Electroweak Physics at the LHC

— Lecture 1 —

Electroweak Issues and Higher-Order Corrections

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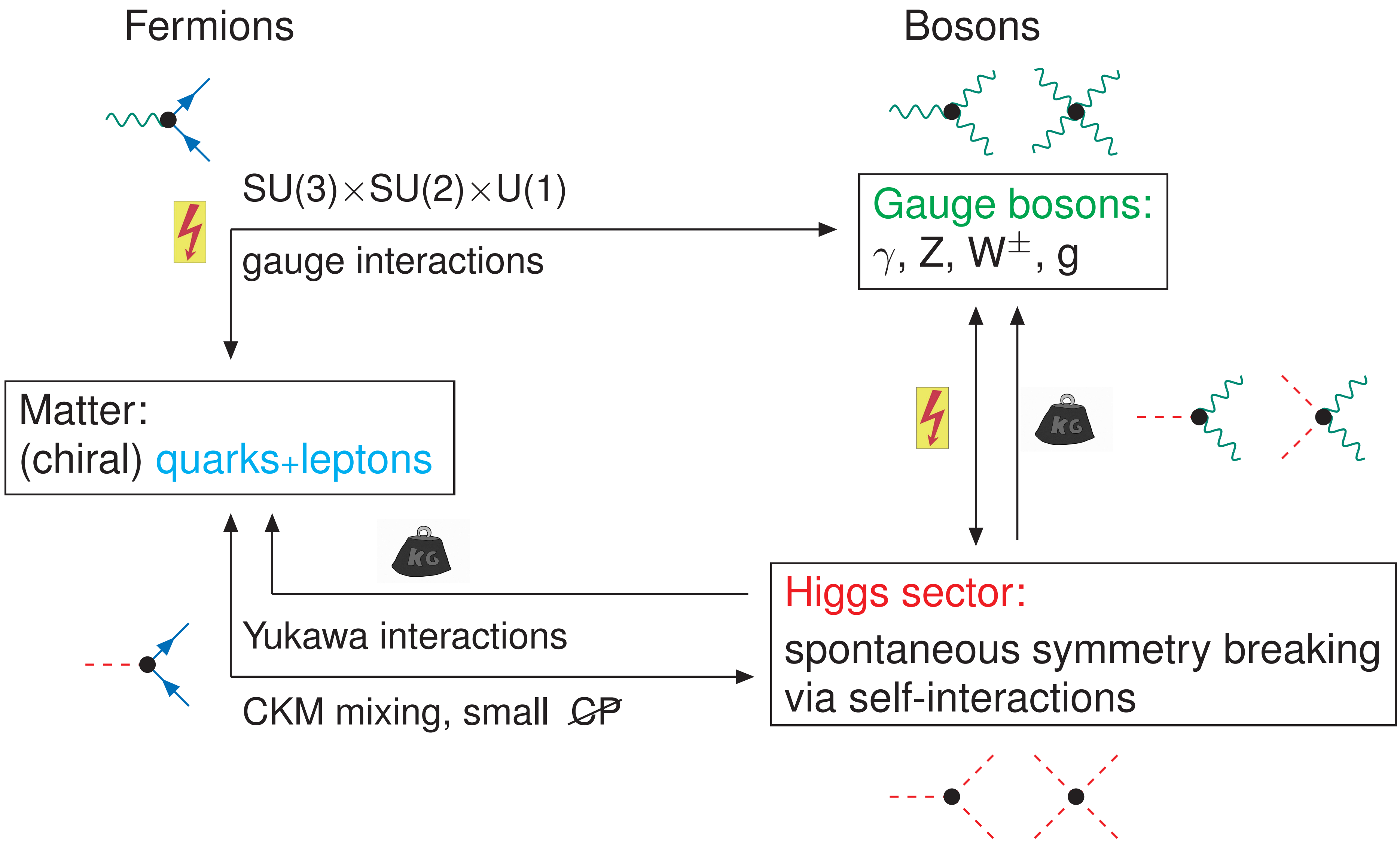
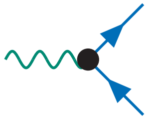
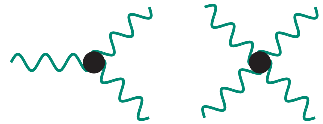
Recapitulation of the Standard Model

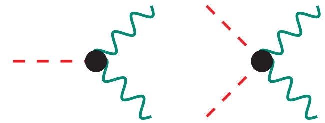
Structure and elementary interactions of the Standard Model

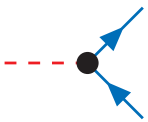


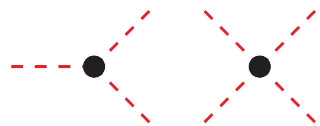




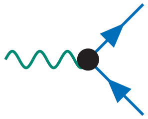
Test of the model
$\Leftrightarrow$ Exp. reconstruction of the $\underbrace{\text{elementary couplings}}_{\text{Feynman rules}}$

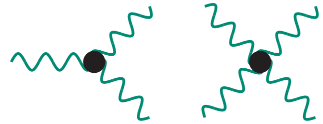






Structure and elementary interactions of the Standard Model

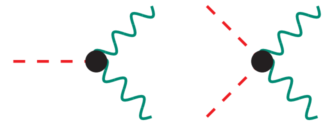


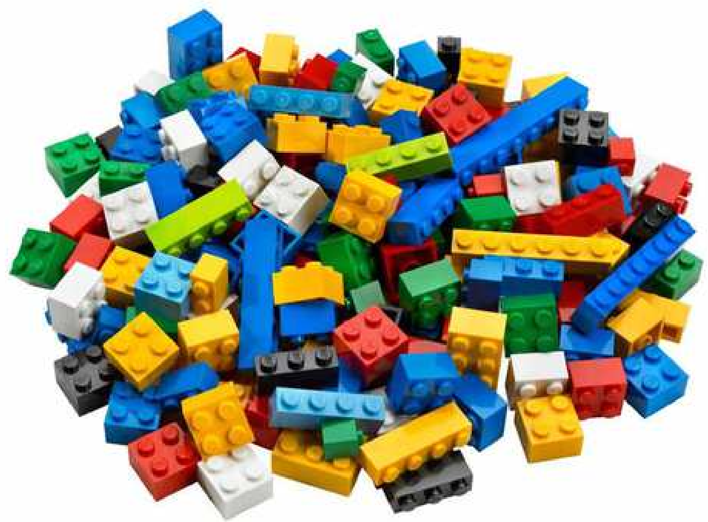


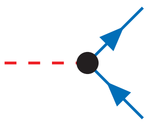
Test of the model
$\Leftrightarrow$ Exp. reconstruction of the elementary couplings

Feynman rules

Building blocks for particle reactions







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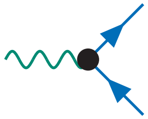

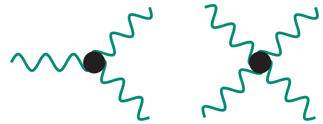

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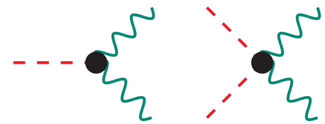
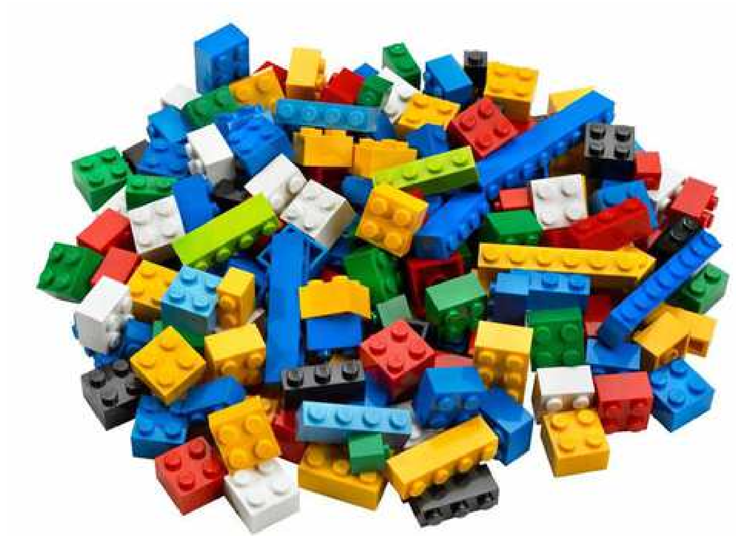
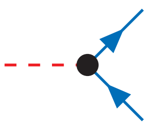
Feynman rules

Building blocks for particle reactions







Standard Model extensions $\rightarrow$ more fields, more particles, more interactions, ...

Feynman rules derived from SM Lagrangian:

$\hookrightarrow$ Recapitulate EW gauge interactions !

Gauge-boson couplings to fermions

$\hookrightarrow$ induced by "minimal substitution" in free Lagrangian $\mathcal{L}_{0,\mathrm{ferm}} = \sum_f \mathrm{i}\overline{\psi_f}\partial\!\!\!/\psi_f$:

$$\partial_\mu \to D_\mu \;=\; \partial_\mu - \mathrm{i}g_2 T_\mathrm{I}^a W_\mu^a + \mathrm{i}g_1 \frac{Y}{2} B_\mu$$

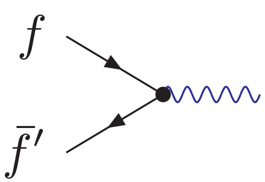
$$T_\mathrm{I}^a \;=\; \text{weak isopsin} = \begin{cases} \sigma^a/2 & \text{for left-handed } f \\ 0 & \text{for right-handed } f \end{cases}$$

$Y \;=\;$ weak hypercharge, fixed by Gell-Mann–Nishijima relation $Q = T_\mathrm{I}^3 + Y/2$

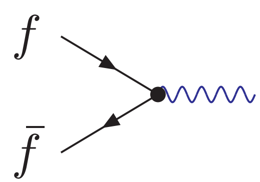
Identification of photon after "Weinberg rotation" about weak mixing angle θ_W:

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} c_\mathrm{W} & s_\mathrm{W} \\ -s_\mathrm{W} & c_\mathrm{W} \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} \quad \text{with } g_2 = \frac{e}{s_\mathrm{W}},\; g_1 = \frac{e}{c_\mathrm{W}}, \quad s_\mathrm{W} \equiv \sin\theta_\mathrm{W}$$

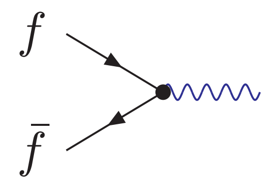
$\Rightarrow$ Interaction vertices:



$W_\mu \quad \dfrac{\mathrm{i}e}{\sqrt{2}s_\mathrm{W}}\gamma_\mu \frac{1}{2}(1-\gamma_5)$

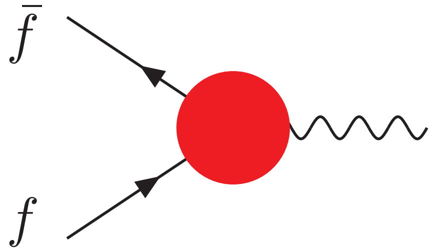


$A_\mu \quad -\mathrm{i}Q_f e\gamma_\mu$



$Z_\mu \quad \mathrm{i}e\gamma_\mu(g_{Vf} - g_{Af}\gamma_5), \qquad g_{Vf} = -\dfrac{s_\mathrm{W}}{c_\mathrm{W}}Q_f + \dfrac{T_{\mathrm{I},f}^3}{2c_\mathrm{W}s_\mathrm{W}}, \quad g_{Af} = \dfrac{T_{\mathrm{I},f}^3}{2c_\mathrm{W}s_\mathrm{W}}$

Effective $Zf\bar{f}$ couplings from $e^+e^- \to Z/\gamma^* \to f\bar{f}$ @ LEP1



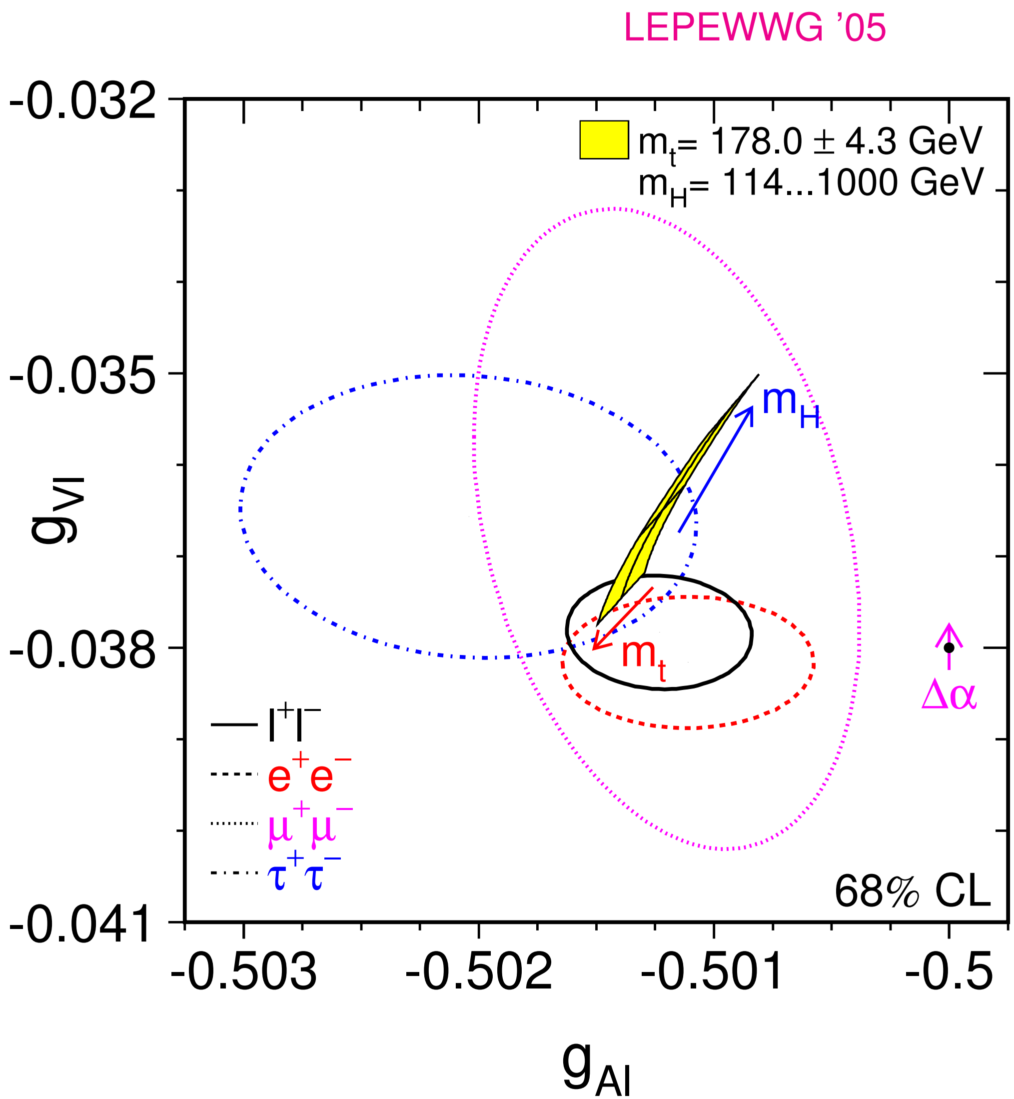
$$Z_\mu \quad = \quad ie\gamma_\mu(\bar{g}_{V\,f} - \bar{g}_{A\,f}\gamma_5)$$

Leptonic couplings from LEP1 asymmetry measurements, e.g.:

$$A_{\mathrm{FB}}^{0,f} = \frac{\sigma^0_{f,\mathrm{F}} - \sigma^0_{f,\mathrm{B}}}{\sigma^0_{f,\mathrm{F}} + \sigma^0_{f,\mathrm{B}}} = \frac{3}{4}\mathcal{A}_{\mathrm{e}}\mathcal{A}_f$$

(F/B = For/Backward hemisphere)

with $\mathcal{A}_f = \dfrac{2\bar{g}_{V\,f}\bar{g}_{A\,f}}{\bar{g}^2_{V\,f} + \bar{g}^2_{A\,f}}$



Good agreement with SM

- lepton universality confirmed
- constraints on m_t and M_H

Translation of effective couplings into effective weak mixing angle

$$\sin^2\theta_{\text{eff}}^{\text{lept}} = \frac{1}{4}\left(1 - \text{Re}\left\{\frac{g_{Vl}}{g_{Al}}\right\}\right)$$

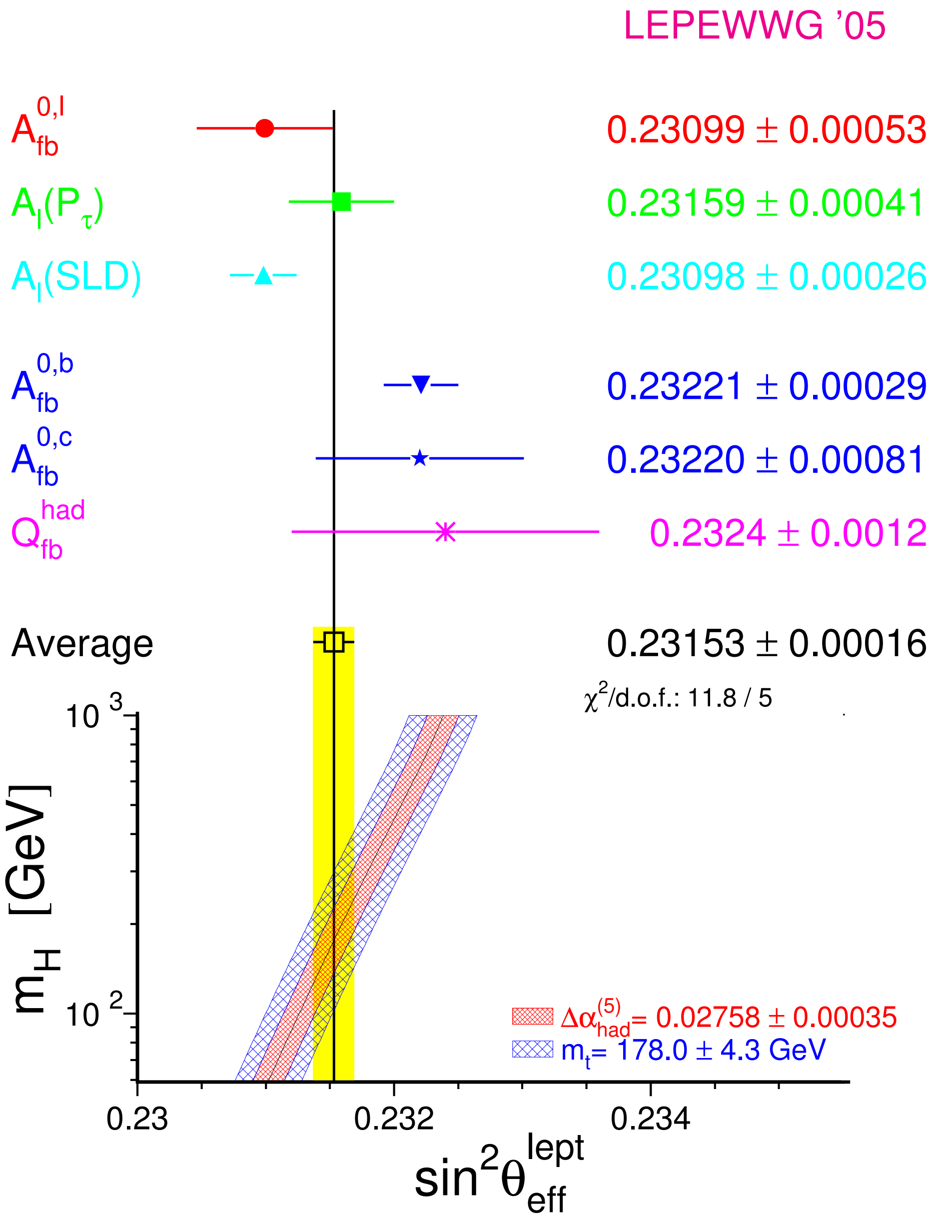
Important features:

- high sensitivity to M_{H}
- combination of very different observables
- $\sim 3\sigma$ difference between $A_{\text{FB}}^{0,b}$(LEP) and $A_{\text{LR}}^{0,l}$(SLD)

with the initial-state pol. asymmetry

$$A_{\text{LR}}^{0,l} = \frac{\sigma_{\text{L}}^0 - \sigma_{\text{R}}^0}{\sigma_{\text{L}}^0 + \sigma_{\text{R}}^0}\,\frac{1}{\langle|\mathcal{P}_{\text{e}}|\rangle}$$

$\Rightarrow$ Precise LHC result on $\sin^2\theta_{\text{eff}}^{\text{lept}}$ highly desirable !



Gauge-boson self-interactions

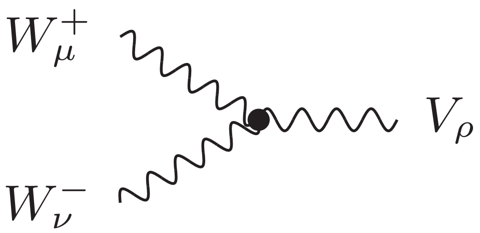
$\hookrightarrow$ induced by gauge-invariant Yang–Mills Lagrangian

$$\mathcal{L}_{\mathrm{YM}} = -\frac{1}{4} W^a_{\mu\nu} W^{a,\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu},$$

with the field-strength tensors

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g_2 \epsilon^{abc} W^b_\mu W^c_\nu, \qquad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

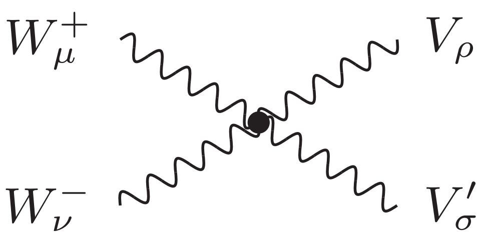
$\Rightarrow$ **Feynman rules for gauge-boson self-interactions:** (fields and momenta incoming)



$$\mathrm{i}eC_{WWV} \left[g_{\mu\nu}(k_+ - k_-)_\rho + g_{\nu\rho}(k_- - k_V)_\mu + g_{\rho\mu}(k_V - k_+)_\nu \right]$$

with $C_{WW\gamma} = 1, \quad C_{WWZ} = -\frac{c_{\mathrm{W}}}{s_{\mathrm{W}}}$

$\rightarrow$ testable in di-boson production $\mathrm{ee/pp} \to VV$



$$\mathrm{i}e^2 C_{WWVV'} \left[2g_{\mu\nu}g_{\rho\sigma} - g_{\mu\rho}g_{\sigma\nu} - g_{\mu\sigma}g_{\nu\rho} \right]$$

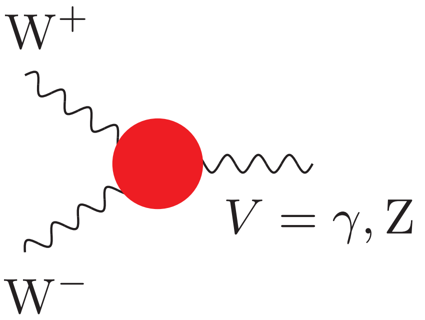
with $C_{W^2\gamma^2} = -1, C_{W^2\gamma Z} = \frac{c_{\mathrm{W}}}{s_{\mathrm{W}}}, C_{W^2Z^2} = -\frac{c^2_{\mathrm{W}}}{s^2_{\mathrm{W}}}, C_{W^4} = \frac{1}{s^2_{\mathrm{W}}}$

$\rightarrow$ testable in tri-boson production $\mathrm{ee/pp} \to VVV$
and vector-boson scattering $\mathrm{pp}(VV \to VV) \to VV + 2\mathrm{jets}$

(Non-)standard TGCs

Gaemers, Gounaris '79; Hagiwara, Hikasa, Peccei, Zeppenfeld '87; Bilenky, Kneur, Renard, Schildknecht '93; etc.

General parametrization (C- and P-conserving):



$$\mathcal{L}_{VWW} = -\mathrm{i}eg_{VWW}\Big\{ g_1^{\mathrm{V}}(W^+_{\mu\nu}W^{-,\mu}V^\nu - W^{-,\mu\nu}W^+_\mu V_\nu) + \kappa_{\mathrm{V}} W^+_\mu W^-_\nu V^{\mu\nu} + \frac{\lambda_{\mathrm{V}}}{M_{\mathrm{W}}^2} W^+_{\rho\mu}W^{-,\mu}_{\nu}V^{\nu\rho}\Big\}$$

Meaning for static W^+ bosons:

$$Q_{\mathrm{W}} = eg_1^\gamma = \text{electric charge } (= e \text{ by charge conservation})$$

$$\mu_{\mathrm{W}} = \frac{e}{2M_{\mathrm{W}}}(g_1^\gamma + \kappa_\gamma + \lambda_\gamma) = \text{magnetic dipole moment}$$

$$q_{\mathrm{W}} = -\frac{e}{M_{\mathrm{W}}^2}(\kappa_\gamma - \lambda_\gamma) = \text{electric quadrupole moment}$$

Standard Model values:

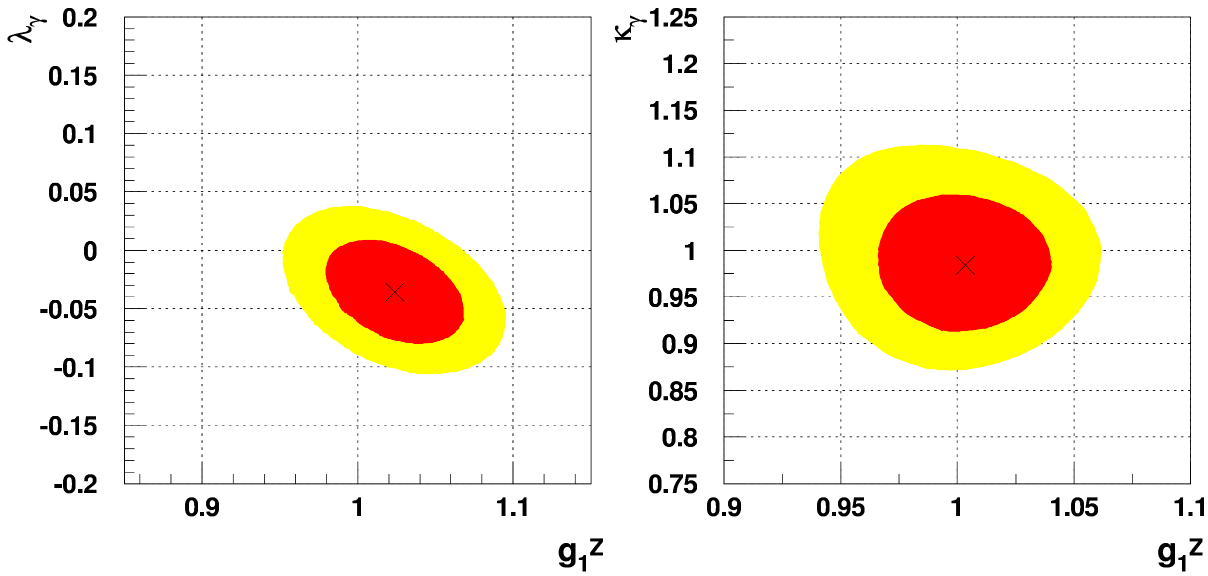
$$g_1^{\mathrm{V}} = \kappa_{\mathrm{V}} = 1, \qquad \lambda_{\mathrm{V}} = 0$$

Restriction to SU(2)×U(1)-symmetric dim-6 operators:

$$\kappa_{\mathrm{Z}} = g_1^{\mathrm{Z}} - (\kappa_\gamma - 1)\tan^2\theta_{\mathrm{W}}, \qquad \lambda_{\mathrm{Z}} = \lambda_\gamma$$

LEP2 constraints on charged TGCs from $e^+e^- \to WW \to 4f$

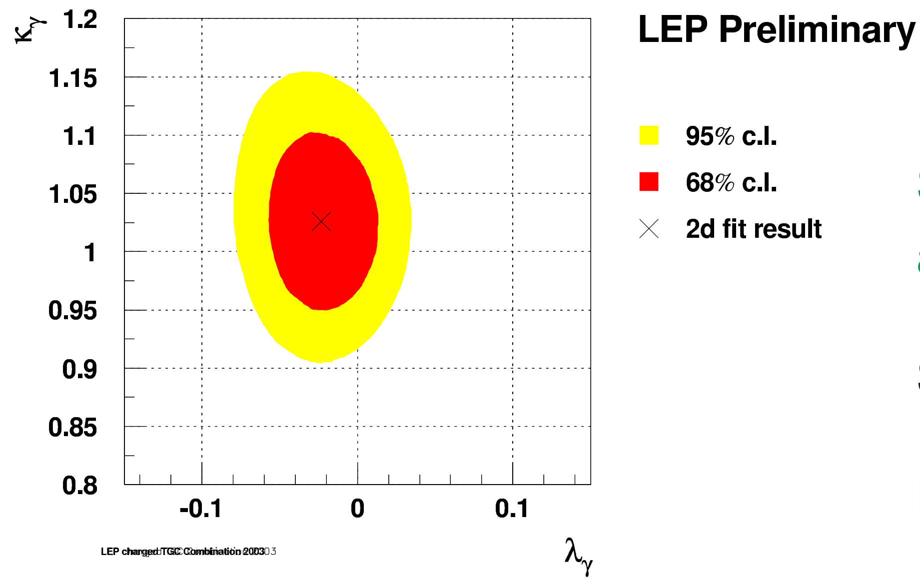
LEPEWWG '04



$$\Delta g_1^Z = -0.009^{+0.022}_{-0.021}$$

$$\Delta\kappa_\gamma = -0.016^{+0.042}_{-0.047}$$

$$\lambda_\gamma = -0.016^{+0.021}_{-0.023}$$



Standard Model values verified at the level of 2–4%

Similar results from Tevatron and LHC Run 1

LHC will tighten limits further !

Generic features of electroweak corrections

Relevance of EW corrections @ LHC

- 2015: LHC restarts @ $13-14\,\text{TeV}$
  $\hookrightarrow$ energy reach extends deeper into TeV range
  $\hookrightarrow$ $\delta_{\text{EW}} \sim \text{some } 10\%$
- integrated LHC luminosity will reach some $100\,\text{fb}^{-1}$
  $\hookrightarrow$ many measurements at several-$\%$ level
  $\hookrightarrow$ typical size of δ_{EW}
- planned high-precision measurements: XS ratios, M_{W}, $\sin^2\theta_{\text{eff}}^{\text{lept}}$
  $\hookrightarrow$ δ_{EW} is crucial ingredient

Spirit of this lecture

- describe salient features of EW corrections, in particular enhancement effects
- prepare the ground for the discussion of W/Z production processes coming in the follow-up lectures

Features of and issues in EW precision calculations

Relevance and size of EW corrections

generic size $\mathcal{O}(\alpha) \sim \mathcal{O}(\alpha_s^2)$ suggests NLO EW $\sim$ NNLO QCD

but systematic enhancements possible, e.g.

- by photon emission
  $\hookrightarrow$ kinematical effects, mass-singular log's $\propto \alpha \ln(m_\mu/Q)$ for bare muons, etc.
- at high energies
  $\hookrightarrow$ EW Sudakov log's $\propto (\alpha/s_W^2)\ln^2(M_W/Q)$ and subleading log's

EW corrections to PDFs at hadron colliders

induced by factorization of collinear initial-state singularities, new: photon PDF

Instability of W and Z bosons

- realistic observables have to be defined via decay products (leptons, γ's, jets)
- off-shell effects $\sim \mathcal{O}(\Gamma/M) \sim \mathcal{O}(\alpha)$ are part of the NLO EW corrections

Combining QCD and EW corrections in predictions

- how to merge results from different calculations
- reweighting procedures in MC's

Input parameter schemes

SM input parameters: (natural choice)

α_s, α, M_W, M_Z, M_H, m_f, V_{CKM}

Issues:

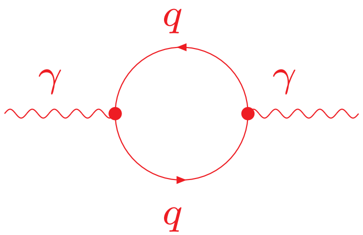
- **Setting of α:** process-specific choice to
  - ◇ avoid sensitivity to non-preturbative light-quark masses
  - ◇ minimize universal EW corrections

  Schemes: fix M_W, M_Z and α
  - ◇ $\alpha(0)$-scheme: relevant for external photon
  - ◇ $\alpha(M_Z)$-scheme: relevant for internal photons at high energies (γ^*)
  - ◇ G_μ-scheme: $\alpha_{G_\mu} = \sqrt{2} G_\mu M_W^2 (1 - M_W^2/M_Z^2)/\pi$, relevant for W, Z

- **Warnings / pitfalls:**
  - ◇ α must not be set diagram by diagram,
    but global factors like $\alpha(0)^m \alpha_{G_\mu}^n$ in gauge-invariant contributions mandatory !
  - ◇ weak mixing angle: $s_W \neq$ free parameter if M_W and M_Z are fixed !
  - ◇ Yukawa couplings are uniquely fixed by fermion masses !

The universal radiative corrections $\Delta\alpha$ and $\Delta\rho$

Running electromagnetic coupling $\alpha(s)$:



becomes sensitive to unphysical quark masses m_q

for $|s|$ in GeV range and below (non-perturbative regime)

$\hookrightarrow$ charge-renormalization constant δZ_e sensitive to m_q

Solution:

fit hadronic part of $\Delta\alpha(s) = -\mathrm{Re}\{\Sigma^{AA}_{\mathrm{T,ren}}(s)/s\}$ and thus of δZ_e

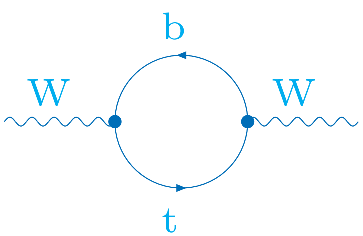
via dispersion relations to $R(s) = \dfrac{\sigma(\mathrm{e^+e^-} \to \text{hadrons})}{\sigma(\mathrm{e^+e^-} \to \mu^+\mu^-)}$ Jegerlehner et al.

$\Rightarrow$ Running elmg. coupling: $\quad \alpha(s) = \dfrac{\alpha(0)}{1 - \Delta\alpha_{\mathrm{ferm}\neq\mathrm{top}}(s)}$

Leading correction to the ρ-parameter:

mass differences in fermion doublets break custodial SU(2) symmetry

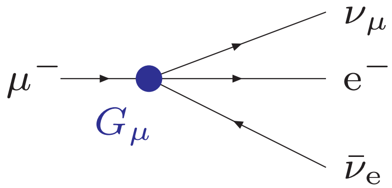
$\hookrightarrow$ large effects from bottom–top loops in W self-energy Veltman '77



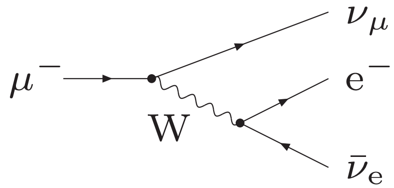
$$\Delta\rho_{\mathrm{top}} \sim \frac{\Sigma^{ZZ}_{\mathrm{T}}(0)}{M_{\mathrm{Z}}^2} - \frac{\Sigma^{WW}_{\mathrm{T}}(0)}{M_{\mathrm{W}}^2} \sim \frac{3G_\mu m_{\mathrm{t}}^2}{8\sqrt{2}\pi^2}$$

Fermi constant G_μ as input parameter – the quantity Δr

μ decay including higher-order corrections



+ QED corrections $\iff$



+ EW corrections

$\hookrightarrow$ Relation between G_μ, $\alpha(0)$, M_W, and M_Z including corrections:

$$\alpha_{G_\mu} \equiv \frac{\sqrt{2}}{\pi} G_\mu M_W^2 \left(1 - \frac{M_W^2}{M_Z^2}\right) = \alpha(0)(1 + \Delta r)$$

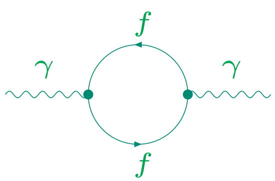
Δr comprises quantum corrections to μ decay

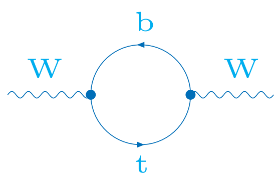
(beyond electromagnetic corrections in Fermi model) Sirlin '80, Marciano, Sirlin '80

$$\Delta r_{1-\mathrm{loop}} = \quad \Delta\alpha(M_Z^2) \quad - \frac{c_W^2}{s_W^2}\Delta\rho_{\mathrm{top}} \quad + \quad \Delta r_{\mathrm{rem}}(M_H)$$

$$\sim 6\% \qquad \sim 3\% \qquad \sim 1\%$$

$$\alpha \ln(m_f/M_Z) \qquad G_\mu m_t^2 \qquad \alpha \ln(M_H/M_Z)$$





Input-parameter schemes including electroweak NLO corrections

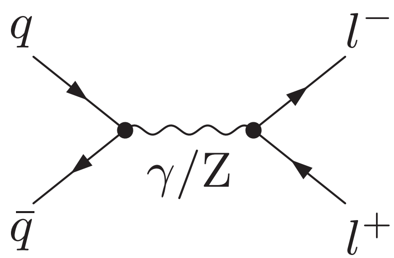
Cross section: $\sigma_{\mathrm{NLO}} = \alpha^N A_{\mathrm{LO}} \, (1 + \delta_{\mathrm{EW}})\,, \qquad \delta_{\mathrm{EW}} = \mathcal{O}(\alpha)$

- $\alpha(0)$-scheme: $\sigma_{\mathrm{LO}} = \alpha(0)^N A_{\mathrm{LO}}$
- $\alpha(M_{\mathrm{Z}})$-scheme: $\sigma_{\mathrm{LO}} = \alpha(M_{\mathrm{Z}})^N A_{\mathrm{LO}}, \quad \delta_{\mathrm{EW}}^{\alpha(M_{\mathrm{Z}})} = \delta_{\mathrm{EW}}^{\alpha(0)} + N\Delta\alpha(M_{\mathrm{Z}}) + \dots$
- G_μ-scheme: $\sigma_{\mathrm{LO}} = \alpha(G_\mu)^N A_{\mathrm{LO}}, \quad \delta_{\mathrm{EW}}^{G_\mu} = \delta_{\mathrm{EW}}^{\alpha(0)} + N\Delta r + \dots$
- Mixed scheme: $N = n + n_\gamma, \quad n_\gamma =$ # external photons

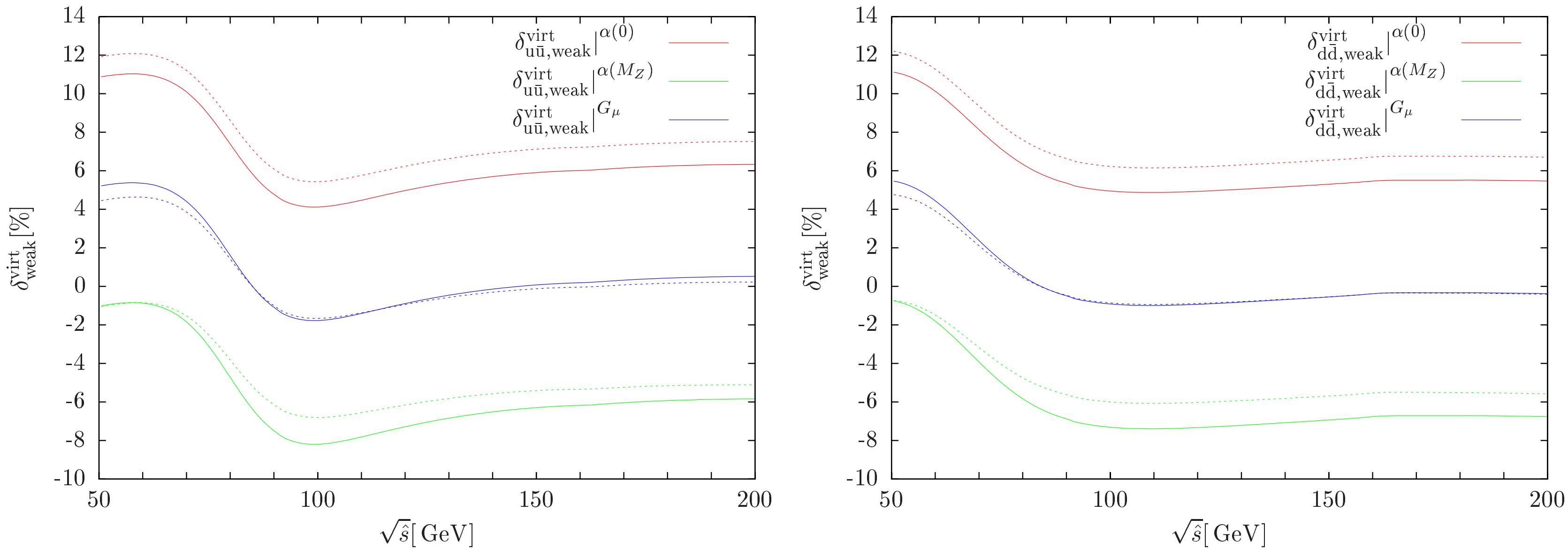
  $\sigma_{\mathrm{LO}} = \alpha(G_\mu)^n \alpha(0)^{n_\gamma} A_{\mathrm{LO}}, \quad \delta_{\mathrm{EW}}^{\mathrm{mix}} = \delta_{\mathrm{EW}}^{\alpha(0)} + n\Delta r + \dots$
  - ◇ absorbs all $\Delta\alpha$ terms in LO to all orders
  - ◇ absorbs $\Delta\rho$ terms in LO (all for Ws up to 2 loops, parts for Zs)
  - ◇ factor α in δ_{EW} can still be adjusted appropriately

    (e.g. $\alpha \to \alpha(0)$ if γ radiation dominates, $\alpha \to \alpha_{G_\mu}$ if weak corrections dominate)
  - ◇ example: $q\bar{q}' \to \mathrm{W}\gamma, \quad n = n_\gamma = 1$

Example: weak corrections to Z production



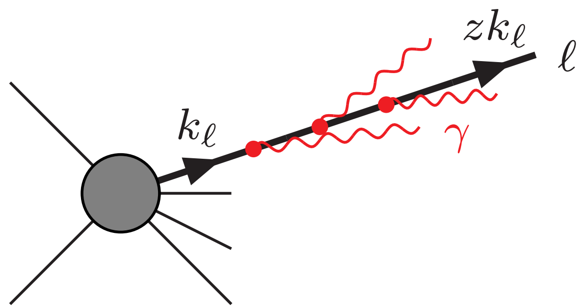
S.D., Huber '09



- off-sets between NLO EW corrections in different schemes
- dashed lines include leading 2-loop effects from $\Delta\alpha$ and $\Delta\rho$
  $\hookrightarrow$ highest stability against h.o. corrections in G_μ scheme here

Photon radiation off leptons

Collinear final-state radiation (FSR) off leptons



Leading logarithmic effect is universal:

$$\sigma_{\rm LL,FSR} = \int \underbrace{{\rm d}\sigma^{\rm LO}(k_l)}_{\text{hard scattering}} \int_0^1 {\rm d}z \underbrace{\Gamma_{\ell\ell}^{\rm LL}(z,Q^2)}_{\substack{\text{leading-log structure} \\ \text{function, } Q \text{ = typ. scale}}} \Theta_{\rm cut}(zk_l)$$

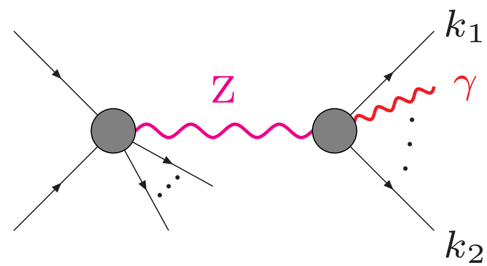
- $\Gamma_{\ell\ell}^{\rm LL}(z,Q^2)$ known to $\mathcal{O}(\alpha^5)$ + soft exponentiation, equivalent description by QED parton showers
- $\mathcal{O}(\alpha)$ approximation: $\Gamma_{\ell\ell}^{\rm LL,1}(z,Q^2) = \frac{\alpha(0)}{2\pi}\left[\ln\left(\frac{Q^2}{m_\ell^2}\right)-1\right]\left(\frac{1+z^2}{1-z}\right)_+$
- Alternative approach: QED parton shower
  $\hookrightarrow$ advantage: photons described with finite $p_{\rm T}$ and definite multiplicity

Impact on predictions:

- log-enhanced corrections for "bare" leptons (muons) $\rightarrow$ large radiative tails
- KLN theorem: mass-singular FSR effects cancel if $(\ell\gamma)$ system is inclusive
  (full integration over z)
- full FSR not universal, in general not even separable from other EW corrections

Radiative tail from final-state radiation

results if resonances reconstructed from decay products



Typical situations: $\mathrm{e^+e^-} \to \mathrm{WW/ZZ} \to 4f,$
$\mathrm{pp} \to \mathrm{Z} \to f\bar{f} + X$

Final-state radiation:

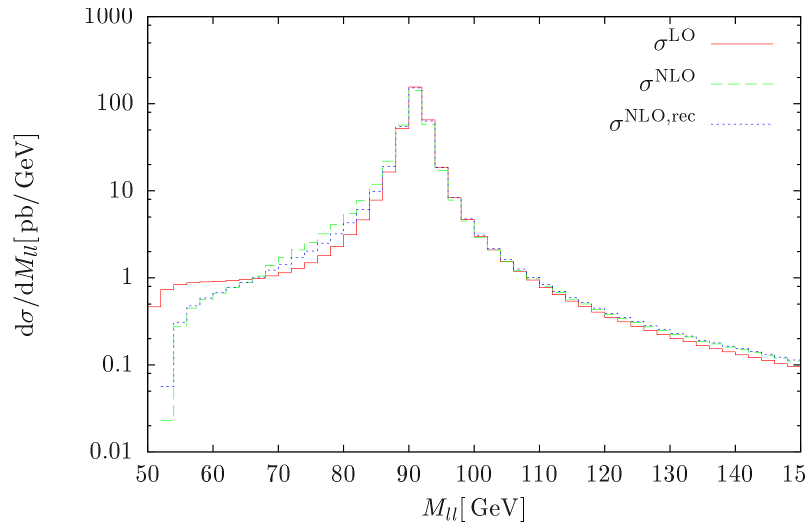
resonance for

$M^2 = (k_1+k_2)^2 < (k_1+k_2+k_\gamma)^2 \sim M_\mathrm{Z}^2$

$\hookrightarrow$ radiative tail in distribution $\frac{\mathrm{d}\sigma}{\mathrm{d}M}$

of reconstructed invariant mass M

for $M < M_\mathrm{Z}$

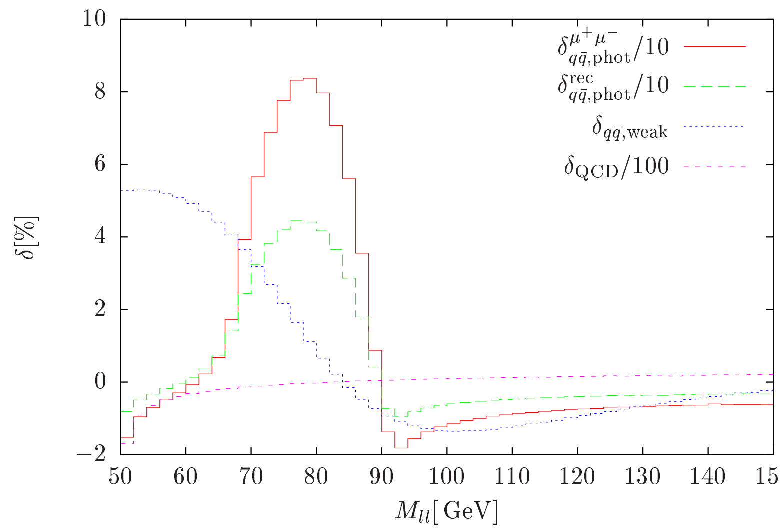


S.D., Huber '09

Example: Single-Z production

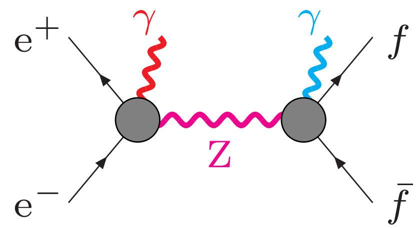
- radiative tail with corrections up to $\sim 80\%$
- FSR effect drastically reduced
  by photon recombination ("rec"):

  If $R_{l\gamma} < 0.1$ then $(l\gamma) \to \tilde{l}$ with $p_{\tilde{l}} = p_l + p_\gamma$.



Comparison with radiative tail from initial-state radiation

appears if initial state is fixed



Typical situations: $e^+e^- \to Z \to f\bar{f}$,
$\mu^+\mu^- \to Z, H, ? \to f\bar{f}$

$\hookrightarrow$ scan over s-channel resonance in $\sigma_{\rm tot}(s)$ by changing CM energy $\sqrt{s}$

Initial-state radiation:

Z can become resonant for $s = (p_+ + p_-)^2 \; > \; (p_+ + p_- - k_\gamma)^2 \sim M_Z^2$

$\hookrightarrow$ radiative tail for $s > M_Z^2$ due to "radiative return"

Final-state radiation:

$s = k_Z^2 \sim M_Z^2$ for FSR

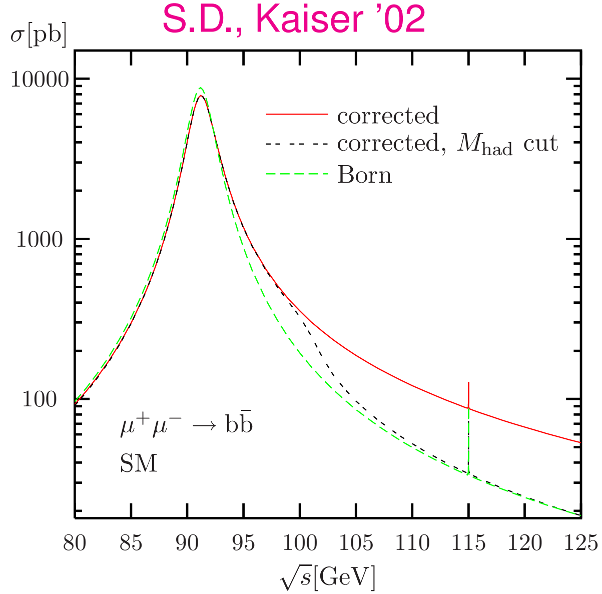
$\hookrightarrow$ only rescaling of resonance

Example:

cross section for $\mu^-\mu^+ \to b\bar{b}$ in lowest order and including photonic and QCD corrections, with and without invariant-mass cut

$\sqrt{s} - M(b\bar{b}) < 10\,\text{GeV}$

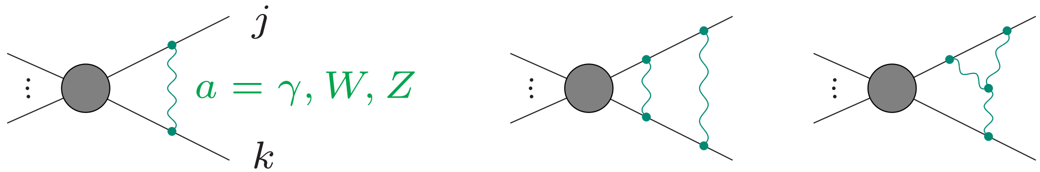
S.D., Kaiser '02



Electroweak corrections at high energies

Electroweak corrections at high energies

Sudakov logarithms induced by soft gauge-boson exchange



etc.

+ sub-leading logarithms from collinear singularities

Typical impact on $2 \to 2$ reactions at $\sqrt{s} \sim 1\,\mathrm{TeV}$:

$$\delta^{1-\mathrm{loop}}_{\mathrm{LL}} \sim -\frac{\alpha}{\pi s_{\mathrm{W}}^2} \ln^2\left(\frac{s}{M_{\mathrm{W}}^2}\right) \simeq -26\%, \qquad \delta^{1-\mathrm{loop}}_{\mathrm{NLL}} \sim +\frac{3\alpha}{\pi s_{\mathrm{W}}^2} \ln\left(\frac{s}{M_{\mathrm{W}}^2}\right) \simeq 16\%$$

$$\delta^{2-\mathrm{loop}}_{\mathrm{LL}} \sim +\frac{\alpha^2}{2\pi^2 s_{\mathrm{W}}^4} \ln^4\left(\frac{s}{M_{\mathrm{W}}^2}\right) \simeq 3.5\%, \qquad \delta^{2-\mathrm{loop}}_{\mathrm{NLL}} \sim -\frac{3\alpha^2}{\pi^2 s_{\mathrm{W}}^4} \ln^3\left(\frac{s}{M_{\mathrm{W}}^2}\right) \simeq -4.2\%$$

$\Rightarrow$ Corrections still relevant at 2-loop level

Note: differences to QED / QCD where Sudakov log's cancel

- massive gauge bosons W, Z can be reconstructed
  $\hookrightarrow$ no need to add "real W, Z radiation"
- non-Abelian charges of W, Z are "open" $\to$ Bloch–Nordsieck theorem not applicable

Extensive theoretical studies at fixed perturbative (1-/2-loop) order and suggested resummations via evolution equations

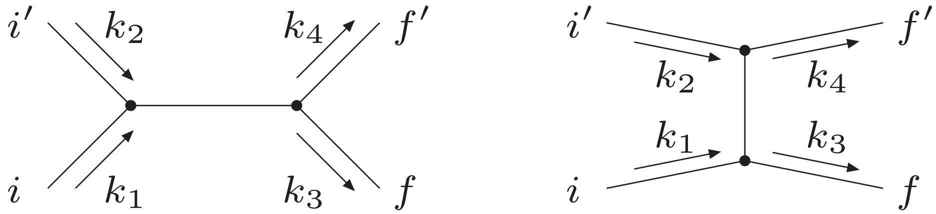
Beccaria et al.; Beenakker, Werthenbach; Ciafaloni, Comelli; Denner, Pozzorini; Fadin et al.; Hori et al.; Melles; Kühn et al., Denner et al.; Manohar et al. '00–

High-energy limit – Sudakow versus Regge regime

Sudakov regime: all invariants $k_i \cdot k_j \gg M_W^2$!

Example:

$2 \to 2$ particle process



Kinematic variables in centre-of-mass frame in high-energy limit ($k_j^2 \to 0$):

$$s = (k_1 + k_2)^2 \sim 4E^2, \qquad E = \text{beam energy},$$

$$t = (k_1 - k_3)^2 \sim -4E^2 \sin^2(\theta/2), \qquad \theta = \text{scattering angle},$$

$$M_{34} = \sqrt{s} \sim 2E,$$

$$k_T = k_{3,T} \sim E \sin\theta$$

High-energy limits in distributions:

- $\dfrac{d\sigma}{dk_T}$: $k_T \gg M_W \quad \Rightarrow \quad s, |t| \gg M_W^2 \quad \Rightarrow$ Sudakov domination
- $\dfrac{d\sigma}{dM_{34}}$: $M_{34} \gg M_W \quad \Rightarrow$ small $|t|$ possible $\quad \Rightarrow$ in general no Sudakov domination
  (i.e. typically smaller corrections)

Example: Drell–Yan production

Neutral current: $pp \to \ell^+\ell^-$ at $\sqrt{s} = 14\,\text{TeV}$ (based on S.D./Huber arXiv:0911.2329)

$M_{\ell\ell}/\text{GeV}$	$50-\infty$	$100-\infty$	$200-\infty$	$500-\infty$	$1000-\infty$	$2000-\infty$
σ_0/pb	738.733(6)	32.7236(3)	1.48479(1)	0.0809420(6)	0.00679953(3)	0.000303744(1)
$\delta^{\text{rec}}_{q\bar{q},\text{phot}}/\%$	-1.81	-4.71	-2.92	-3.36	-4.24	-5.66
$\delta_{q\bar{q},\text{weak}}/\%$	-0.71	-1.02	-0.14	-2.38	-5.87	-11.12
$\delta^{(1)}_{\text{Sudakov}}/\%$	0.27	0.54	-1.43	-7.93	-15.52	-25.50
$\delta^{(2)}_{\text{Sudakov}}/\%$	-0.00046	-0.0067	-0.035	0.23	1.14	3.38

no Sudakov domination!

Charged current: $pp \to \ell^+\nu_\ell$ at $\sqrt{s} = 14\,\text{TeV}$ (based on Brensing et al. arXiv:0710.3309)

$M_{T,\nu_\ell\ell}/\text{GeV}$	$50-\infty$	$100-\infty$	$200-\infty$	$500-\infty$	$1000-\infty$	$2000-\infty$
σ_0/pb	4495.7(2)	27.589(2)	1.7906(1)	0.084697(4)	0.0065222(4)	0.00027322(1)
$\delta^{\mu^+\nu_\mu}_{q\bar{q}}/\%$	$-2.9(1)$	$-5.2(1)$	$-8.1(1)$	$-14.8(1)$	$-22.6(1)$	$-33.2(1)$
$\delta^{\text{rec}}_{q\bar{q}}/\%$	$-1.8(1)$	$-3.5(1)$	$-6.5(1)$	$-12.7(1)$	$-20.0(1)$	$-29.6(1)$
$\delta^{(1)}_{\text{Sudakov}}/\%$	0.0005	0.5	-1.9	-9.5	-18.5	-29.7
$\delta^{(2)}_{\text{Sudakov}}/\%$	-0.0002	-0.023	-0.082	0.21	1.3	3.8

Sudakov domination!

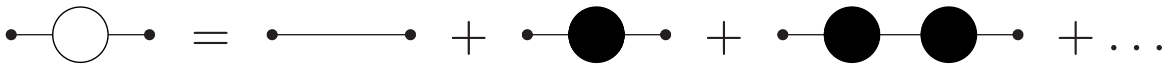
Unstable particles in QFT

Problem of unstable particles:

description of resonances requires resummation of propagator corrections

$\hookrightarrow$ mixing of perturbative orders potentially violates gauge invariance

Dyson series and propagator poles (scalar example)



$$G^{\phi\phi}(p) = \frac{\mathrm{i}}{p^2-m^2} + \frac{\mathrm{i}}{p^2-m^2}\,\mathrm{i}\Sigma(p^2)\,\frac{\mathrm{i}}{p^2-m^2} + \ldots = \frac{\mathrm{i}}{p^2-m^2+\Sigma(p^2)}$$

$\Sigma(p^2) =$ renormalized self-energy, $m =$ ren. mass

stable particle: $\mathrm{Im}\{\Sigma(p^2)\} = 0$ at $p^2 \sim m^2$

$\hookrightarrow$ propagator pole for real value of p^2,
renormalization condition for physical mass m: $\Sigma(m^2) = 0$

unstable particle: $\mathrm{Im}\{\Sigma(p^2)\} \neq 0$ at $p^2 \sim m^2$

$\hookrightarrow$ location μ^2 of propagator pole is complex,
possible definition of mass M and width Γ: $\mu^2 = M^2 - \mathrm{i}M\Gamma$

Different proposals:

- **Naive fixed-width schemes:**

$$\frac{1}{p^2 - M^2} \to \frac{1}{p^2 - M^2 + \mathrm{i}M\Gamma}$$ in all or at least in resonant propagators

$\hookrightarrow$ breaks gauge invariance only mildly (?),
but partial inclusion of widths in loops screws up singularity structure

- **Pole scheme** Stuart '91; Aeppli et al. '93, '94; etc.

Isolate resonance pole and introduce width Γ only there.

$\hookrightarrow$ consistent, gauge invariant, but involves subtleties

Pole approximation: isolate and keep only leading (=resonant) terms

$\hookrightarrow$ consistent, gauge invariant,
but not reliable at threshold or in off-shell tails of resonances

- **Effective field theory approach** Beneke et al. '04; Hoang, Reisser '04

$\hookrightarrow$ gauge invariant, involves pole expansions,
but can be combined with threshold expansions

- **Complex-mass scheme** Denner, S.D., Roth, Wackeroth '99; Denner, S.D., Roth, Wieders '05

$\hookrightarrow$ gauge invariant, valid everywhere in phase space

The complex-mass scheme at NLO

Basic idea: mass^2 = location of propagator pole in complex p^2 plane

$\hookrightarrow$ consistent use of complex masses everywhere !

Application to gauge-boson resonances:

- replace $M_\mathrm{W}^2 \to \mu_\mathrm{W}^2 = M_\mathrm{W}^2 - \mathrm{i}M_\mathrm{W}\Gamma_\mathrm{W}, \qquad M_\mathrm{Z}^2 \to \mu_\mathrm{Z}^2 = M_\mathrm{Z}^2 - \mathrm{i}M_\mathrm{Z}\Gamma_\mathrm{Z}$

  and define (complex) weak mixing angle via $c_\mathrm{W}^2 = 1 - s_\mathrm{W}^2 = \frac{\mu_\mathrm{W}^2}{\mu_\mathrm{Z}^2}$

- virtues:
  - ◇ gauge-invariant result (Slavnov–Taylor identities, gauge-parameter independence)
    $\hookrightarrow$ unitarity cancellations respected !
  - ◇ perturbative calculations as usual (loops and counterterms)
  - ◇ no double counting of contributions (bare Lagrangian unchanged !)

- drawbacks:
  - ◇ unitarity-violating spurious terms of $\mathcal{O}(\alpha^2)$ $\to$ but beyond NLO accuracy !
    (from t-channel/off-shell propagators and complex mixing angle)
  - ◇ complex gauge-boson masses also in loop integrals

Commonly used mass/width definitions:

- "on-shell mass/width" $M_{\rm OS}/\Gamma_{\rm OS}$: $\quad M_{\rm OS}^2 - m^2 + {\rm Re}\{\Sigma(M_{\rm OS}^2)\} \stackrel{!}{=} 0$

$$\hookrightarrow \; G^{\phi\phi}(p) \underset{p^2\to M_{\rm OS}^2}{\sim} \frac{1}{(p^2 - M_{\rm OS}^2)(1+{\rm Re}\{\Sigma'(M_{\rm OS}^2)\}) + {\rm i}\,{\rm Im}\{\Sigma(p^2)\}}$$

comparison with form of Breit–Wigner resonance $\dfrac{R_{\rm OS}}{p^2 - m^2 + {\rm i}m\Gamma}$

yields: $\quad M_{\rm OS}\Gamma_{\rm OS} \equiv {\rm Im}\{\Sigma(M_{\rm OS}^2)\} \,/\, (1+{\rm Re}\{\Sigma'(M_{\rm OS}^2)\}), \qquad \Sigma'(p^2) \equiv \frac{\partial\Sigma(p^2)}{\partial p^2}$

- "pole mass/width" M/Γ: $\quad \mu^2 - m^2 + \Sigma(\mu^2) \stackrel{!}{=} 0$

complex pole position: $\quad \mu^2 \equiv M^2 - {\rm i}M\Gamma$

$$\hookrightarrow \; G^{\phi\phi}(p) \underset{p^2\to \mu^2}{\sim} \frac{1}{(p^2-\mu^2)[1+\Sigma'(\mu^2)]} = \frac{R}{p^2 - M^2 + {\rm i}M\Gamma}$$

Note: $\quad \mu$ = gauge independent for any particle (pole location is property of S-matrix)

$M_{\rm OS}$ = gauge dependent at 2-loop order

Sirlin '91; Stuart '91; Gambino, Grassi '99; Grassi, Kniehl, Sirlin '01

Relation between “on-shell” and “pole” definitions:

Subtraction of defining equations yields:

$$M_{\mathrm{OS}}^2 + \mathrm{Re}\{\Sigma(M_{\mathrm{OS}}^2)\} \;=\; M^2 - \mathrm{i}M\Gamma + \Sigma(M^2 - \mathrm{i}M\Gamma)$$

Equation can be uniquely solved via recursion in powers of coupling α:

ansatz: $$M_{\mathrm{OS}}^2 = M^2 + c_1\alpha^1 + c_2\alpha^2 + \dots$$
$$M_{\mathrm{OS}}\Gamma_{\mathrm{OS}} = M\Gamma + d_2\alpha^2 + d_3\alpha^3 + \dots, \qquad c_i, d_i = \text{real}$$

counting in α: $$M_{\mathrm{OS}}, M = \mathcal{O}(\alpha^0), \quad \Gamma_{\mathrm{OS}}, \Gamma, \Sigma(p^2) = \mathcal{O}(\alpha^1)$$

Result:

$$M_{\mathrm{OS}}^2 \;=\; M^2 + \mathrm{Im}\{\Sigma(M^2)\}\,\mathrm{Im}\{\Sigma'(M^2)\} \;+\; \mathcal{O}(\alpha^3)$$

$$M_{\mathrm{OS}}\Gamma_{\mathrm{OS}} \;=\; M\Gamma + \mathrm{Im}\{\Sigma(M^2)\}\,\mathrm{Im}\{\Sigma'(M^2)\}^2 + \tfrac{1}{2}\,\mathrm{Im}\{\Sigma(M^2)\}^2\,\mathrm{Im}\{\Sigma''(M^2)\} \;+\; \mathcal{O}(\alpha^4)$$

i.e. $\{M_{\mathrm{OS}}, \Gamma_{\mathrm{OS}}\} \;=\; \{M, \Gamma\} \;+$ gauge-dependent 2-loop corrections

Important examples: W and Z bosons

In good approximation: $\mathrm{W} \to f\bar{f}'$, $\mathrm{Z} \to f\bar{f}$ with masses fermions f, f'

so that: $\mathrm{Im}\{\Sigma_\mathrm{T}^\mathrm{V}(p^2)\} = p^2 \times \frac{\Gamma_\mathrm{V}}{M_\mathrm{V}}\,\theta(p^2), \qquad \mathrm{V}=\mathrm{W},\mathrm{Z}$

$\hookrightarrow \; M_\mathrm{OS}^2 = M^2 + \Gamma^2 + \mathcal{O}(\alpha^3) \qquad M_\mathrm{OS}\Gamma_\mathrm{OS} = M\Gamma + \frac{\Gamma^3}{M} + \mathcal{O}(\alpha^4)$

In terms of measured numbers:

W boson: $M_\mathrm{W} \approx 80\,\mathrm{GeV}, \qquad \Gamma_\mathrm{W} \approx 2.1\,\mathrm{GeV}$

$\hookrightarrow \; M_\mathrm{W,OS} - M_\mathrm{W,pole} \approx 28\,\mathrm{MeV}$

Z boson: $M_\mathrm{Z} \approx 91\,\mathrm{GeV}, \qquad \Gamma_\mathrm{Z} \approx 2.5\,\mathrm{GeV}$

$\hookrightarrow \; M_\mathrm{Z,OS} - M_\mathrm{Z,pole} \approx 34\,\mathrm{MeV}$

Exp. accuracy: $\Delta M_\mathrm{W,exp} = 29\,\mathrm{MeV}, \quad \Delta M_\mathrm{Z,exp} = 2.1\,\mathrm{MeV}$

$\hookrightarrow$ Difference in definitions phenomenologically important !

Example of W and Z bosons continued:

Approximation of massless decay fermions:

$$\Gamma_{\mathrm{V,OS}}(p^2) \;=\; \Gamma_{\mathrm{V,OS}} \times \frac{p^2}{M^2_{\mathrm{V,OS}}}\,\theta(p^2), \qquad \mathrm{V} = \mathrm{W}, \mathrm{Z}$$

Fit of W/Z resonance shapes to experimental data:

- ansatz $\left|\frac{R'}{p^2 - m'^2 + \mathrm{i}\gamma' p^2/m'}\right|^2$ yields: $m' = M_{\mathrm{V,OS}}, \quad \gamma' = \Gamma_{\mathrm{V,OS}}$
- ansatz $\left|\frac{R}{p^2 - m^2 + \mathrm{i}\gamma m}\right|^2$ yields: $m = M_{\mathrm{V,pole}}, \quad \gamma = \Gamma_{\mathrm{V,pole}}$

Note: the two forms are equivalent:

$$R = \frac{R'}{1+\mathrm{i}\gamma'/m'}, \quad m^2 = \frac{m'^2}{1+\gamma'^2/m'^2}, \quad m\gamma = \frac{m'\gamma'}{1+\gamma'^2/m'^2}$$

$\hookrightarrow$ consistent with relation between “on-shell” and “pole” definitions !

Literature

For more details see “Dictionary for electroweak corrections” in

J. Butterworth, *et al.*, “Les Houches 2013: Physics at TeV Colliders: Standard Model Working Group Report,” arXiv:1405.1067 [hep-ph], page 11,

and original references therein.