

Neutrino physics and implications

José W F Valle

Lecture I



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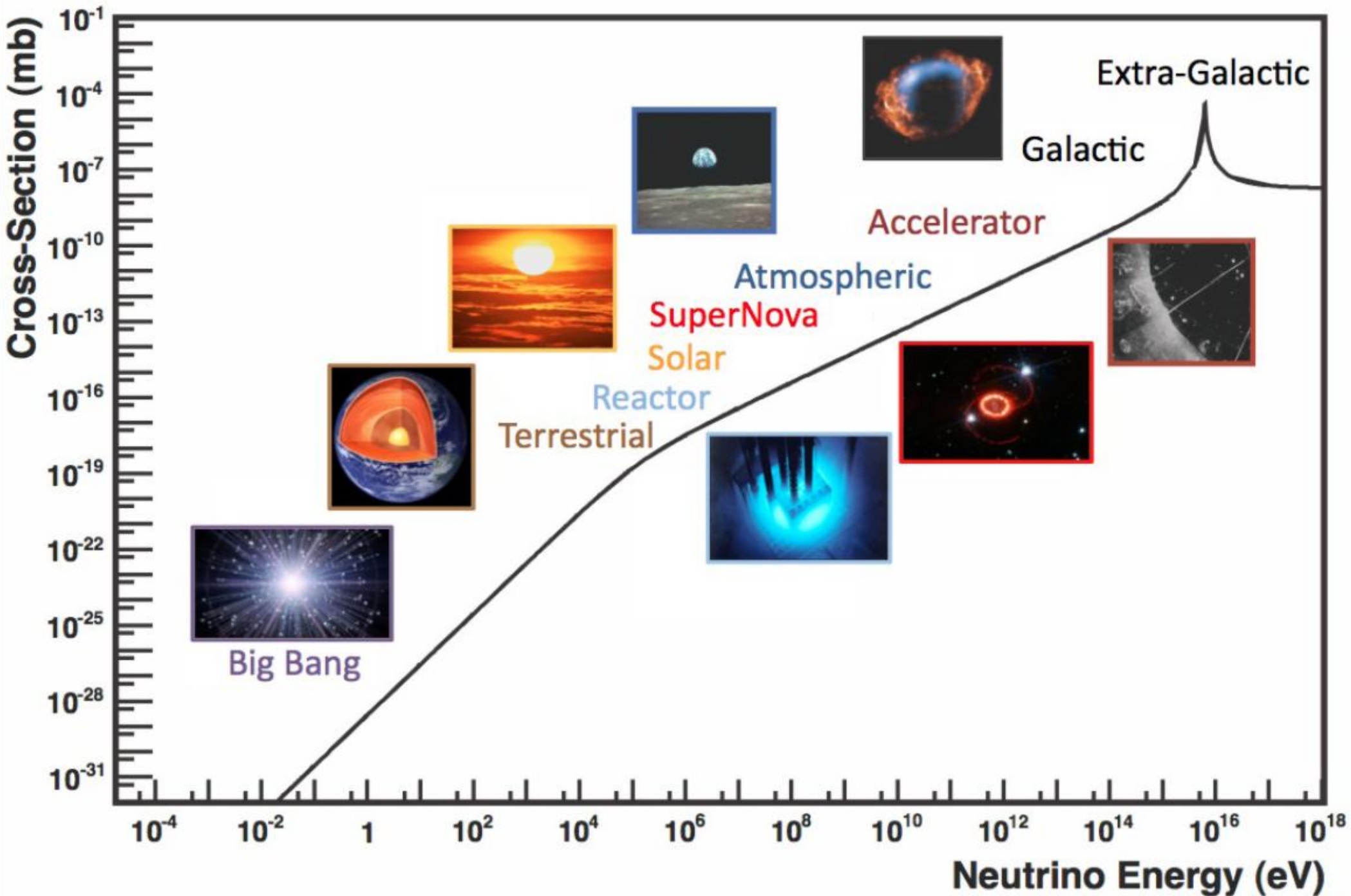
Summer School and Workshop on Standard Model and Beyond – Corfu 2016

Outline

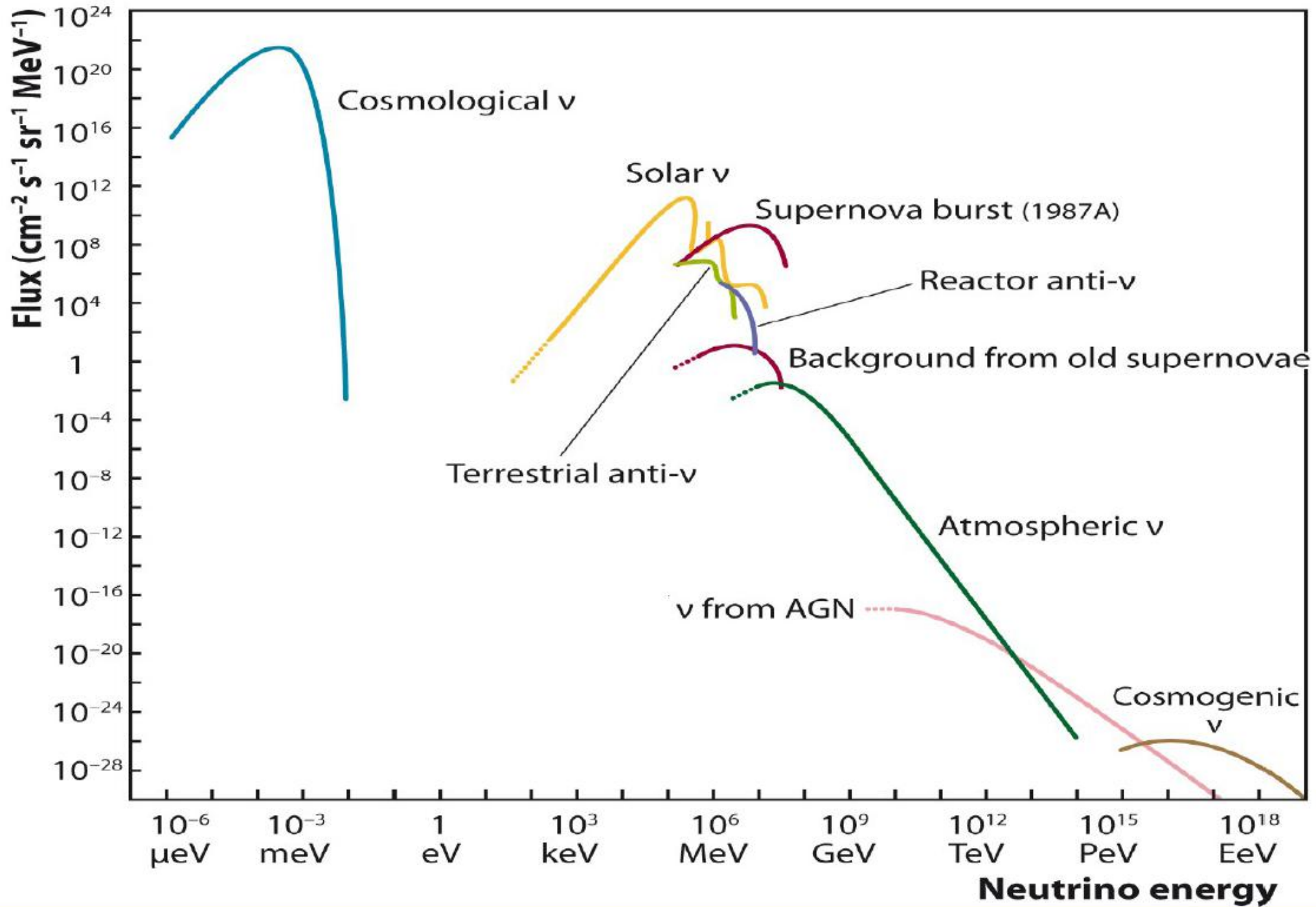
Lecture I

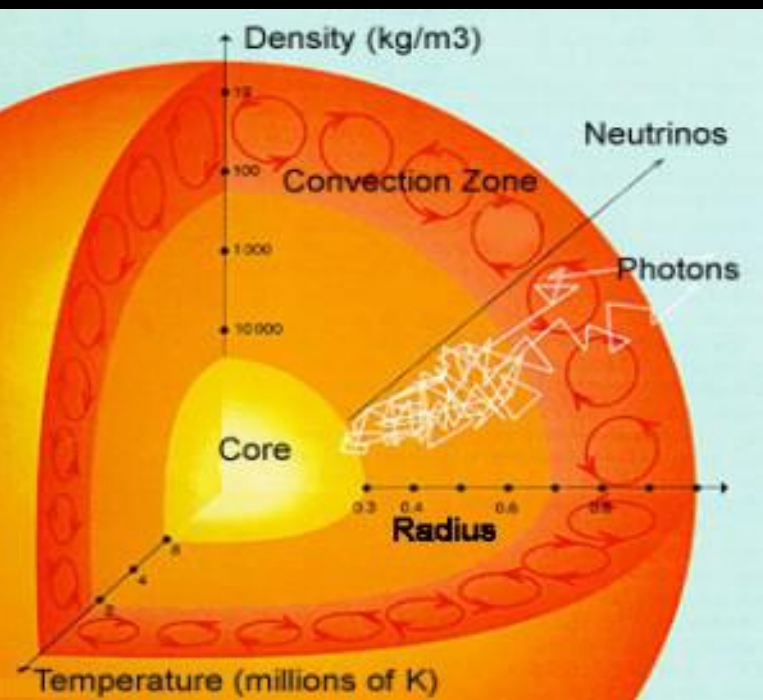
- Why neutrinos
- Neutrinos in the Standard model
- Neutrino oscillations
- Origin of neutrino mass
- Seesaw mechanisms
- Radiative mechanisms
- $SU(3)_c \otimes SU(3)_L \otimes U(1)$ electroweak model
- The flavor problem
 - Symmetry approaches
 - The role of neutrinos
- Neutrinos and unification

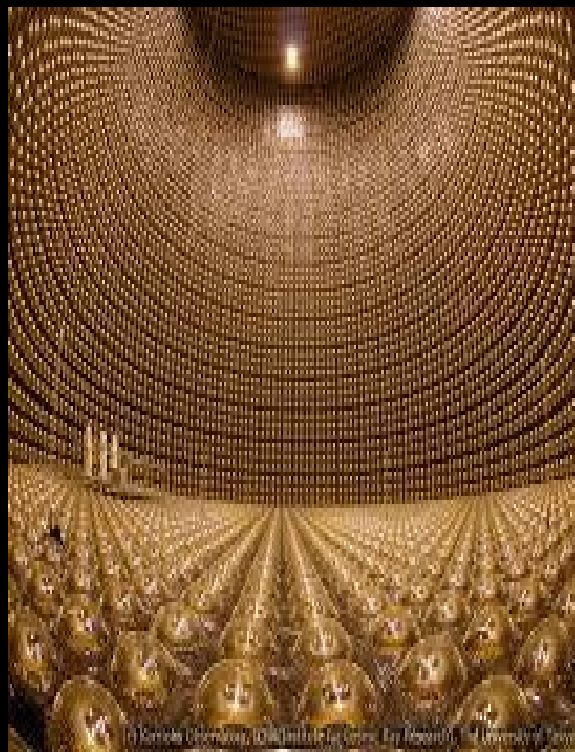
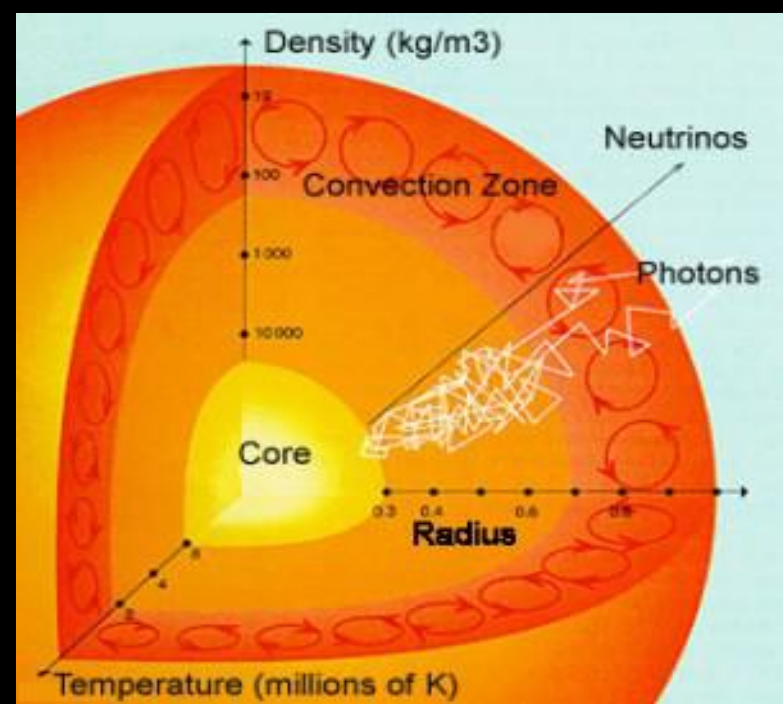
neutrino cross sections

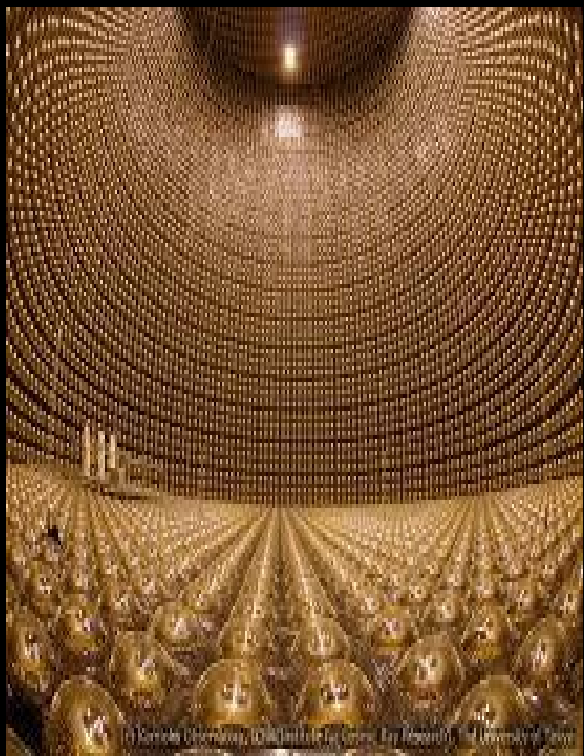
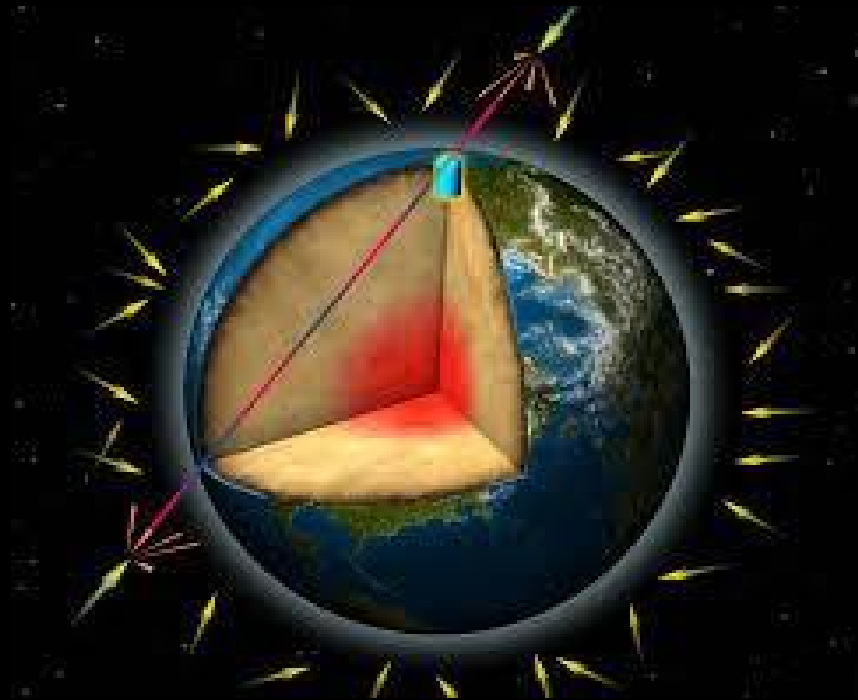
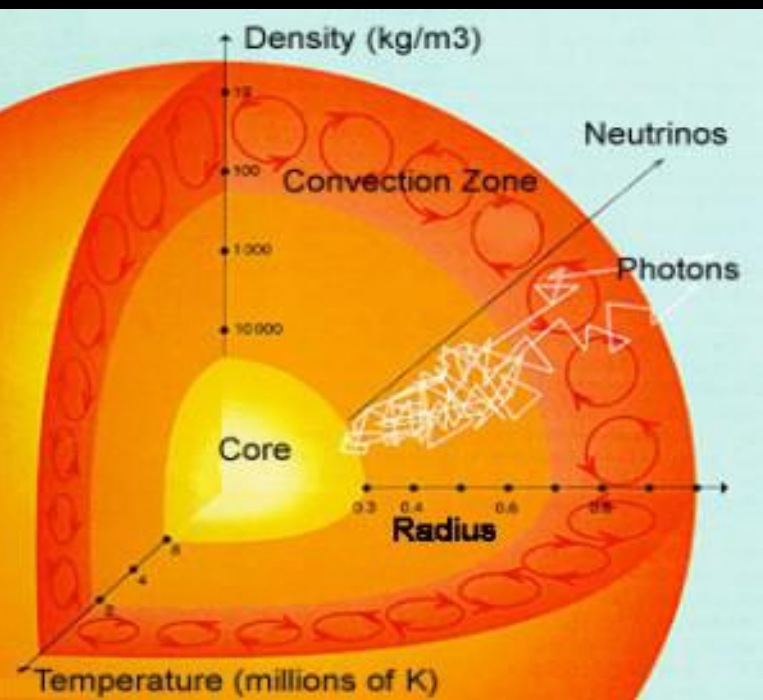


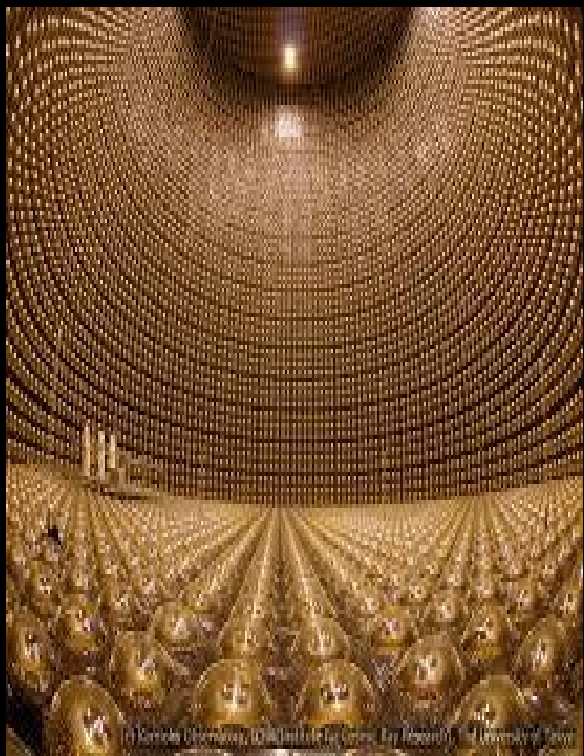
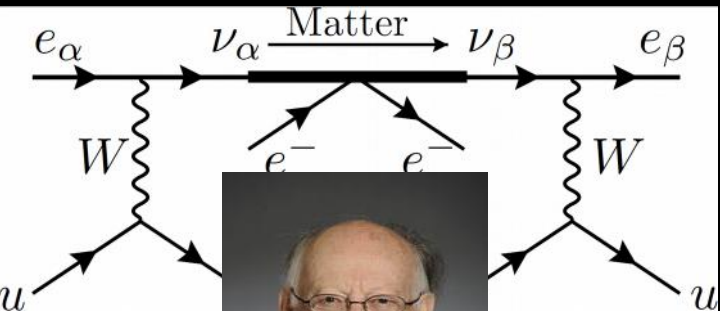
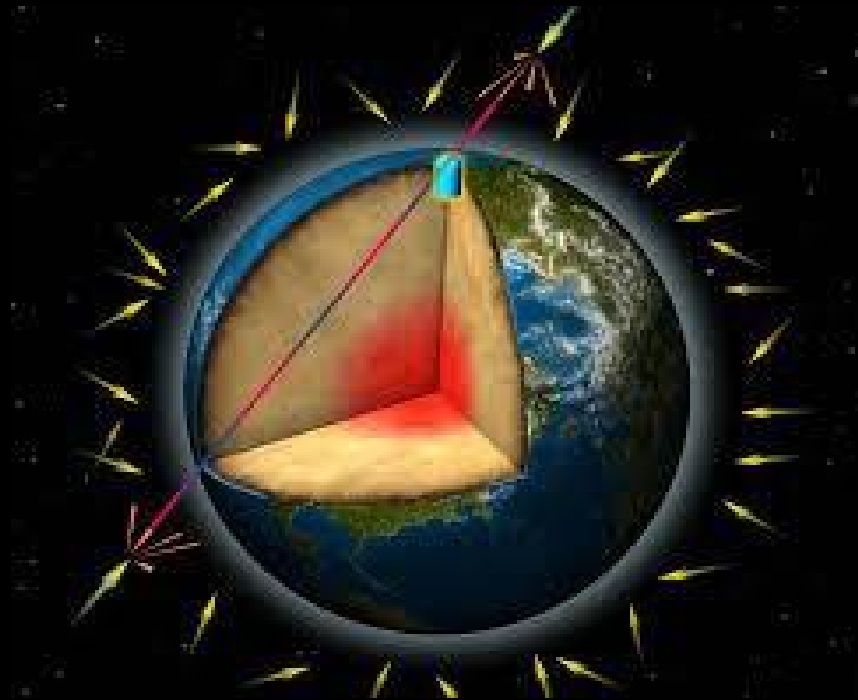
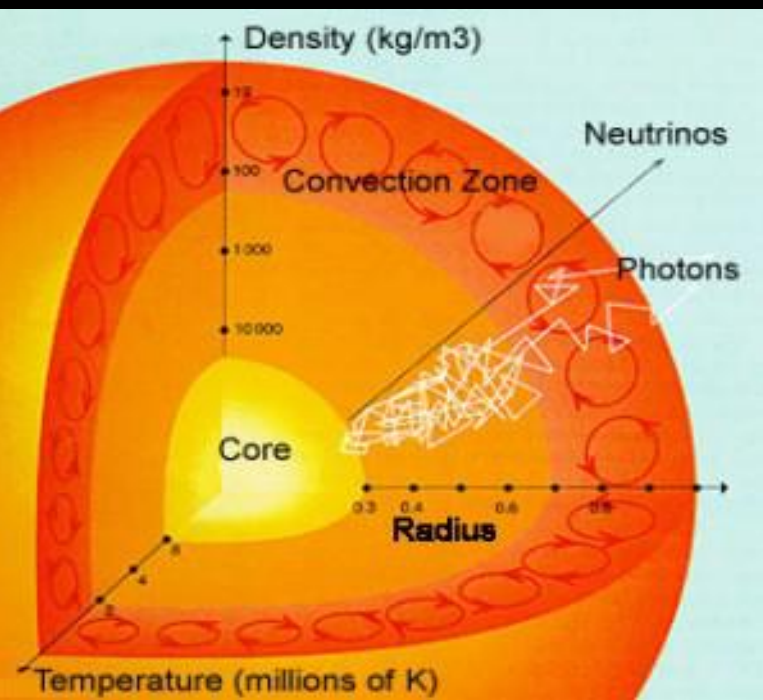
neutrino sources

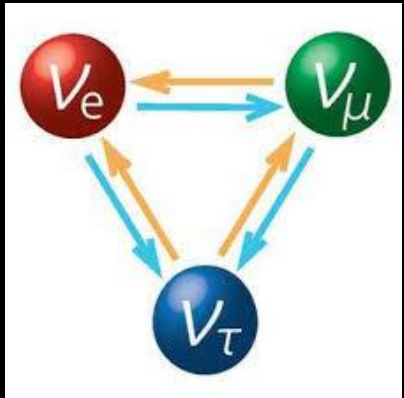
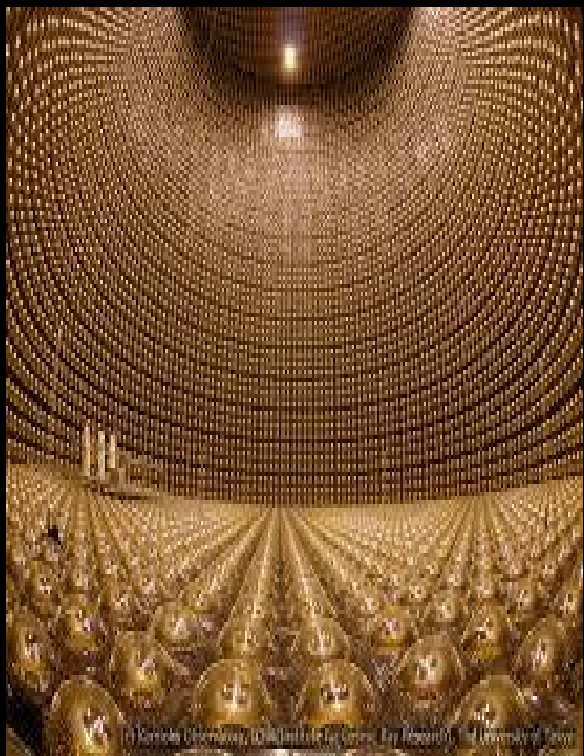
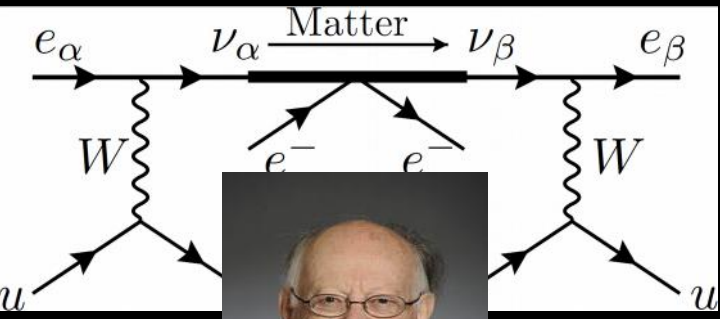
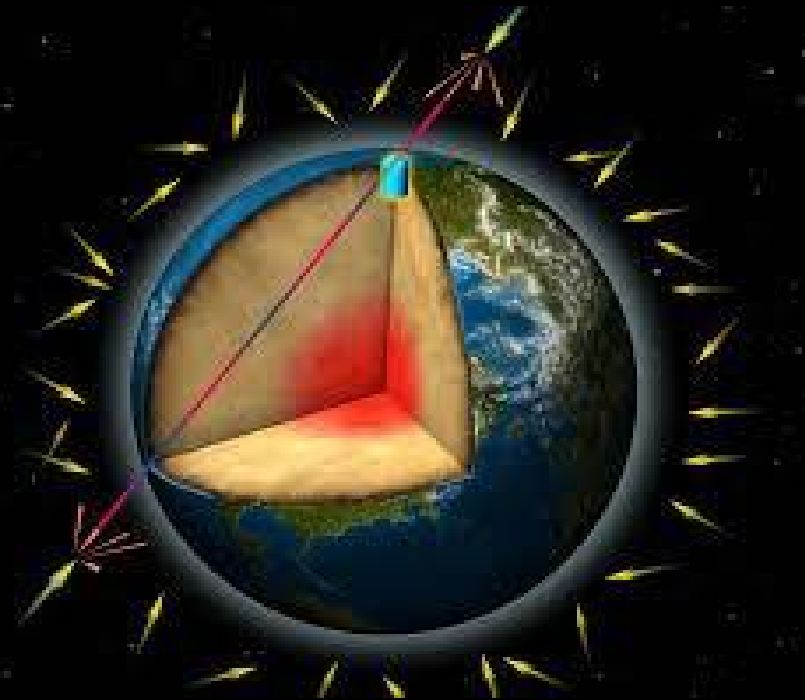
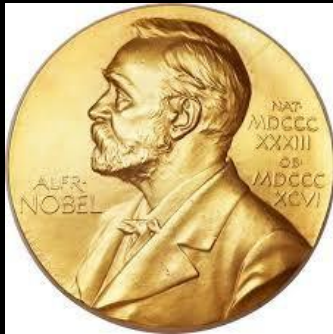
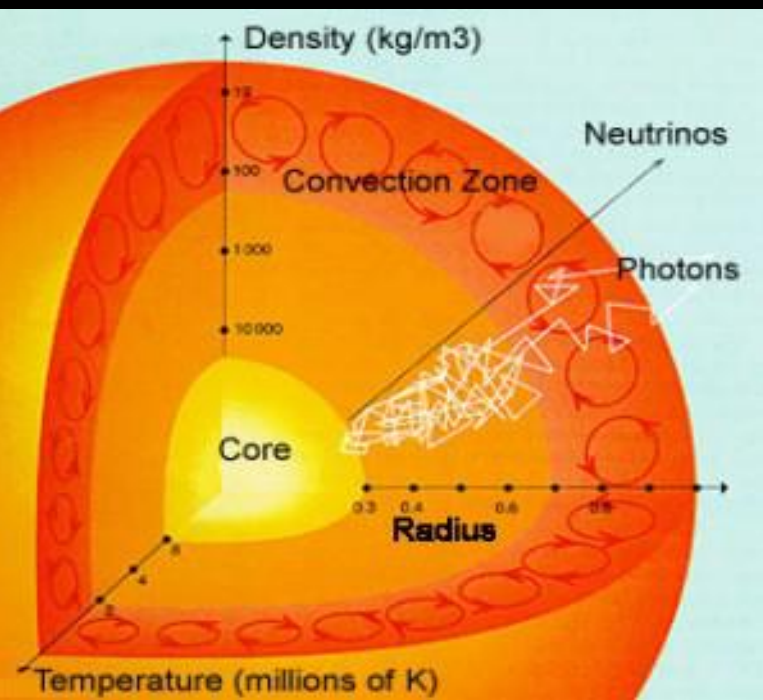












Why the fuss about neutrinos

Can not do without neutrinos

Basic cosmological and astro probe

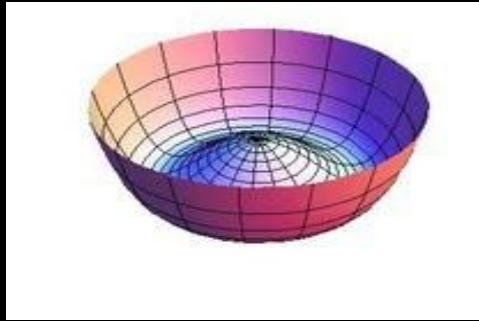
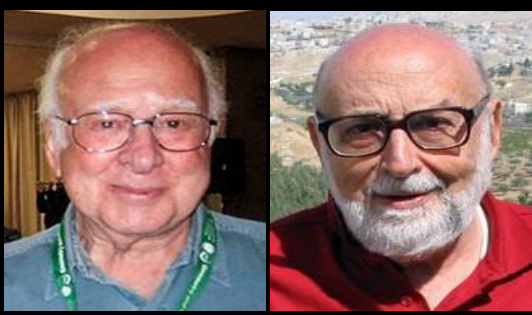
key building block of the Standard Model

Standard model

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$$

Three Generations of Matter (Fermions) spin 1/2

	I	II	III	
mass →	2.4 MeV	1.27 GeV	171.2 GeV	0
charge →	2/3	2/3	2/3	0
name →	u up	c charm	t top	g gluon
	Left Right	Left Right	Left Right	Left Right
Quarks	4.8 MeV -1/3 d down	104 MeV -1/3 s strange	4.2 GeV -1/3 b bottom	0 0 γ photon
	Left Right	Left Right	Left Right	Left Right
	0 eV 0 ν_e electron neutrino	0 eV 0 ν_μ muon neutrino	0 eV 0 ν_τ tau neutrino	91.2 GeV 0 Z⁰ weak force
	Left Right	Left Right	Left Right	Left Right
Leptons	0.511 MeV -1 e electron	105.7 MeV -1 μ muon	1.777 GeV -1 τ tau	80.4 GeV ±1 W[±] weak force
	Left Right	Left Right	Left Right	Left Right
				Bosons (Forces) spin 1



Standard model

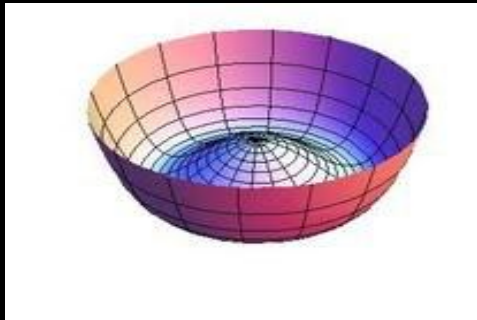
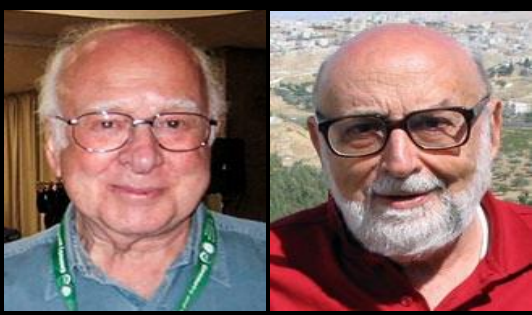
$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$$

125 GeV
H
Higgs boson
spin 0

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Bosons (Forces) spin 1



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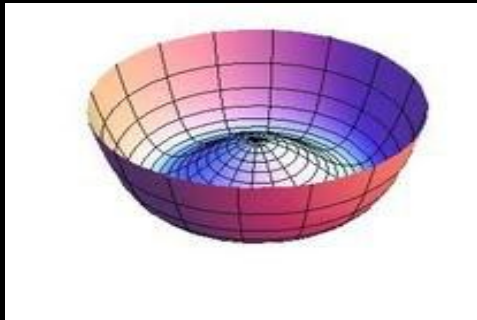
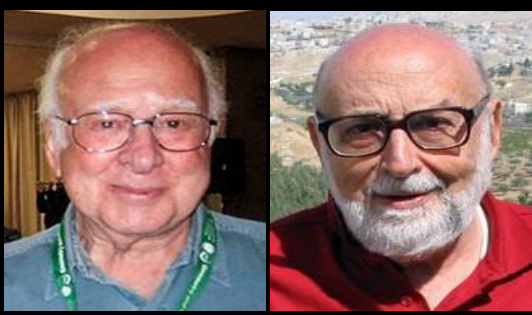
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Last brick ? ...

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	4.8 MeV	104 MeV	4.2 GeV	0
Quarks	-1/3 d Left down Right	-1/3 s Left strange Right	-1/3 b Left bottom Right	0 γ photon
	0 eV ν_e Left electron neutrino Right	0 eV ν_μ Left muon neutrino Right	0 eV ν_τ Left tau neutrino Right	91.2 GeV Z⁰ weak force
Leptons	0.511 MeV e Left electron Right	105.7 MeV μ Left muon Right	1.777 GeV τ Left tau Right	80.4 GeV W[±] weak force

Bosons (Forces) spin 1



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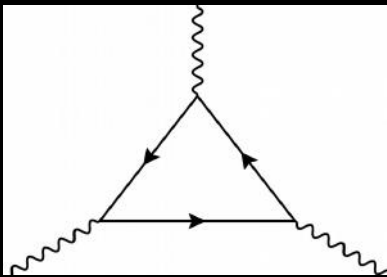
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Quarks	Left Right	Left Right	Left Right	0
	0 eV	0 eV	0 eV	91.2 GeV
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Bosons (Forces) spin 1

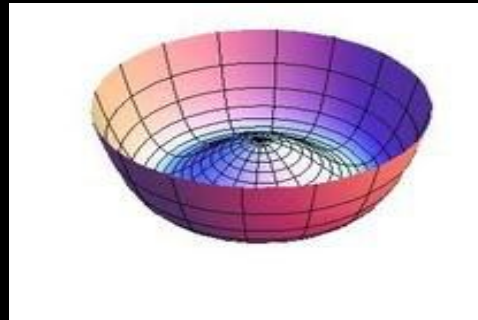
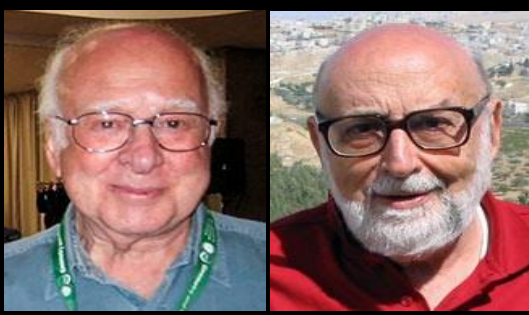
$$-\frac{1}{2} \quad -\frac{1}{2} \quad \frac{1}{6} \quad \frac{1}{6} \quad -1 \quad \frac{2}{3} \quad -\frac{1}{3}$$

anomalies



Charge quantization

Standard model



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Last brick? ...

Three Generations of Matter (Fermions) spin 1/2

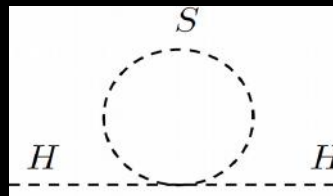
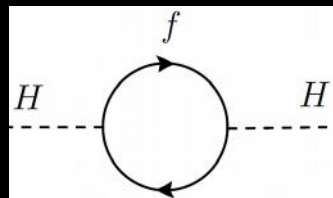
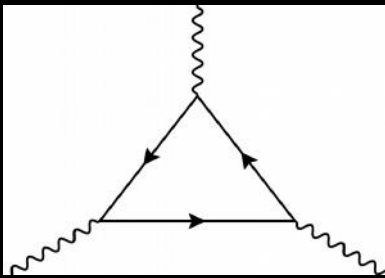
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Bosons (Forces) spin 1

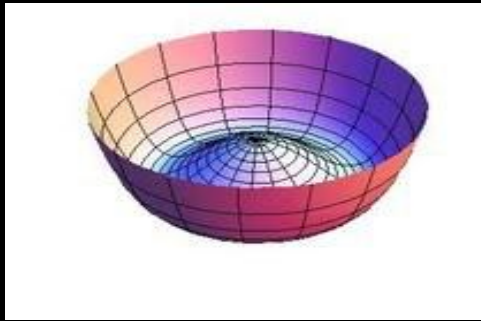
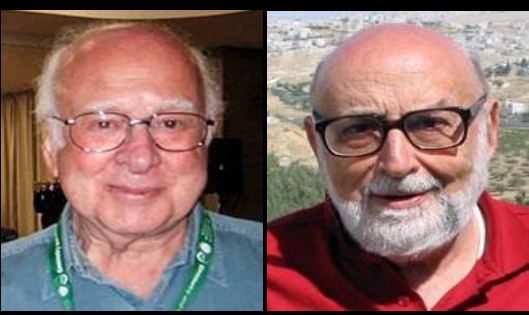
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anomalies

Consistency of SSB



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Bosons (Forces) spin 1

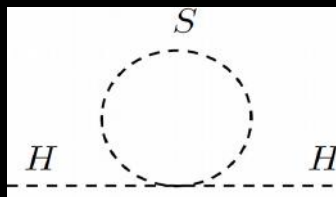
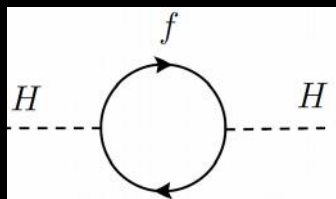
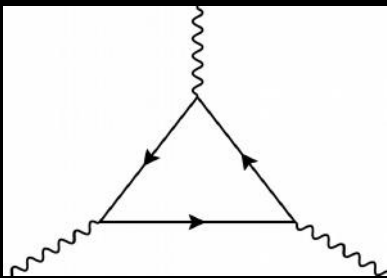
Last brick? ...

$$-\frac{1}{2} \quad -\frac{1}{2} \quad \frac{1}{6} \quad \frac{1}{6} \quad -1 \quad \frac{2}{3} \quad -\frac{1}{3}$$

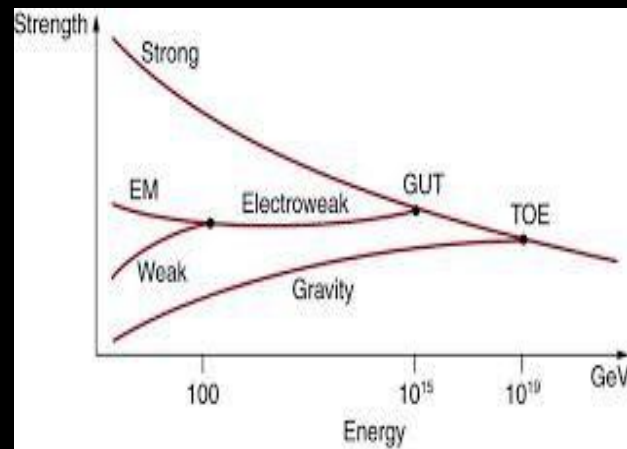
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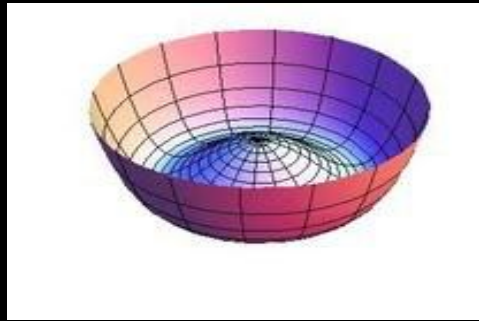
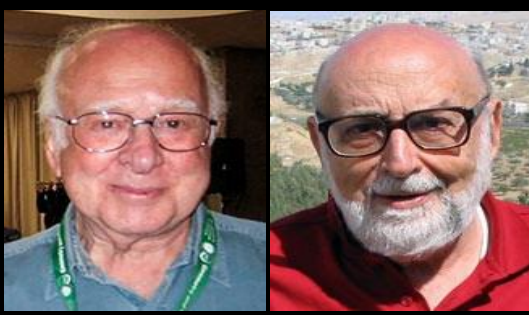
Consistency of SSB

coupling unification



Charge quantization





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Leptons	Left Right	Left Right	Left Right	+1

Bosons (Forces) spin 1

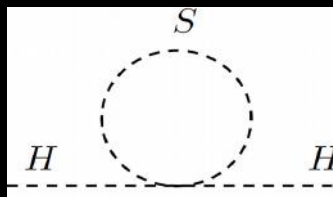
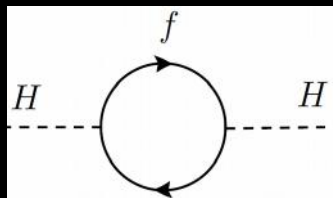
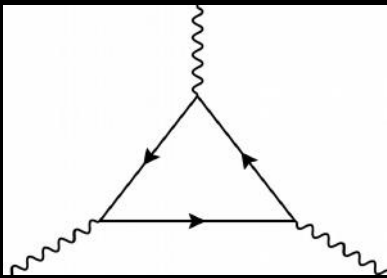
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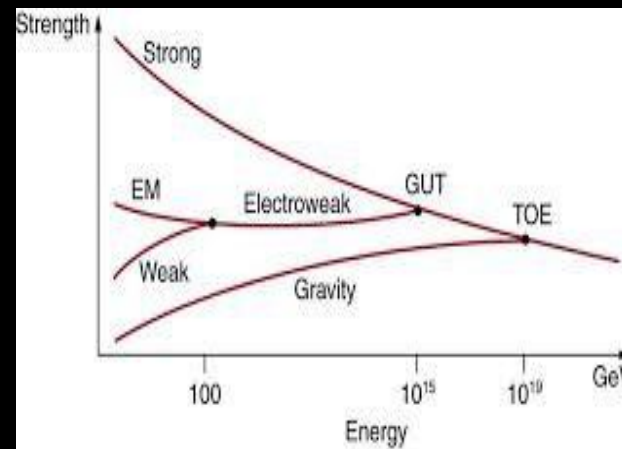
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Consistency of SSB

coupling unification



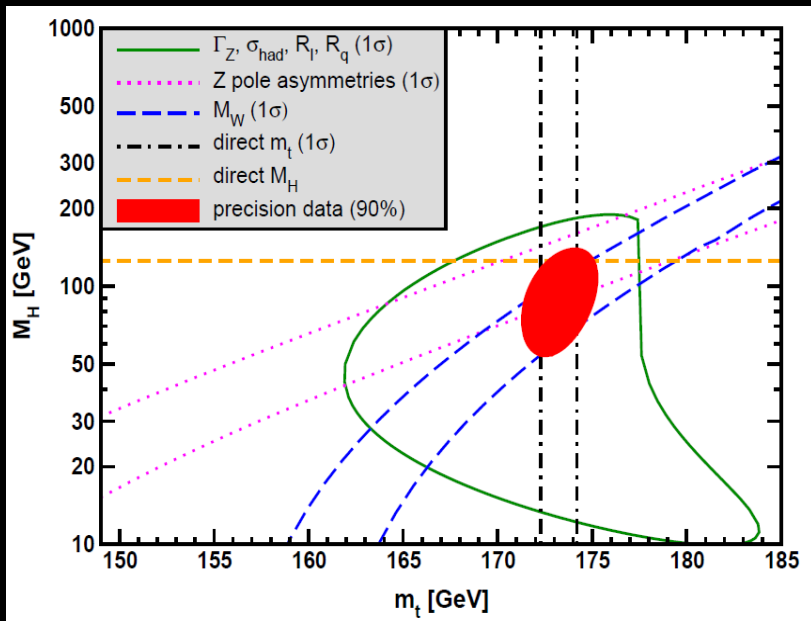
Charge quantization



Gravity ...

Neutrino mass

Why 3 families



THE STANDARD MODEL

FERMIONS (matter)

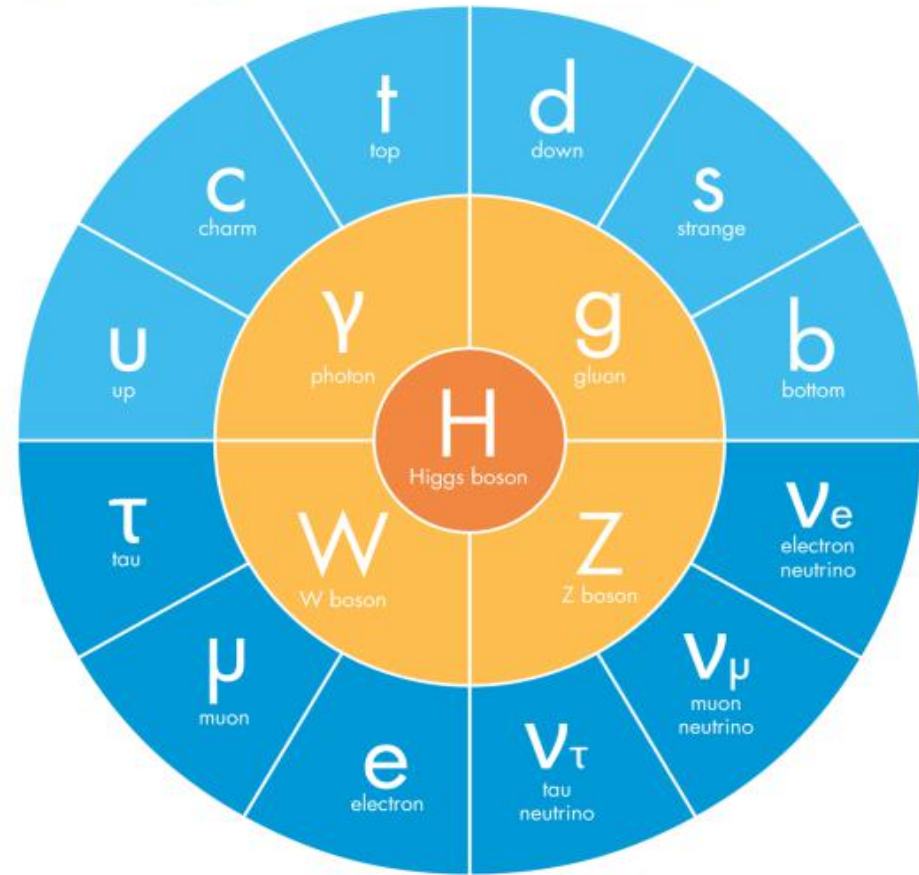
BOSONS (force carriers)

● Quarks

● Leptons

● Gauge bosons

● Higgs boson



Higgs not the last brick! ...

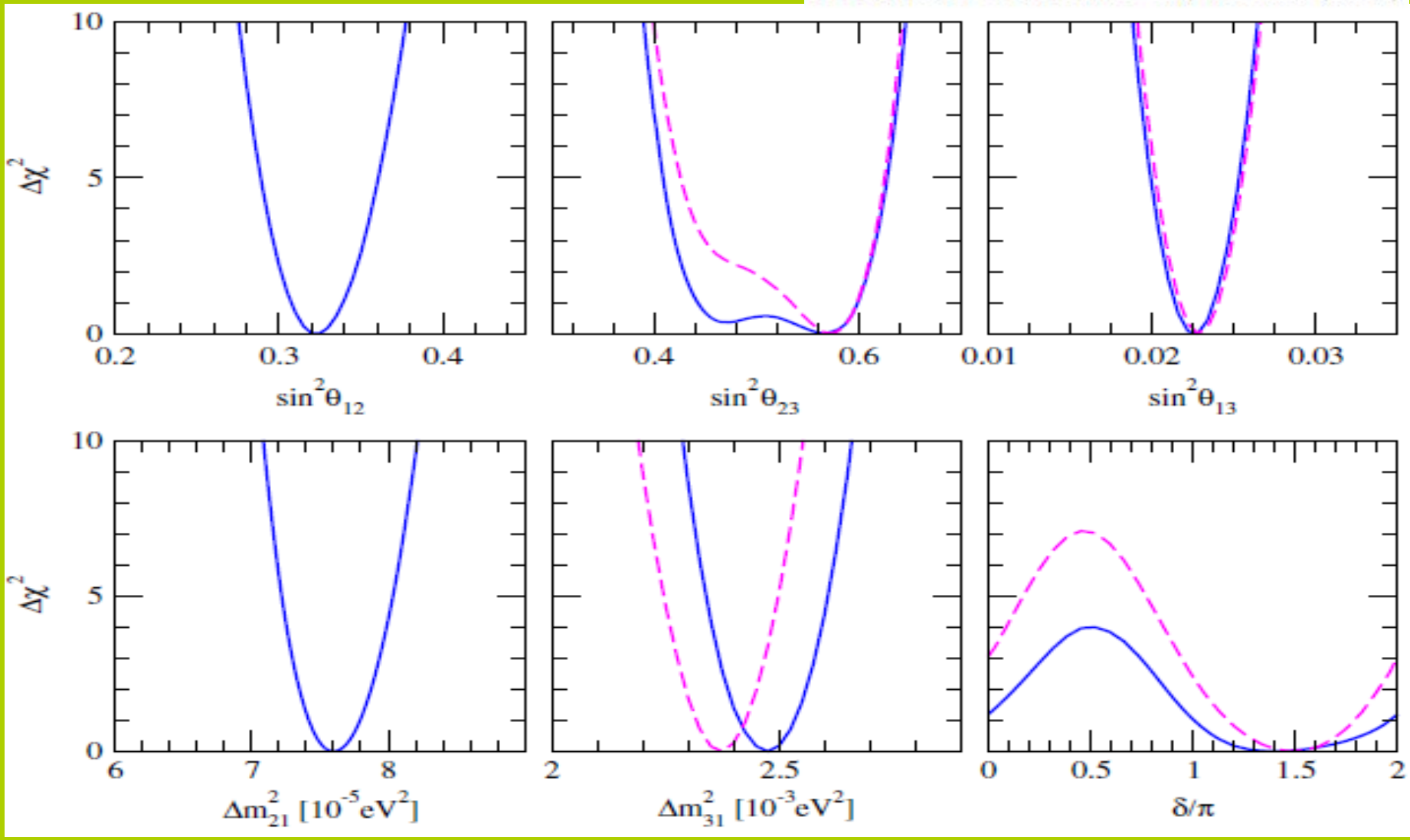
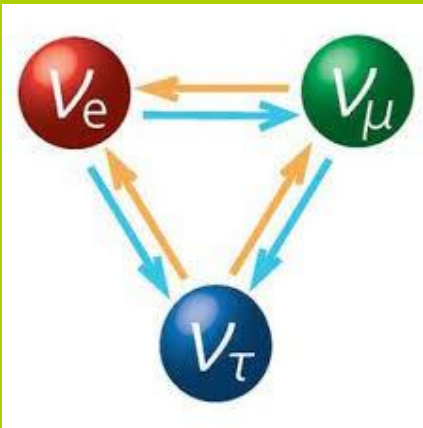


SM can not explain neutrinos

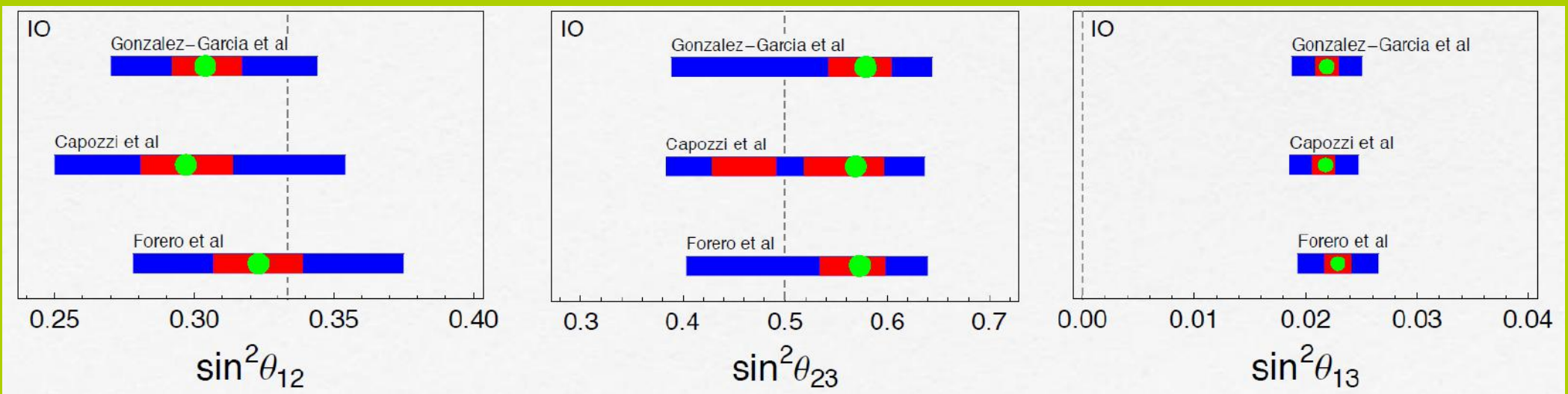
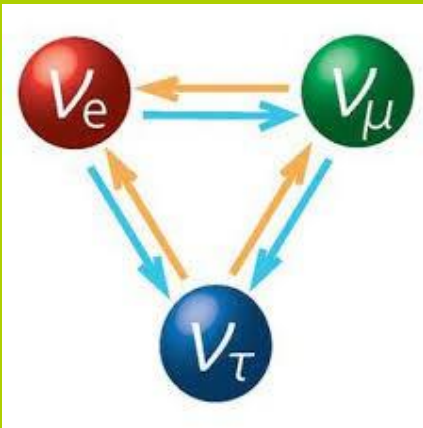
Oscillation parameters

Precision era starts

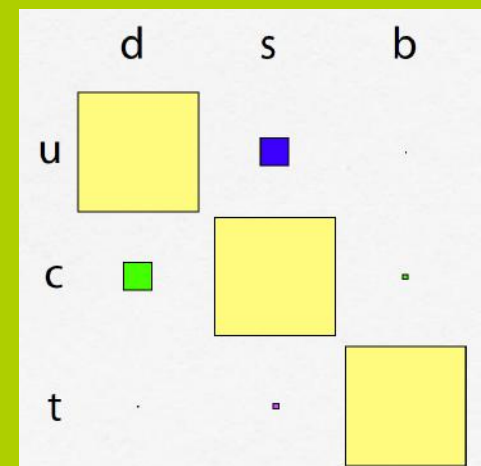
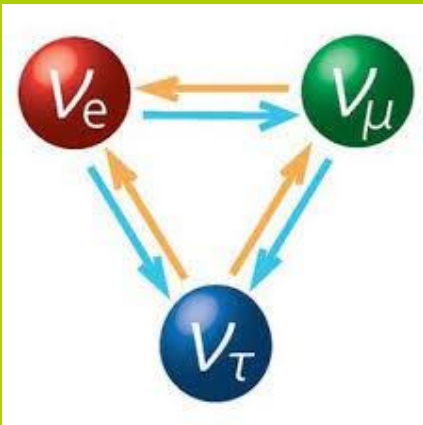
PHYSICAL REVIEW D 90, 093006 (2014)



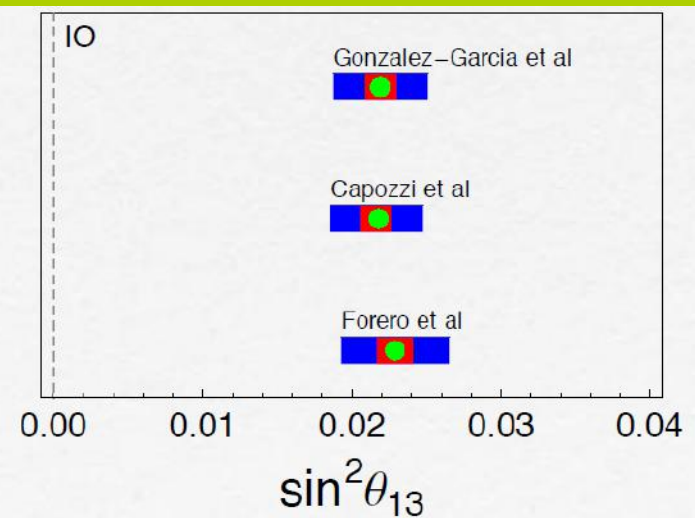
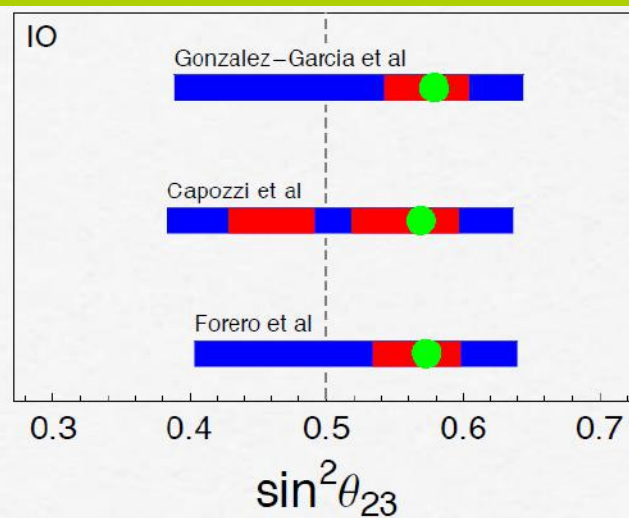
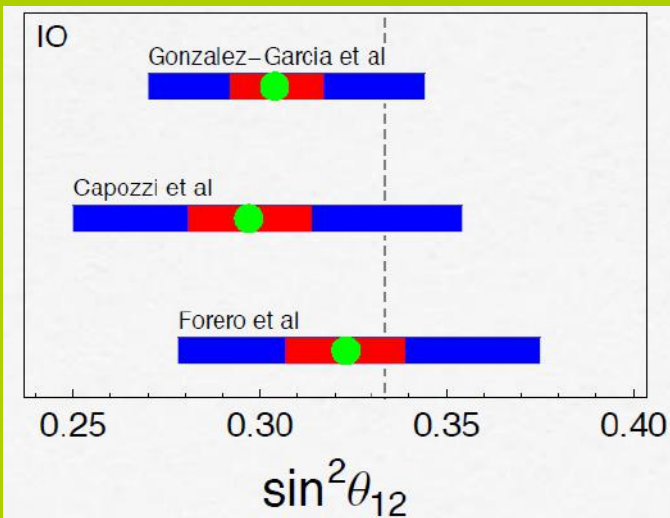
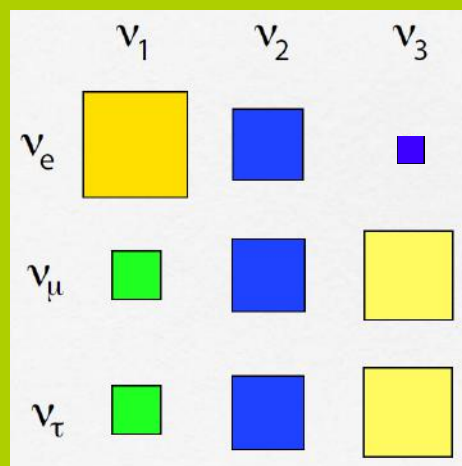
Neutrino oscillation parameters



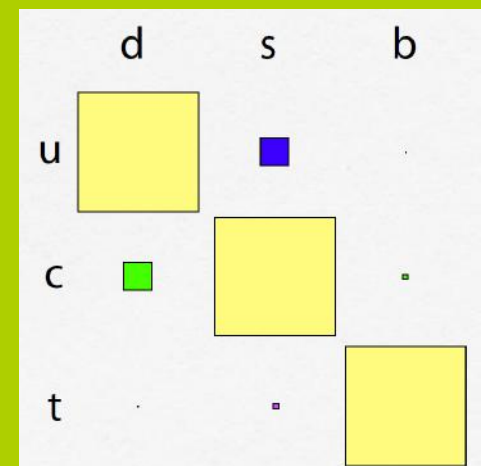
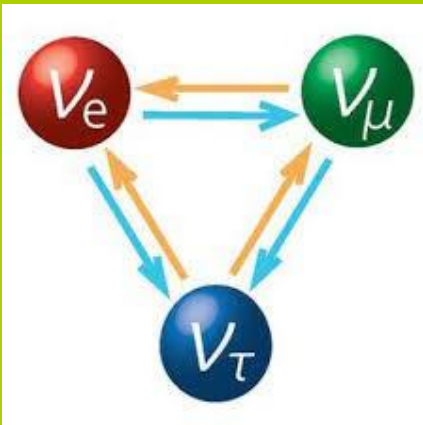
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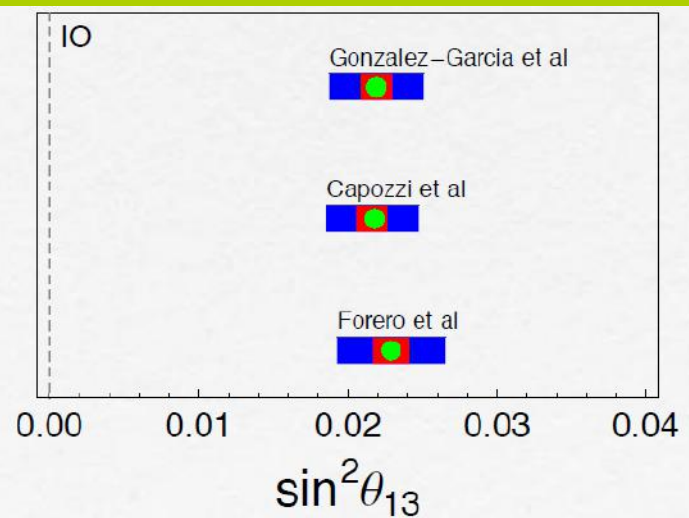
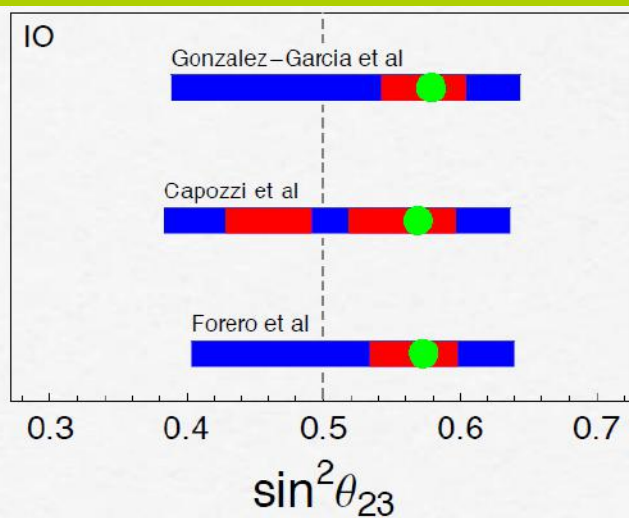
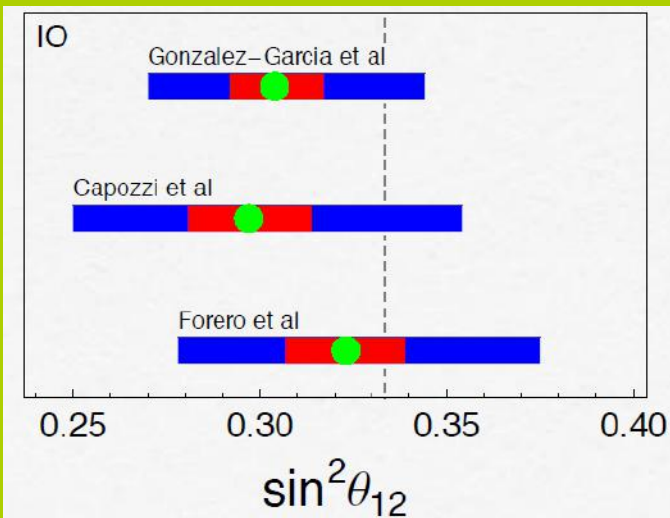
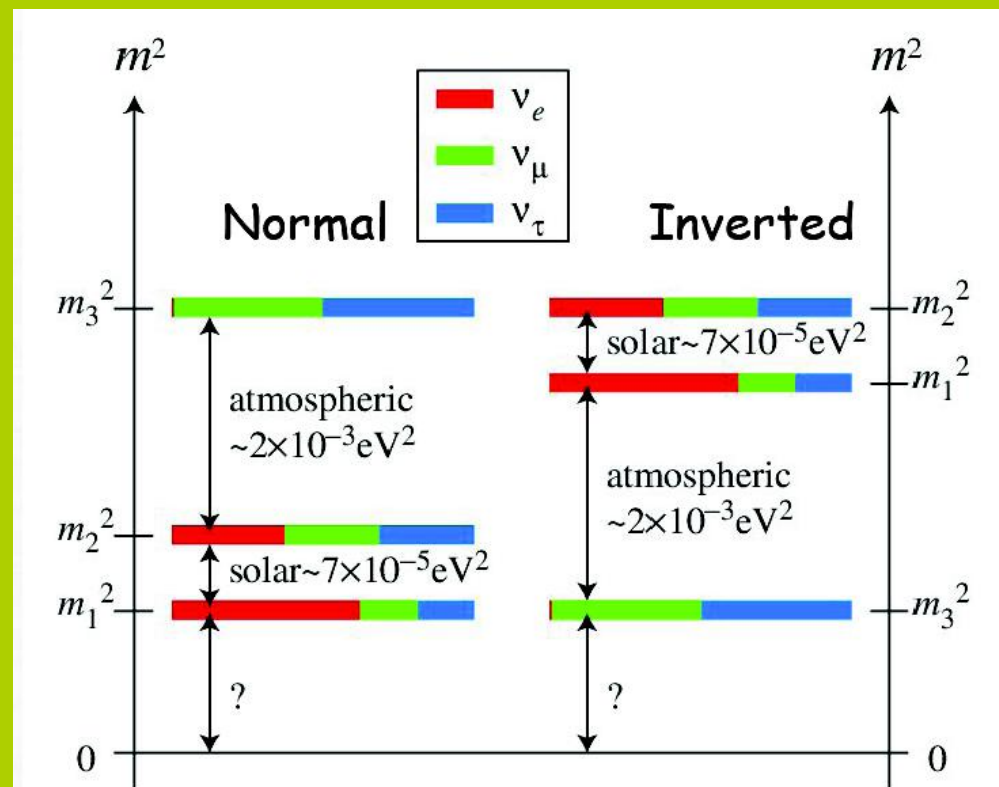
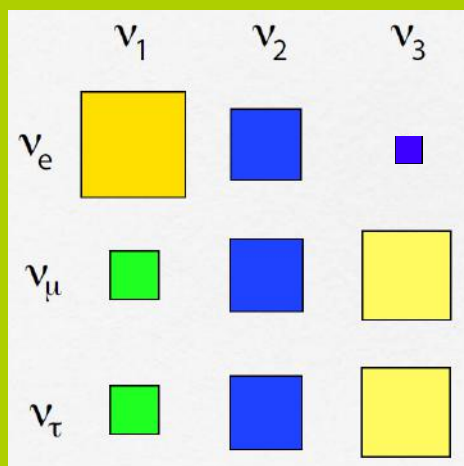
νS



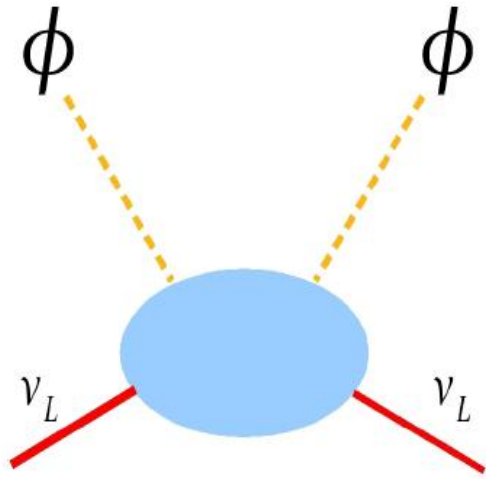
Neutrino oscillation parameters



νS



The origin of neutrino mass

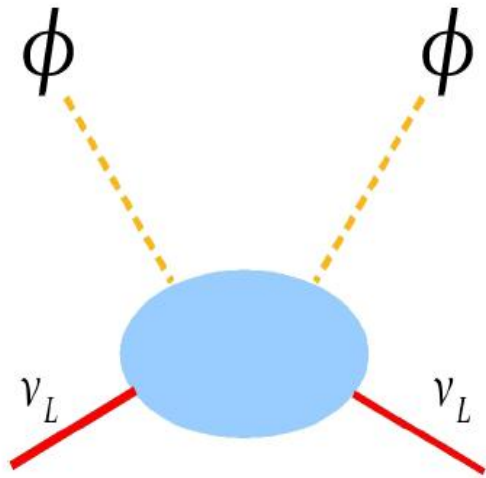


MECHANISM

SCALE

FLAVOR STRUCTURE

The origin of neutrino mass



MECHANISM

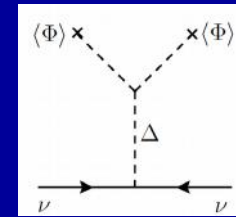
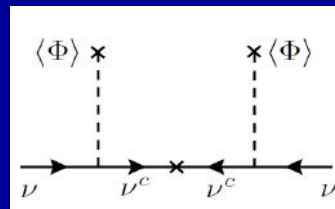
SCALE

FLAVOR STRUCTURE

Seesaw

$$v_3 v_1 \sim v_2^2$$

The origin of neutrino mass

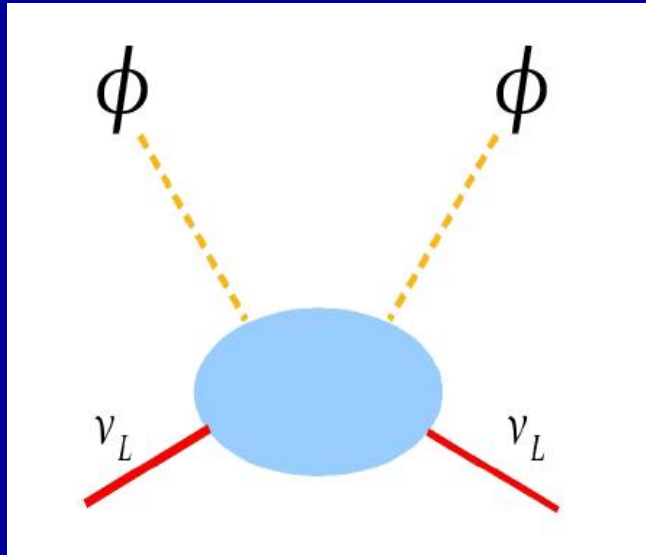


TYPE I

Minkowski 77
 Gellman Ramond Slansky 80
 Glashow, Yanagida 79
 Mohapatra Senjanovic 80
 Lazarides Shafi Weterrich 81
 Schechter-Valle, 80 & 82

TYPE II

Schechter-Valle 80/82



Seesaw

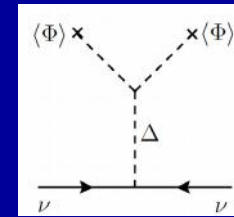
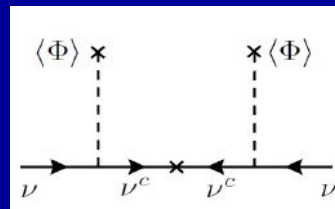
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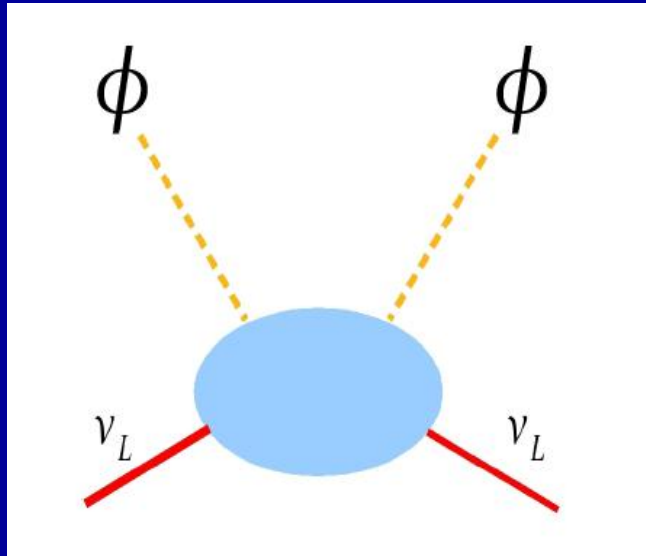


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MECHANISM

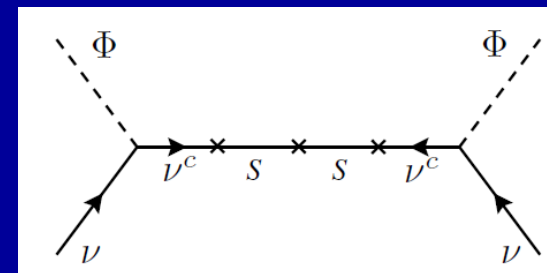
SCALE

FLAVOR STRUCTURE

Number & properties of messengers

LOW-SCALE SEESAW

Mohapatra-Valle 86
 Akhmedov et al PRD53 (1996) 2752
 Malinsky et al PRL95(2005)161801
 Bazzocchi et al, PRD81 (2010) 051701



Non unitary lepton mixing & CP

$$\begin{pmatrix} \alpha_{11} & 0 & 0 \\ \alpha_{21} & \alpha_{22} & 0 \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix} U$$

$$\alpha_{11}^2 \geq 0.989, \quad \alpha_{22}^2 \geq 0.999, \quad |\alpha_{21}|^2 \leq 6.6 \times 10^{-4}$$

Schechter & JV PRD22 (1980) 2227 & PDG
Rodejohann, JV Phys.Rev. D84 (2011) 073011

non-unitary propagation hints associated
(relatively low-mass) type-I seesaw
messenger responsible for inducing neutrino mass

<http://dx.doi.org/10.1103/PhysRevD.92.053009>

<http://journals.aps.org/prl/abstract/10.1103/PhysRevLett.117.061804>

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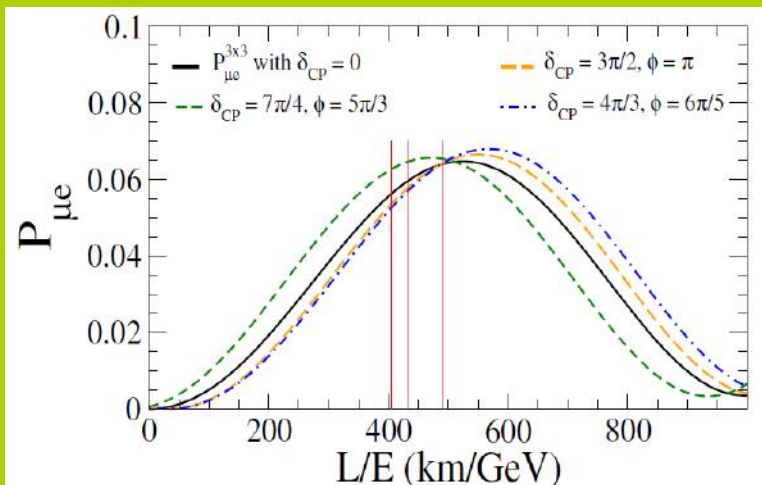
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<http://journals.aps.org/prl/abstract/10.1103/PhysRevLett.117.061804>

$$P_{\mu e} = \alpha_{11}^2 \alpha_{22}^2 P_{\mu e}^{3 \times 3} + \alpha_{11}^2 \alpha_{22} |\alpha_{21}| P_{\mu e}^I + \alpha_{11}^2 |\alpha_{21}|^2$$

$$P_{\mu e}^I = -2 \sin 2\theta_{13} \sin \theta_{23} \sin \Delta_{31} \sin (\Delta_{31} + \delta_{CP} + \phi) - \cos \theta_{13} \cos \theta_{23} \sin 2\theta_{12} \sin 2\Delta_{21} \sin \phi,$$



Non unitary lepton mixing & CP

$$\begin{pmatrix} \alpha_{11} & 0 & 0 \\ \alpha_{21} & \alpha_{22} & 0 \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix} U$$

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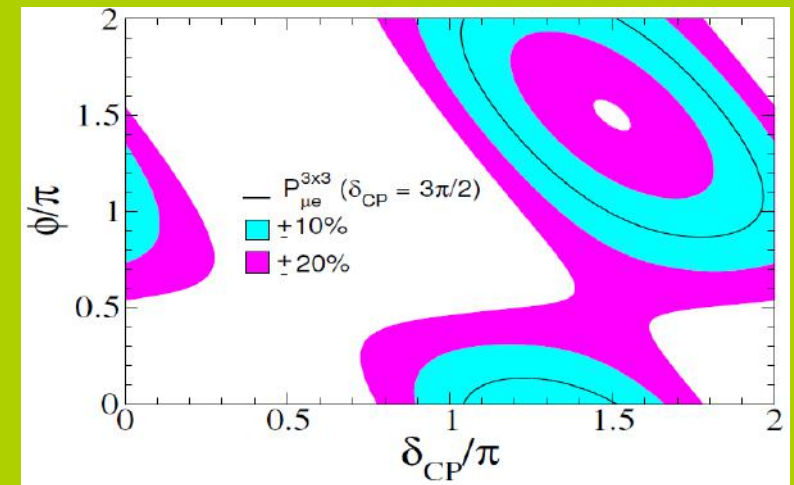
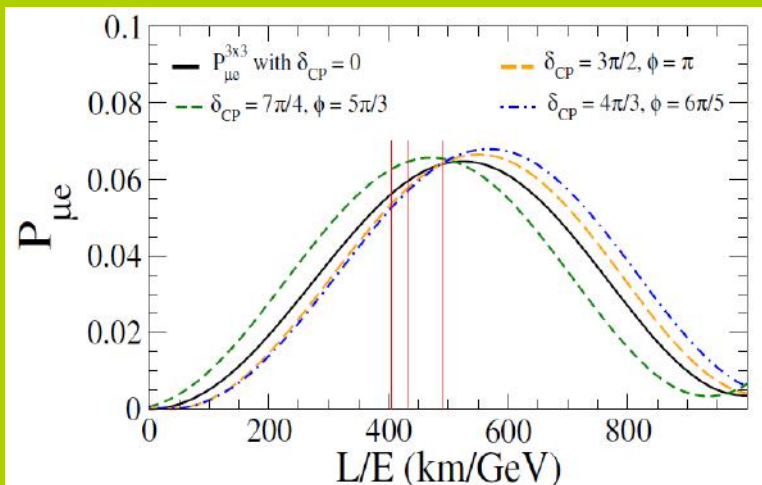
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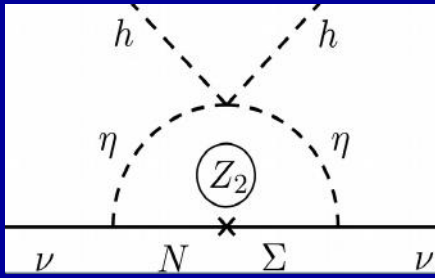
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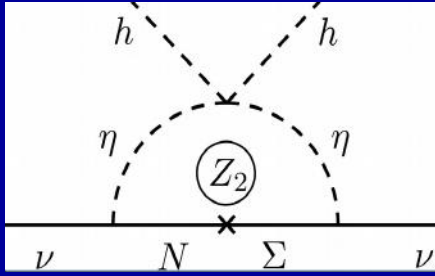




Radiative neutrino mass

many low-scale neutrino mass schemes ...

arXiv:1404.3751



Radiative neutrino mass

many low-scale neutrino mass schemes ...

arXiv:1404.3751

331 electroweak theory # generations = # colours

Singer, Valle, Schechter, Phys.Rev. D22 (1980) 738

Radiative neutrino mass

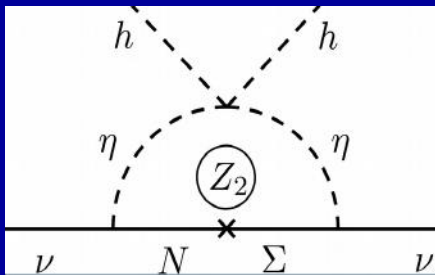
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arXiv:1404.3751

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Singer, Valle, Schechter, Phys.Rev. D22 (1980) 738

Gauge vs Higgs



BOUCENNA, MORISI, AND VALLE

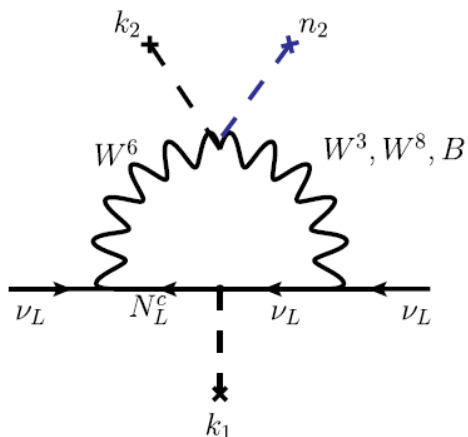
TABLE I. Matter content of the model, where $\hat{u}_R \equiv (u_R, c_R, t_R, t'_R)$ and $\hat{d}_R \equiv (d_R, s_R, b_R, d'_R, s'_R)$ (see text).

	ψ_L^ℓ	ℓ_R	$Q_L^{1,2}$	Q_L^3	\hat{u}_R	\hat{d}_R	S	ϕ_1	ϕ_2	ϕ_3
$SU(3)_c$	1	1	3	3	3	3	1	1	1	1
$SU(3)_L$	3*	1	3	3*	1	1	1	3*	3*	3*
$U(1)_X$	$-\frac{1}{3}$	-1	0	$+\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	0	$+\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$
\mathcal{L}	$-\frac{1}{3}$	-1	$-\frac{2}{3}$	$+\frac{2}{3}$	0	0	1	$+\frac{2}{3}$	$-\frac{4}{3}$	$+\frac{2}{3}$

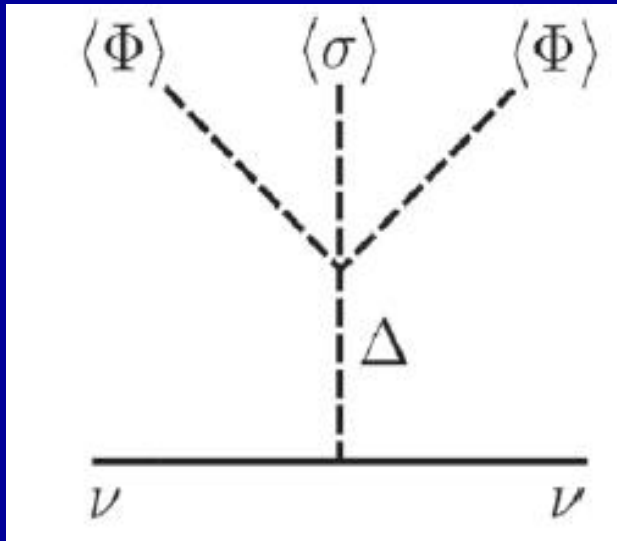
$$Q = T_3 + \frac{1}{\sqrt{3}}T_8 + X, \quad (2)$$

$$L = \frac{4}{\sqrt{3}}T_8 + \mathcal{L}. \quad (3)$$

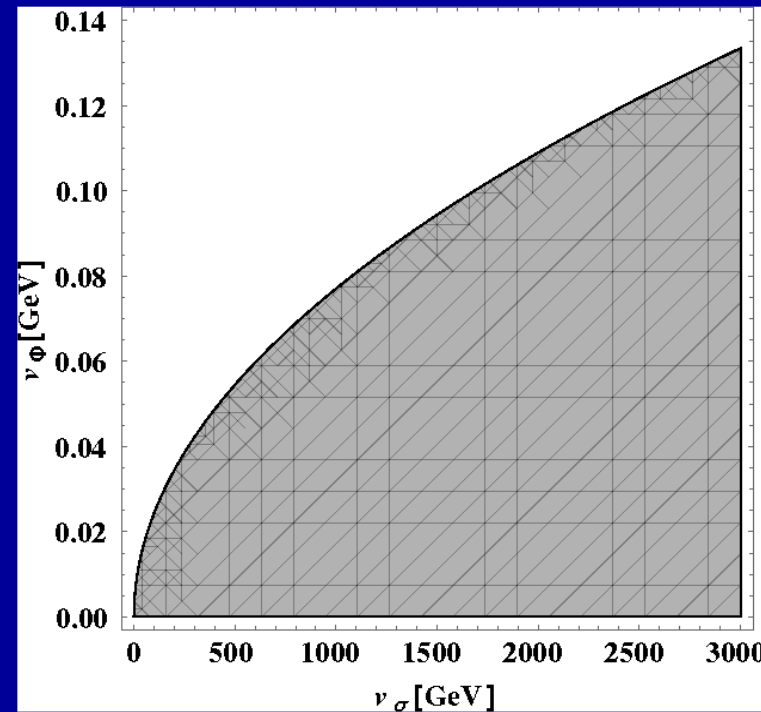
PHYSICAL REVIEW D **90**, 013005 (2014)



Majoron & Dirac type-II seesaw

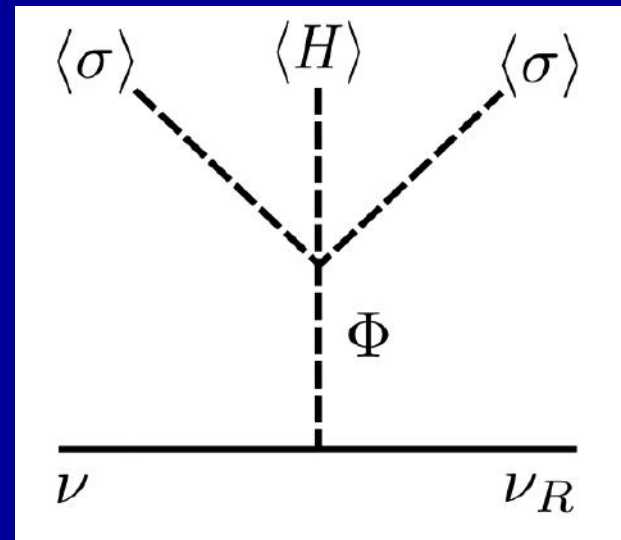
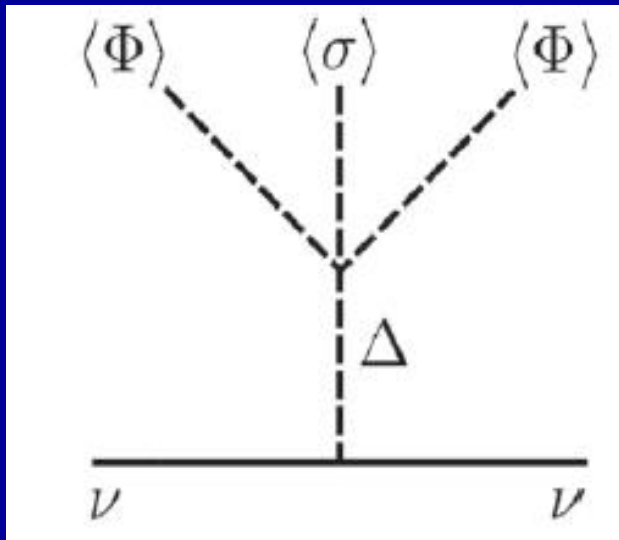


- Astrophysical limit

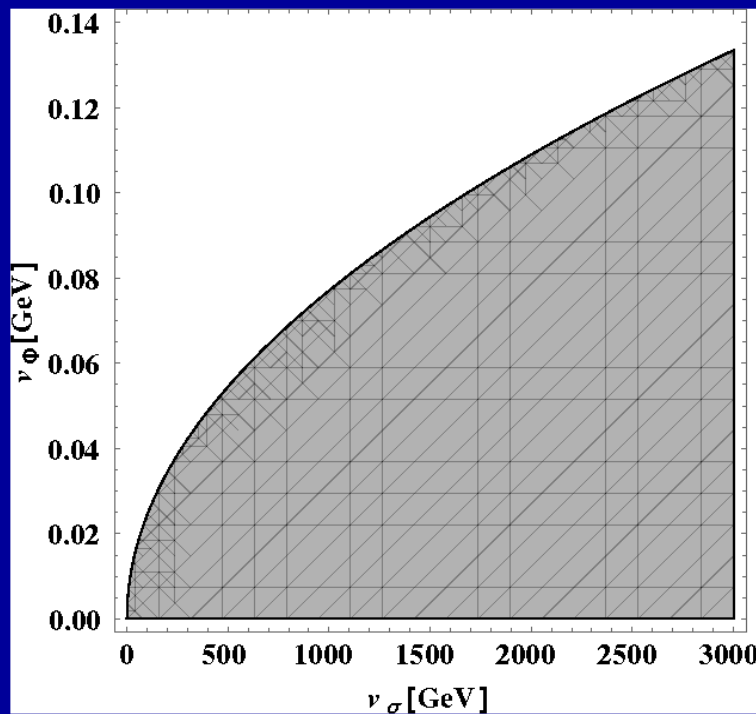


Majoron & Dirac type-II seesaw

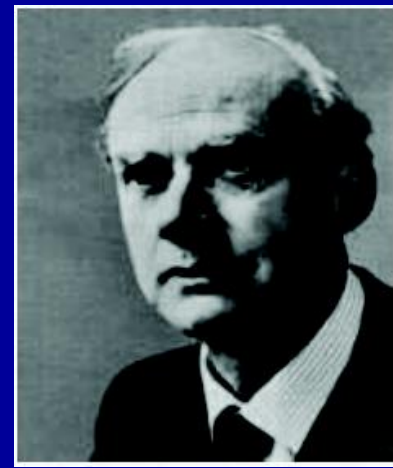
- Lepton # from accidental global U(1)
- Naturally small induced vev



- Astrophysical limit

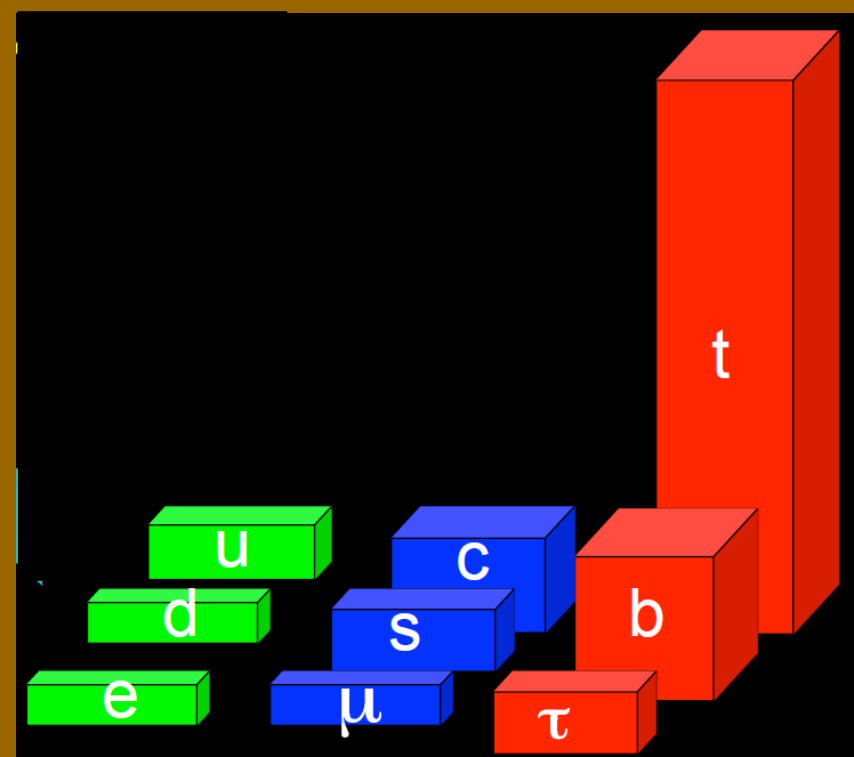


ArXiv:1605.08362 PLB



Flavor puzzle

masses



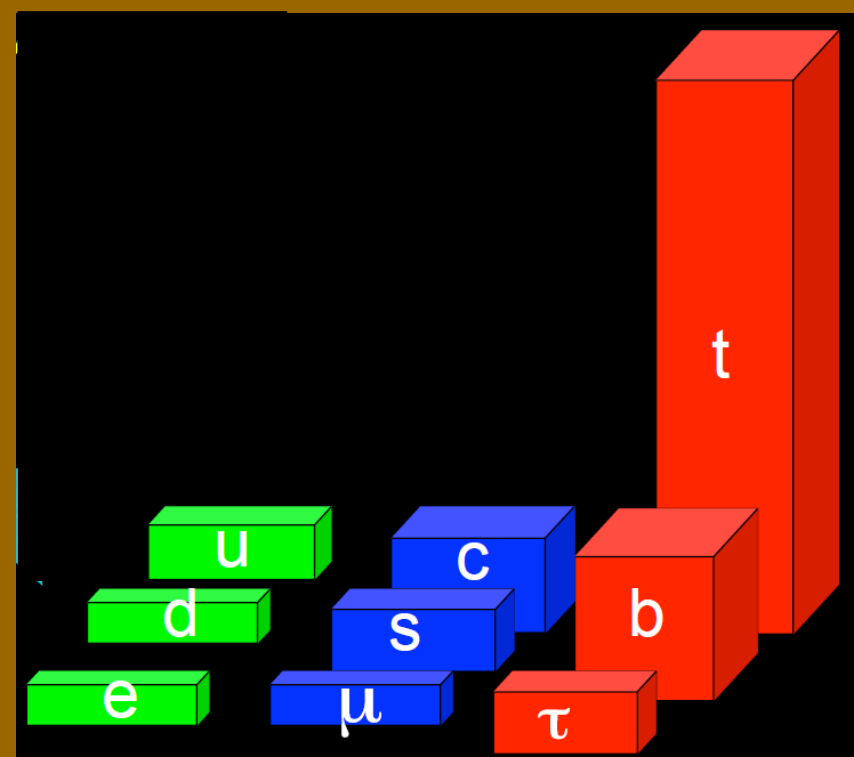
Neutrinos : Lepton number?

mixings

Flavor puzzle

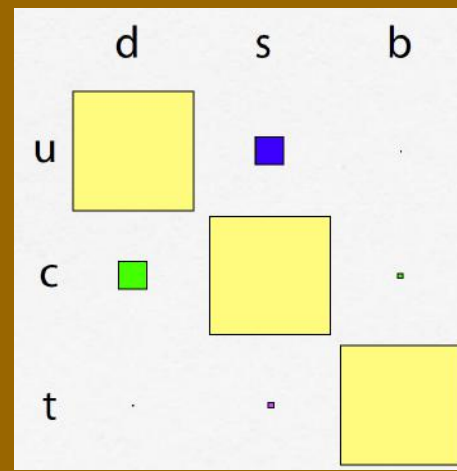
masses

b-tau unification

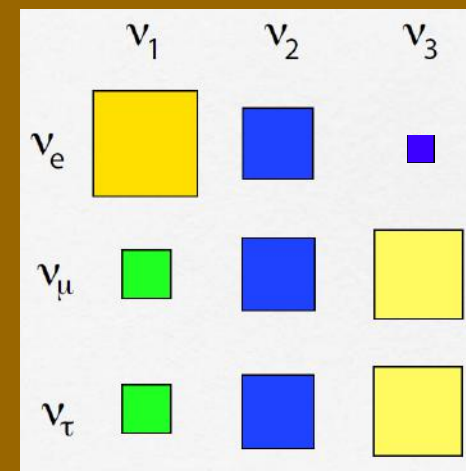


Neutrinos : Lepton number?

mixings

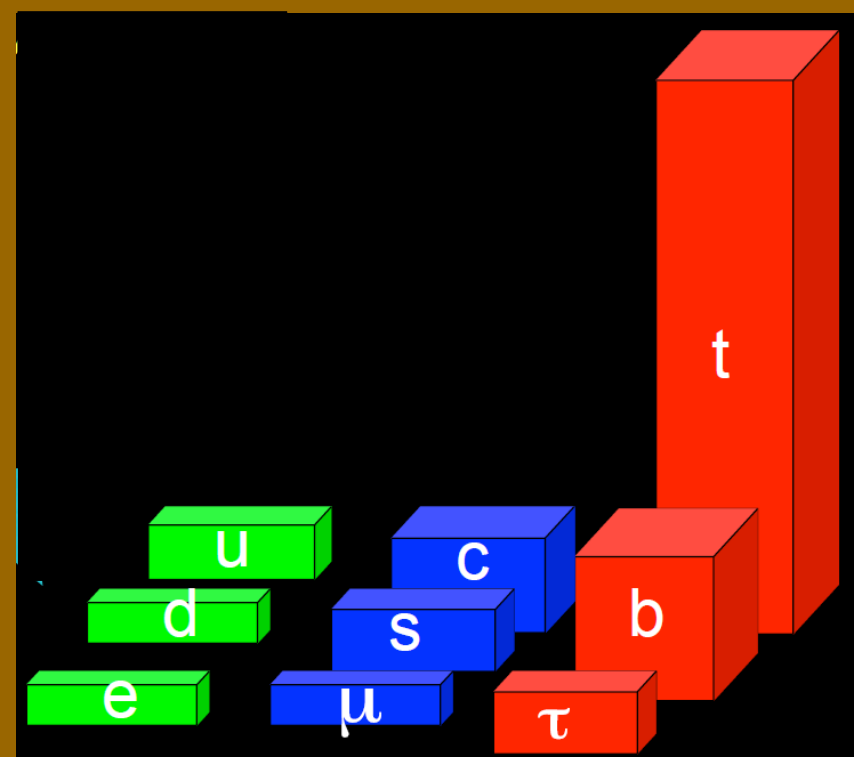


VS



Flavor puzzle

masses



b-tau unification without GUTS

Neutrinos : Lepton number?

$$\frac{m_\tau}{\sqrt{m_e m_\mu}} \approx \frac{m_b}{\sqrt{m_d m_s}}$$

mixings

Morisi et al Phys.Rev. D84 (2011) 036003

King et al

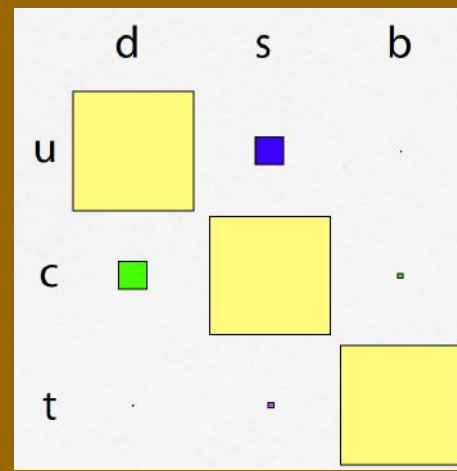
Phys. Lett. B 724 (2013) 68

Morisi et al

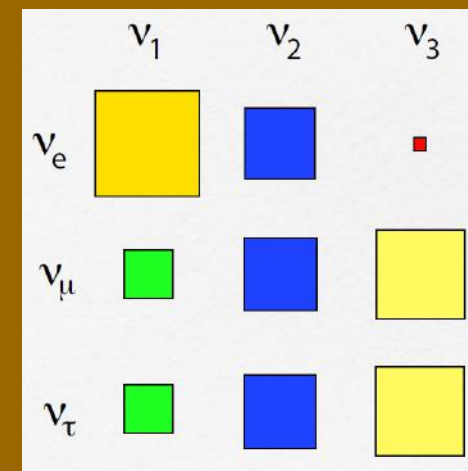
Phys.Rev. D88 (2013) 036001

Bonilla et al

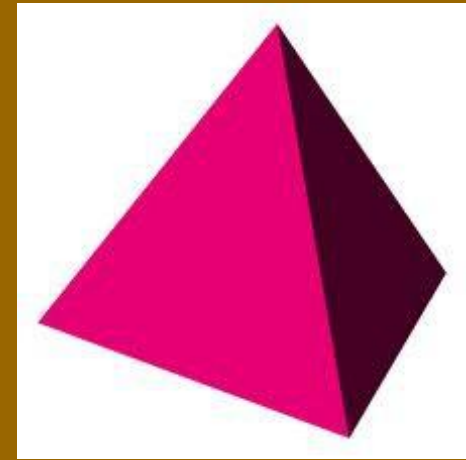
Phys.Lett. B742 (2015) 99



νs



Flavor Symmetry



$\begin{pmatrix} \nu_e \\ e \\ e_R \end{pmatrix}_L$	$\begin{pmatrix} \nu_\mu \\ \mu \\ \mu_R \end{pmatrix}_L$	$\begin{pmatrix} \nu_\tau \\ \tau \\ \tau_R \end{pmatrix}_L$
$\begin{pmatrix} u \\ d \\ u_R \\ d_R \end{pmatrix}_L$	$\begin{pmatrix} c \\ s \\ c_R \\ s_R \end{pmatrix}_L$	$\begin{pmatrix} t \\ b \\ t_R \\ b_R \end{pmatrix}_L$

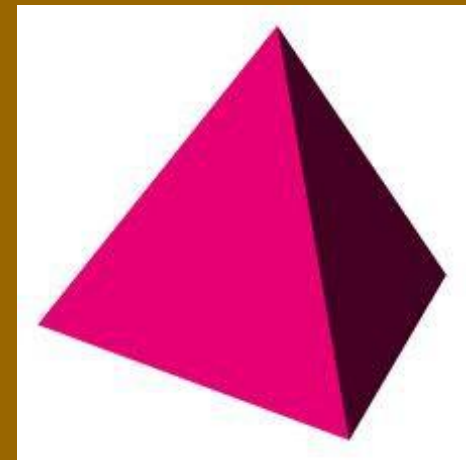
A4

Babu-Ma-Valle PLB552 (2003) 207
Hirsch et al PRD69 (2004) 093006

$$\sin^2 \theta_{23} = 0.5$$

$$\sin^2 \theta_{13} = 0$$

Flavor Symmetry



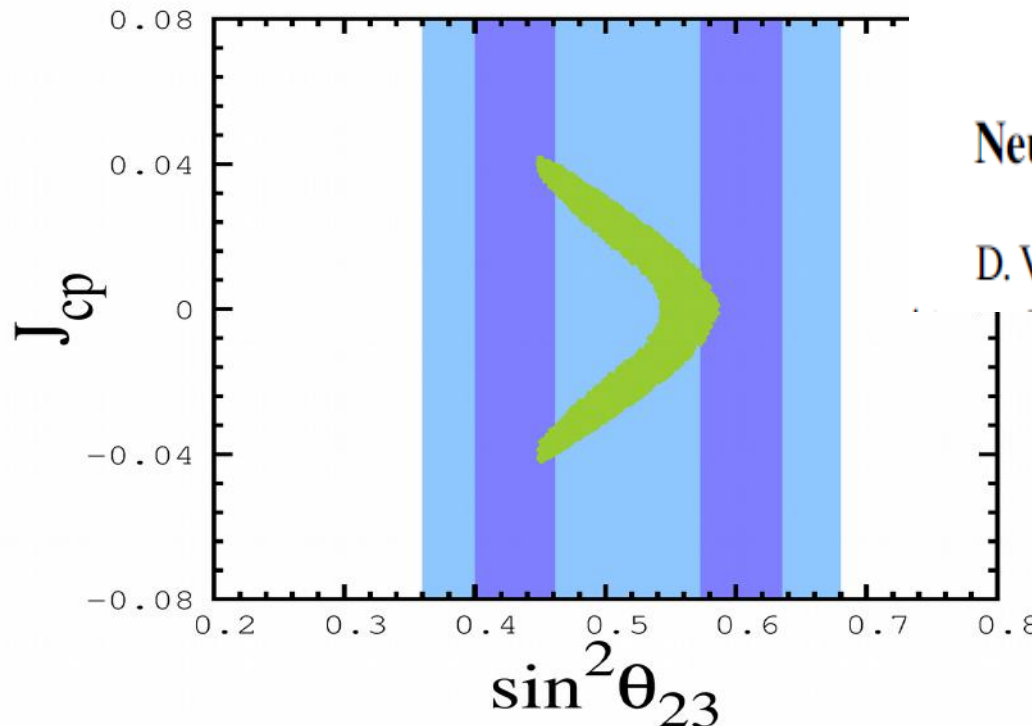
$$\begin{array}{ccc}
 \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L & \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L & \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L \\
 e_R & \mu_R & \tau_R \\
 \begin{pmatrix} u \\ d \end{pmatrix}_L & \begin{pmatrix} c \\ s \end{pmatrix}_L & \begin{pmatrix} t \\ b \end{pmatrix}_L \\
 u_R & c_R & t_R \\
 d_R & s_R & b_R
 \end{array}$$

A4

Babu-Ma-Valle PLB552 (2003) 207
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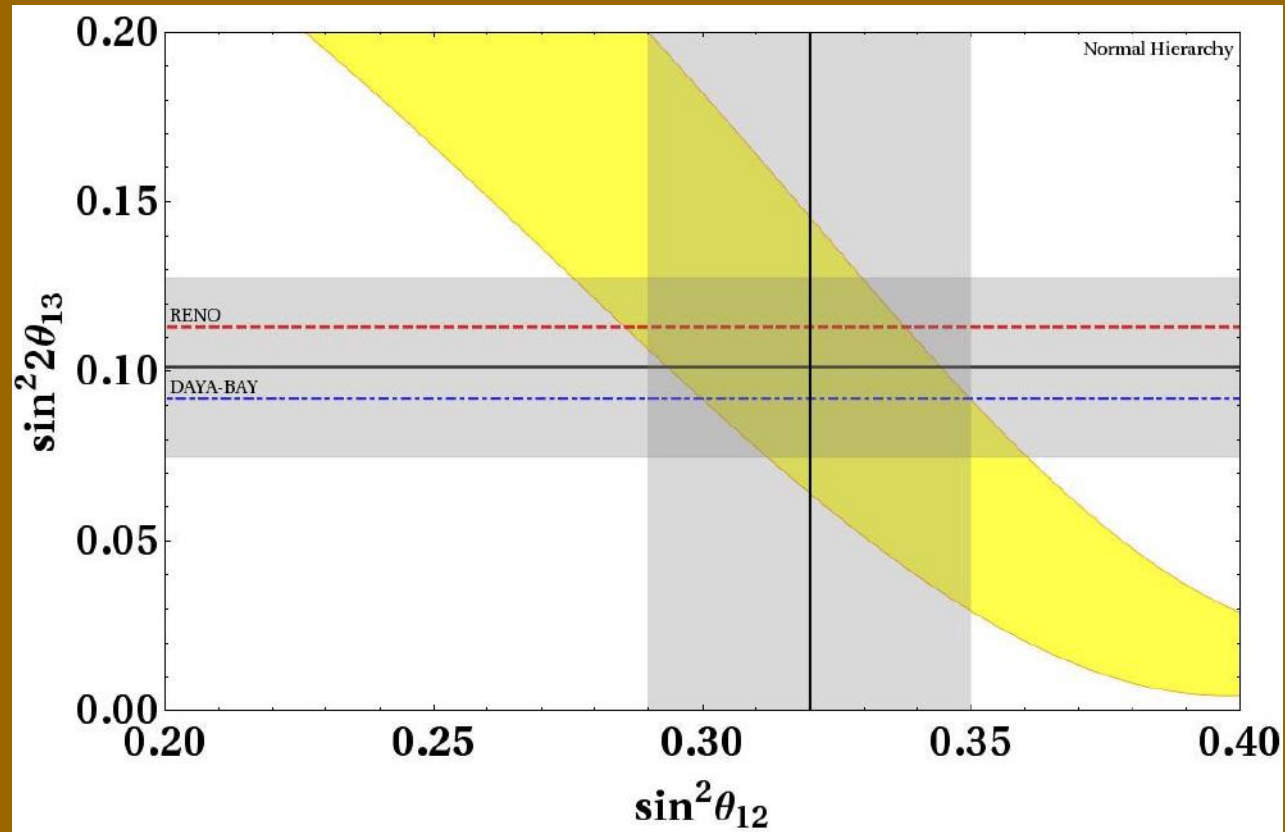
PHYSICAL REVIEW D 88, 016003 (2013)

Neutrino mixing with revamped A4 flavor symmetry

D. V. Forero,^{1,2,*} S. Morisi,^{3,†} J. C. Romão,^{1,‡} and J. W. F. Valle^{2,§}

Flavor correlations

Boucenna et al
PhysRevD.86.073008



Model-independent flavor approach

$$\mathbf{X}^T \mathbf{m}_\nu \mathbf{X} = \mathbf{m}_\nu^*$$

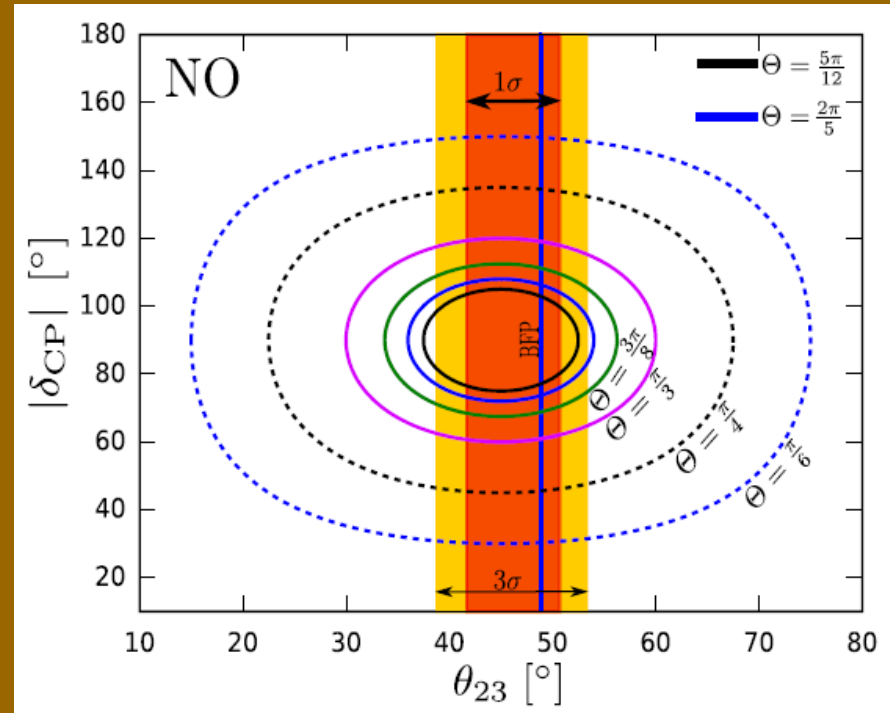
*Predicting neutrino mixing
from residual CP symmetries*

Model-independent flavor approach

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*Predicting neutrino mixing
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P. Chen et al. / Physics Letters B 753 (2016) 644–652



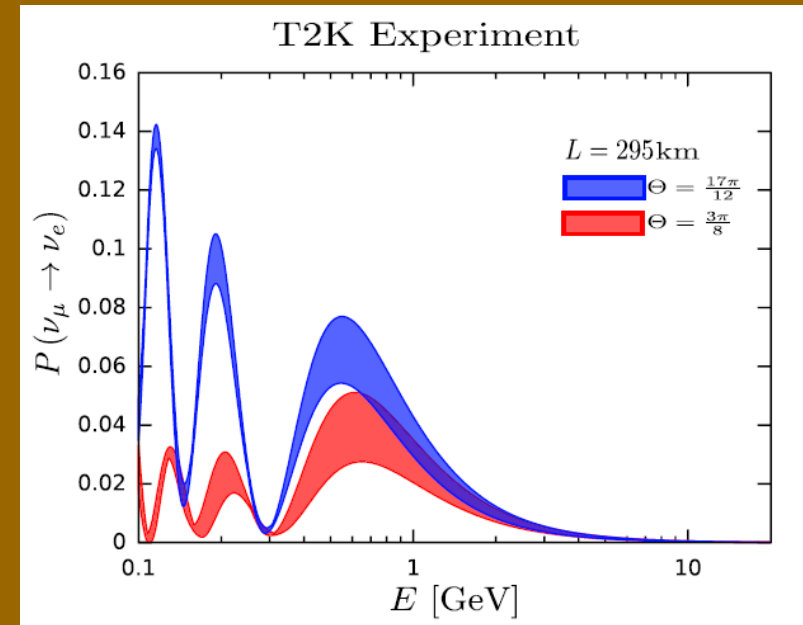
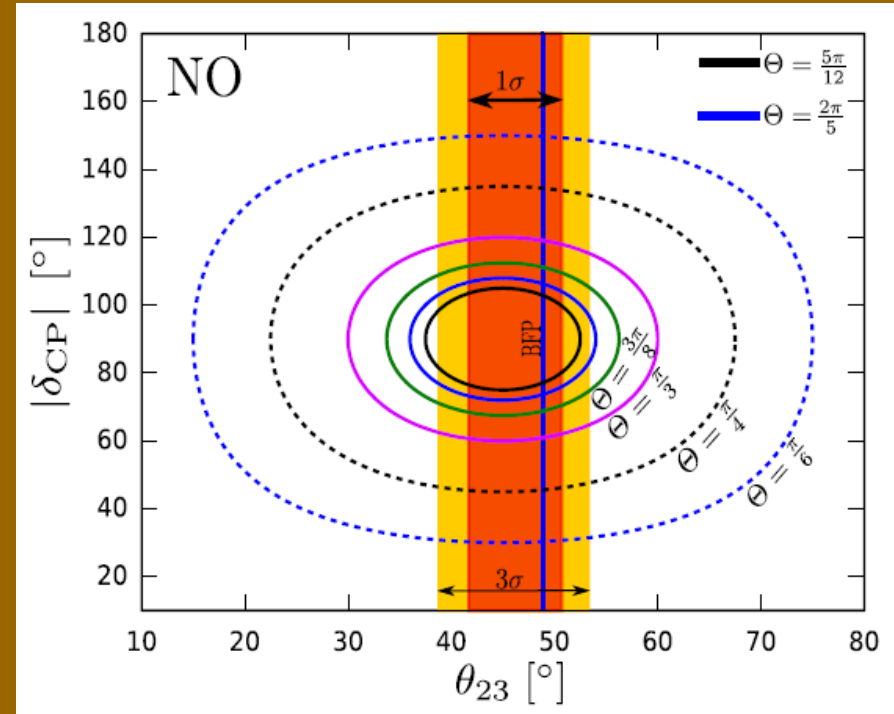
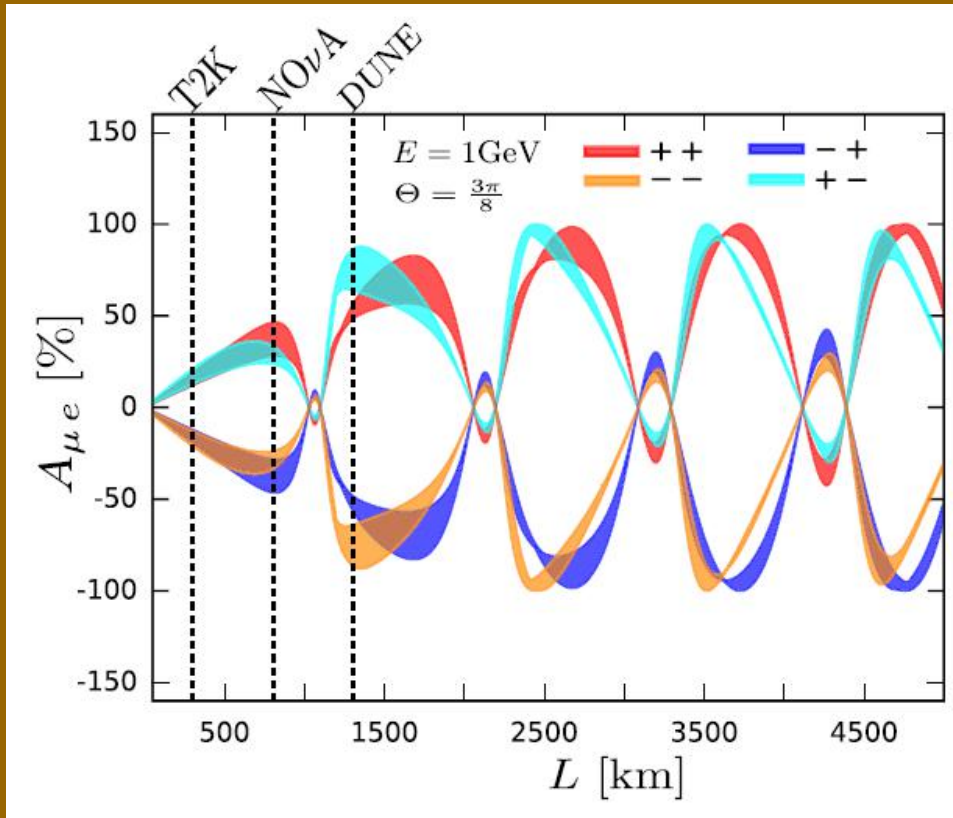
Model-independent flavor approach

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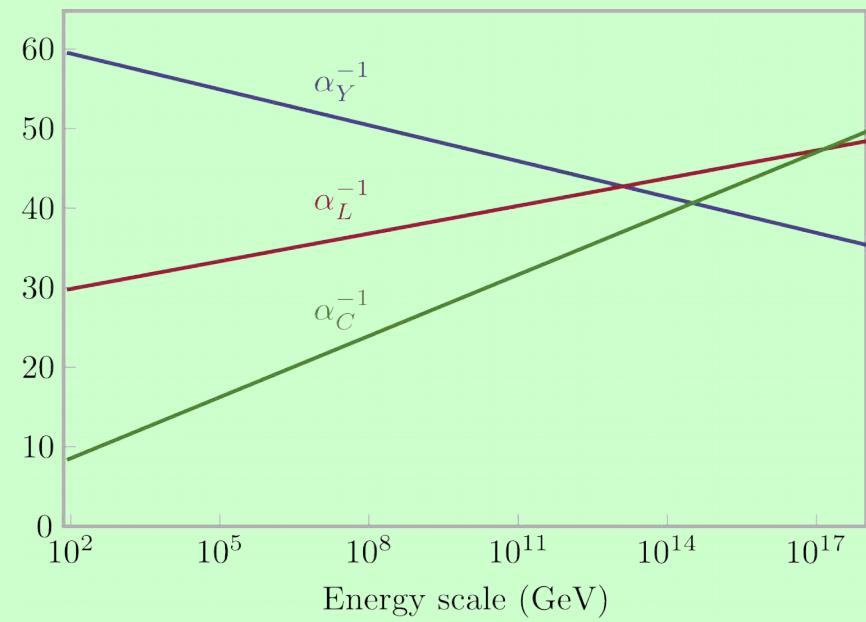
*Predicting neutrino mixing
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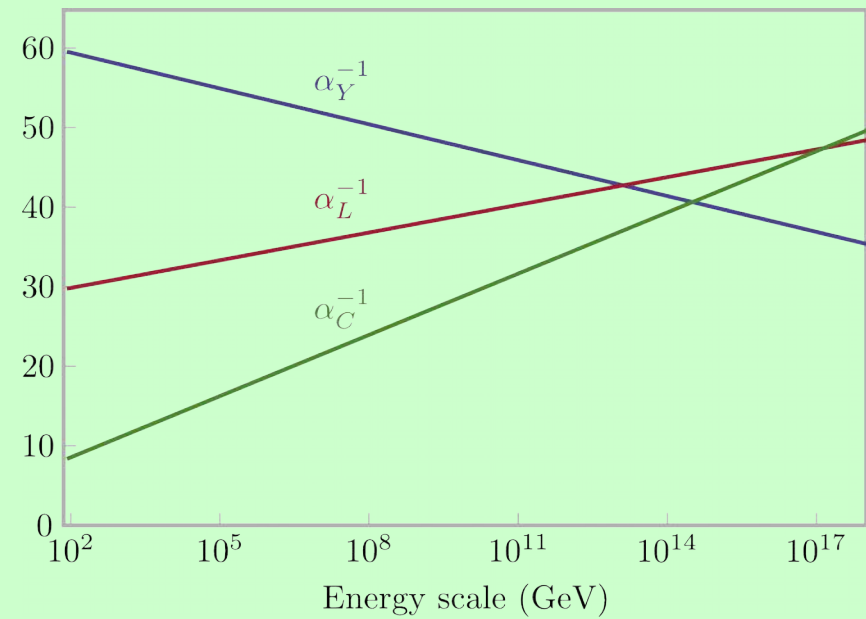
Gauge coupling unification

a near miss ...



Gauge coupling unification

a near miss ...

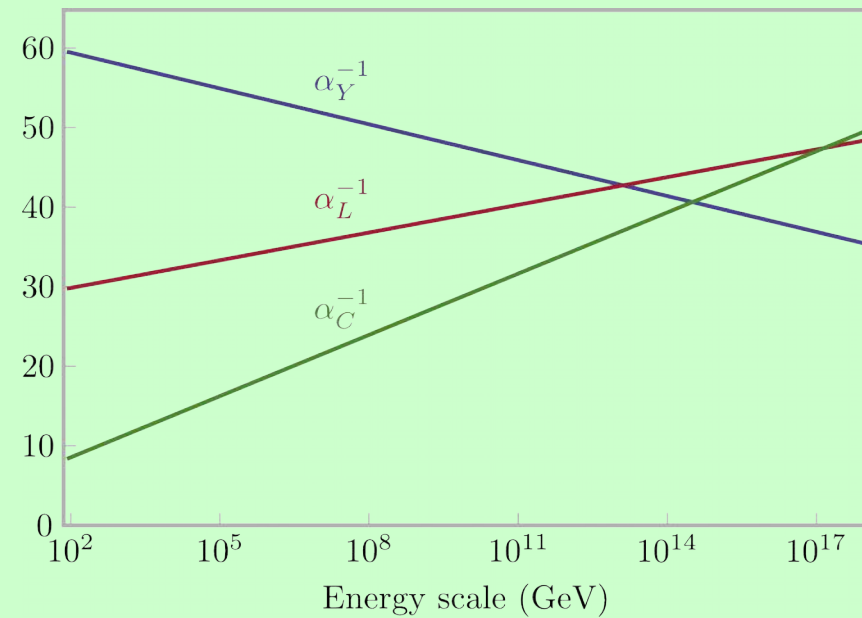


What makes the gauge couplings unify? **SUSYGUT**

But ... p decay, sparticles ...

Gauge coupling unification

a near miss ...



What makes the gauge couplings unify? SUSYGUT

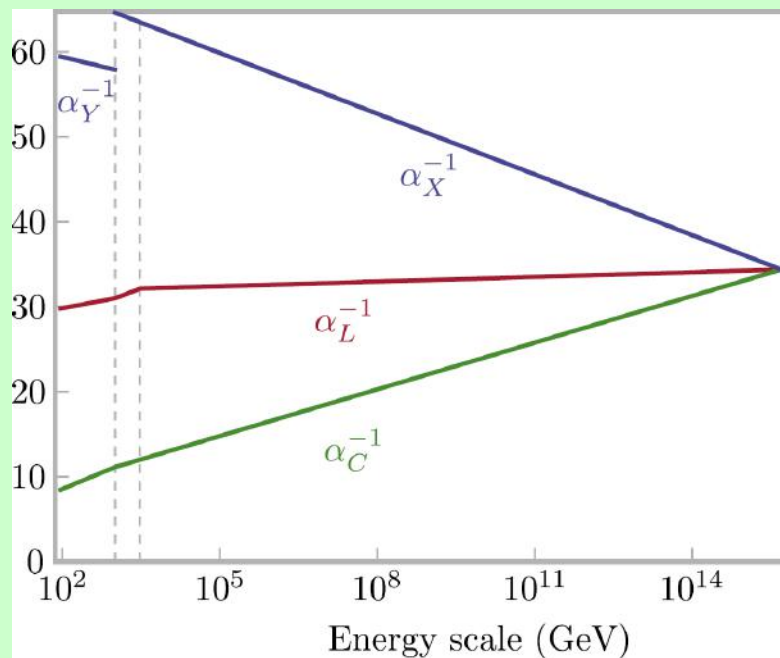
But ... p decay, sparticles ...

neutrino

The physics responsible for gauge coupling unification may also induce small neutrino masses

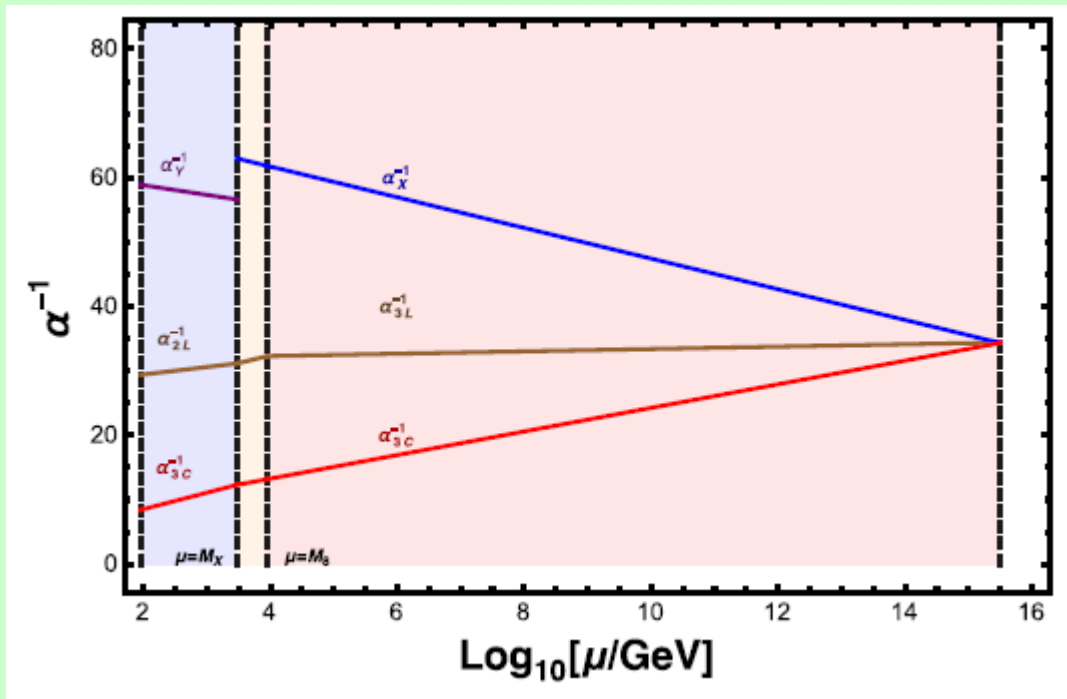
Phys. Rev. D 91, 031702 (2015)

Boucenna, Fonseca, Gonzalez-Canales, JV



unification without conventional GUT embedding

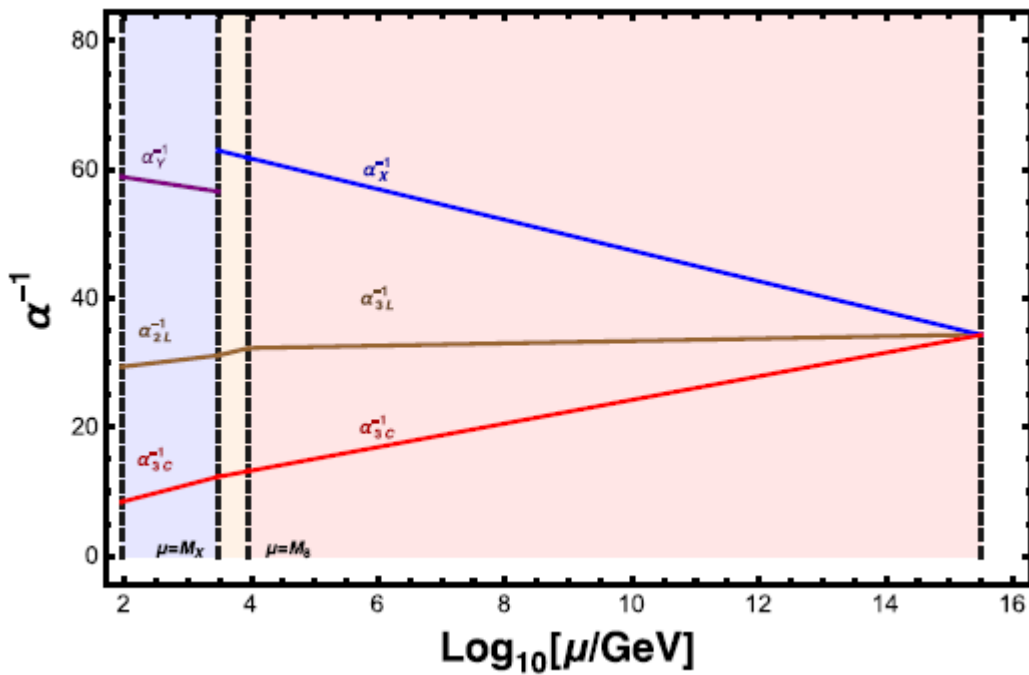
Deppisch et al 1608.05334



SVS case: F-theory GUTs
King, Leontaris et al

unification without conventional GUT embedding

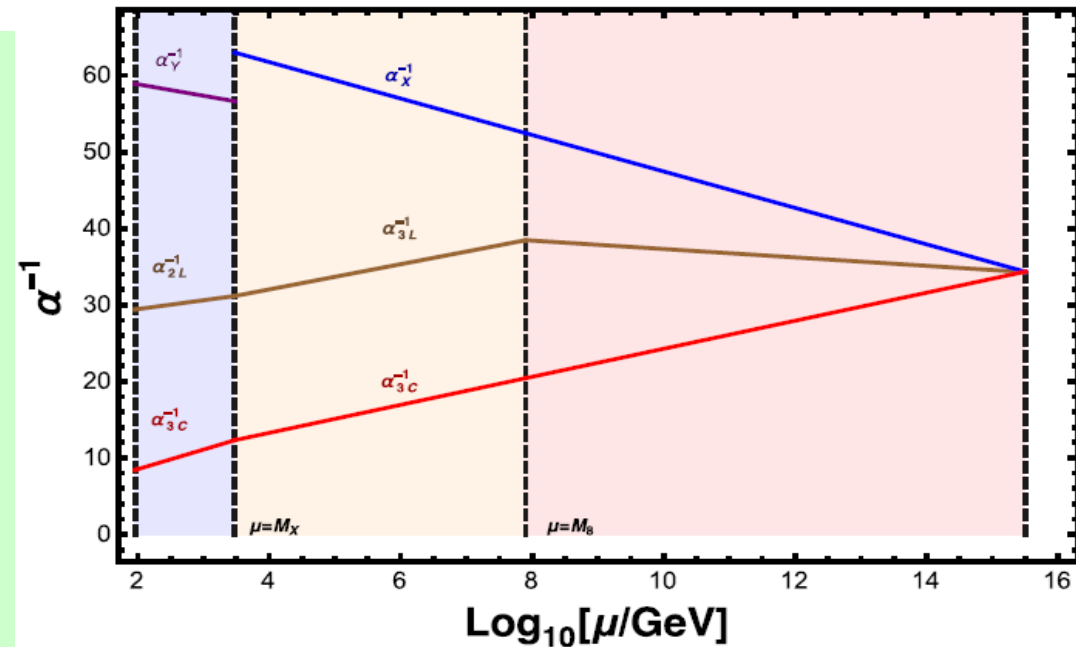
Deppisch et al 1608.05334



SU(6) unification of 331 EW model

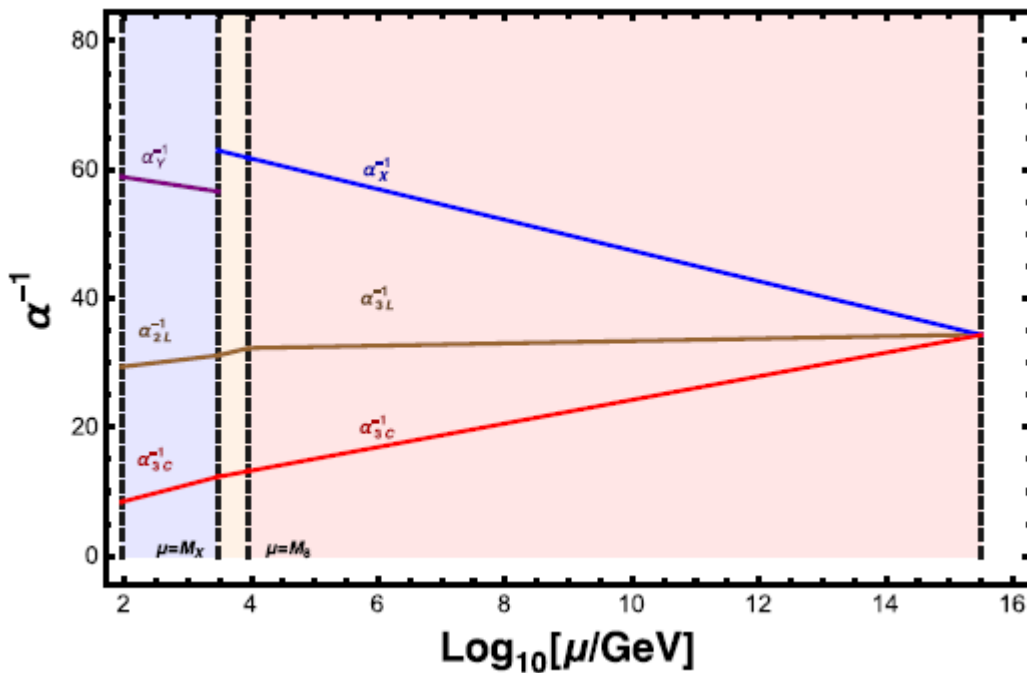
Sequential case

SVS case: F-theory GUTs
King, Leontaris et al



unification without conventional GUT embedding

Deppisch et al 1608.05334



SU(6) unification of 331 EW model

Sequential case

SVS case: F-theory GUTs
King, Leontaris et al

$$\Gamma^{-1}(p \rightarrow e^+ \pi^0) \sim 10^{36} \text{ yrs} \left(\frac{\alpha_{\text{GUT}}^{-1}}{35} \right)^2 \left(\frac{M_U}{10^{16} \text{ GeV}} \right)^4$$

