Scale symmetry without the anomaly

Paweł Olszewski

Based on:

D.M. Ghilencea, Z. Lalak, PO 1608.05336 [hep-th]
Two-loop scale-invariant potential and quantum effective operators

D.M. Ghilencea 1508.00595 [hep-ph]
Manifest scale-invariant regularisation and quantum effective operators

UNIVERSITY OF WARSAW
UW 2 Two centuries Good beginning

Plan

1) Scale symmetry vs quantum corrections

2) Scale invariant (SI) regularisation

3) We did an exercise! A toy-model:

   a) Spontaneous scale symmetry breaking

   b) Effective potential

   c) Callan-Symanzik

4) Conclusions

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Purpose:

Comparison between the two methods of regularisation

Scale symmetry:

$$\Phi \rightarrow s^d \Phi$$
$$x^\mu \rightarrow \frac{1}{s} x^\mu$$

→ Classically, dimensionfull parameters are forbidden.

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$$\mu^{1-\epsilon} = \sigma(x)$$

$$\mu^{2\epsilon}\mathcal{L}[\Phi,\sigma] \rightarrow \sigma^{\frac{2\epsilon}{1-\epsilon}}\mathcal{L}[\Phi,\sigma]$$

M. Shaposhnikov, D. Zenhäusern, arXiv:*0809.3406v3*
M. Shaposhnikov, F.V. Tkachov, arXiv:*0905.4857v1*

Simple setup

Scale invariant SM + σ,

$$\mathcal{L}_{SM}\Big|_{m^2=0} + \frac{1}{2}\left(\partial\sigma\right)^2 - \lambda_m|H|^2\sigma^2 - \frac{\lambda_\sigma}{4}\sigma^4$$

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Accomodating S(Scale)SB & tuning *C.C.* in the effective potential

$$\lambda_\sigma - \frac{9\lambda_m^2}{\lambda_\phi}$$

$$V(\phi,\sigma) =$$

$$= \frac{\lambda_\phi}{4!}\left(\phi^2 + \frac{3\lambda_m}{\lambda_\phi}\sigma^2\right)^2 + \frac{1}{4!}\,\zeta\,\sigma^4$$

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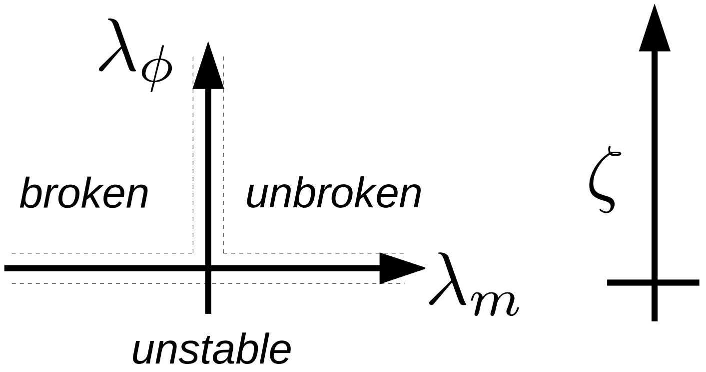

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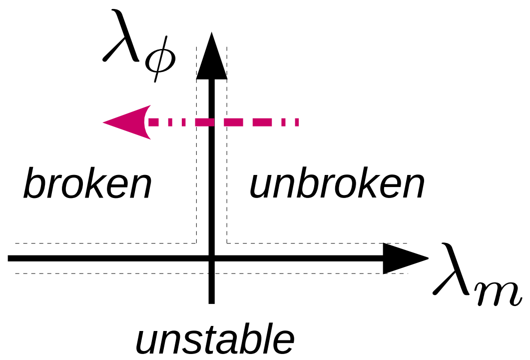

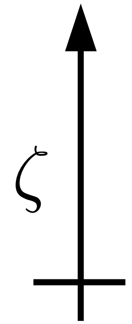

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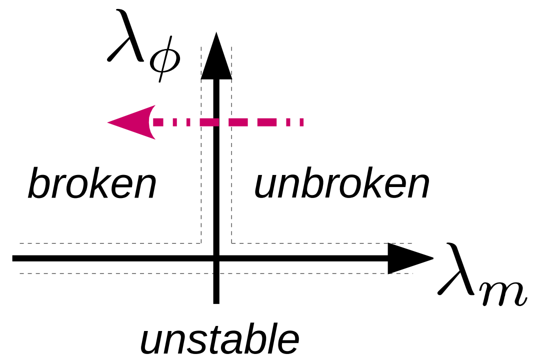

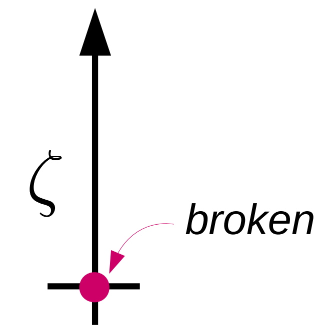

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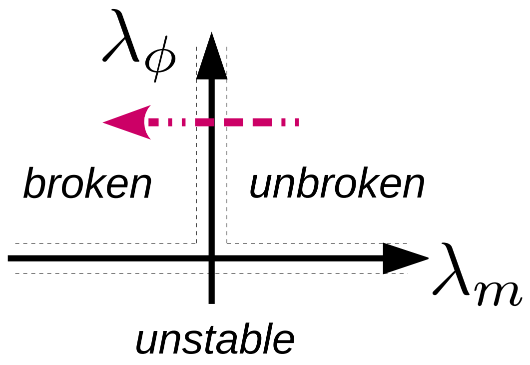

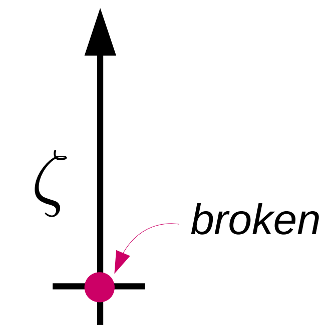

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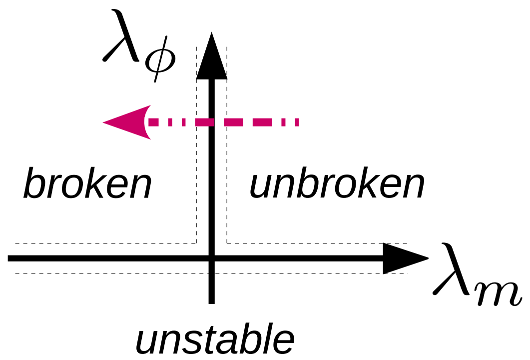

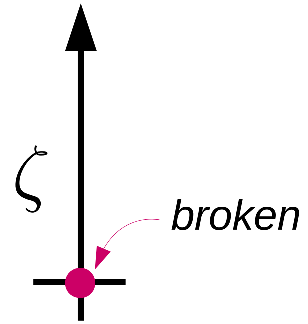

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e.g. $\frac{\lambda_\phi}{(4\pi)^2}\log\lambda_\phi$

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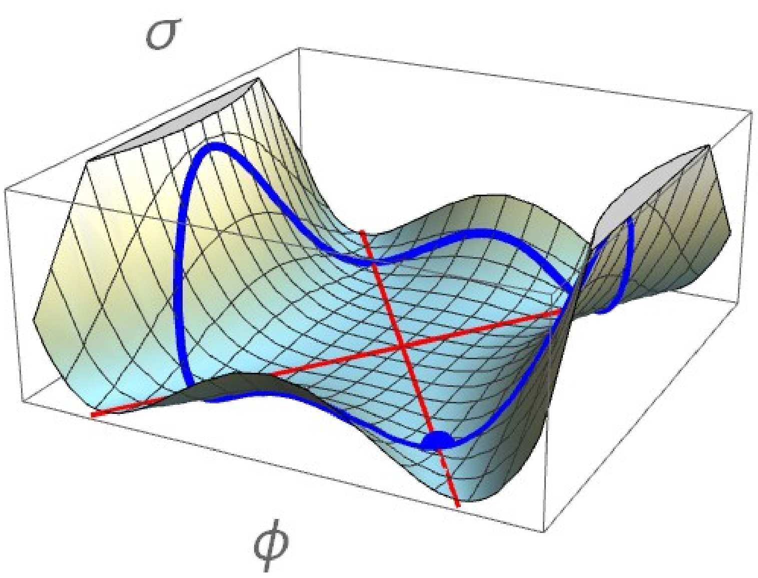

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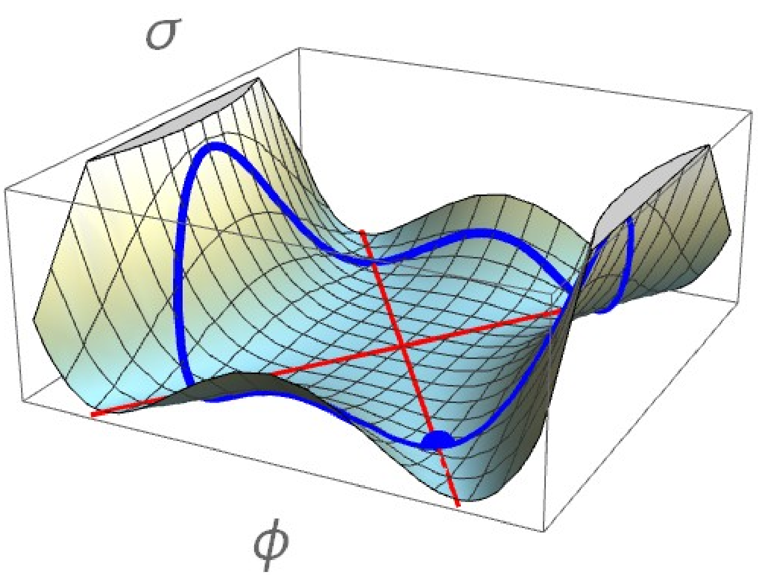

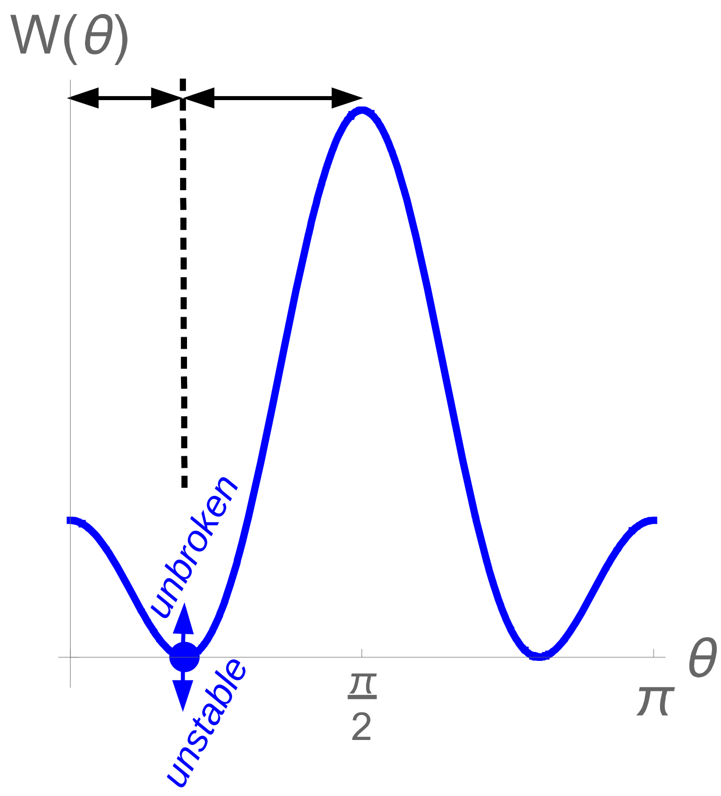

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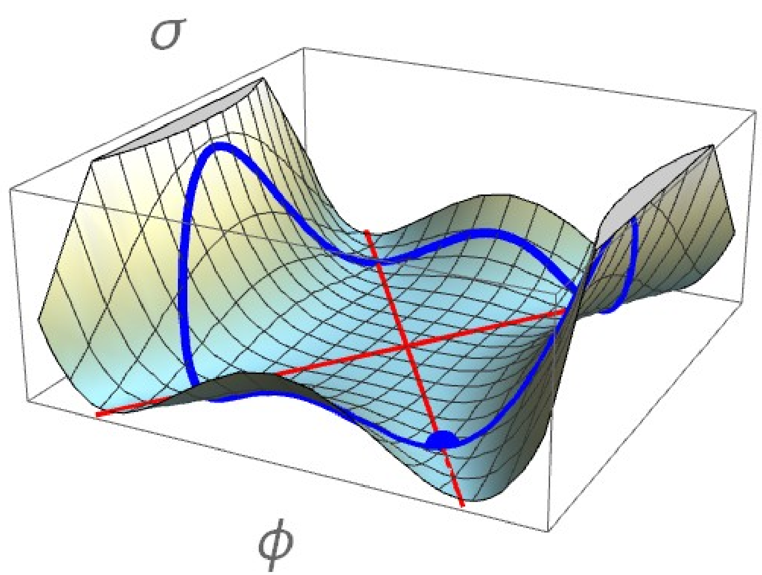

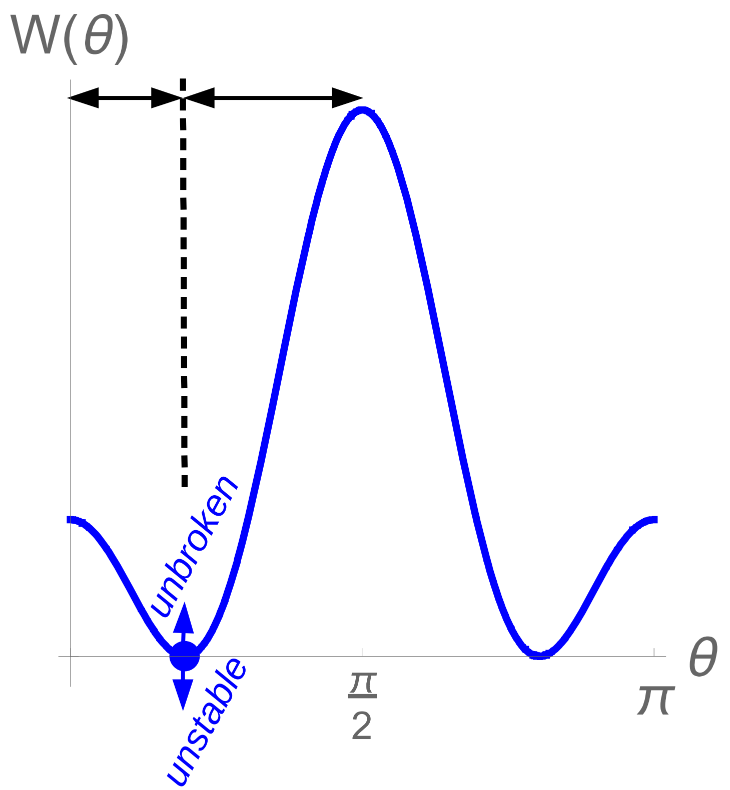

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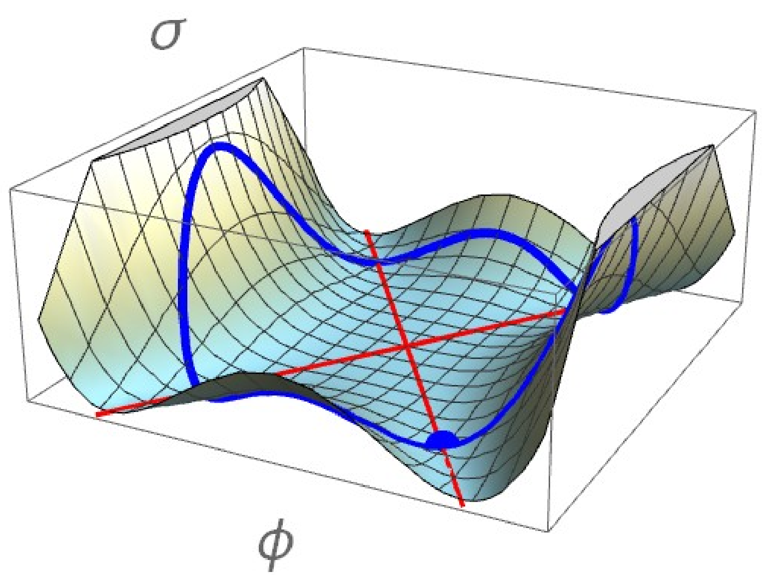

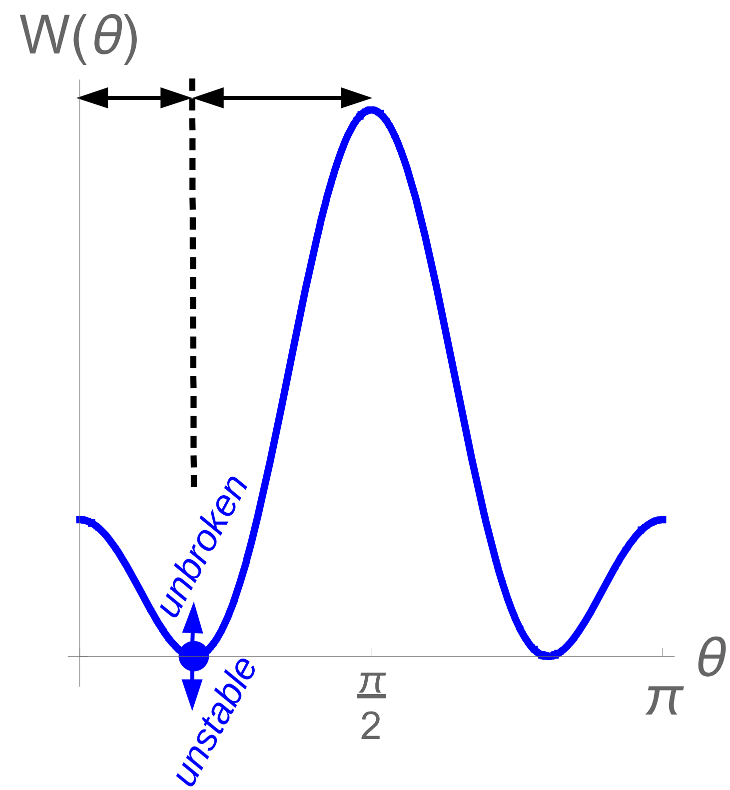

$$\langle \tan^2\theta\rangle = \frac{\langle\phi\rangle^2}{\langle\sigma\rangle^2} = -\frac{3\lambda_m}{\lambda_\phi}\,[1 + loops] \approx (10^{-16})^2\,?$$

← tree-level tuning

K. Allison, G. Ross, arXiv:1404.6268

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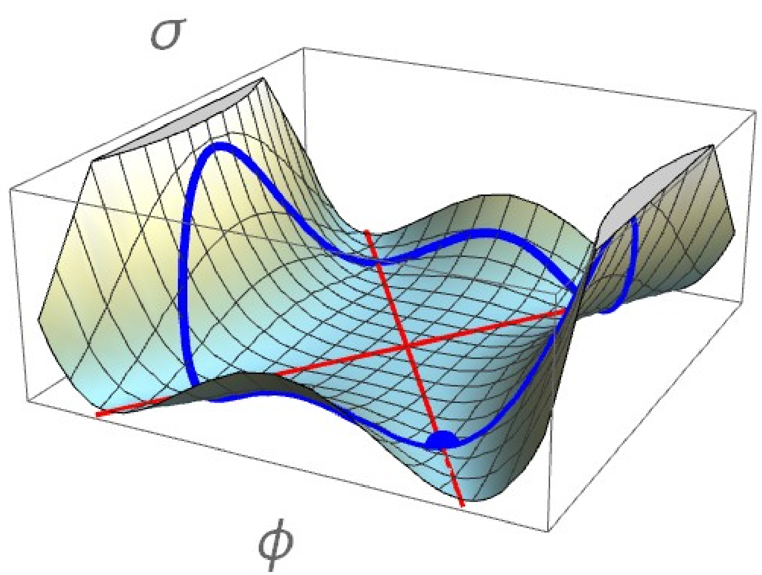

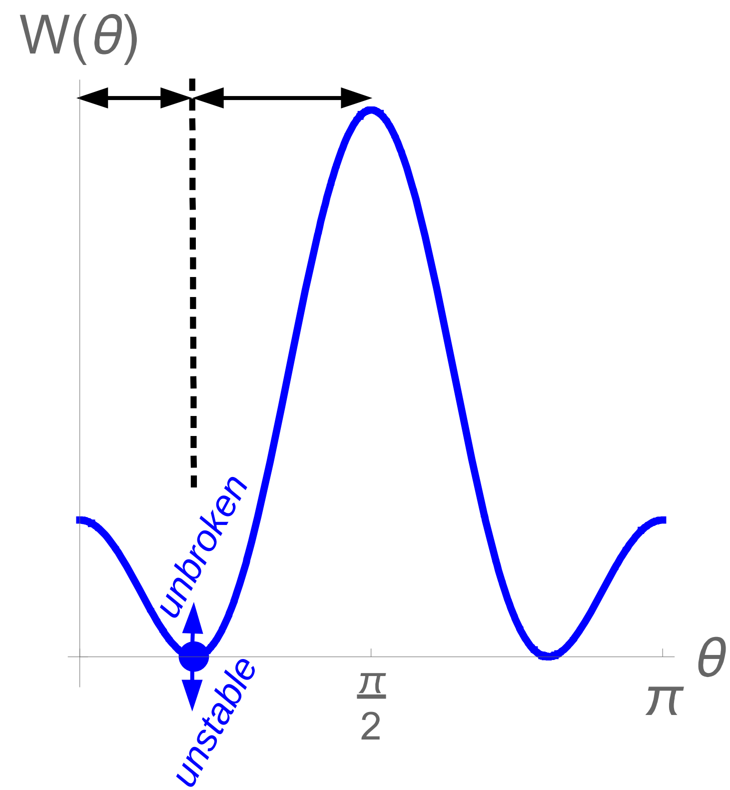

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the massive scalar mode →

$$\mathrm{m}_+^2 = -\lambda_m\sigma^2\left[1-\frac{3\lambda_m}{\lambda_\phi}+loops\right]$$

Calculating corrections with evanescent interactions

$$\lambda_\phi \, \sigma^{\frac{2\epsilon}{1-\epsilon}} \phi^4 \;\to\; \lambda_\phi \, \sigma_0^{\frac{2\epsilon}{1-\epsilon}} \left(1 + \frac{\delta\sigma}{\sigma_0}\right)^{\frac{2\epsilon}{1-\epsilon}} (\phi_0 + \delta\phi)^4 \;\to\; \ldots$$

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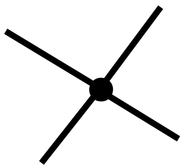
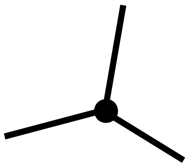
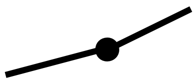


$\sim \lambda_\phi$



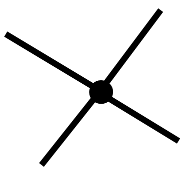
$\sim \lambda_\phi \phi_0$



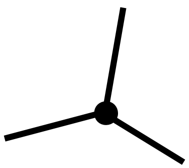
$\sim \lambda_\phi \phi_0^2$

Calculating corrections with evanescent interactions

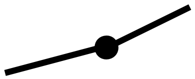
$$\lambda_\phi \, \sigma^{\frac{2\epsilon}{1-\epsilon}} \phi^4 \; \to \; \lambda_\phi \, \sigma_0^{\frac{2\epsilon}{1-\epsilon}} \left(1 + \frac{\delta\sigma}{\sigma_0}\right)^{\frac{2\epsilon}{1-\epsilon}} (\phi_0 + \delta\phi)^4 \; \to \; \dots$$



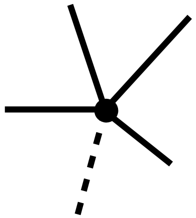
$\sim \lambda_\phi$



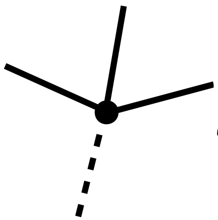
$\sim \lambda_\phi \phi_0$



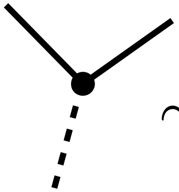
$\sim \lambda_\phi \phi_0^2$



$\sim \frac{\lambda_\phi}{\sigma_0} \left(\epsilon + \epsilon^2 + \dots\right)$



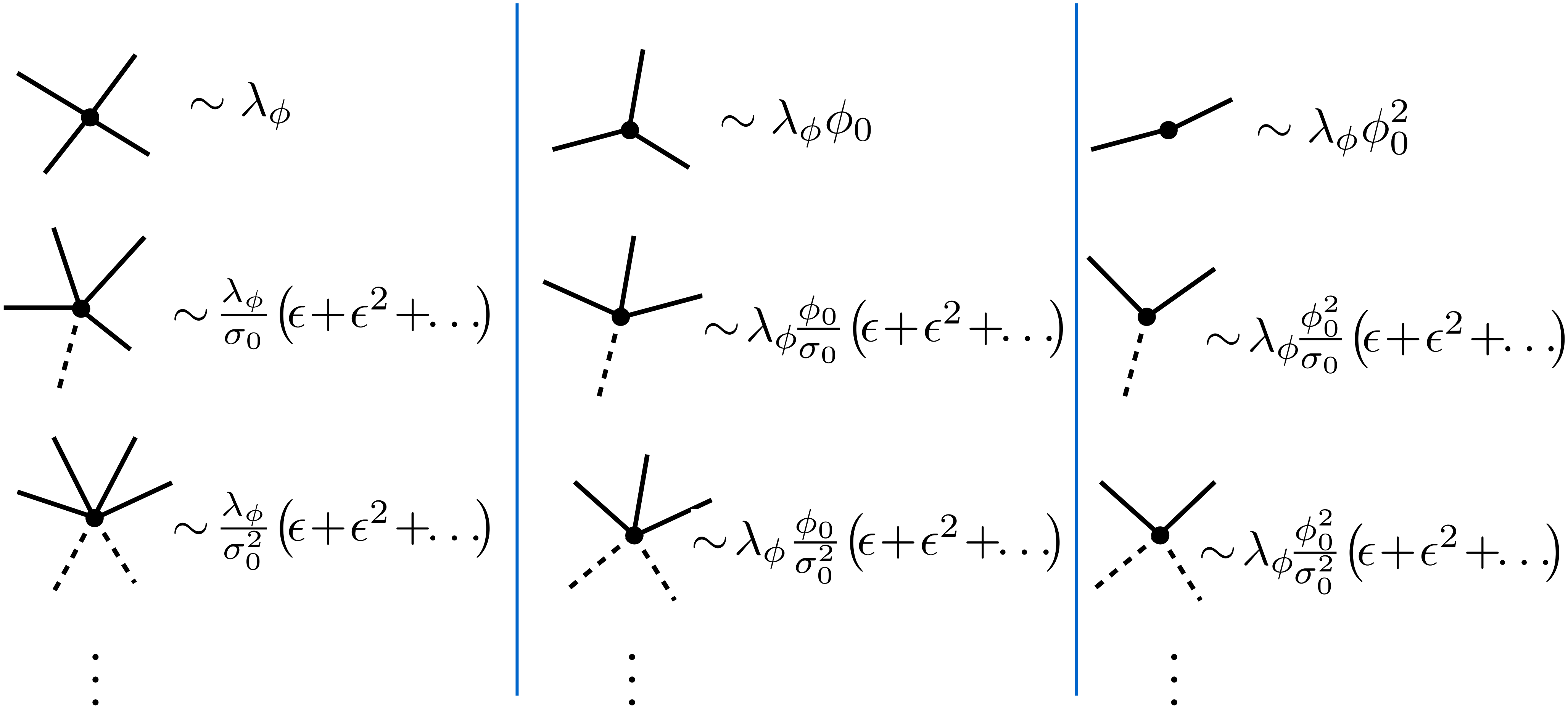
$\sim \lambda_\phi \frac{\phi_0}{\sigma_0} \left(\epsilon + \epsilon^2 + \dots\right)$



$\sim \lambda_\phi \frac{\phi_0^2}{\sigma_0} \left(\epsilon + \epsilon^2 + \dots\right)$

Calculating corrections with evanescent interactions

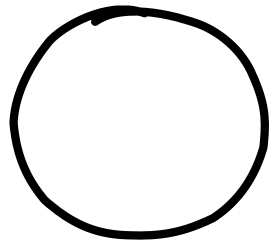
$$\lambda_\phi\, \sigma^{\frac{2\epsilon}{1-\epsilon}} \phi^4 \;\rightarrow\; \lambda_\phi\, \sigma_0^{\frac{2\epsilon}{1-\epsilon}} \left(1 + \frac{\delta\sigma}{\sigma_0}\right)^{\frac{2\epsilon}{1-\epsilon}} (\phi_0 + \delta\phi)^4 \;\rightarrow\; \ldots$$



Effective potential (by the background field method)

0-subscripts will be omitted:
$\phi_0 \to \phi,\ \sigma_0 \to \sigma$

- 1-loop



$$\frac{[\mu(\sigma)]^{2\epsilon}}{64\pi^2}\sum_{s=(+,-)} m_s^4 \left(\frac{1}{\epsilon} - \log\frac{m_s^2}{[\mu(\sigma)]^{2(1-\epsilon)}} + \frac{3}{2}\right) + V_{1\text{-loop}}^{new}$$

Effective potential (by the background field method)

0-subscripts will be omitted:
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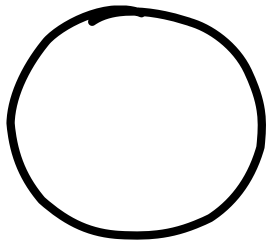
- 1-loop



the usual 1-loop counterterms

$$\frac{[\mu(\sigma)]^{2\epsilon}}{64\pi^2} \sum_{s=(+,-)} m_s^4 \left(\frac{1}{\epsilon} - \log \frac{m_s^2}{[\mu(\sigma)]^{2(1-\epsilon)}} + \frac{3}{2} \right) + V_{1\text{-loop}}^{new}$$

Field-dependent μ

Effective potential (by the background field method)

0-subscripts will be omitted:
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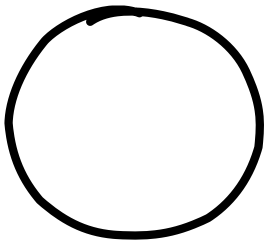
- 1-loop



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Field-dependent μ

$$V_{1\text{-loop}}^{new}=\frac{1}{48(4\pi)^2}\left[\left(\ldots\phi^4+\ldots\phi^2\sigma^2+\ldots\sigma^4\right)+\lambda_\phi\lambda_m\frac{\phi^6}{\sigma^2}\right]$$

Effective potential (by the background field method)

0-subscripts will be omitted:
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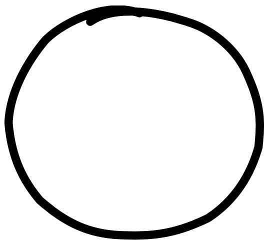

the usual 1-loop counterterms

$$\frac{[\mu(\sigma)]^{2\epsilon}}{64\pi^2}\sum_{s=(+,-)} m_s^4\left(\frac{1}{\epsilon}-\log\frac{m_s^2}{[\mu(\sigma)]^{2(1-\epsilon)}}+\frac{3}{2}\right)+V^{new}_{\text{1-loop}}$$

Field-dependent *μ*

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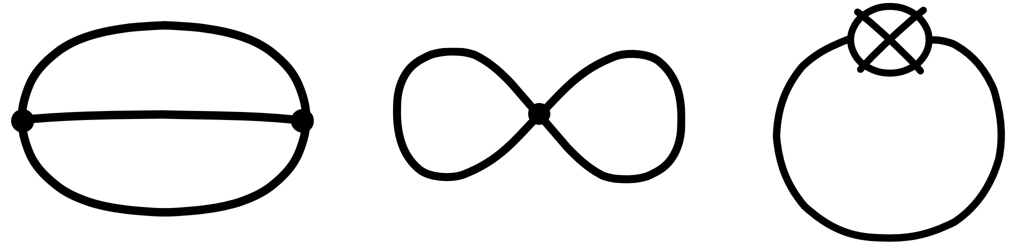
Can you renormalise it away by a finite counterterm?

Effective potential (by the background field method)

- 2-loop

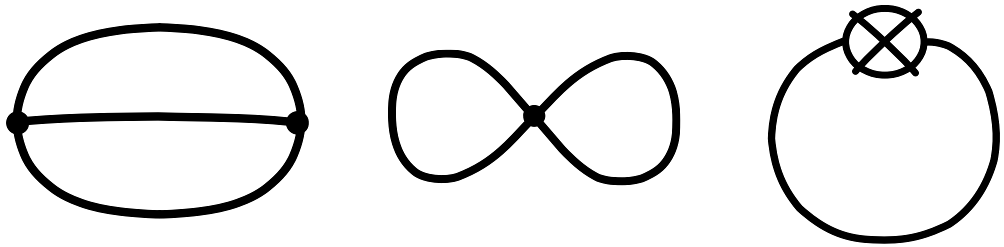


$$V^{usual}_{\text{2-loop}} + V^{new}_{\text{2-loop}}$$

Davydychev, Tausk, *„Two loop selfenergy diagrams with different masses and the momentum expansion", Nucl. Phys. B 397 (1993) 123*

Effective potential (by the background field method)

- 2-loop



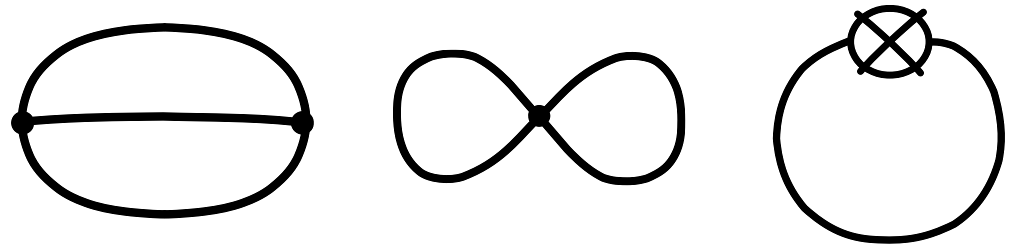
$$V_{2\text{-loop}}^{usual} + V_{2\text{-loop}}^{new}$$

Davydychev, Tausk, *„Two loop selfenergy diagrams with different masses and the momentum expansion", Nucl. Phys. B 397 (1993) 123*

$$V_{2\text{-loop}}^{new} \supset \frac{1}{\epsilon}\,\frac{[\mu(\sigma)]^{2\epsilon}}{16(4\pi)^2}\left[(\sim\lambda^2)\,\frac{\phi^6}{\sigma^2} + (\sim\lambda^2)\,\frac{\phi^8}{\sigma^4}\right]$$

Effective potential (by the background field method)

- 2-loop



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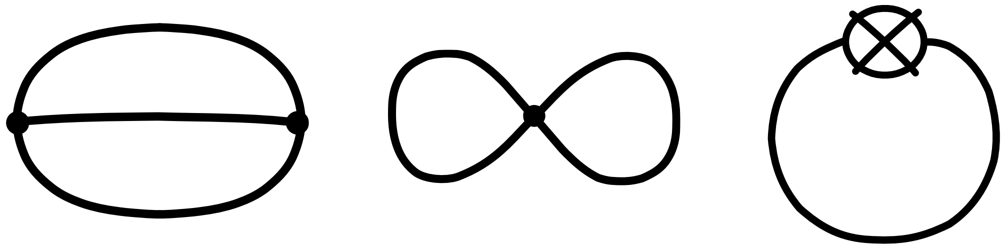

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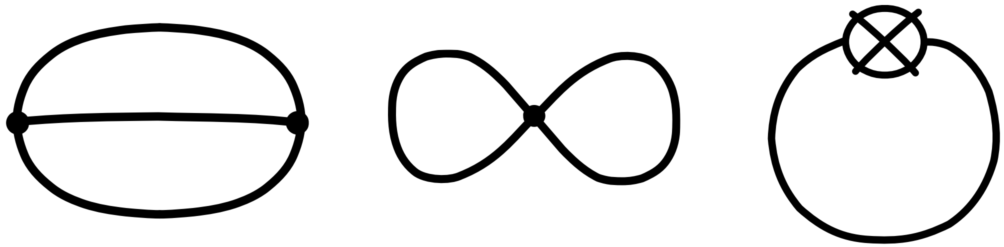
Infinitely many new terms

$$\lambda_{(4+2n)} \frac{\phi^{4+2n}}{\sigma^{2n}}\,, \;\; n=1,2,\ldots$$

Effective potential (by the background field method)

- 2-loop



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Infinitely many new terms

$$\lambda_{(4+2n)} \frac{\phi^{4+2n}}{\sigma^{2n}}\,, \quad n=1,2,\ldots$$

New couplings run away from zero!

$$\beta_{\lambda_6} = \frac{1}{4(4\pi)^2}\lambda_\phi\lambda_m\,(7\lambda_\phi - 14\lambda_m + \lambda_\sigma)$$

$$\beta_{\lambda_8} = \frac{1}{2(4\pi)^2}\lambda_\phi\lambda_m^2\,, \qquad \text{at } \lambda_{(4+2n)} = 0$$

Callan-Symanzik equation (at 2-loop)

Recall that $\mu(\sigma) = z\,\sigma^{\frac{1}{1-\epsilon}}$

a number

$$0 = \frac{\mathrm{d}\,V(\lambda,\, z)}{\mathrm{d}\log z}$$

Callan-Symanzik equation (at 2-loop)

Recall that $\mu(\sigma) = z\,\sigma^{\frac{1}{1-\epsilon}}$

a number

$$0 = \frac{\mathrm{d}\,V(\lambda,\, z)}{\mathrm{d}\log z}$$

$$0 = \left(z\frac{\partial}{\partial z} + \beta_{\lambda_j}\frac{\partial}{\partial \lambda_j} - \phi\gamma_\phi\frac{\partial}{\partial \phi} - \sigma\gamma_\sigma\frac{\partial}{\partial \sigma}\right) V(\lambda, z)$$

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assumed to contain

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Summation over lambda's, including

λ_6 and λ_8

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Summation over lambda's, including

λ_6 and λ_8

New RGE's

New corrections to *V*

C-S eq. holds anew!

Conclusions

1) You may use a field as a μ to preserve scale symmetry at the quantum level.

2) The price to pay: infinitely many nonpolynomial operators and corresponding couplings.

3) All RGE's are different. They can tell you, whether the model is scale invariant at quantum level.

4) Callan-Symanzik still holds, and nontrivially incorporates the modifications.

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Outlook

1) SM

2) Gravity K.G. Fereira, C.T. Hill, G.G. Ross, arXiv:1404.6268

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Stay tuned!
Thank You!

Appendix