Electroweak Physics at the LHC

— Lecture 3 —

Electroweak Di-boson Production

Stefan Dittmaier

Albert-Ludwigs-Universität Freiburg

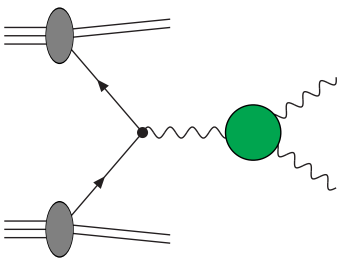
Contents

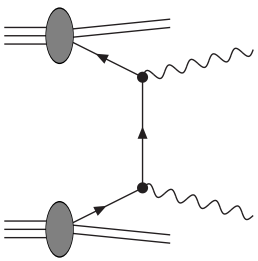
Electroweak di-boson production

—

brief overview

EW di-boson production





$$\mathrm{V}, \mathrm{V}' = \gamma, \mathrm{Z}, \mathrm{W}^{\pm}$$

Physics issues:

- triple-gauge-boson couplings, especially at high momentum transfer
  - ◇ EW corrections significant
  - ◇ anomalous TGC: "formfactor approach" to switch off unitarity violation
    $\hookrightarrow$ element of arbitrariness, avoid when possible
- important background processes
  - ◇ to Higgs production, $\mathrm{H} \to \mathrm{WW}^*/\mathrm{ZZ}^* \to 4f$
    $\hookrightarrow$ invariant masses below VV thresholds,
    proper description of off-shell $V^*V^* \to 4f$ production required !
  - ◇ to searches at high invariant masses
    $\hookrightarrow$ EW corrections

State-of-the-art predictions

$W\gamma/Z\gamma$ (with leptonic decays)

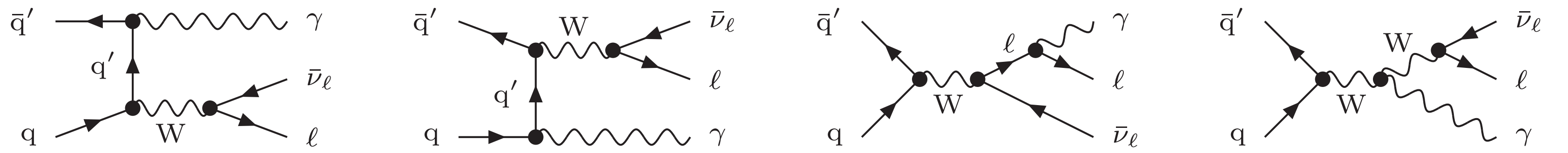
- NNLO QCD Grazzini, Kallweit, Rathlev '14,'15
- NLO EW Denner, S.D., Hecht, Pasold '14,'15

WW, WZ, ZZ

- NNLO QCD
  - ZZ (on-shell and off-shell) Cascioli et al. '14; Grazzini, Kallweit, Rathlev '15
  - WW (on-shell) Gehrmann et al. '14
  - $gg \to VV \to 4$ leptons LO: Binoth et al. '05,'06; NLO: Caola et al. '15,'16
- NLO EW
  - stable W/Z bosons Bierweiler, Kasprzik, Kühn '12/'13; Baglio, Le, Weber '13
  - $pp \to WW \to 4$ leptons in DPA Billoni, S.D., Jäger, Speckner '13
  - approximative inclusion in HERWIG++ Gieseke, Kasprzik, Kühn '14
  - $pp \to WW/ZZ \to 4$ leptons fully off-shell Biedermann et al. '16

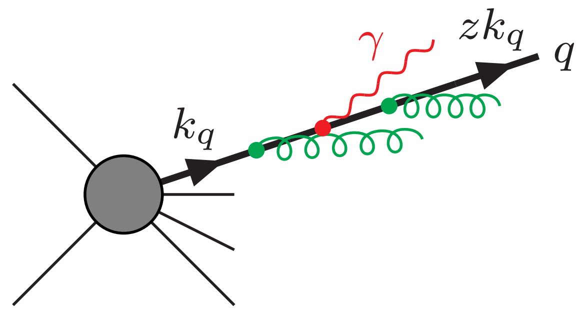
Wγ / Zγ production

Example of $\mathrm{W}\gamma$ production

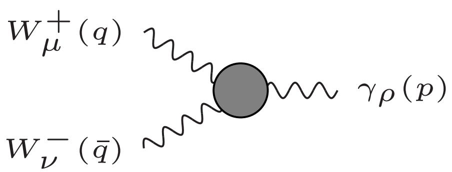


Issues / physics goals:

- clean photon–jet separation
  $\hookrightarrow$ quark-to-photon fragmentation function Glover, Morgan '94
  or Frixione isolation Frixione '98



- stronger bounds on anomalous $\mathrm{WW}\gamma$ coupling:

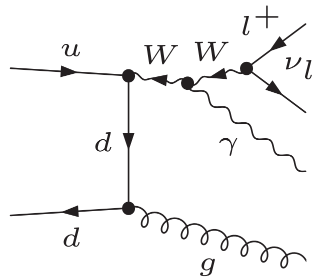


$$= e\left\{\bar{q}^\mu g^{\nu\rho}\left(\Delta\kappa^\gamma + \lambda^\gamma \frac{q^2}{M_\mathrm{W}^2}\right) - q^\nu g^{\mu\rho}\left(\Delta\kappa^\gamma + \lambda^\gamma \frac{\bar{q}^2}{M_\mathrm{W}^2}\right) + (\bar{q}^\rho - q^\rho)\frac{\lambda^\gamma}{M_\mathrm{W}^2}\left(p^\mu p^\nu - \frac{1}{2}g^{\mu\nu}p^2\right)\right\} \times \left(1 + \frac{M_{\mathrm{W}\gamma}^2}{\Lambda^2}\right)^2$$

ATLAS limits '12: $\Delta\kappa^\gamma = 0.41, \quad \lambda^\gamma = 0.074$ for $\Lambda = 2\,\mathrm{TeV}$

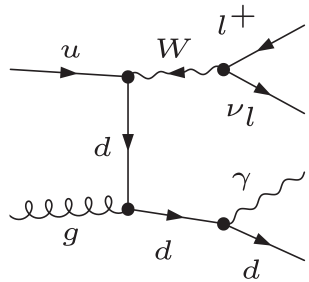
Photon–jet separation via photon fragmentation function $D_{q\to\gamma}$ Glover, Morgan '94

Why?



- QCD radiation cannot be suppressed by cuts

  $\hookrightarrow$ treat at least soft/collinear jets inclusively



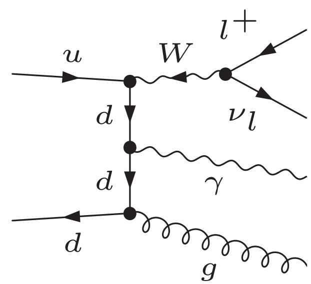
- separation of collinear quarks and photons leads to IR-unstable corrections $\propto \ln(m_q^2/Q^2)$

  $\hookrightarrow$ recombine collinear quarks and photons

- quark and gluon jets cannot be distinguished event by event

  $\hookrightarrow$ common recombination required for quarks/gluons with photons

  $\Rightarrow$ $\underbrace{(g_{\mathrm{hard}} + \gamma_{\mathrm{soft}})}_{\text{EW corr. to X+jet}}$ and $\underbrace{(g_{\mathrm{soft}} + \gamma_{\mathrm{hard}})}_{\text{QCD corr. to X+}\gamma}$ both appear as 1 jet



Problem: signatures of X+jet and X+γ overlap !

Photon–jet separation via photon fragmentation function $D_{q\to\gamma}$ Glover, Morgan '94

Solution:

- idea: declare photon/jet systems as photon or jet according to energy share

- determine photon energy fraction $z_\gamma = \dfrac{E_\gamma}{E_{\rm jet} + E_\gamma}$ of photon/jet system

  $\hookrightarrow$ event selection:
  $z_\gamma > z_0$: photon
  $z_\gamma < z_0$: jet (typical value $z_0 = 0.7$)

- but: cut on z_γ destroys inclusiveness needed for KLN theorem
  $\hookrightarrow$ collinear singularity $\propto \alpha \ln m_q$ remains (but are universal!)

- absorb universal collinear singularity in "fragmentation function" $D_{q\to\gamma}(z_\gamma)$

  $\hookrightarrow$ subtract convolution of LO cross section with

$$D^{\overline{\rm MS}}_{q\to\gamma}(z_\gamma, \mu_{\rm fact})\Big|_{\rm mass.reg.} = \frac{\alpha Q_q^2}{2\pi} P_{q\to\gamma}(z_\gamma) \left[\ln \frac{m_q^2}{\mu_{\rm fact}^2} + 2\ln z_\gamma + 1 \right]$$ $\leftarrow$ cancels coll. singularities

$$+ \; D^{\rm ALEPH}_{q\to\gamma}(z_\gamma, \mu_{\rm fact})$$ $\leftarrow$ non-perturbative part fitted to ALEPH data

where $P_{q\to\gamma}(z_\gamma) = \frac{1+(1-z_\gamma)^2}{z_\gamma}$ = quark-to-photon splitting function

Alternative: photon–jet separation via Frixione isolation

Frixione '98

Idea: suppress jets inside collinear cone around photons:

$$p_{\mathrm{T,jet}} < \varepsilon\, p_{\mathrm{T},\gamma} \left(\frac{1-\cos R_{\gamma\mathrm{jet}}}{1-\cos R_0} \right) \qquad (R_0 = \text{fixed cone size})$$

- photon and jet collinear ($R_{\gamma\mathrm{jet}} \to 0$) $\to$ event discarded
- photon soft or collinear to beams ($p_{\mathrm{T},\gamma} \to 0$) $\to$ event discarded
- jet soft or collinear beams ($p_{\mathrm{T,jet}} \to 0$) $\to$ event kept $\Rightarrow$ IR safety

Comments:

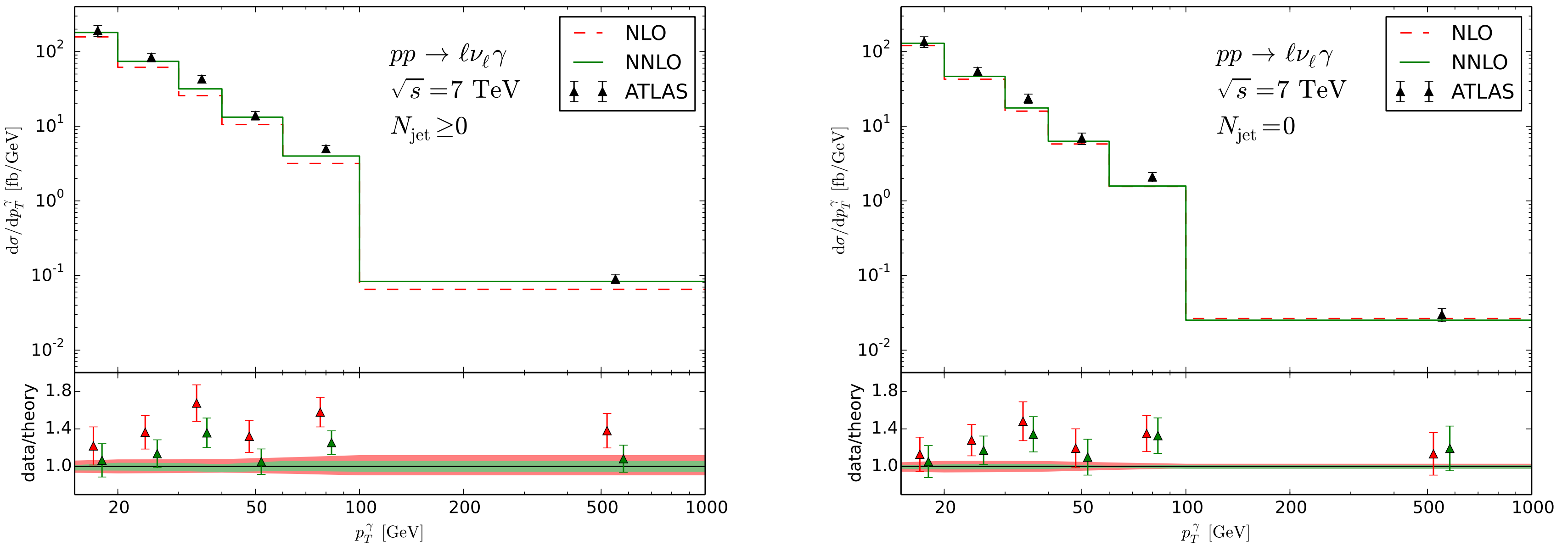
- Frixione isolation simple to implement theoretically, but problematic experimentally
- cleaner isolation of non-perturbative effects by fragmentation function
- approximate relation between the two methods:

$$z_\gamma \sim \frac{p_{\mathrm{T},\gamma}}{p_{\mathrm{T},\gamma}+p_{\mathrm{T,jet}}} > \frac{1}{1+\varepsilon\frac{1-\cos R_{\gamma\mathrm{jet}}}{1-\cos R_0}} \sim \frac{1}{1+\varepsilon} \quad \text{for } R_{\gamma\mathrm{jet}} \sim R_0$$

$\hookrightarrow$ methods yield quite similar results for $z_0 \sim \frac{1}{1+\varepsilon}$

$\mathrm{W}\gamma$ production – QCD theory versus experiment

Grazzini, Kallweit, Rathlev ’15

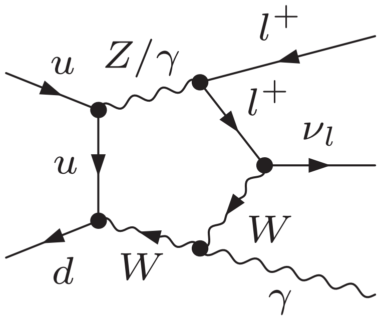


- good agreement of experimental results with NNLO QCD
  (no EW corrections included)
- QCD uncertainties: (for small/moderate $p_{\mathrm{T},\gamma}$)
  scale: $4{-}5\%$, PDF: $1{-}2\%$ (increasing with $p_{\mathrm{T},\gamma}$)
- LHC run 2: higher energy reach & higher statistics
  $\hookrightarrow$ EW corrections important

Wγ production – EW corrections

Denner, S.D., Hecht, Pasold '14

- NLO EW corrections calculated with full W off-shell/decay effects (complex-mass scheme)
  $\hookrightarrow$ more + more complicated diagrams than in QCD

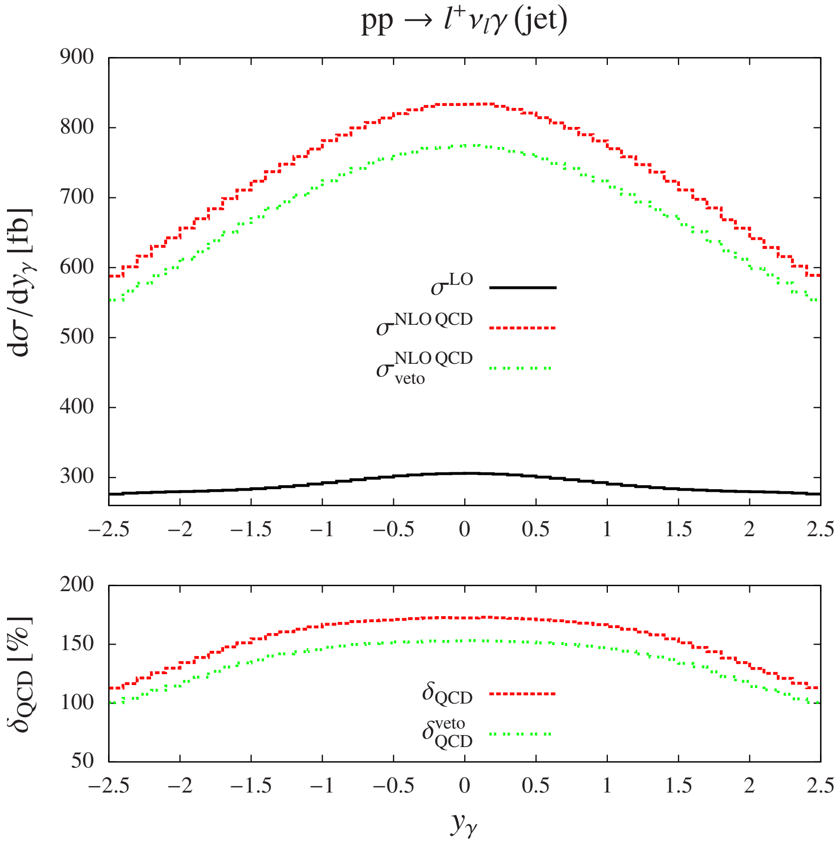


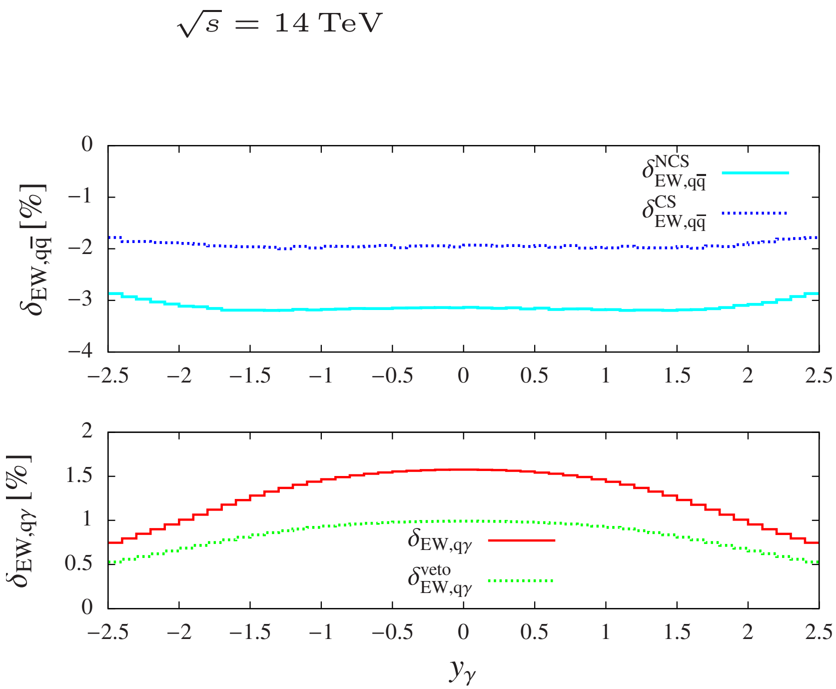
etc.

- particular focus on:
  - ◇ high energies (e.g. large p_{T}):
    large EW corrections $\leftrightarrow$ sensitivity to anomalous couplings
    $\hookrightarrow$ missing corrections could fake anomalous couplings
  - ◇ photon-induced contributions

Rapidity distributions in $\mathrm{W}\gamma$ production

Denner, S.D., Hecht, Pasold '14

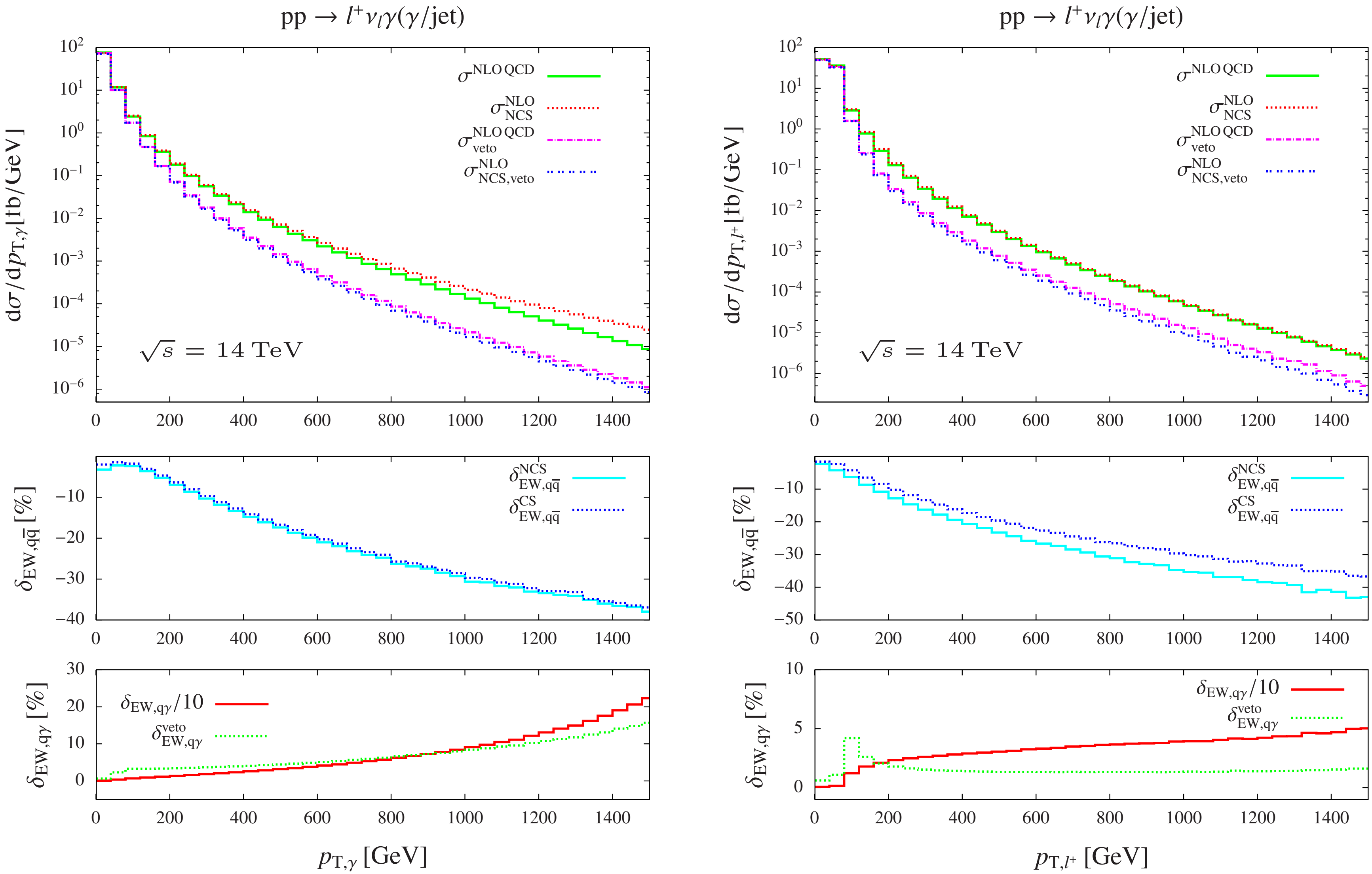




- huge QCD corrections ($\sim 100\%$), only mildly reduced by jet veto $p_{\mathrm{T,jet}} < 100\,\mathrm{GeV}$
- EW corrections and $q\gamma$ channels (few $\%$) small and flat (CS=collinear-safe, NCS=non-collinear-safe)
  $\hookrightarrow$ resemble corrections to integrated cross section

p_{T} distributions in $\mathrm{W}\gamma$ production – EW corrections

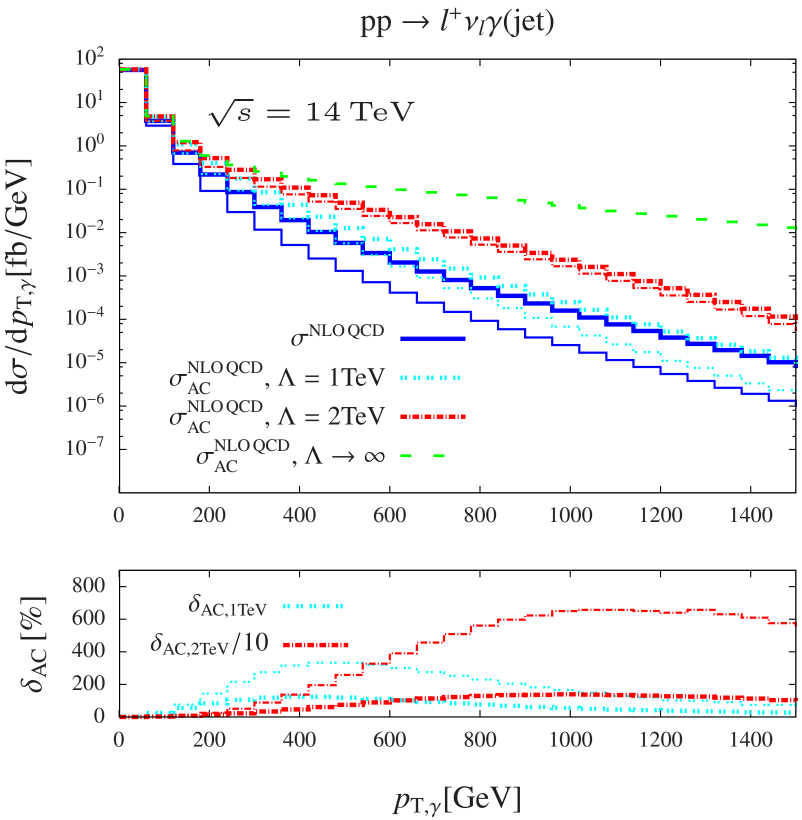
Denner, S.D., Hecht, Pasold '14

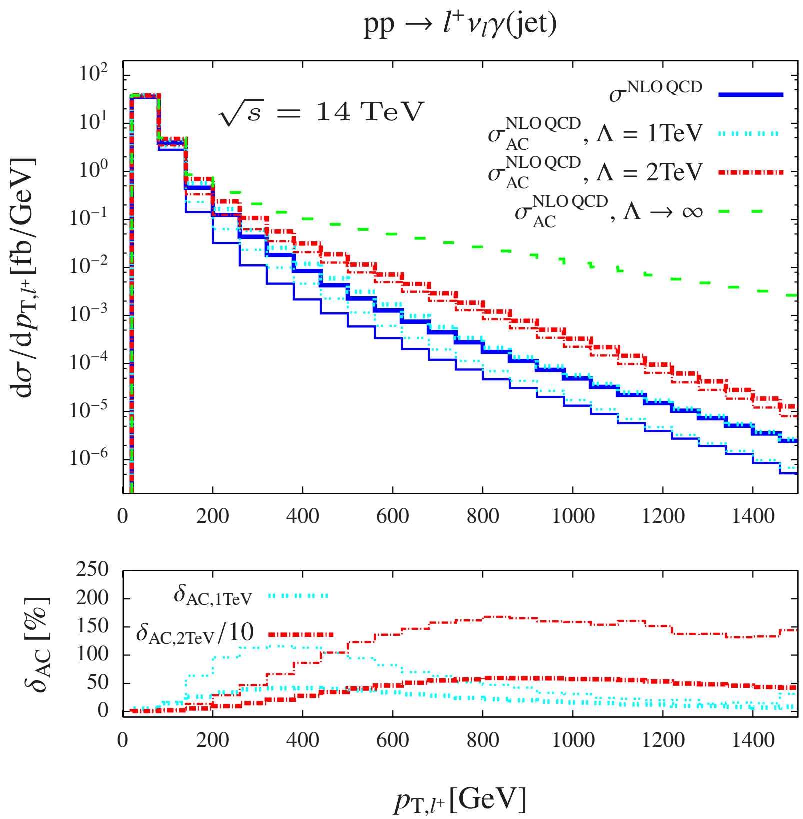


- EW corrections $\sim -30\%$ in TeV range (CS=collinear-safe, NCS=non-collinear-safe)
- γ-induced corrections non-negligible in TeV range (even with jet veto)
  $\hookrightarrow$ reduction of γ PDF uncertainties mandatory !

$\mathrm{W}\gamma$ production – anomalous couplings

Denner, S.D., Hecht, Pasold ’14





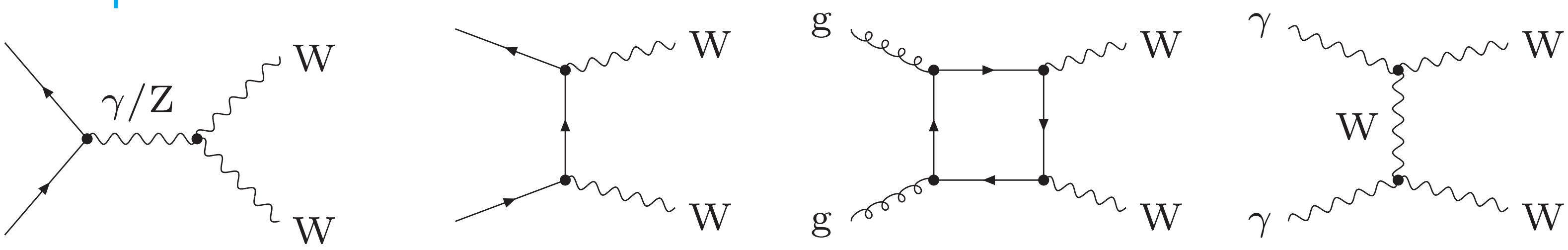
- results shown without and with jet veto on $p_{\mathrm{T,jet}} > 100\,\mathrm{GeV}$
- ATLAS values of 2012 used: $\Delta\kappa^\gamma = 0.41$, $\lambda^\gamma = 0.074$

  $\hookrightarrow$ much tighter limits expected at LHC run 2

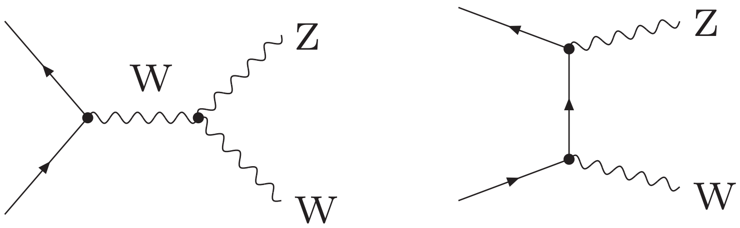
WW / WZ / ZZ production

Complementarity in WW / WZ / ZZ production

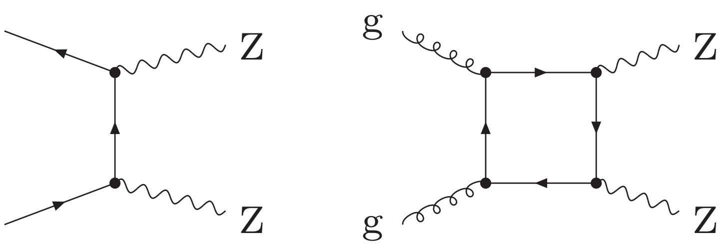
WW production:



WZ production:

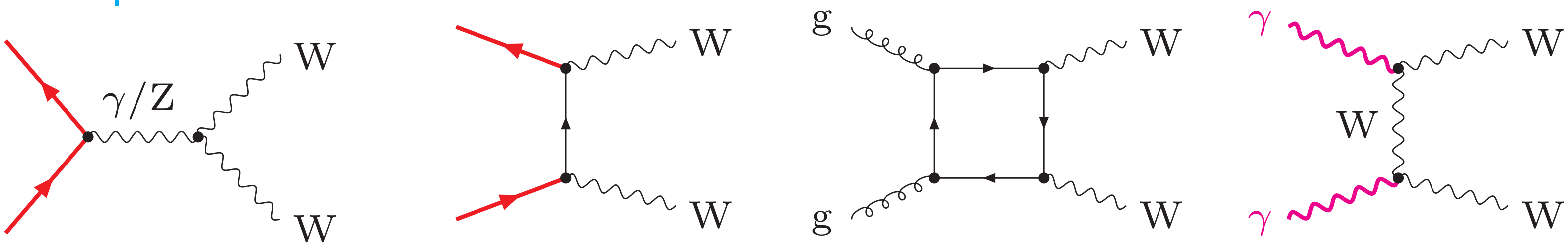


ZZ production:

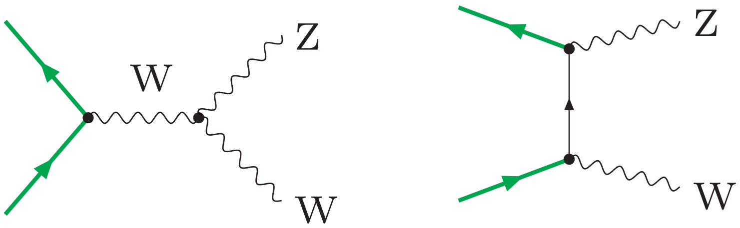


Complementarity in WW / WZ / ZZ production

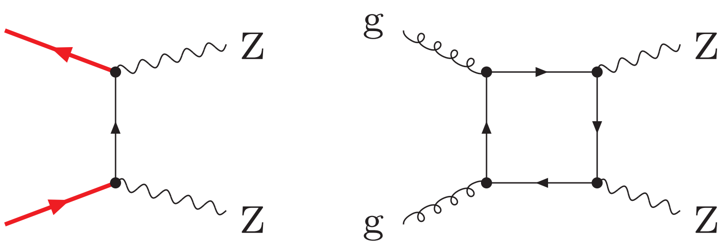
WW production:



WZ production:



ZZ production:

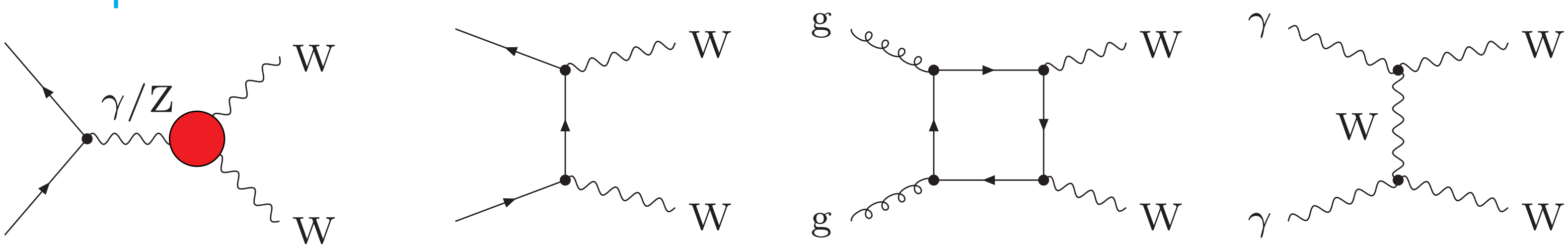


Sensitivity to different PDF combinations:

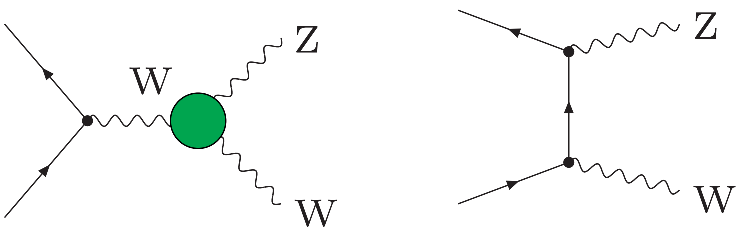
- $q\bar{q}$ in $\mathrm{WW/ZZ}$
- $u\bar{d}/d\bar{u}$ in $\mathrm{W^+Z/W^-Z}$
- $\gamma\gamma$ in WW

Complementarity in WW / WZ / ZZ production

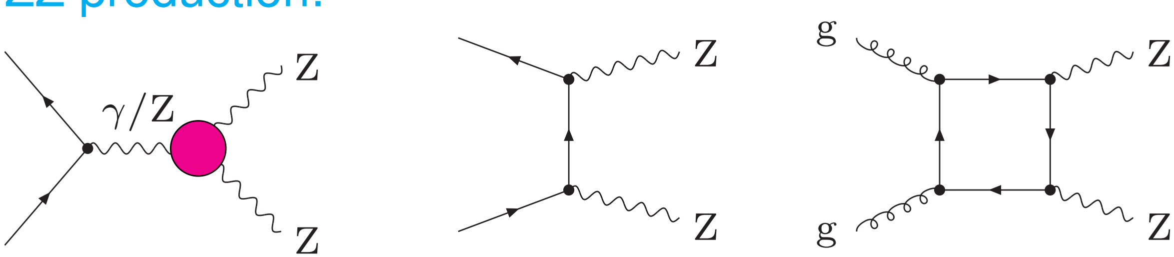
WW production:



WZ production:



ZZ production:

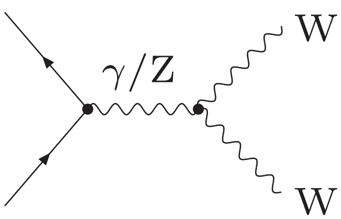


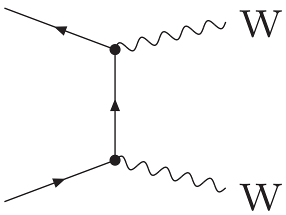
Sensitivity to different anomalous TGCs:

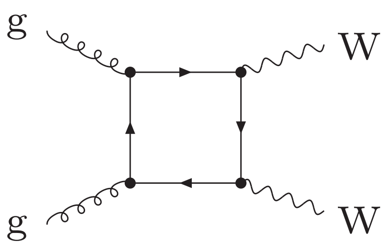
- overlay of $\gamma\mathrm{WW}/\mathrm{ZWW}$ in WW
- only ZWW in WZ
- $\gamma\mathrm{ZZ}/\mathrm{ZZZ}$ in ZZ

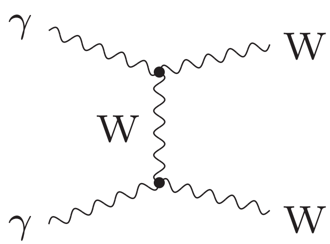
Complementarity in WW / WZ / ZZ production

WW production:

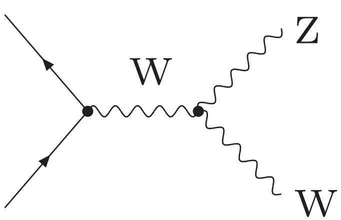


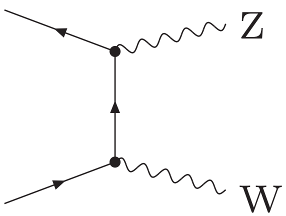




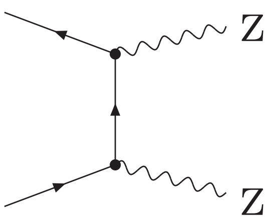


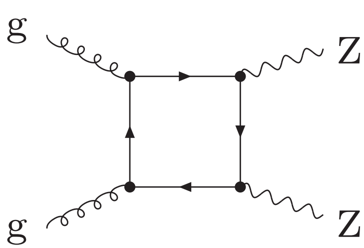
WZ production:





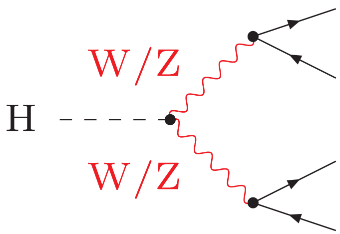
ZZ production:





Background to Higgs production
in channel $\mathrm{H} \to \mathrm{WW}^*/\mathrm{ZZ}^* \to 4f$

$\hookrightarrow$ off-shell calculation
particularly important for WW/ZZ !

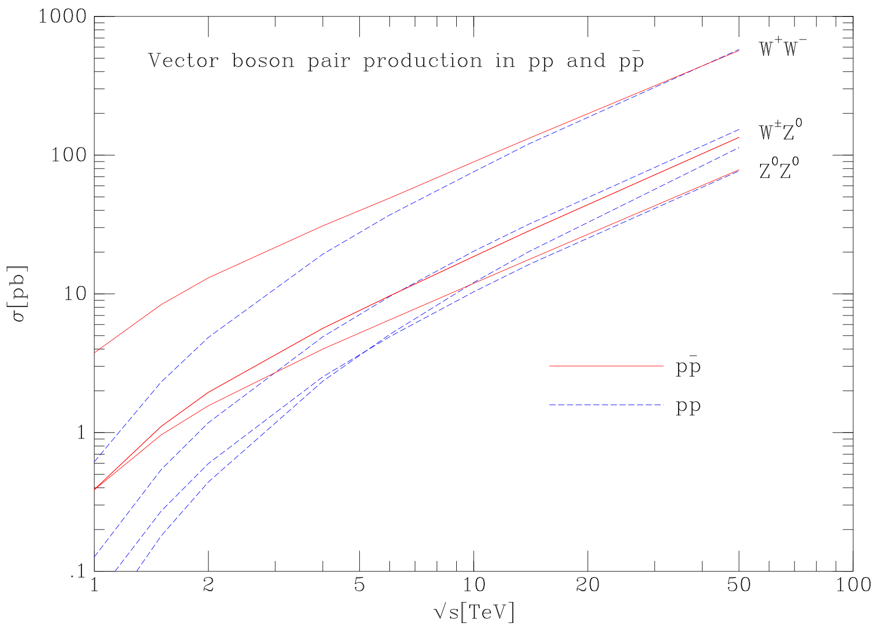


QCD corrections to $\mathrm{WW}, \mathrm{WZ}, \mathrm{ZZ}, \mathrm{W}\gamma, \mathrm{Z}\gamma$ production

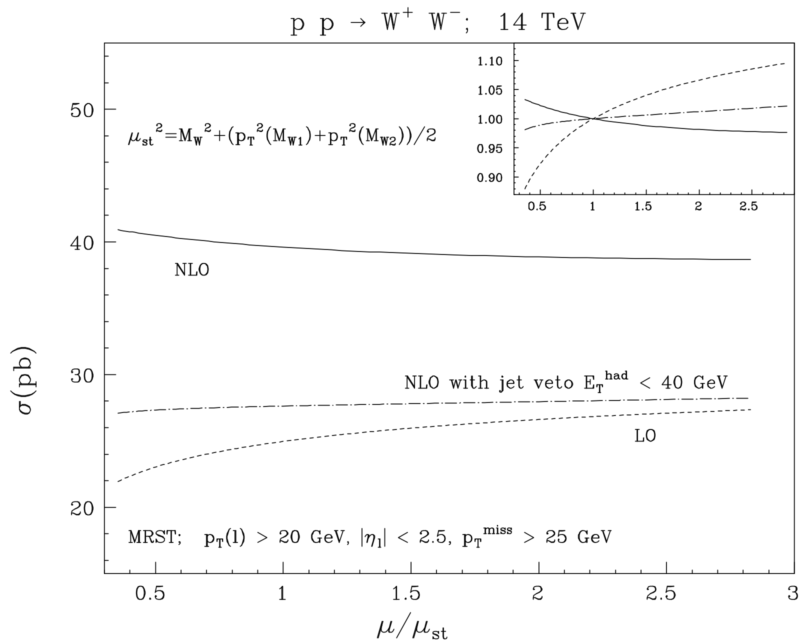
NLO QCD calculated (including leptonic W/Z decays)

Baur, Han, Ohnemus '93-'98
Dixon, Kunszt, Signer '99
Campbell, R.K.Ellis '99
DeFlorian, Signer '00

Campbell, R.K.Ellis et al. '99



Haywood et al. '00

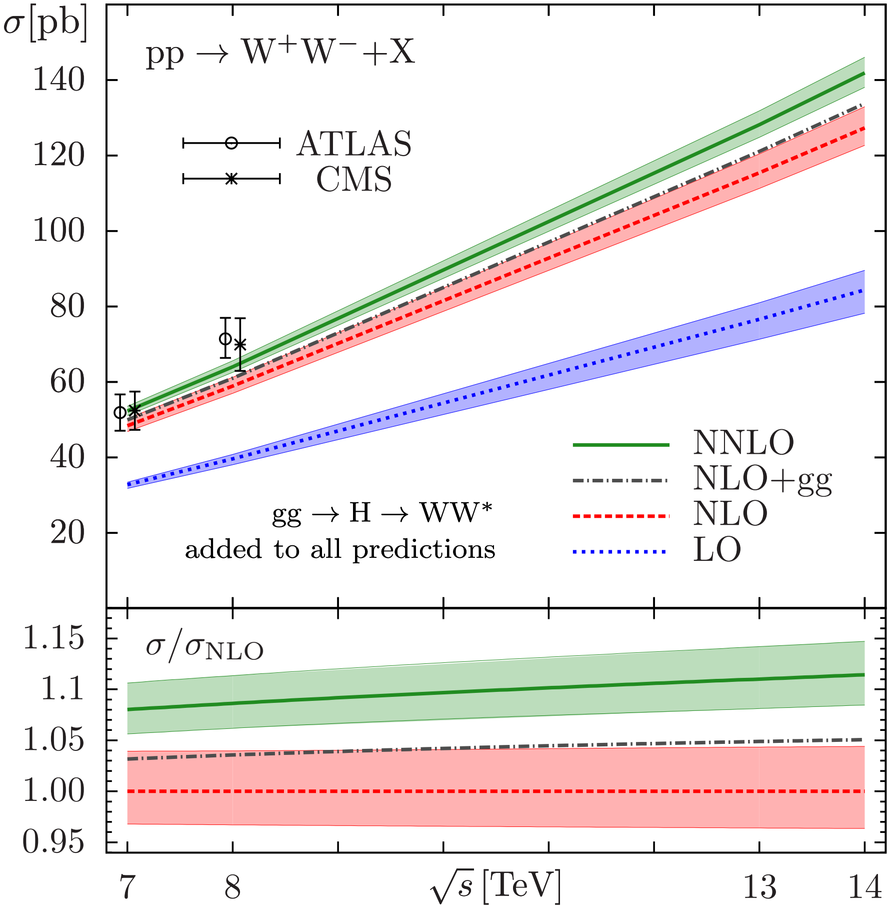


Large positive corrections due to jet radiation, i.e. $VV + \mathrm{jet}$ production

- reduction of corrections and scale dependence by jet veto: $p_{\mathrm{T,jet}} < \mathrm{cut}$?
  $\hookrightarrow$ include QCD resummation for veto
- NNLO QCD corrections important

WW production – NNLO QCD theory versus experiment

Gehrmann et al. '14



Subtlety:

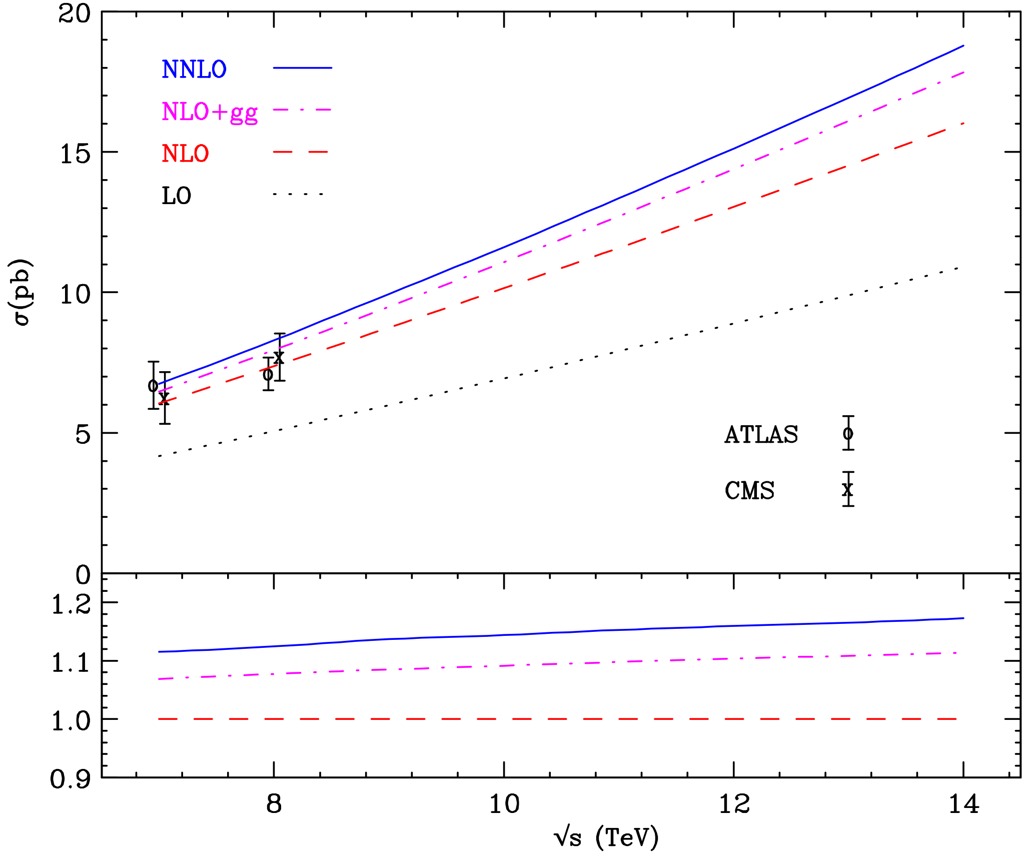
Separation of single-t and $t\bar{t}$ contributions @ NNLO QCD

↪ b-jet veto, etc.

- good agreement of experimental results with NNLO QCD
- NNLO QCD correction $\sim 7(12)\%$ @ $8(13)\,\mathrm{TeV}$, scale uncertainty $\lesssim 3\%$
- gg contribution $\sim 7(8)\%$ @ $8(13)\,\mathrm{TeV}$
- LHC run 2: higher energy & higher statistics → EW corrections important

ZZ production – NNLO QCD theory versus experiment

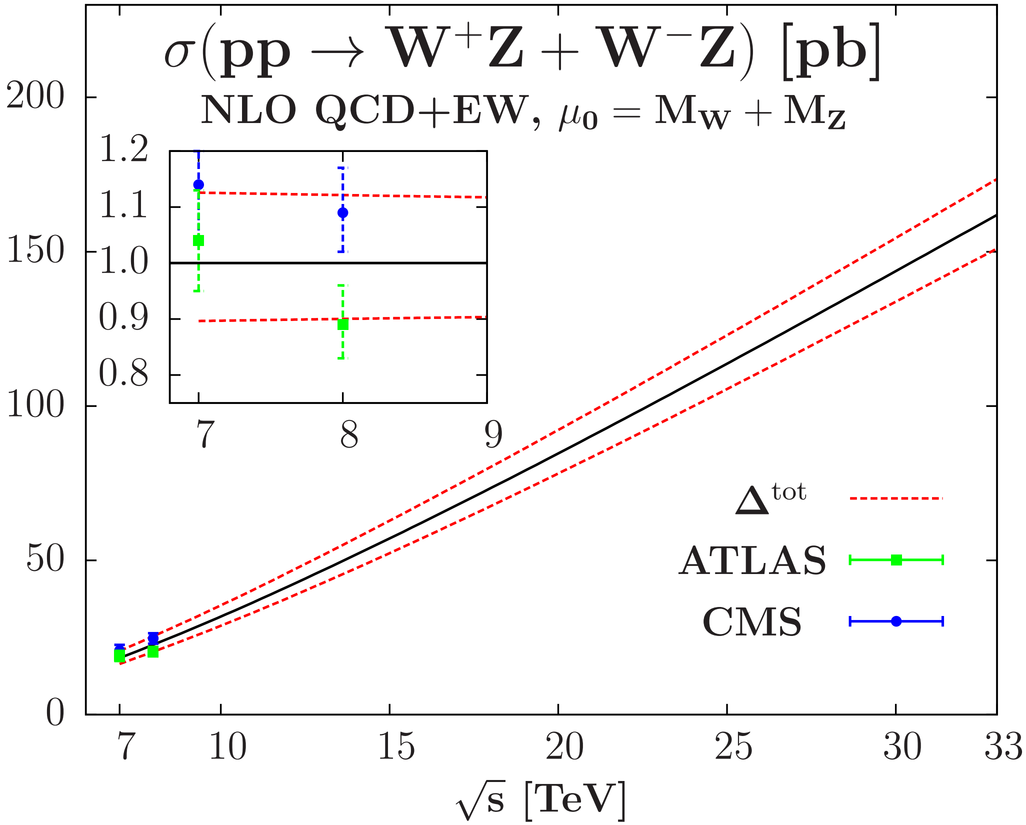
Cascioli et al. '14



- good agreement of experimental results with NNLO QCD
- NNLO QCD correction $\sim 12(17)\%$ @ $8(13)\,\mathrm{TeV}$, scale uncertainty $\lesssim 3\%$
- gg contribution $\sim 7(10)\%$ @ $8(13)\,\mathrm{TeV}$
- LHC run 2: higher energy & higher statistics $\rightarrow$ EW corrections important

WZ production – NLO QCD theory versus experiment

Baglio, Le, Weber et al. '13

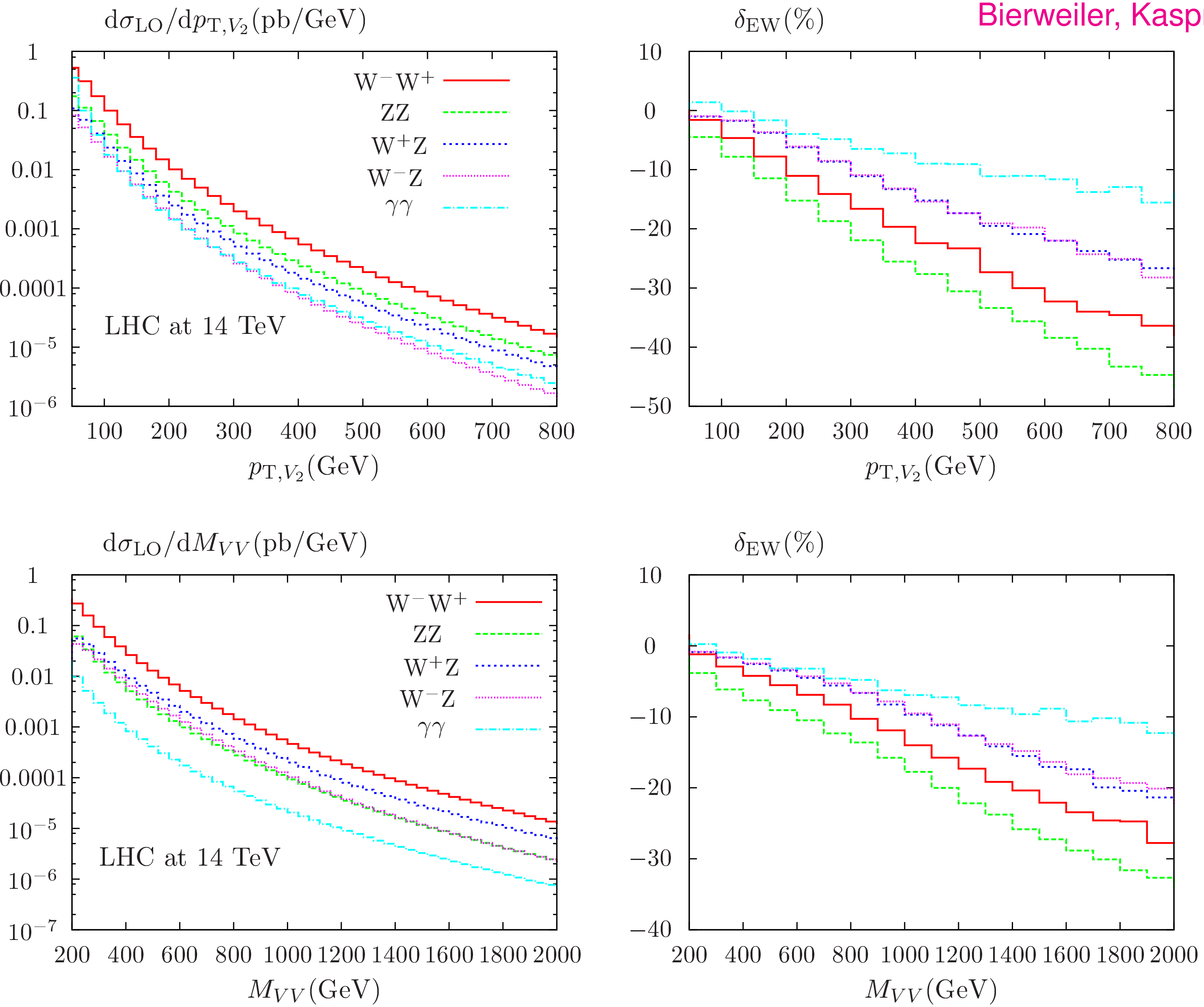


- good agreement of experimental results with NLO QCD
- NLO QCD scale uncertainty $\sim 3\%$, $\quad \Delta_{\mathrm{PDF}+\alpha_s} \sim 4\%$
- LHC run 2: higher energy & higher statistics

  $\hookrightarrow$ NNLO QCD and NLO EW corrections important

EW corrections to massive di-boson production (stable/on-shell W bosons)

Bierweiler, Kasprzik, Kühn '13



EW corrections

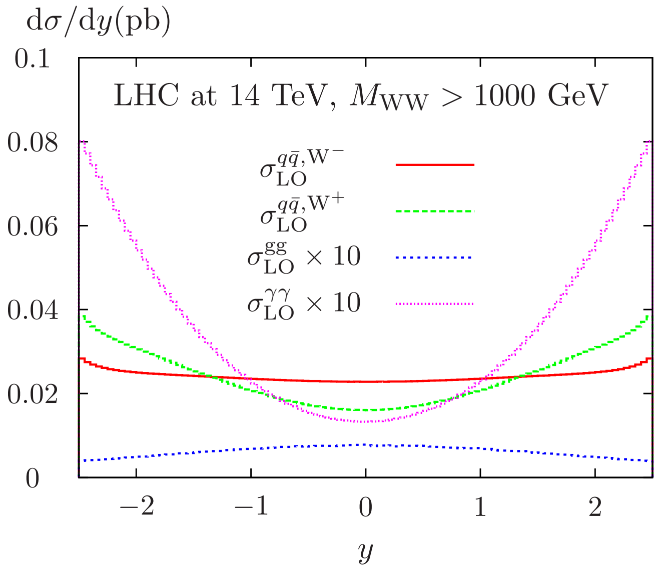
- small for integrated XS
- growing in distributions for larger scales

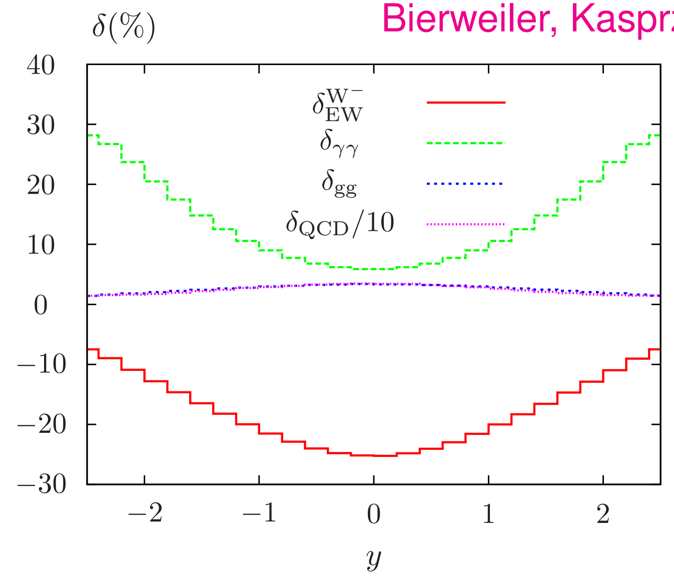
Note:

- M_{VV} not accessible for W final states
- on-shell approximation not applicable for $M_{VV} < M_{V_1} + M_{V_2}$

Survey of corrections to WW production (stable/on-shell W bosons)

Bierweiler, Kasprzik, Kühn '12

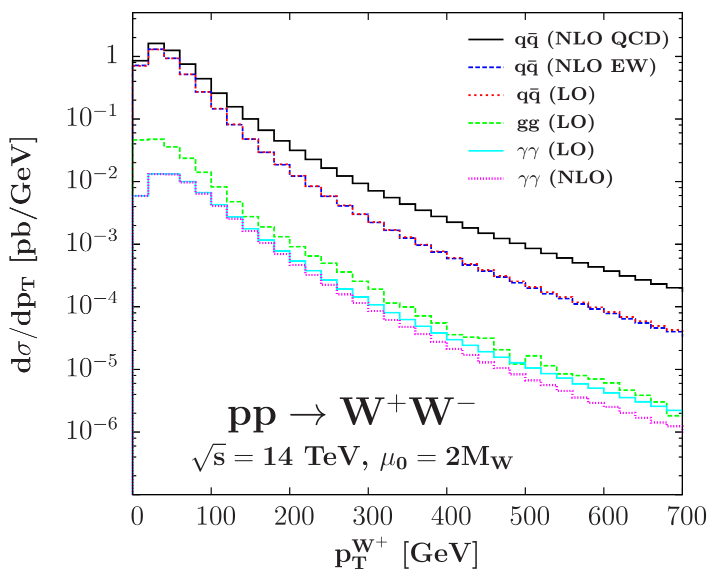


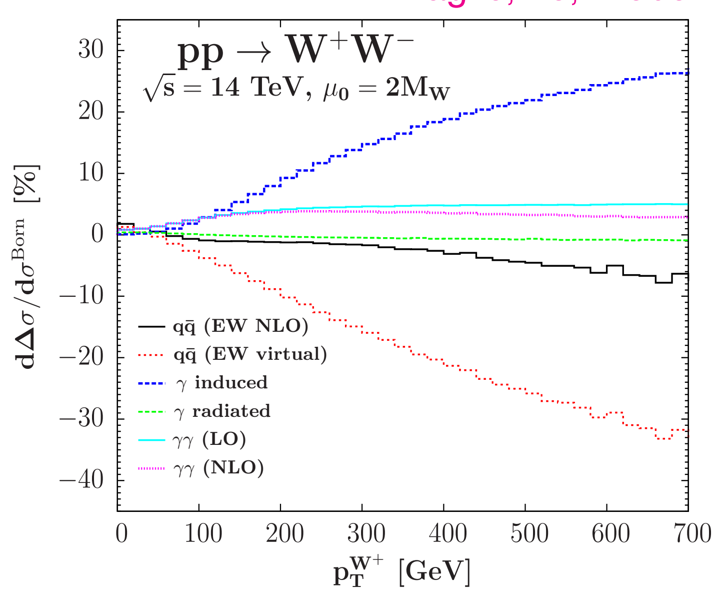


Note:

large contribution by $\gamma\gamma$ channel for high invariant WW masses !

Baglio, Le, Weber '13

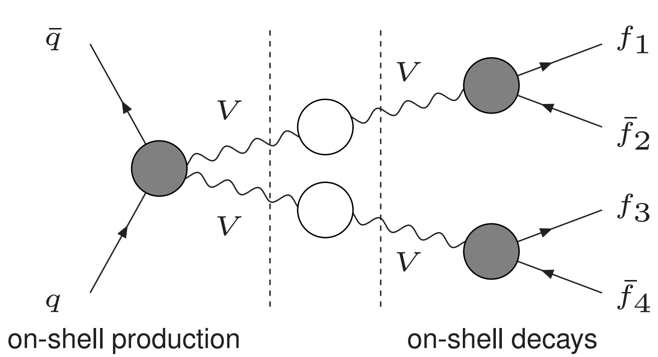


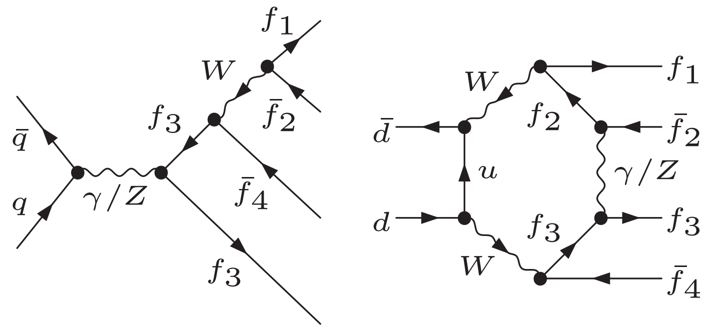


Large impact of $q\gamma$ collisions ?

EW corrections with leptonic W/Z decays

Double-pole approximation (DPA) vs. Full off-shell $q\bar{q} \to 4f$ calculation





- expansion about resonance poles
  $\hookrightarrow$ factorizable & non-factorizable corrs.
- not many diagrams ($2 \to 2$ production)

\+ numerically fast

– validity only for $\sqrt{\hat{s}} > 2M_V + \mathcal{O}(\Gamma_V)$

- error estimate for $\sqrt{\hat{s}} \lesssim 0.5{-}1\,\mathrm{TeV}$:
  $\Delta \sim \frac{\alpha}{\pi}\frac{\Gamma_V}{M_V}\log(\ldots) \sim 0.5{-}2\%$

- off-shell calculation with complex-mass scheme
- many off-shell diagrams ($\sim 10^3$/channel)

– CPU intensive

\+ NLO accuracy everywhere

- global error estimate:
  $\Delta \sim \delta_{\mathrm{NNLO\,EW}} \sim \delta^2_{\mathrm{NLO\,EW}}$

Approaches compared for $\mathrm{e^+e^-} \to \mathrm{WW} \to 4f$ Denner, S.D., Roth, Wieders '05

New: $\mathrm{pp} \to \mathrm{WW} \to 4f$ Biedermann et al. '16

Details of the full $4f$ NLO calculation Biedermann et al. '16

Virtual corrections

- one version diagrammatically as for $\mathrm{e^+e^-} \to \mathrm{WW} \to 4f$ Denner et al. '05
- another version based on recursive method with RECOLA Actis et al. '13
- some checks done with FEYNARTS/FORMCALC in the framework of POLE Accomando et al. '05
- $\mathrm{W/Z}$ resonances treated in the *complex-mass scheme*
- loop integrals evaluated with COLLIER

Real corrections and Monte Carlo integration

- IR singularities treated with dipole subtraction Catani, Seymour '96; S.D. '99; S.D. et al. '08
- collinear-unsafe ("bare") and "dressed" leptons supported
- multi-channel Monte Carlo integration

γ-induced contributions

- $\gamma\gamma$ collisions included in LO (small contributions)
- $q\gamma$ contributions taken into account

Two independent calculations of all ingredients

Details of

Virtual co

- one ve
- anoth
- some
- $\mathrm{W/Z}$ r
- loop in

Real corre

- IR sing
- colline
- multi-c

γ-induced

- $\gamma\gamma$ col
- $q\gamma$ con

Two indep

'05

'13

POLE

et al. '05

'99; S.D. et al. '08

Collier is hosted by Hepforge, IPPP Durham

COLLIER

A Complex One-Loop LIbrary with Extended Regularizations

Released on April 25!

Authors

Ansgar Denner *Universität Würzburg, Germany*

Stefan Dittmaier *Universität Freiburg, Germany*

Lars Hofer *Universitat de Barcelona, Spain*

Features of the library

COLLIER is a fortran library for the numerical evaluation of one-loop scalar and tensor integrals appearing in perturbative relativistic quantum field theory with the following features:

- scalar and tensor integrals for high particle multiplicities
- dimensional regularization for ultraviolet divergences
- dimensional regularization for soft infrared divergences (mass regularization for abelian soft divergences is supported as well)
- dimensional regularization or mass regularization for collinear mass singularities
- complex internal masses (for unstable particles) fully supported (external momenta and virtualities are expected to be real)
- numerically dangerous regions (small Gram or other kinematical determinants) cured by dedicated expansions
- two independent implementations of all basic building blocks allow for internal cross-checks
- cache system to speed up calculations

If you use Collier for a publication, please cite all the references listed here!

Details of the full $4f$ NLO calculation Biedermann et al. '16

Virtual corrections

- one version diagrammatically as for $\mathrm{e^+e^-} \to \mathrm{WW} \to 4f$ Denner et al. '05
- another version based on recursive method with RECOLA Actis et al. '13
- some checks done with FEYNARTS/FORMCALC in the framework of POLE Accomando et al. '05
- $\mathrm{W/Z}$ resonances treated in the *complex-mass scheme*
- loop integrals evaluated with COLLIER

Real corrections and Monte Carlo integration

- IR singularities treated with dipole subtraction Catani, Seymour '96; S.D. '99; S.D. et al. '08
- collinear-unsafe ("bare") and "dressed" leptons supported
- multi-channel Monte Carlo integration

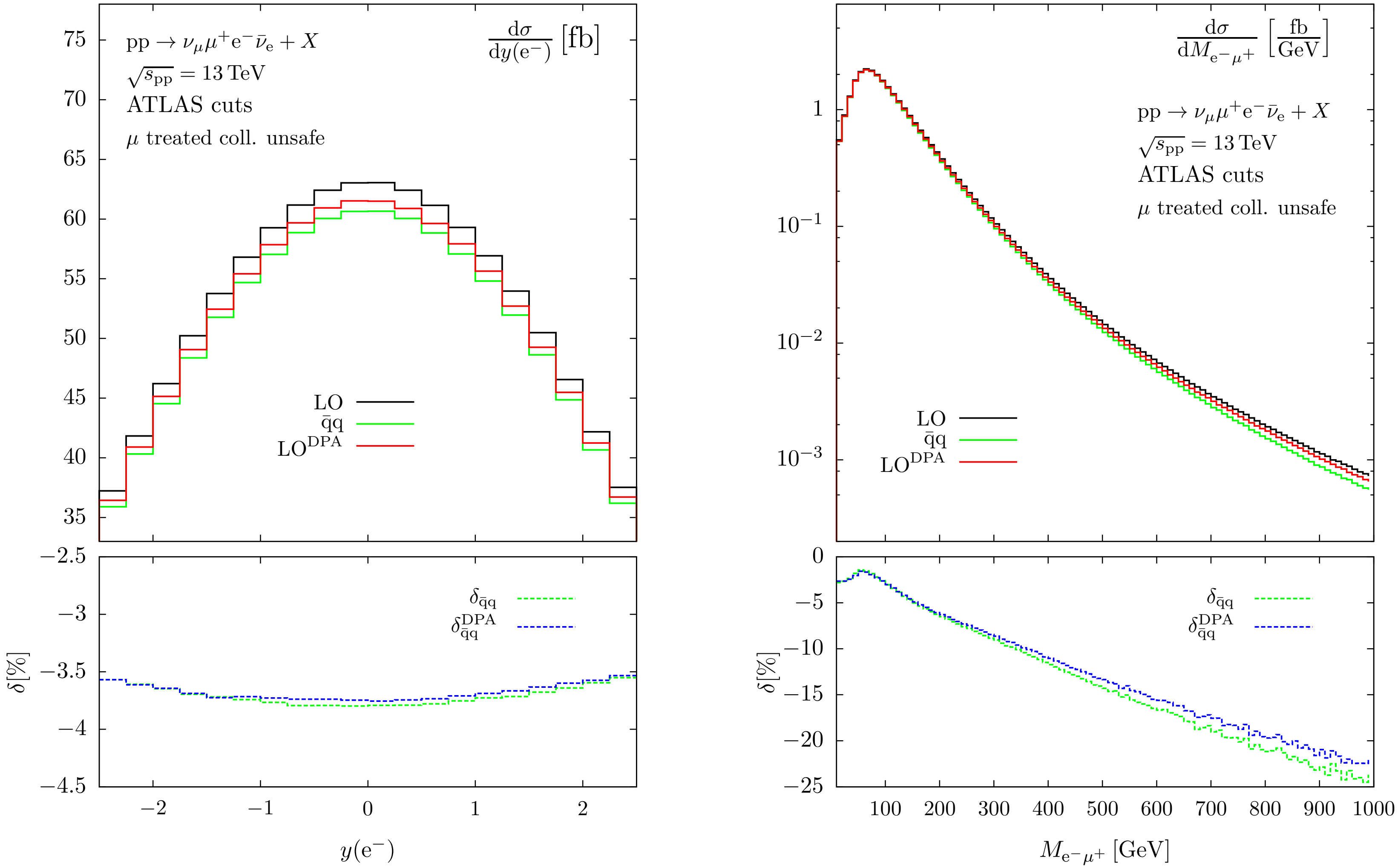
γ-induced contributions

- $\gamma\gamma$ collisions included in LO (small contributions)
- $q\gamma$ contributions taken into account

Two independent calculations of all ingredients

DPA versus full off-shell EW correction in $\mathrm{pp} \to \nu_\mu \mu^+ \mathrm{e}^- \bar{\nu}_\mathrm{e} + X$ Biedermann et al. '16

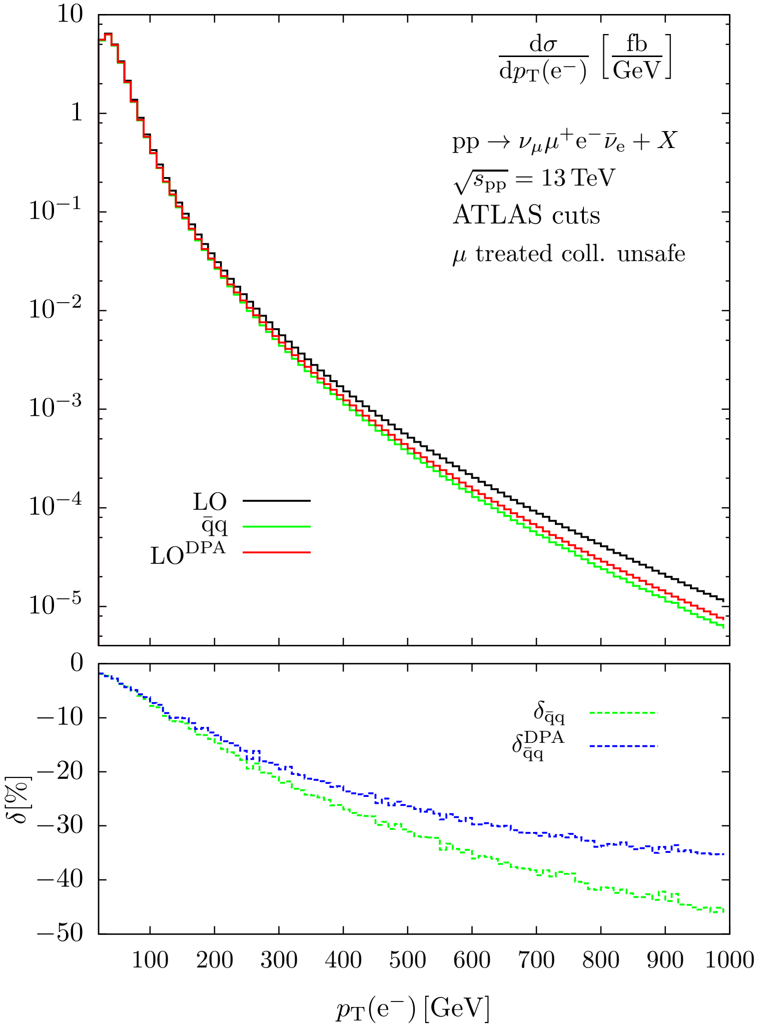
Rapidity and invariant-mass distributions

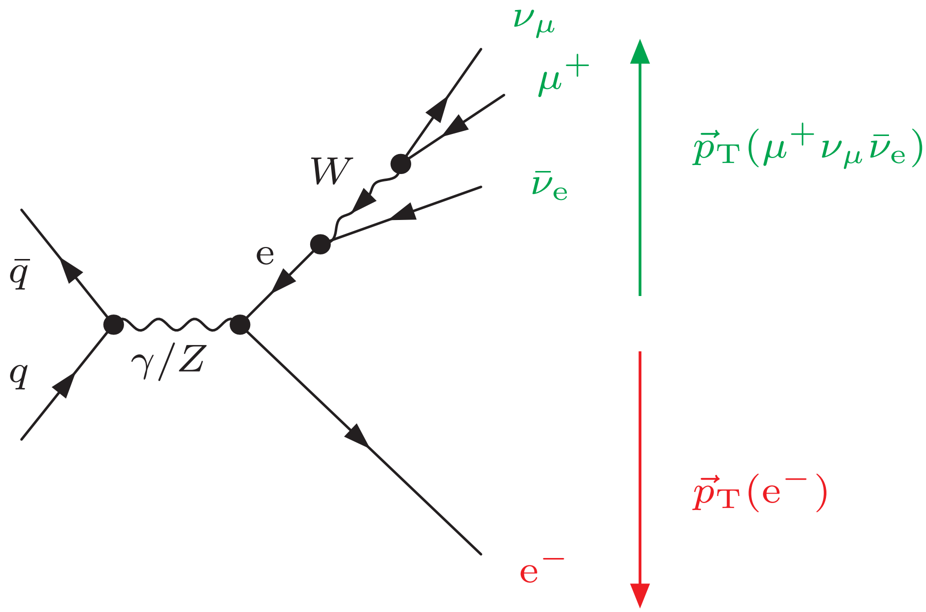


Level of agreement as expected (dominance of doubly-resonant diagrams)

DPA versus full off-shell EW correction in $\mathrm{pp} \to \nu_\mu \mu^+ \mathrm{e}^- \bar{\nu}_\mathrm{e} + X$ Biedermann et al. '16

Transverse-momentum distribution of a single lepton



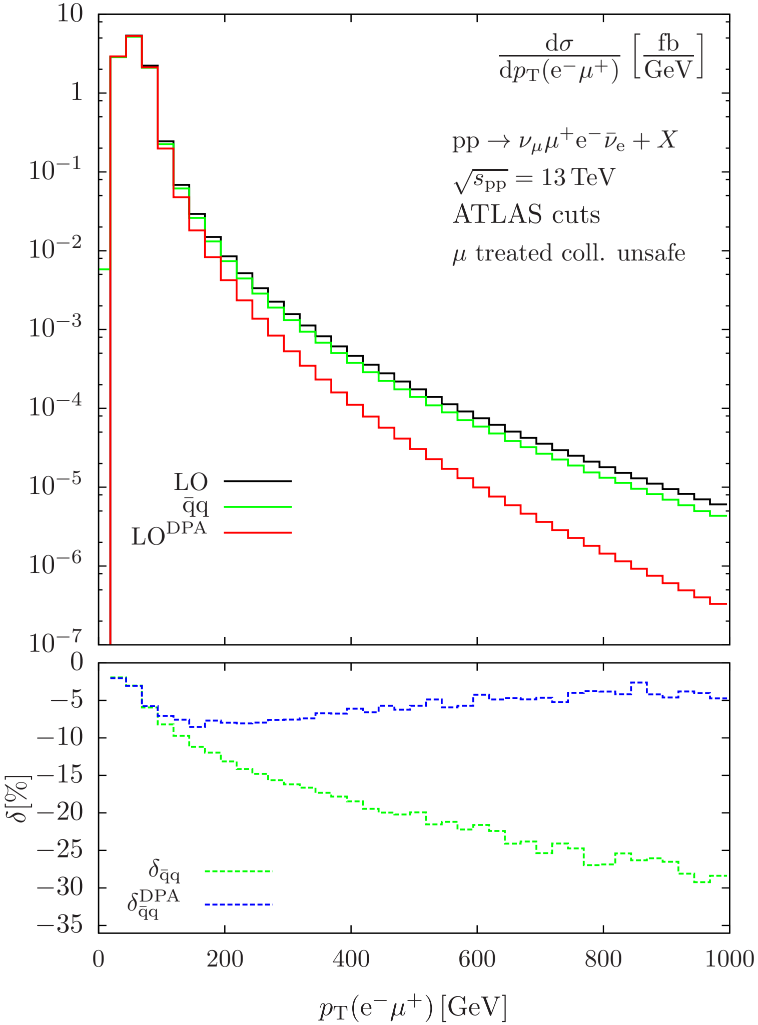


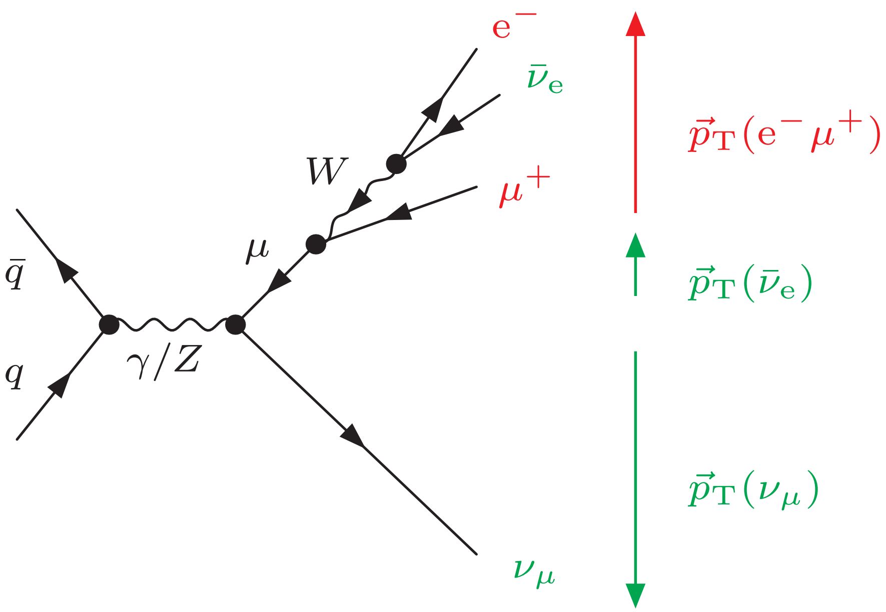
Impact of singly-resonant diagrams where e^- takes recoil from $(\mu^+\nu_\mu\bar{\nu}_\mathrm{e})$

Agreement degrades for $p_\mathrm{T} \gtrsim 300\,\mathrm{GeV}$, since off-shell diagrams get enhanced

DPA versus full off-shell EW correction in $\mathrm{pp} \to \nu_\mu \mu^+ \mathrm{e}^- \bar{\nu}_\mathrm{e} + X$ Biedermann et al. '16

Transverse-momentum distribution of the charged lepton pair





- Double resonance extremely suppressed !
- Dominance of singly-resonant diagrams where $(\mathrm{e}^-\mu^+)$ recoil against $(\nu_\mu \bar{\nu}_\mathrm{e})$

DPA fails for $p_\mathrm{T} \gtrsim 200\,\mathrm{GeV}$, since off-shell production dominates!

Gauge-invariance issues in EW multi-boson production

Gauge invariance implies...

- Slavnov–Taylor or Ward identites
  = algebraic relations of or between Greens functions
  $\hookrightarrow$ guarantee cancellation of unitarity-violating terms,
  crucial for proof of unitarity of S-matrix
- Nielsen identities (compensation of gauge-fixing artefacts)
  $\hookrightarrow$ gauge-parameter independence of S-matrix
  although Greens function (e.g. self-energies) are gauge dependent

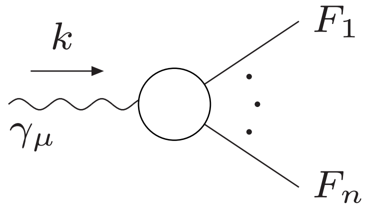
Both statements hold order by order in standard perturbation theory !

Implications:

- Resonances require Dyson summation of resonant propagators
  $\hookrightarrow$ perturbative orders mixed $\to$ gauge invariance jeopardized !
  Gauge-invariance-violating terms $\propto \Gamma$ are formally of higher order,
  but can be dramatically enhanced if unitarity cancellations disturbed
- Anomalous couplings potentially enhanced
  if effective operator not gauge invariant

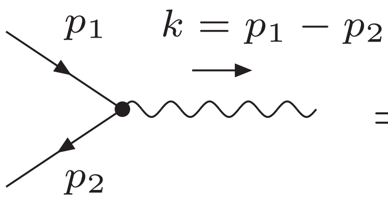
Important Ward identities for processes with EW gauge bosons:

Elmg. U(1) gauge invariance implies

k^μ 

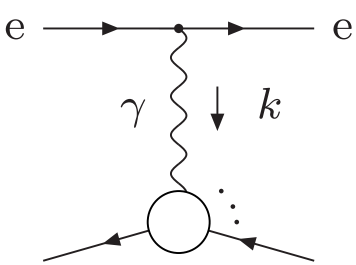
$= \quad 0$ for any on-shell fields F_l

$\hookrightarrow$ Identity becomes crucial for collinear light fermions:

for fermion momenta $p_1 \sim c\, p_2$: 

$= \bar{u}_2(p_2)\gamma^\mu u_1(p_1) \;\propto\; k^\mu$

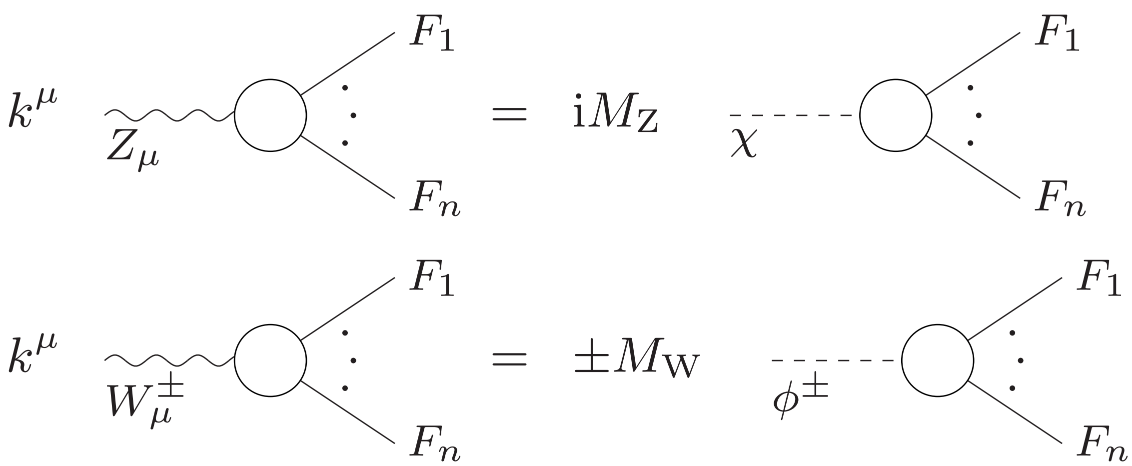
A typical situation: quasi-real space-like photons



$\sim \dfrac{1}{k^2}\, k^\mu\, T^\gamma_\mu$ for $k^2 \to \mathcal{O}(m_\mathrm{e}^2) \ll E^2$

Identity $k^\mu\, T^\gamma_\mu = 0$ needed to cancel $1/k^2$,
otherwise gauge-invariance-breaking terms enhanced by E^2/m_e^2 ($\sim 10^{10}$ for LEP2)

Electroweak SU(2) gauge invariance implies

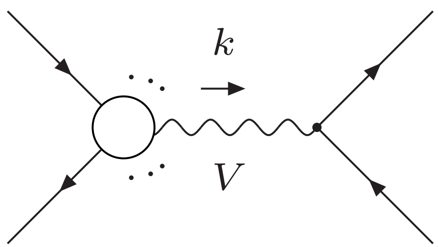


F_l = on-shell fields

$\chi, \phi^\pm$ = would-be Goldstone fields

A typical situation: high-energetic quasi-real longitudinal vector bosons

$\hookrightarrow$ fermion current attached to $\mathrm{V}(k)$ again $\propto k^\mu$



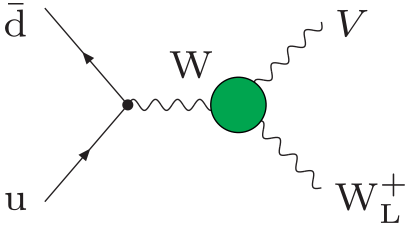
$$\sim \frac{1}{k^2 - M_\mathrm{V}^2} \, k^\mu \, T^V_\mu \qquad \text{for} \quad k^0 \gg M_\mathrm{V}$$

Identity $k^\mu T^V_\mu = c_V M_\mathrm{V} T^S$ needed to cancel factor k^0,
otherwise gauge-invariance/unitarity-breaking terms enhanced by k^0/M_V

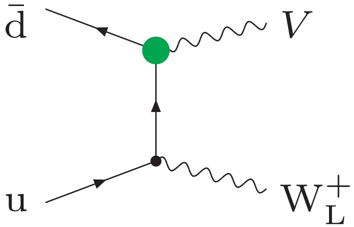
For on-shell V: $\quad \varepsilon^\mu_{V_\mathrm{L}}(k) = \dfrac{k^\mu}{M_V} + \mathcal{O}(M_V/k^0)$

Illustration of unitarity cancellations for $\mathrm{W}V$ production ($V = \mathrm{Z}/\gamma$)

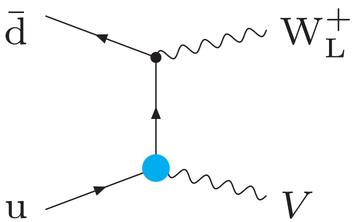
Leading behaviour of amplitudes with $\varepsilon^{\mu}_{\mathrm{W}^+_\mathrm{L}}(k) = \dfrac{k^\mu}{M_V} + \dots$ for $k^0 \gg M_\mathrm{W}$:



$$\sim \frac{-\mathrm{i}e^2 g_{VWW}}{2\sqrt{2} s_\mathrm{W} M_\mathrm{W}} \left[\bar{v}_{\bar{\mathrm{d}}}\gamma_\mu \omega_- u_\mathrm{u}\right] \left\{ g_1^V \left[\varepsilon_V^{*\mu} - k^\mu \frac{\varepsilon_V^* \cdot k}{s}\right] + \kappa_V \left[\varepsilon_V^{*\mu} + k^\mu \frac{\varepsilon_V^* \cdot k}{s}\right]\right\}$$



$$\sim \frac{\mathrm{i}e^2 g^-_{Vdd}}{\sqrt{2} s_\mathrm{W} M_\mathrm{W}} \left[\bar{v}_{\bar{\mathrm{d}}} \not{\varepsilon}^*_V \omega_- u_\mathrm{u}\right], \qquad g^-_{Zdd} = -\frac{s_\mathrm{W}}{c_\mathrm{W}} Q_\mathrm{d} - \frac{1}{2 s_\mathrm{W} c_\mathrm{W}}, \qquad g^-_{\gamma dd} = -Q_\mathrm{d}$$



$$\sim \frac{-\mathrm{i}e^2 g^-_{Vuu}}{\sqrt{2} s_\mathrm{W} M_\mathrm{W}} \left[\bar{v}_{\bar{\mathrm{d}}} \not{\varepsilon}^*_V \omega_- u_\mathrm{u}\right], \qquad g^-_{Zuu} = -\frac{s_\mathrm{W}}{c_\mathrm{W}} Q_\mathrm{u} + \frac{1}{2 s_\mathrm{W} c_\mathrm{W}}, \qquad g^-_{\gamma uu} = -Q_\mathrm{u}$$

Cancellation (unitarity!) of sum demands:

$$g^-_{Vdd} - g^-_{Vuu} - \frac{g_{VWW}}{2}\left(g_1^V + \kappa_V\right) \overset{!}{=} 0, \qquad g_1^V \overset{!}{=} \kappa_V$$

$\hookrightarrow$ SM provides unique solution: $\quad g_1^\mathrm{Z} = \kappa_\mathrm{Z} = g_1^\gamma = \kappa_\gamma = 1$

Note: no constraint on coupling λ_V, since effective operator gauge invariant !

Width schemes for LO calculations and gauge invariance

Naive propagator substitutions in full tree-level amplitudes:

$$\frac{1}{k^2-m^2} \to \frac{1}{k^2-m^2+\mathrm{i}m\Gamma(k^2)} \qquad \text{in all propagators}$$

- constant width $\Gamma(k^2) = \text{const.}$ $\to$ U(1) respected, SU(2) “mildly” violated
- running width $\Gamma(k^2) \neq \text{const.}$ $\to$ U(1) and SU(2) violated

  $\hookrightarrow$ results can be totally wrong !

Fudge factor approaches:

Multiply full amplitudes without widths with

factors $\dfrac{p^2-m^2}{p^2-m^2+\mathrm{i}m\Gamma}$ for each potentially resonant propagator

$\hookrightarrow$ gauge invariant, but spurious factors of $\mathcal{O}(\Gamma/m)$

Complex-mass scheme: (see lecture 1)

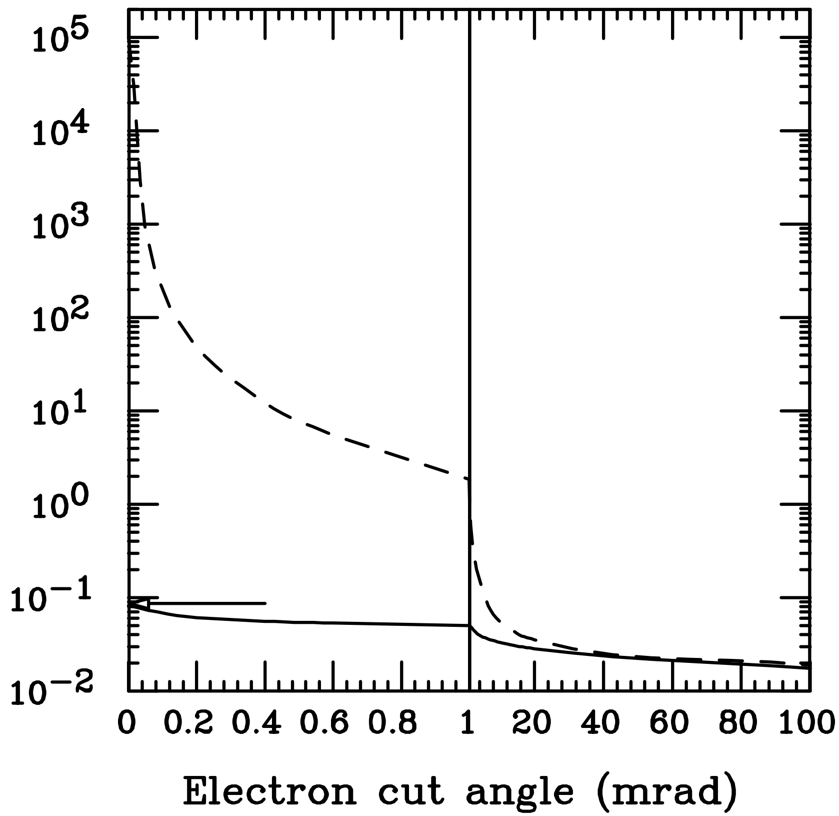
Consistent use of complex masses everywhere (including couplings)

For W/Z bosons: $M_V^2 \to \mu_V^2 = M_V^2 - \mathrm{i}M_V\Gamma_V, \quad V = \mathrm{W}, \mathrm{Z}$

complex weak mixing angle: $c_\mathrm{W}^2 = 1 - s_\mathrm{W}^2 = \dfrac{\mu_\mathrm{W}^2}{\mu_\mathrm{Z}^2}$

$\hookrightarrow$ gauge invariance fully respected

An example: $e^-e^+ \to e^-\bar{\nu}_e u\bar{d}$ result of Kurihara, Perret-Gallix, Shimizu '95



$\sqrt{s} = 180\,\text{GeV}$

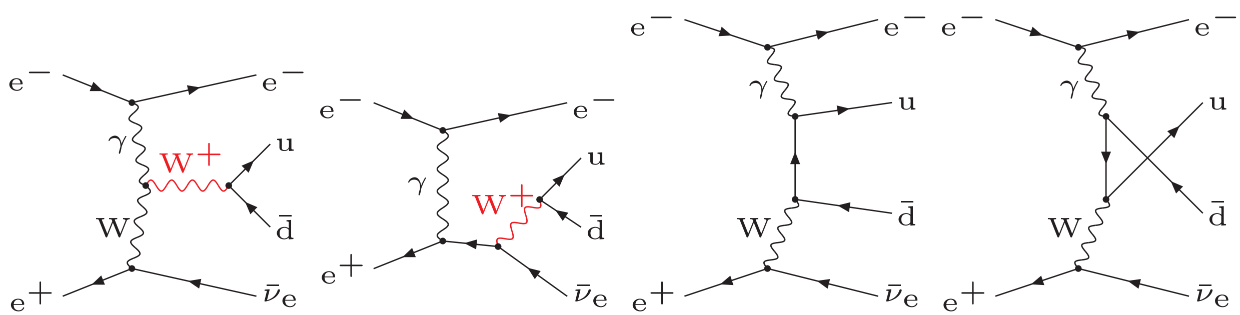
solid: gauge-invariant (fudge factor) scheme

dashed: constant width only in resonant propagator

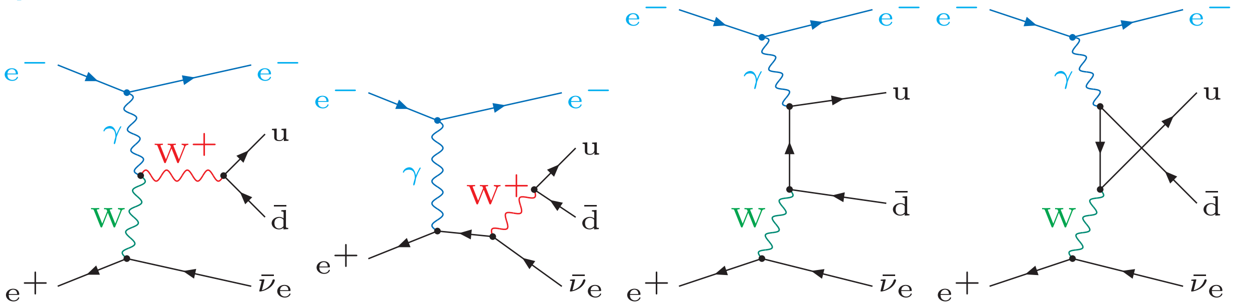
$\hookrightarrow$ crude U(1) gauge-invariance violation

Dominant diagrams:

nearly real photon !



Example continued:



Partial amplitude from above "photon diagrams":

$$\mathcal{M}_\gamma = Q_e e\, \bar{u}_e(k_e)\gamma^\mu u_e(p_e)\, \frac{1}{k_\gamma^2}\, T_\mu^\gamma$$

Elmg. Ward identity:

$$0 \stackrel{!}{=} k_\gamma^\mu T_\mu^\gamma \propto (p_+^2 - p_-^2) Q_W P_W(p_+^2) P_W(p_-^2) + Q_e P_W(p_+^2) - (Q_d - Q_u) P_W(p_-^2)$$

With $Q_W = Q_e = Q_d - Q_u$ and $P_W(p^2) = [p^2 - M_W^2 + iM_W\Gamma_W(p^2)]^{-1}$

one obtains: $\Gamma_W(p_+^2) \stackrel{!}{=} \Gamma_W(p_-^2)$

↪ Elmg. gauge invariance demands
common width on s- and t-channel propagators in "naive fixed width scheme"

Examples from e^+e^- physics: RacoonWW (Denner et al. '99-'01) and Lusifer (S.D.,Roth '02)

- σ[fb] for $e^+e^- \to u\bar{d}\mu^-\bar{\nu}_\mu$

$\sqrt{s}$	189 GeV	500 GeV	2 TeV	10 TeV
constant width	703.5(3)	237.4(1)	13.99(2)	0.624(3)
running width	703.4(3)	238.9(1)	34.39(3)	498.8(1)
complex mass	703.1(3)	237.3(1)	13.98(2)	0.624(3)

- σ[fb] for $e^+e^- \to u\bar{d}\mu^-\bar{\nu}_\mu + \gamma$ (separation cuts for "visible" γ: $E_\gamma, \theta_{\gamma f} >$ cut)

$\sqrt{s} =$	189 GeV	500 GeV	2 TeV	10 TeV
constant width	224.0(4)	83.4(3)	6.98(5)	0.457(6)
running width	224.6(4)	84.2(3)	19.2(1)	368(6)
complex mass	223.9(4)	83.3(3)	6.98(5)	0.460(6)

- σ[fb] for $e^+e^- \to \nu_e\bar{\nu}_e\mu^-\bar{\nu}_\mu u\bar{d}$ (phase-space cuts applied)

$\sqrt{s}$	500 GeV	800 GeV	2 TeV	10 TeV
constant width	1.633(1)	4.105(4)	11.74(2)	26.38(6)
running width	1.640(1)	4.132(4)	12.88(1)	12965(12)
complex mass	1.633(1)	4.104(3)	11.73(1)	26.39(6)

Gauge-invariant width schemes @ NLO

Problem much more complicated than at LO ! (would fill own lectures)

Complex-Mass Scheme (CMS) Denner, S.D., Roth, Wieders '05

- complex, but straightforward renormalization
- NLO everywhere in phase space
- loop integrals with complex masses

Pole Approximation (PA) (= leading term of pole expansion)

- corrections decomposed into two types
  - ◇ factorizable: corrections to on-shell production / decay
  - ◇ non-factorizable: soft photon/gluon exchange between production / decays
- NLO in neighbourhood of resonances
- PA involves less diagrams than CMS $\rightarrow$ higher multiplicities possible

Effective Field Theories Beneke et al. '03,'04; Hoang,Reisser '04

- involves pole expansions $\rightarrow$ NLO in neighbourhood of resonances
- formal elegance $\rightarrow$ e.g. combination with resummations

$\hookrightarrow$ **For details & examples see literature ...**