dot{\rho}}{\rho}\dot{\phi} = \frac{\partial V}{\partial \phi}$$

$$\dot{\rho} = \sqrt{1 + \frac{\rho^2}{3}\left(\frac{1}{2}\dot{\phi}^2 - V\right)}$$

Boundary conditions

$$\begin{aligned} \dot{\phi}(0) &= \dot{\phi}(\tau_{\rm end}) = 0 \\ \rho(0) &= 0 \\ \rho(\tau_{\rm end}) &= 0, \quad \text{(for dS)} \\ \rho(\tau_{\rm end}) &= \rho_{\rm end} \neq 0, \quad \text{(for Minkowski)} \end{aligned}$$

Coleman and De Luccia formalism (CDL) with ξ

True vacuum Euclidean action

$$S^E[\phi_{CDL}] = 2\pi^2 \int d\tau \rho^3 \left(\tfrac{1}{2}\dot{\phi}^2 + V - \tfrac{R}{2\kappa}(1 - \kappa\xi\phi^2) \right) =$$

$$= 4\pi^2 \int d\tau \left[\rho^3 V - \tfrac{3\rho}{\kappa} \left(1 - \xi\kappa\phi^2\right)\right] + \tfrac{6\pi}{\kappa} \left(1 - \kappa\xi\phi^2\right) \rho^2 \dot{\rho}\Big|_0^{\tau_{max}}$$

where

$$R = -6 \left(\frac{\ddot{\phi}}{\phi} + \frac{\dot{\phi}^2}{\phi^2} - \frac{1}{\phi^2} \right)$$

EOM

$$\ddot{\phi} + 3\frac{\dot{\rho}}{\rho}\dot{\phi} - \xi\phi R = \frac{\partial V}{\partial \phi}$$

False vacuum Euclidean action

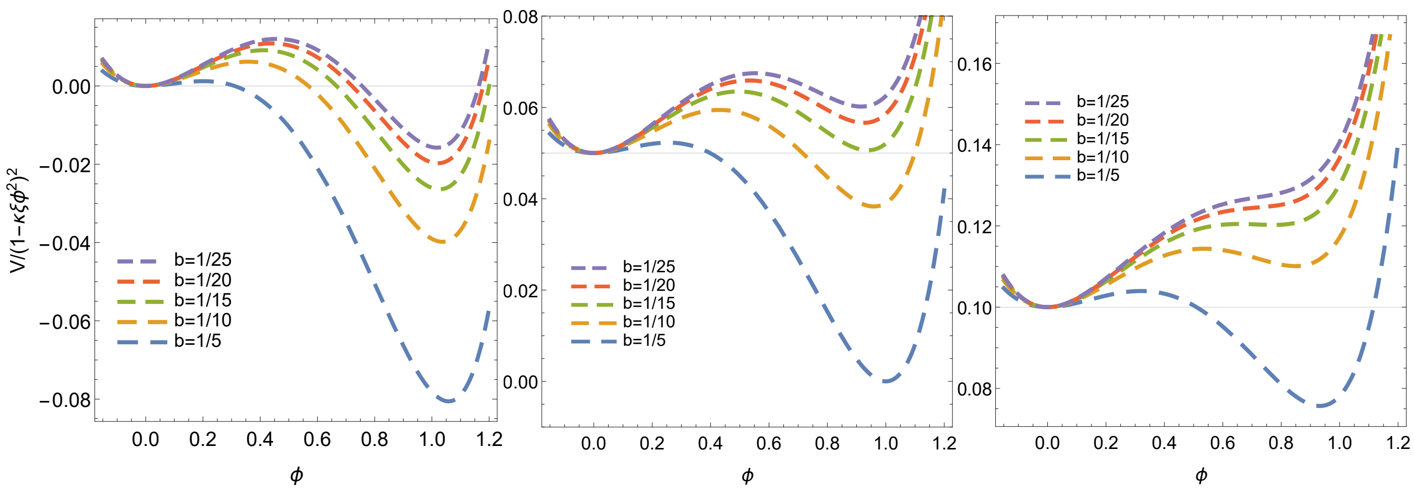
$$S[\phi_{\rm fv}] = -\frac{24\pi^2(1 - \kappa\xi\phi_{\rm fv}^2)^2}{\kappa^2 V_{\rm fv}} \quad \text{(dS)}$$

$$S[\phi_{\rm fv}] = 0 \qquad \text{(Minkowski)}$$

Modification of the vacuum energy by ξ

- fv: $\phi_{\text{fv}} = 0$, no modification of V_{fv}
- tv: modification!
  - V_{tv} can be bigger than V_{fv} making our false vacuum stable
  - true vacuum can disappear altogether - we neglect tunnelling then

    Bubble profile can sometimes be calculated still but such bubble is not energetically favourable and would not grow after nucleation.



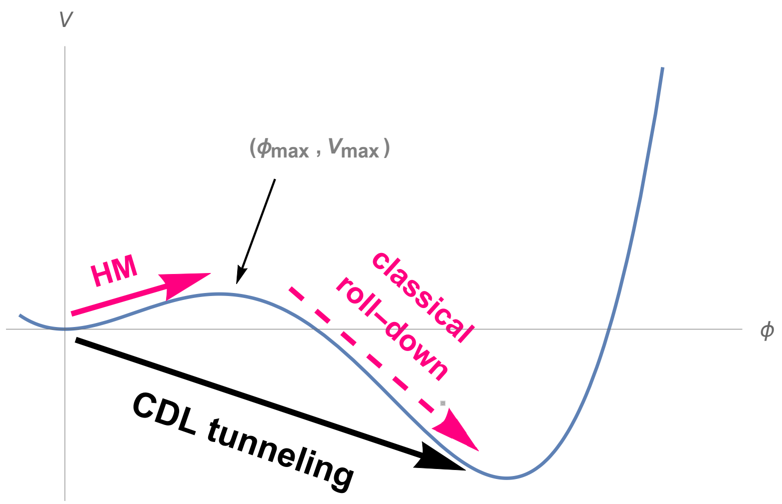
Modified potential (connected with the real for acting on our field) for different choices of the vacuum energy with $c = (0, 0.05, 0.1)$ and $\xi = 0.2$.

Hawking-Moss solution

Simpler HM solution S.W. Hawking, I.G. Moss, Phys.Lett. B 110 (1982) describes the probability for a whole spacetime volume to transition simultaneously to the top of the barrier (max) and continue by a classical roll-down:

$$S_{HM} = S_{\rm max} - S_{\rm fv} =$$
$$= -\frac{24\pi^2(1-\kappa\xi\phi_{\rm max}^2)^2}{\kappa^2 V_{\rm max}} + \frac{24\pi^2(1-\kappa\xi\phi_{\rm fv}^2)^2}{\kappa^2 V_{\rm fv}}$$

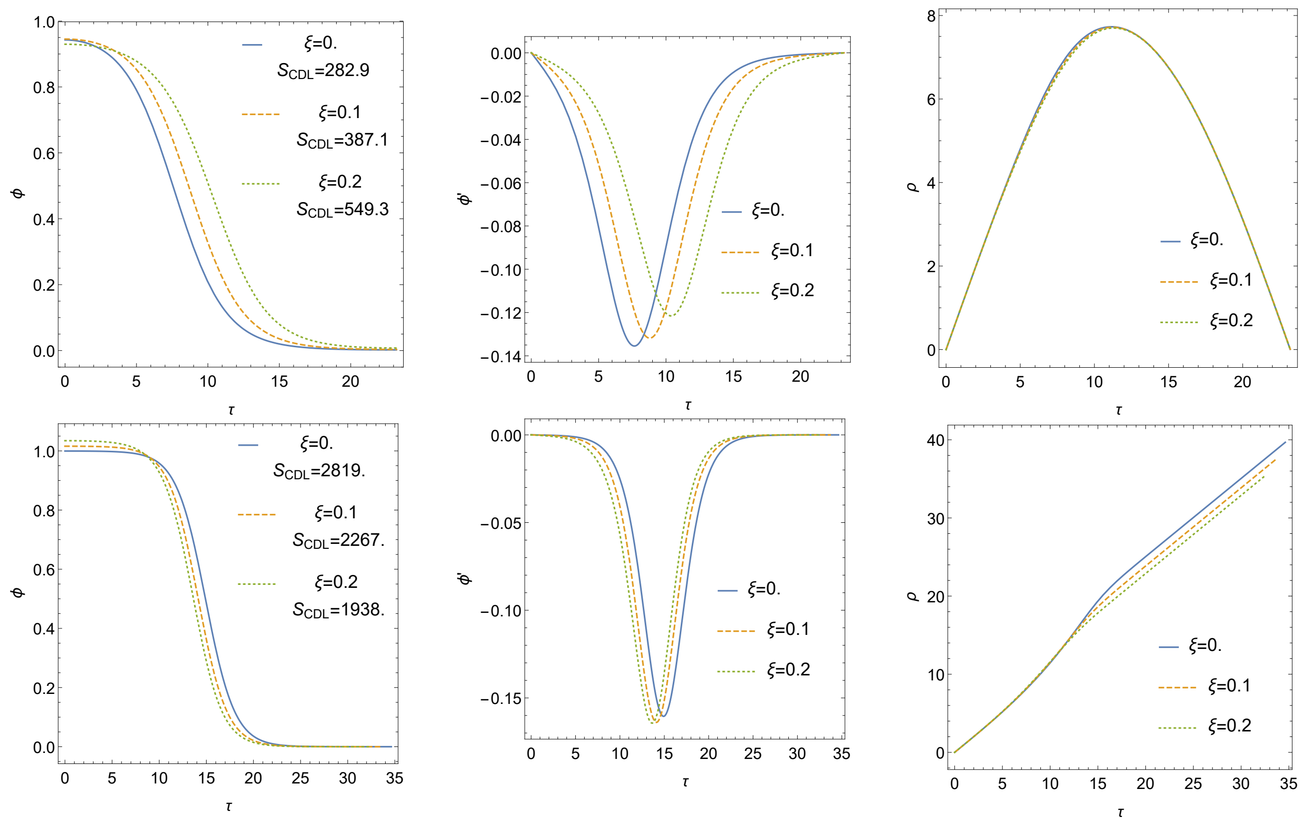
including the modification coming from ξ



Numerical calculation

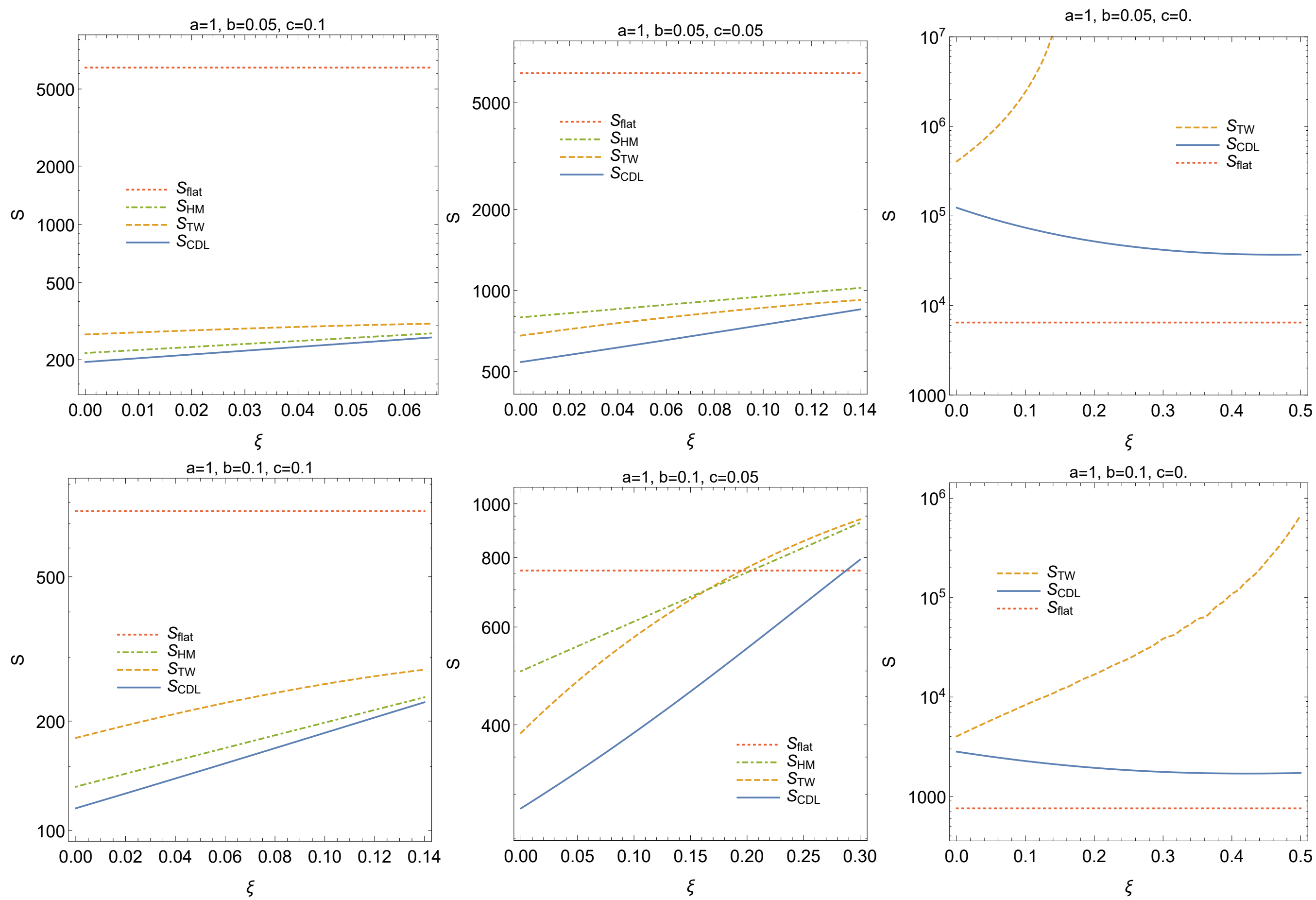
$c = 0.05$
(dS)

$c = 0$
(Minkowski)



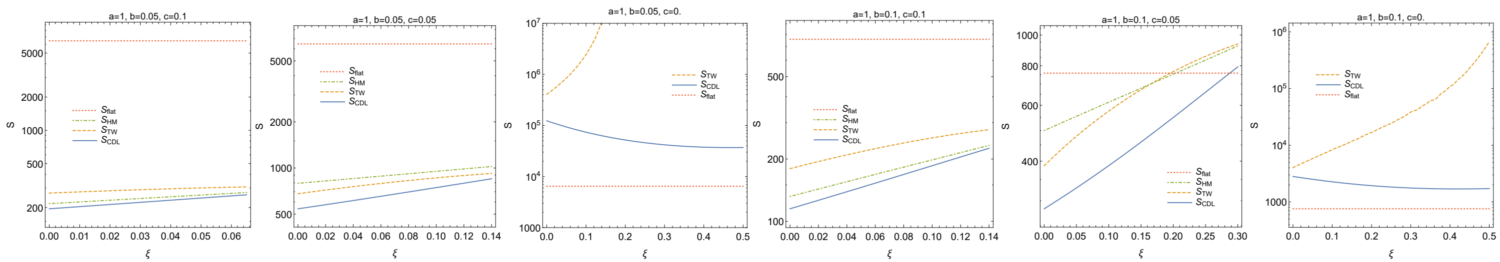
Bubble profile, field "velocity" and scale factor and their modification due to the non-minimal coupling for $b = 1/10$. Boundary term's influence can be observed in $\rho(\tau)$ for Minkowski case.

Comparison of the results



Tunnelling action as a function of non-minimal coupling obtained using four different methods with $c = (0.1, 0.05, c = 0$ (Minkowski)$)$.

Conclusions CDL



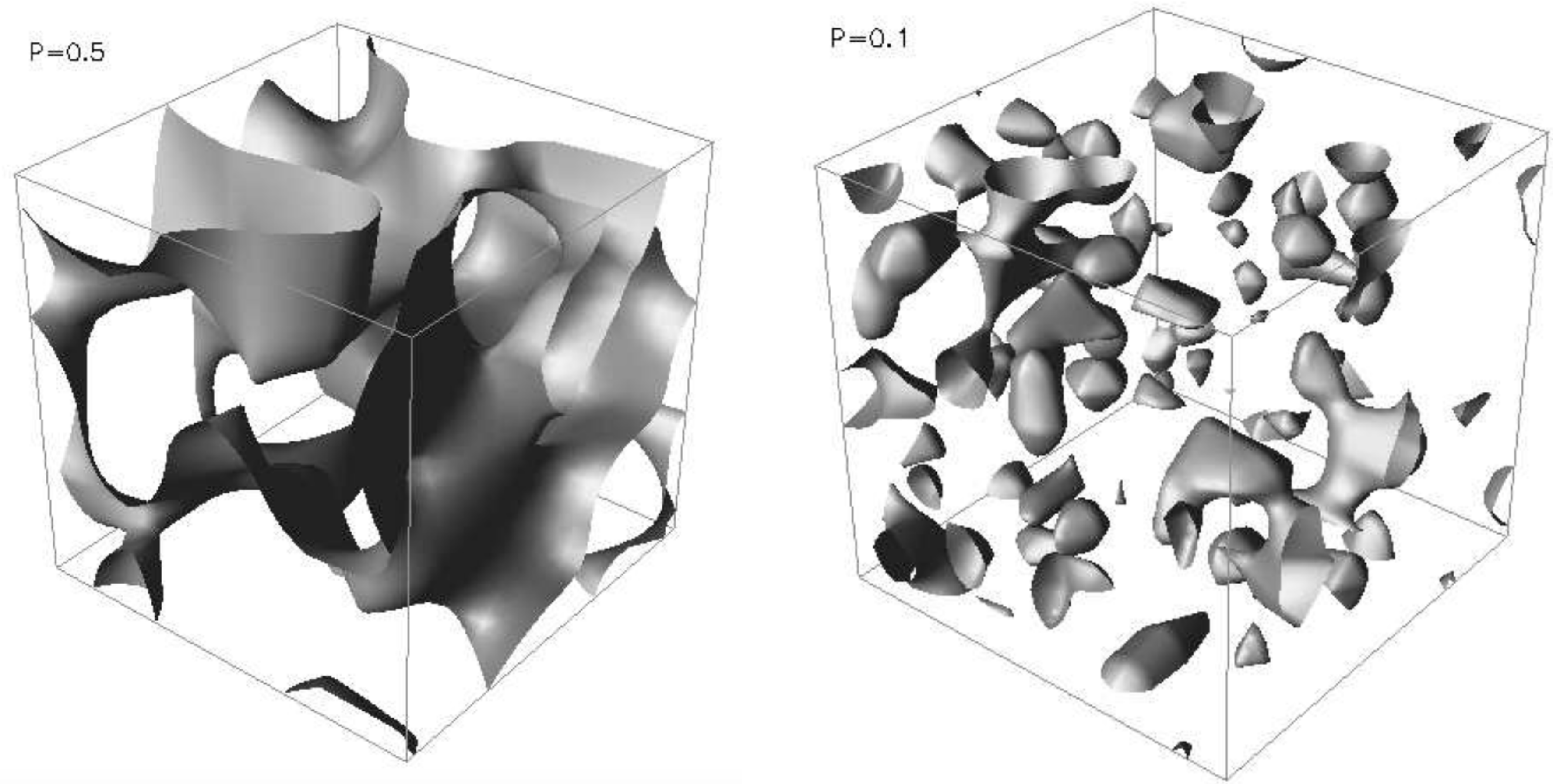
The influence of non-minimal coupling to gravity is very different in cases of Minkowski and dS vacua:

- dS - the decay probability quickly decreases as the coupling grows, vacuum can be made absolutely stable
- Minkowski - effect is much weaker, the decay rate increases for small values, TW approximation works worse significantly overestimating the increase in action due to ξ

Even though TW approximation may not give a precise result in a specific model, the order of magnitude is right (especially in dS case where gravitational correction decreases the stability).

Domain walls and gravitational waves

Network of walls prefers the true vacuum!



About our simulation

- We modeled the Higgs field with a positive, real scalar ϕ.
- The evolution of ϕ is given by EOM:

$$\frac{\partial^2\phi}{\partial\eta^2} + \frac{\alpha}{\eta}\left(\frac{d\ln a}{d\ln\eta}\right)\frac{\partial\phi}{\partial\eta} - \Delta\phi = -a^\beta\frac{\partial V}{\partial\phi},$$

  with a potential $V(\phi)$ equal to the RG improved potential of the SM Higgs $V_{\mathrm{SM}}(|h|)$.
- The PRS algorithm[2] (with $\alpha = 3$, $\beta = 0$) was used.
- We used the optimization of a time step[3].
- Our simulations were run on a lattice of the size 512^3.

[2] William H. Press, Barbara S. Ryden, and David N. Spergel. "Dynamical Evolution of Domain Walls in an Expanding Universe". In: *Astrophys. J.* 347 (1989), pp. 590–604. DOI: 10.1086/168151.

[3] Z. Lalak, S. Lola, and P. Magnowski. "Dynamics of domain walls for split and runaway potentials". In: *Phys. Rev.* D78 (2008), p. 085020. DOI: 10.1103/PhysRevD.78.085020. arXiv: 0710.1233 [hep-ph].

Initial conditions

Following the general considerations[4] we assumed that the initial distribution of field strength is given by probability distribution:

$$P(h) = \frac{1}{\sqrt{2\pi}\sigma_I} e^{-\frac{(h-\theta)^2}{2\sigma_I^2}} \quad \sigma_I \sim \frac{\sqrt{N} H_I}{2\pi}$$

We considered various combinations of values of σ and θ in order to cover the set of initial conditions which can be predicted by models of the early Universe.
Our simulations were initialized at different conformal times η_{start} ranging from $10^{-14}\ \mathrm{GeV}^{-1}$ to $10^{-10}\ \mathrm{GeV}^{-1}$.

[4] Z. Lalak et al. "Large scale structure from biased nonequilibrium phase transitions: Percolation theory picture". In: *Nucl. Phys.* B434 (1995), pp. 675–696. DOI: 10.1016/0550-3213(94)00557-U. arXiv: hep-ph/9404218 [hep-ph].

Dependence on the initialization time

For nearly equal contributions of both vacua at the initialization, late domain walls decay longer leading to domination of the EW vacuum even if the fraction of lattice sites occupied by this vacuum decreases initially.

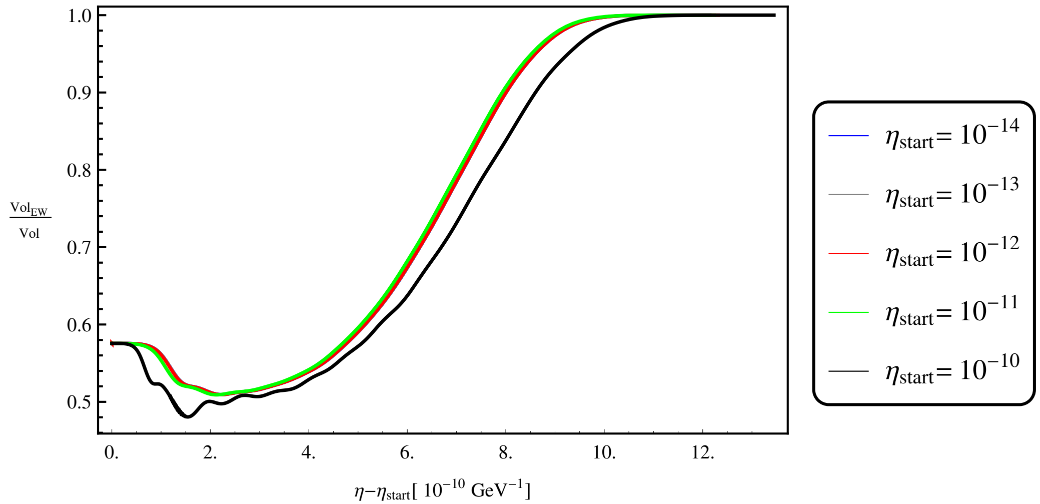


Figure: The fraction $\frac{Vol_{EW}}{Vol}$ as a function of conformal time η for different initialization times η_{start}.

Dependence on the initialization time

The decay of domain walls ending in the state without the EW vacuum is possible even for the initial configuration with a slight dominance of the EW vacuum.

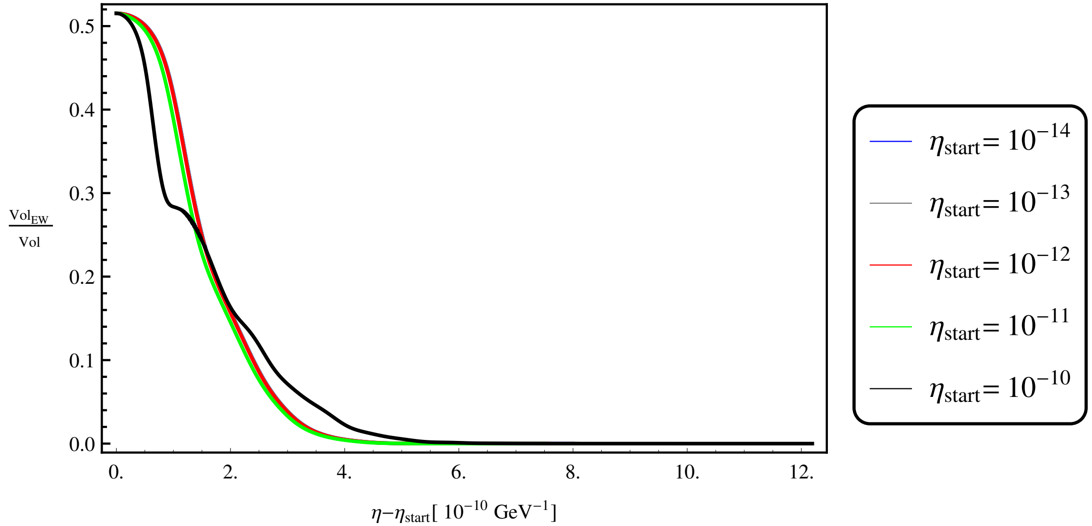


Figure: The fraction $\frac{Vol_{EW}}{Vol}$ as a function of conformal time η for different initialization times η_{start}.

Dependence on standard deviation σ

We investigated the initial conditions satisfying $\theta + \sigma = v_{max}$, where v_{max} is position of the local maximum of the potential. In this case the evolution of networks displays the weak dependence on the value of σ and for all simulations the final state is the EW vacuum.

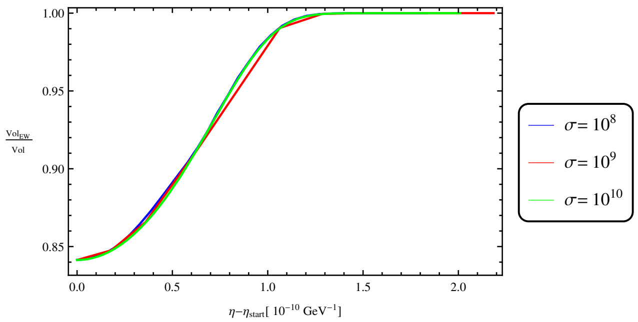


Figure: The fraction $\frac{Vol_{EW}}{Vol}$ as a function of conformal time η for initialization time $\eta_{start} = 10^{-\mathbf{13}}\ \mathrm{GeV}^{-\mathbf{1}}$ and different values of standard deviation σ.

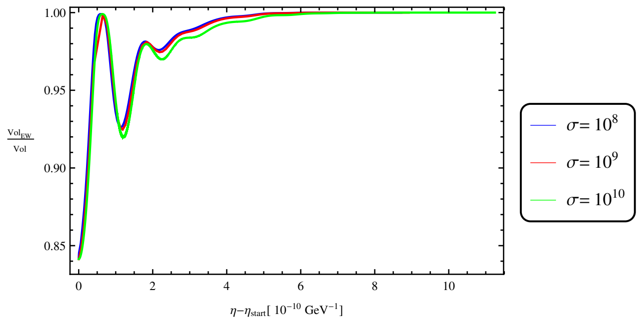


Figure: The fraction $\frac{Vol_{EW}}{Vol}$ as a function of conformal time η for initialization time $\eta_{start} = 10^{-\mathbf{10}}\ \mathrm{GeV}^{-\mathbf{1}}$ and different values of standard deviation σ.

Gravitational waves from domain walls

Energy density generated by one mode $\varrho_{gw}(\eta, k)$ can be expressed as:

$$\varrho_{gw}(\eta, k) = \frac{1}{16\pi^3 {M_{Pl}}^2 a(\eta)^4 V} \sum_{i,j} \left[\left| \int_{\eta_i}^{\eta_f} d\eta' \cos\left(|k|\left(\eta - \eta'\right)\right) a(\eta') \widehat{T^{TT}}_{ij}(\eta', k) \right|^2 \right.$$
$$\left. + \left| \int_{\eta_i}^{\eta_f} d\eta' \sin\left(|k|\left(\eta - \eta'\right)\right) a(\eta') \widehat{T^{TT}}_{ij}(\eta', k) \right|^2 \right],$$

after redshift

$$\frac{d\rho_{gw}}{d\log|k|}(\eta_0, k) = (1 + z_{EQ})^{-4} \frac{a(\eta_{dec})^4}{a(\eta_{EQ})^4} \frac{d\rho_{gw}}{d\log|k|}(\eta_{dec}, k),$$

$$f_0 = \frac{a(\eta_{dec})}{a(\eta_0)} \frac{k}{2\pi} = 5.07 \times 10^6 \left(\frac{10^{19} \frac{\text{eV}}{\hbar}}{H_{dec}} \right)^{\frac{1}{2}} \left(\frac{k}{10^{10} \ \frac{\text{GeV}}{\hbar c}} \right) \text{Hz}.$$

Expectations:

N. Kitajima and F. Takahashi, *Gravitational waves from Higgs domain walls*, *Phys. Lett.* **B745** (2015) 112–117, [1502.03725].

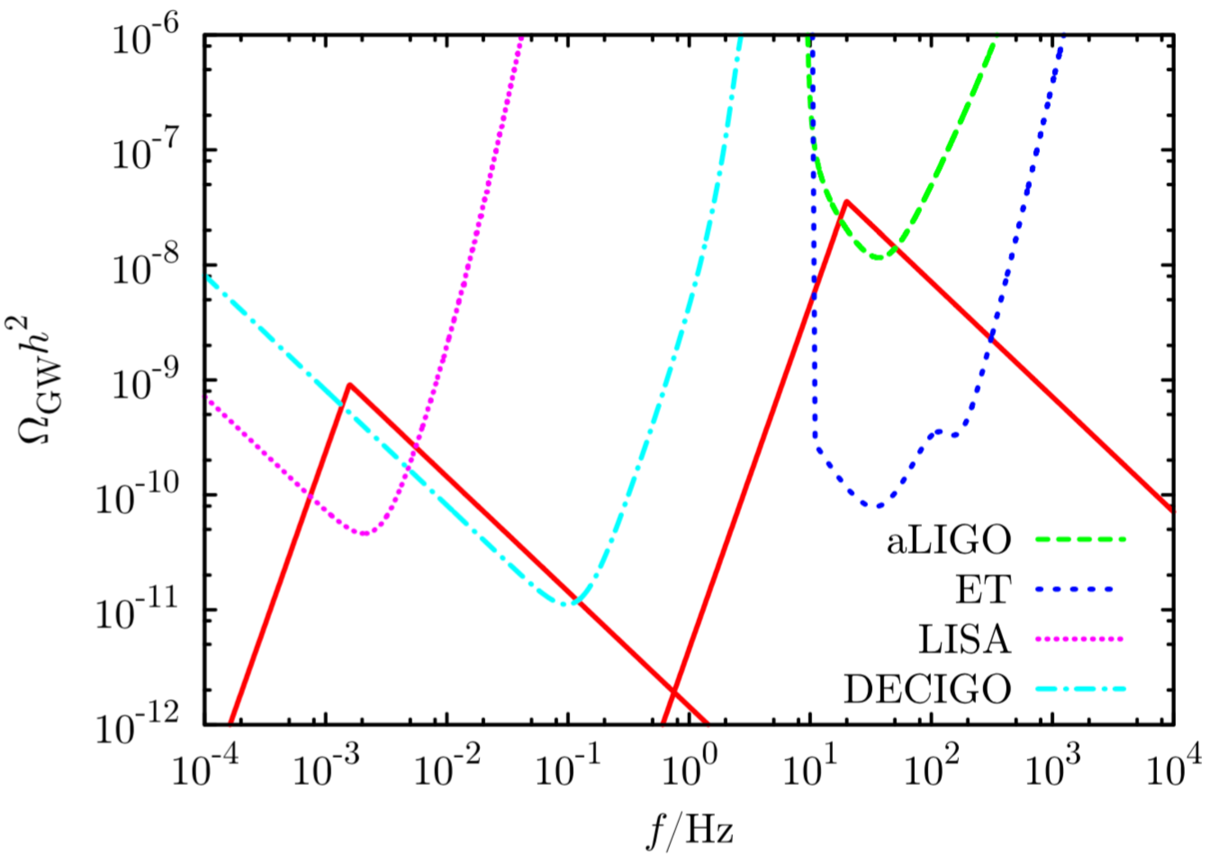


Fig. 3. The typical spectrum of the gravitational waves is shown by the solid (red) lines. We have taken $\varphi_f = 2\times 10^9$ GeV and $(V_f/V_{\rm max})^{1/4} = 5\times 10^{-5}$ for the left line and $\varphi_f = 2\times 10^{12}$ GeV and $(V_f/V_{\rm max})^{1/4} = 10^{-3}$ for the right line.

Numerical simulations:

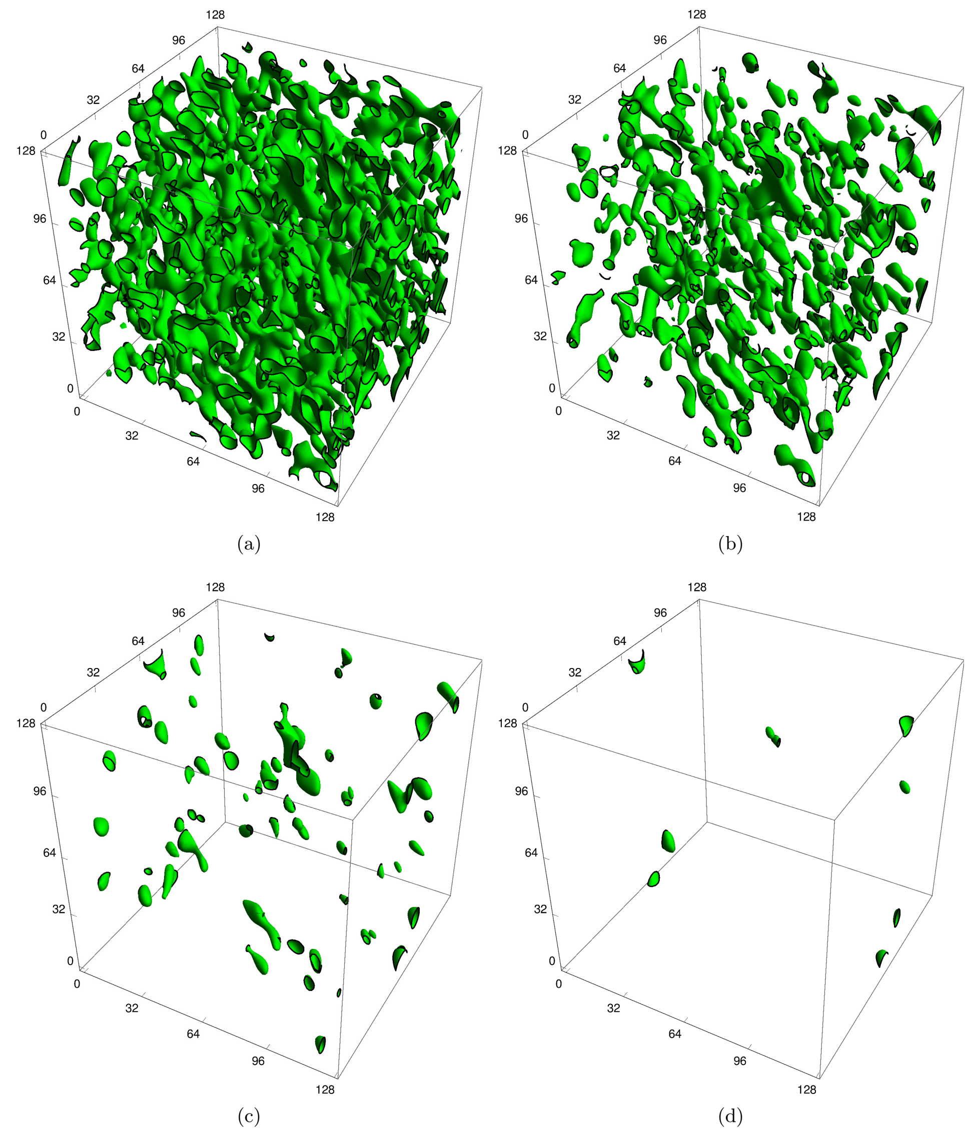


Figure 11: Visualization of the isosurface of the field strength ϕ corresponding the value v_{max} at four different conformal times: $\eta = 10^{-9}$ GeV^{-1} (a) and $\eta = 1.2 \times 10^{-9}$ GeV^{-1} (b), $\eta = 1.3 \times 10^{-9}$ GeV^{-1} (c), $\eta = 1.4 \times 10^{-9}$ GeV^{-1}(d). Lengths are given in units of the lattice spacing i.e. 10^{-10} GeV^{-1}.

Spectrum of GWs after emission

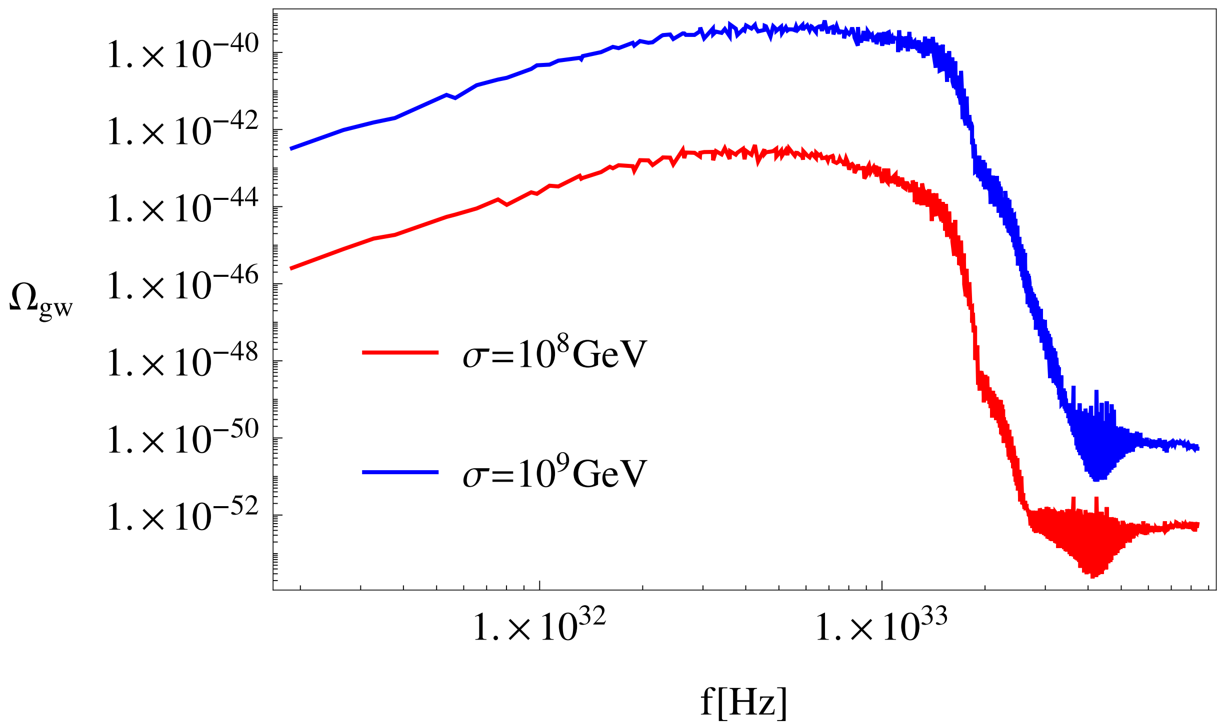


Figure: Spectrum of gravitational waves Ω_{gw} emitted from SM domain walls at the time of the decay.

Present spectrum of GWs

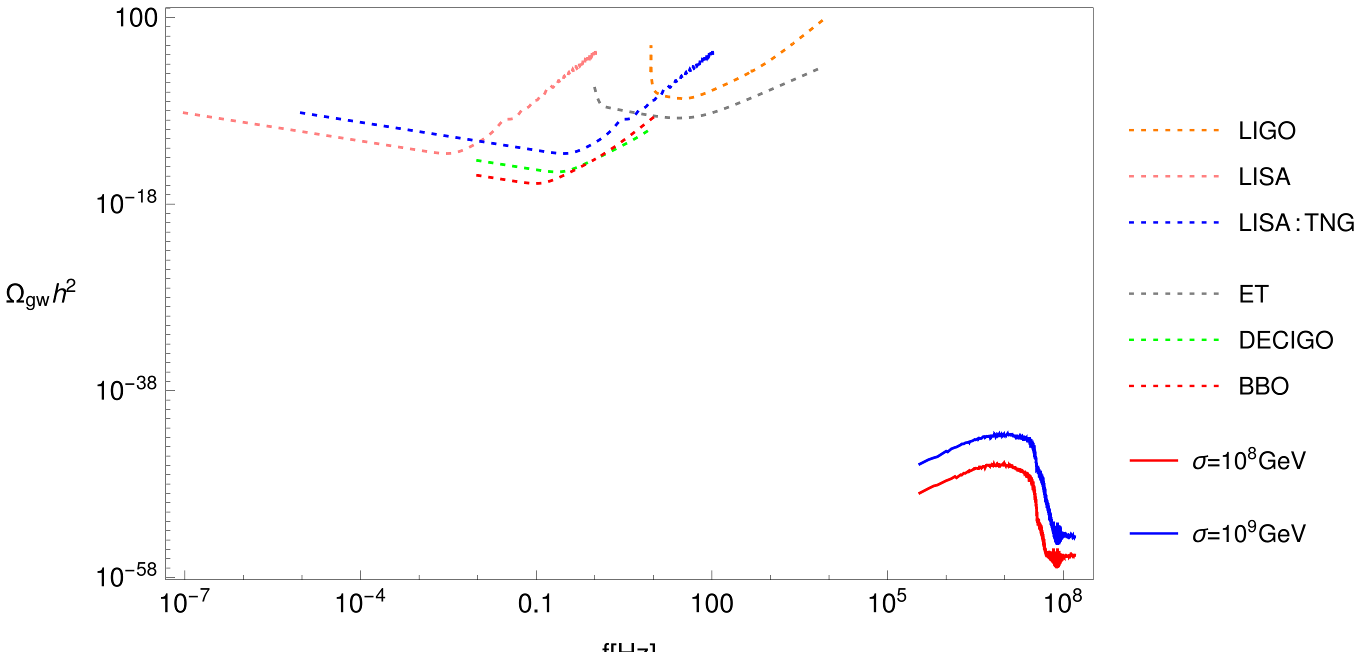
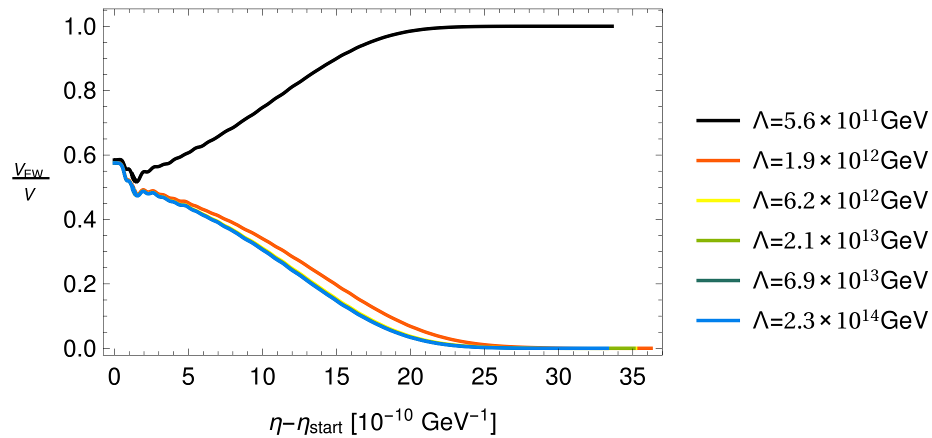
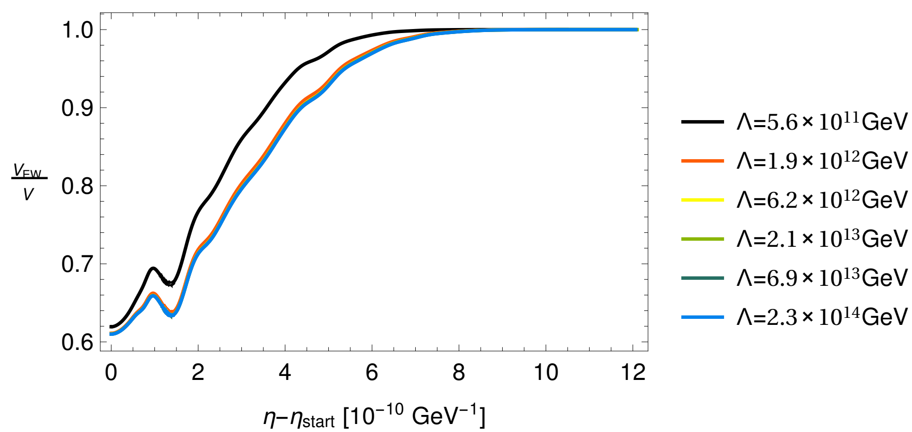


Figure: Predicted sensitivities (dashed) for future GWs detectors: aLIGO, ET, LISA, LISA:TNG, DECIGO and BBO compared with the spectrum of GWs (solid) calculated in lattice simulations for the initial values of $\sigma = 10^{\mathbf{8}},\ 10^{\mathbf{9}}\ \mathrm{GeV}$ and the standard cosmology.

New physics $\delta V^{\Lambda}_{SM}(h) = \frac{\lambda_6}{6\,!}\frac{h^6}{\Lambda^2_{NP}}$





$$V_{SM}^{\Lambda}(h) = \tilde{V}_{SM} + \frac{\lambda_6}{6\,!}\frac{h^6}{\Lambda^2}$$

$$\tilde{V}_{SM}(h_{EW}) - \tilde{V}_{SM}(h_{UV}) \approx 0$$

$$\frac{1}{2}\frac{m^2(h_{EW})}{h_{EW}^2} + \frac{1}{4}\lambda_{\rm eff}(h_{EW};h_{EW}) + \frac{1}{6!}\lambda_6(h_{EW};h_{EW})\frac{h_{EW}^2}{M^2} =$$
$$= (1+\varepsilon)^{-1}\left(\frac{h_{UV}}{h_{EW}}\right)^4\left[\frac{1}{2}\frac{m^2(h_{UV})}{h_{EW}^2}\left(\frac{h_{EW}}{h_{UV}}\right)^2 + \frac{1}{4}\lambda_{\rm eff}(h_{UV};h_{UV}) + \frac{1}{6!}\lambda_6(h_{UV};h_{UV})\frac{h_{UV}^2}{M^2}\right]$$

$$\Lambda_{DEG} = 1.88\,\times\,10^{11}\,\mathrm{GeV},\ h_{UV} \approx 10^{10}\,\mathrm{GeV}$$

so LHS ~ 0.1 and $[\,] \sim 10^{-32}$ on RHS

This implies severe fine-tuning of λ_6 which implies trouble with numerics!

Width of domain walls

Furthermore the potential energy density $\sigma(x_1, x_2)$ of the solution (3.4) is given by the following integral:

$$\sigma^{\Lambda}(x_1, x_2) := \int_{x_1}^{x_2} V_{\rm SM}^{\Lambda}(\varphi(x))dx = \int_{\varphi(x_1)}^{\varphi(x_2)} \frac{V_{\rm SM}^{\Lambda}(\varphi)d\varphi}{\sqrt{2\left(V_{\rm SM}^{\Lambda}(\varphi) - V_{\rm SM}^{\Lambda}(h_{EW})\right)}}. \tag{3.5}$$

Finally, we have found values $\tilde{\varphi}_1$ and $\tilde{\varphi}_2$ such that $V_{\rm SM}^{\Lambda}(\tilde{\varphi}_1) = V_{\rm SM}^{\Lambda}(\tilde{\varphi}_2)$ and the majority of the potential energy density of the solution (3.4) is stored between $x(\tilde{\varphi}_1)$ and $x(\tilde{\varphi}_2)$:

$$\frac{\sigma^{\Lambda}(x(\tilde{\varphi}_1), x(\tilde{\varphi}_2))}{\sigma^{\Lambda}(-\infty, +\infty)} \approx 97\%. \tag{3.6}$$

Our estimation of the width of domain walls is then given by:[1]

$$w(\Lambda) := x^{\Lambda}(\tilde{\varphi}_2) - x^{\Lambda}(\tilde{\varphi}_1). \tag{3.7}$$

The estimated width of domain walls as a function of the suppression scale Λ of the non-renormalizable operator h^6 is presented in the figure 2. Resulting values of the width lay in the range $3.5 \times 10^{-9}\ \mathrm{GeV}^{-1} \leq w(\Lambda) \leq 5.0 \times 10^{-9}\ \mathrm{GeV}^{-1}$.

We have chosen the physical lattice spacing l to be equal to $10^{-10}\ \mathrm{GeV}^{-1}$ which leads to widths of walls contained in the range $35\,l \leq w(\Lambda) \leq 50\,l$. Henceforth, we will often use $\frac{1}{l} = 10^{10}$ GeV as our default unit of energy and inverse distance in space-time.

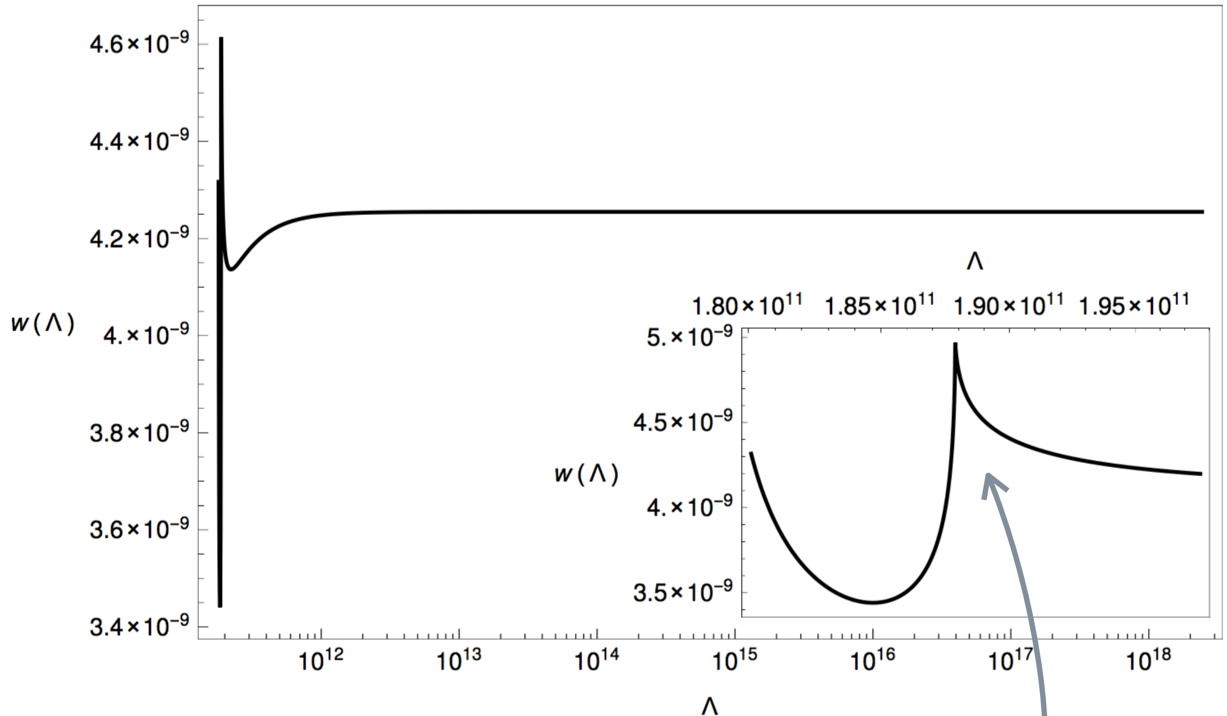


Figure 2: The width of domain walls as a function of the scale of new physics Λ.

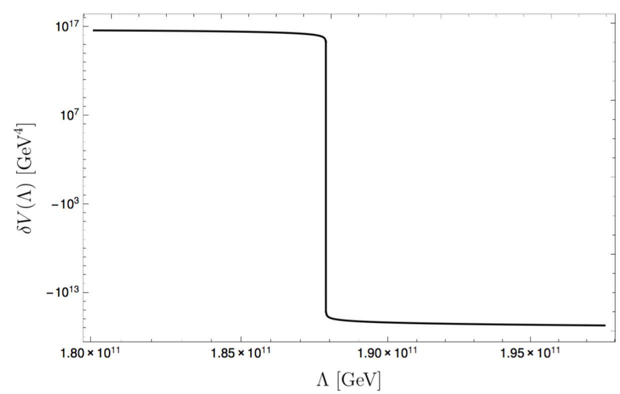


Figure 1: The difference $\delta V := V_{\rm SM}^{\Lambda}(h_{UV}) - V_{\rm SM}^{\Lambda}(h_{EW})$ of values of the RG improved potential $V_{\rm SM}^{\Lambda}$ at two minima as a function of the scale of new physics Λ.

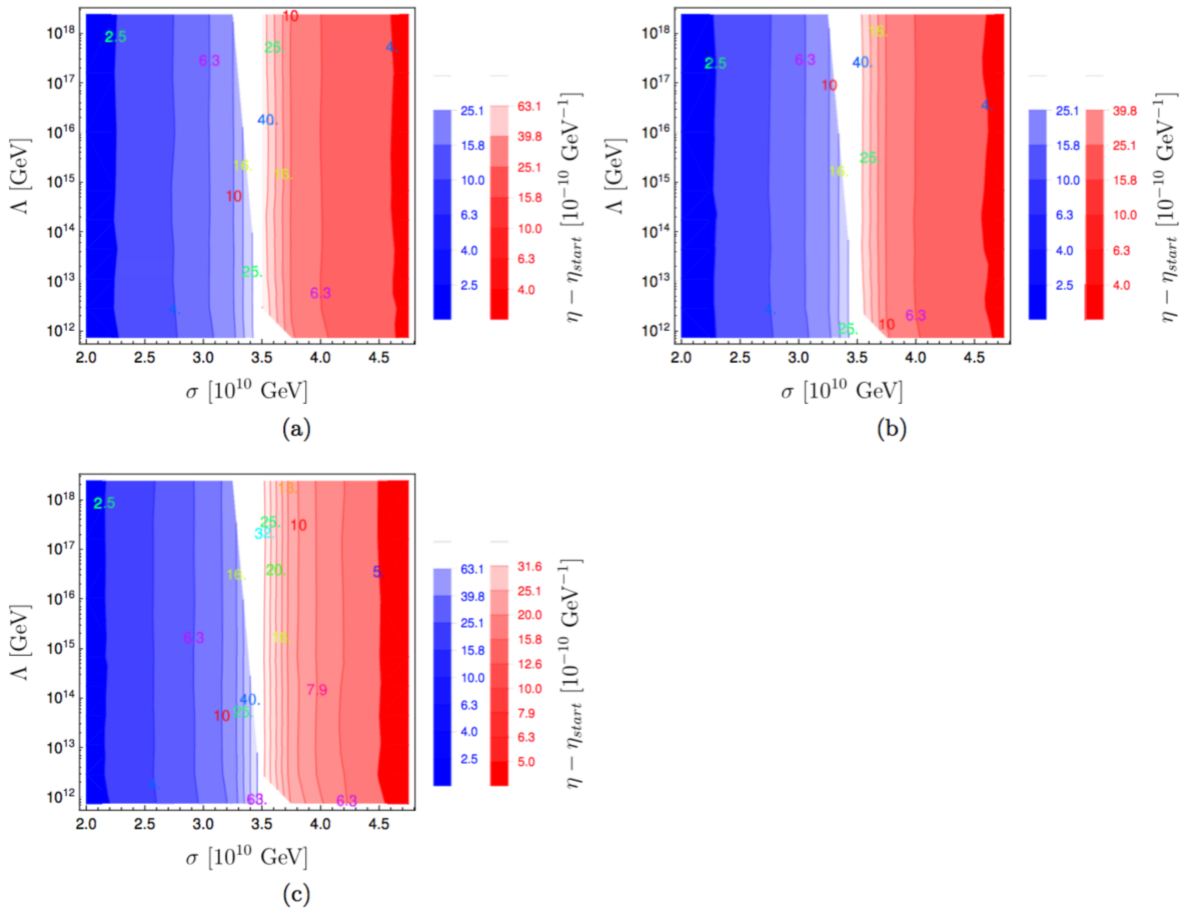


Figure 4: The dependence of the decay time of networks of Higgs domain walls as a function of the standard deviation σ of the initialization probability distribution and the suppression scale of the nonrenormalizable operator h^6 for three different values of the conformal initialization time $\eta_{start} = 10^{-12}\text{GeV}^{-1}$ (a), $\eta_{start} = 10^{-11}\text{GeV}^{-1}$ (b) and $\eta_{start} = 10^{-10}\text{GeV}^{-1}$ (c). Blue regions corresponds to networks decaying to the EWSB vacuum and red ones to networks decaying to the high field strength minimum.

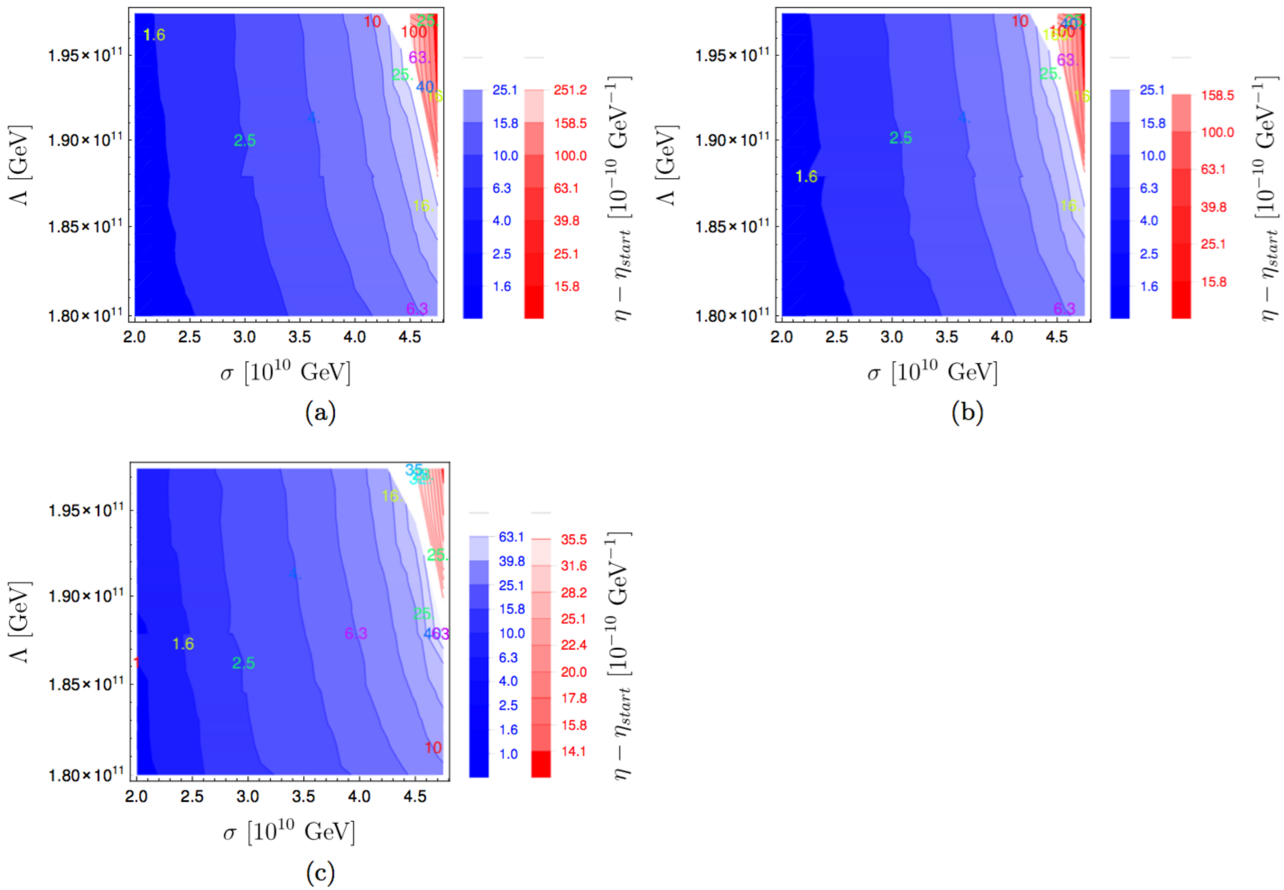


Figure 5: The dependence of the decay time of networks of Higgs domain walls as a function of the standard deviation σ of the initialization probability distribution and the suppression scale of the nonrenormalizable operator h^6 for three different values of the conformal initialization time $\eta_{start} = 10^{-12}$ GeV^{-1} (a), $\eta_{start} = 10^{-11}$ GeV^{-1} (b) and $\eta_{start} = 10^{-10}$ GeV^{-1} (c). Blue regions corresponds to networks decaying to the EWSB vacuum and red to networks decaying to the high field strength minimum.

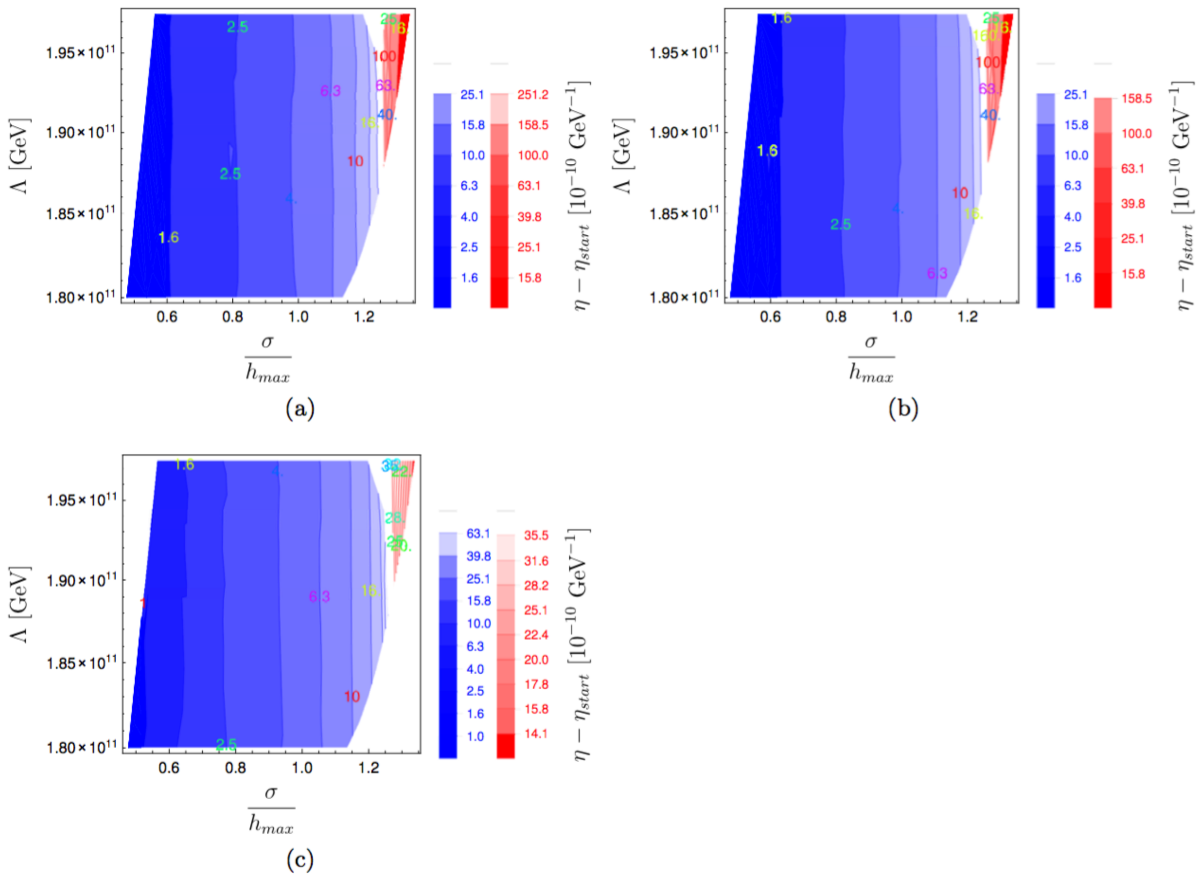


Figure 7: The dependence of the decay time of networks of Higgs domain walls as a function of the ratio $\frac{\sigma}{h_{max}}$ of the standard deviation of initialization probability distribution σ over the position of the local maximum separating the two minima h_{max} and the suppression scale of the nonrenormalizable operator h^6 for three different values of the conformal initialization time $\eta_{start} = 10^{-12}$ GeV^{-1} (a), $\eta_{start} = 10^{-11}$ GeV^{-1} (b) and $\eta_{start} = 10^{-10}$ GeV^{-1} (c)and the standard deviation $\sigma = 3.25 \times 10^{10}$GeV at initialization. Blue regions corresponds to networks decaying to the EWSB vacuum and red to networks decaying to the high field strength minimum.

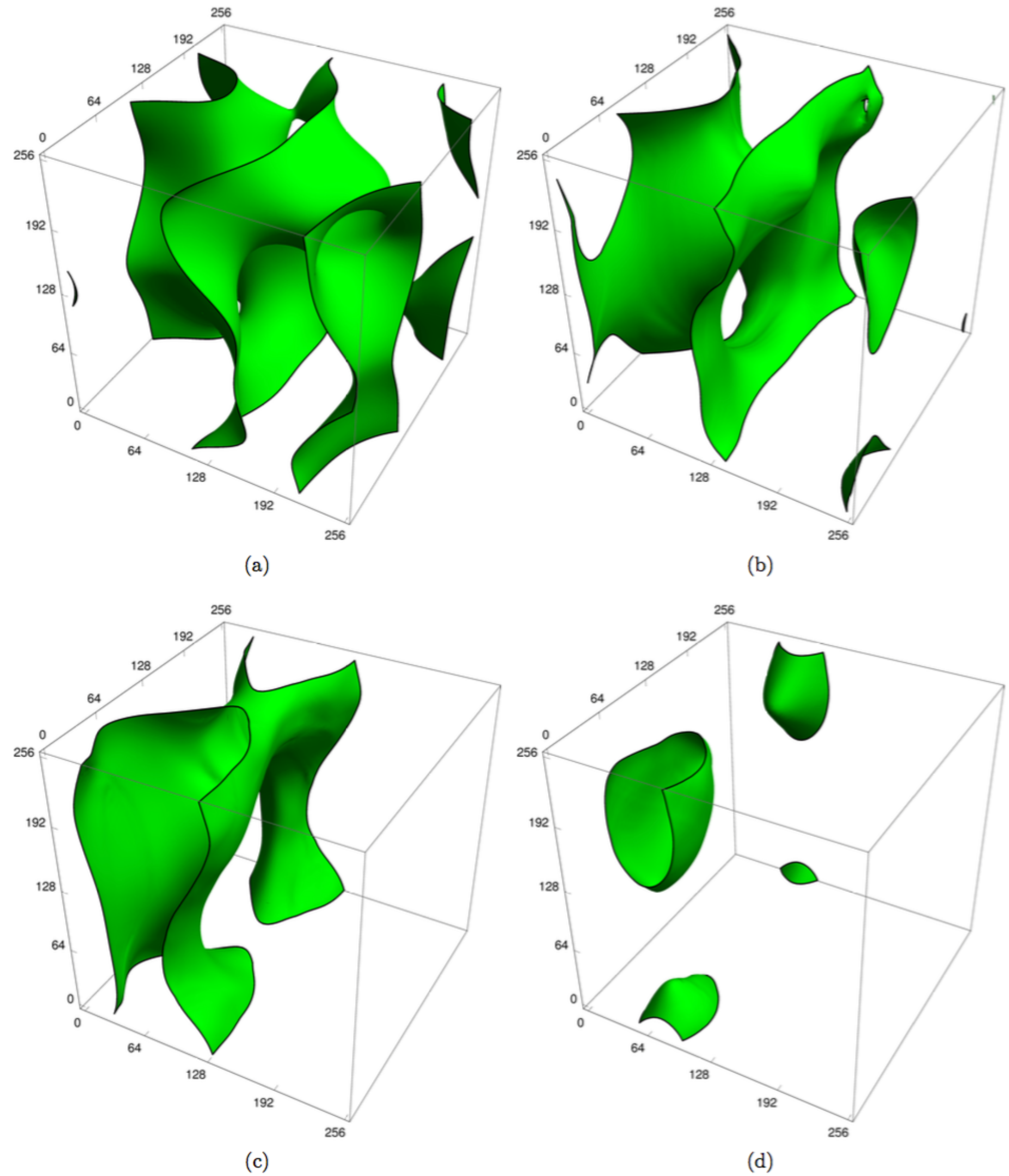


Figure 10: Visualization of the isosurface of the field strength ϕ corresponding to the value h_{max} at four different conformal times: $\eta = 1.41 \times 10^{-8}$ GeV^{-8} (a) and $\eta = 2.11 \times 10^{-8}$ GeV^{-1} (b), $\eta = 2.51 \times 10^{-8}$ GeV^{-1} (c), $\eta = 3.02 \times 10^{-8}$ GeV^{-1}(d). Lengths are given in units of the lattice spacing i.e. 10^{-10} GeV^{-1}.

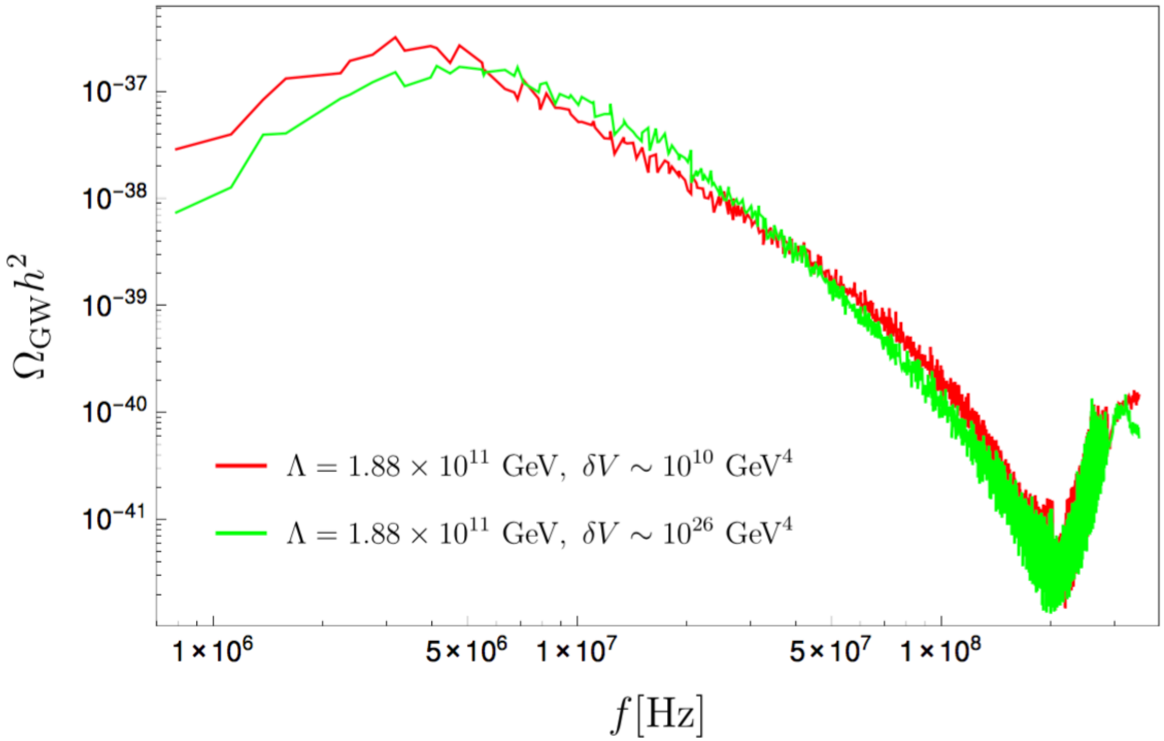
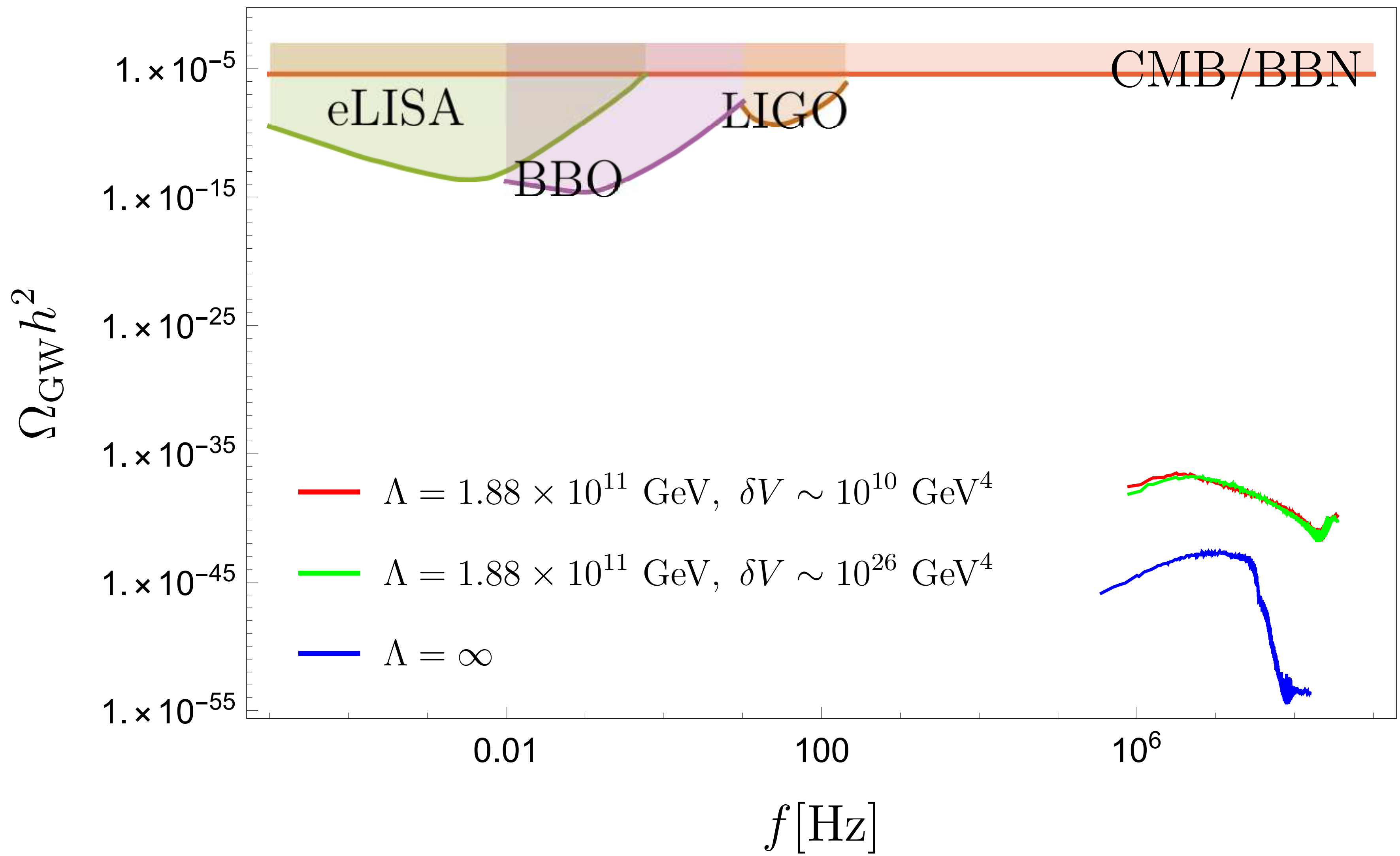


Figure 11: Present day spectrum of gravitational waves Ω_{gw} emitted from Higgs domain walls in the case of the Higgs potential with nearly degenerate minima with the difference in values of potential in minima of the order of $\delta V \sim 10^{10}$ GeV^4 (red) and $\delta V \sim 10^{26}$ GeV^4 (green).



CMB/BBN
eLISA
BBO
LIGO
$1.\times 10^{-5}$
$1.\times 10^{-15}$
$1.\times 10^{-25}$
$1.\times 10^{-35}$
$1.\times 10^{-45}$
$1.\times 10^{-55}$
$\Omega_{\mathrm{GW}} h^2$
$\Lambda = 1.88 \times 10^{11}$ GeV, $\delta V \sim 10^{10}$ GeV^4
$\Lambda = 1.88 \times 10^{11}$ GeV, $\delta V \sim 10^{26}$ GeV^4
$\Lambda = \infty$
0.01
100
10^6
$f\,[\mathrm{Hz}]$

Summary GW

1. Domain walls which separate regions with different VEVs of the Higgs field could be formed in the early Universe.
2. We observed the evolution of networks of domain walls which ends in the electroweak vacuum. We found that only small dominance of the EW vacuum at initialization is needed to reach the final state with EW vacuum.
3. The decay time of SM domain walls ranges from $8 \times 10^{-11}\ \mathrm{GeV}^{-1}$ to $5 \times 10^{-9}\ \mathrm{GeV}^{-1}$.
4. Models of the early Universe predicting the validity of SM up to high scales and Higgs field strengths of the order of the local maximum can lead to an unphysical final state.
5. Decaying networks of domain walls produce gravitational waves too weak to be detected in the upcoming years.

Summary

- SM vacuum can be stabilized by higher order operators if they appear at suffciently low energy scale $10^{10} - 10^{11}\ \mathrm{GeV}$

- SM vacuum lifetime can be dramatically shortened by higher order operators for any suppression scale

- Beyond the leading order one needs to define proper expansion of the action to demonstrate perturbatively the cancellation of gauge-dependent contributions to the lifetime of the EW vacuum. In the abelian Higgs model such a procedure can be carried out at the level of the renormalized effective action

- Peoperties of the electroweak vacuum - critical temperature and lifetime - can be modified by a fast expansion of the gravitational background

- Tunneling from Minkowski suppressed by gravity but tunnelling from dS enhanced by CDL bounces

- Decaying networks of domain walls produce a signal in the form of gravitational waves - too weak to be detected anytime soon - if a signal is detected then either fine-tuning or non-standard cosmology have occurred