The impact of non-minimally coupled gravity on vacuum stability

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in collaboration with
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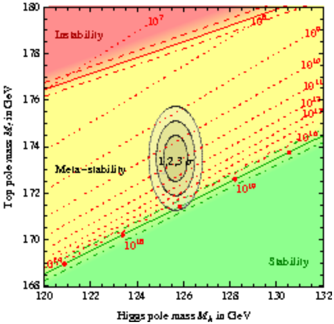
Introduction

- discovery of the Higgs boson gave rise to the important question of vacuum stability in the Standard Model G. Degrassi: arXiv:1205.6497, D. Buttazzo et al: 1307.3536
- observed data indicate that the EW minimum in the SM effective potential is metastable so it may decay to the global one
- investigation of the features of such a process - also impact of gravity
  S. R. Coleman, F. De Luccia: PRD 21 (1980)
  F. Loebbert, J. Plefka: 1502.03093
  J.R. Espinosa, J.F. Fortin, M. Trépanier: 1508.05343
  OC, ZL, Ł. Nakonieczny: 1512.07396
  A. Masoumi, S. Paban, E. J. Weinberg: 1603.07679...



- the role of the non-minimal coupling ξ between the scalar field and scalar curvature
  - required for the renormalizability of the scalar field in curved spacetime
  - crucial feature of the Higgs inflation model still allowed by the experimental data Planck collaboration: 1502.02114, F.L. Bezrukov, M. Shaposhnikov: 0710.3755
  - so far investigated in case of the inflationary background M. Herranen et al.: 1407.3141, 1506.04065, J.R. Espinosa et al.: 1505.04825 and in the Standard Model case G. Isidori et al.: 0712.0242, A. Rajantie, S. Stopyra: 1606.00849

Model

General toy model

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - V + \frac{R}{2\kappa}\left(1 - \xi\kappa\phi^2\right)$$

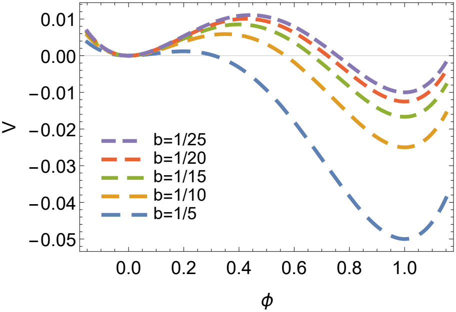
with

$$V = -\frac{1}{4}a^2(3b-1)\phi^2 + \frac{1}{2}a(b-1)\phi^3 + \frac{1}{4}\phi^4 + a^4 c$$

Parameters:

- a - the only dimensionful parameter: decreasing it simply corresponds to pushing the Planck scale further away and decreasing the gravitational effects, bringing our results closer to flat spacetime case
- b - degeneration of the vacua
- c - character of the initial false vacuum (dS: $c > 0$, Minkowski: $c = 0$)

Usually we consider: $a = 1$ (true vacuum positioned at the Planck scale)



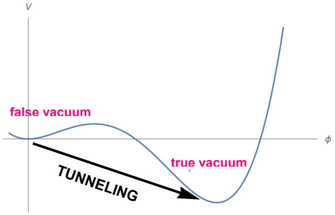
Potential for $a = 1$ and $c = 0$

Tunneling

Our potential has two minima:

- $\phi = 0$: false vacuum (fv)
- $\phi = a$: true vacuum (tv)

Possibility of tunneling from fv to tv!



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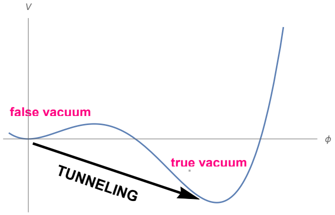

Vacuum decay probability via bubble nucleation S.R. Coleman, PRD 15 (1977), C.G. Callan, S.R. Coleman, PRD 16 (1977)

$$\Gamma = Ae^{-S} \qquad S \uparrow : \Gamma \downarrow$$

with $S = S_{\text{final}} - S_{\text{initial}}$, in our case: $S = S[\phi_{\text{tv}}] - S[\phi_{\text{fv}}]$.

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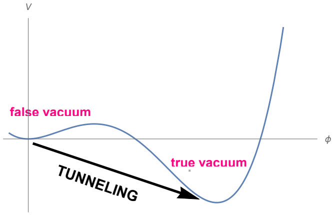

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Different methods to obtain S:

- S_{CDL} - numerically including gravity fully
- S_{flat} - numerical flat spacetime result (completely neglecting gravity)
- S_{TW} - thin-wall approximation
- S_{HM} - Hawking-Moss solution

Coleman and De Luccia formalism (CDL)

Our work is based on the standard formalism of CDL S.R. Coleman, F. De Luccia, PRD 21 (1980), which assumes that vacuum decay proceeds through nucleation of tv bubbles within our fv with

$$S_{CDL} = S[\phi_{CDL}] - S[\phi_{\text{fv}}]$$

Assuming $\phi = \phi(\tau)$ and metric $ds^2 = d\tau^2 + \rho(\tau)^2 (d\Omega)^2$, Euclidean action takes the form

$$S^E[\phi_{CDL}] = 4\pi^2 \int d\tau \left[\rho^3 V - \frac{3\rho}{\kappa}\left(1 - \xi\kappa\phi^2\right)\right] + \frac{6\pi}{\kappa}\left(1 - \kappa\xi\phi^2\right)\rho^2\dot{\rho}\Big|_0^{\tau_{max}}$$

where $\rho(\tau)$ is the radius of the bubble.

Gravitational background energy

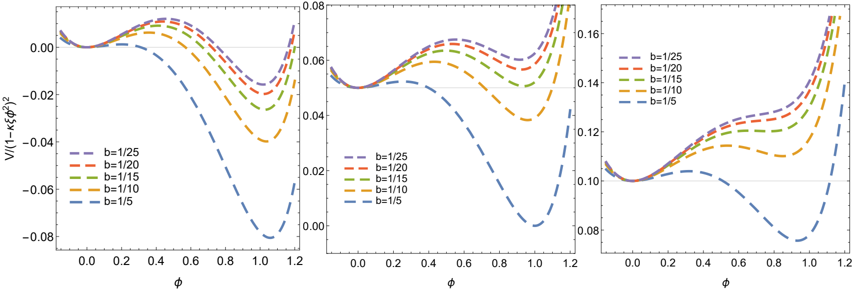
$$S[\phi_{\text{fv}}] = -\frac{24\pi^2(1 - \kappa\xi\phi_{\text{fv}}^2)^2}{\kappa^2 V_{\text{fv}}} \quad \text{(dS)}$$

$$S[\phi_{\text{fv}}] = 0 \qquad\qquad \text{(Minkowski)}$$

Modification of the vacuum energy by ξ

- fv: $\phi_{\text{fv}} = 0$, no modification of V_{fv}
- tv: modification!
  - V_{tv} can be bigger than V_{fv} making our false vacuum stable
  - true vacuum can disappear altogether - we neglect tunnelling then

    Bubble profile can sometimes be calculated still but such bubble is not energetically favourable and would not grow after nucleation.



Modified potential for different choices of the vacuum energy with $c = (0, 0.05, 0.1)$ and $\xi = 0.2$.

Thin-wall approximation
S.R. Coleman: PRD 15 (1977), S.R. Coleman, F. De Luccia: PRD 21 (1980)

TW approximation assumes the true vacuum bubble stretches to some $\bar{\rho}$ having a constant value V_{tv} and on the outside our solution is identical to the false vacuum V_{fv}.

Our input:
including effects coming from the non-minimal coupling to gravity ξ (linear terms)

Overall actions reads:

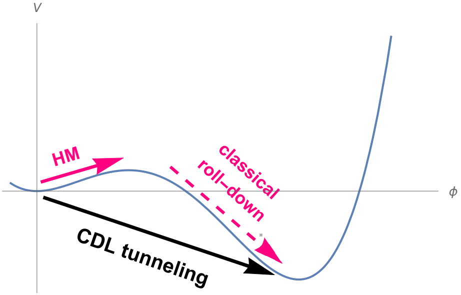
$$S_{TW} = S_{wall} + S[\phi_{tv}] - S[\phi_{fv}] =$$
$$= 2\pi^2 \left(\bar{\rho}^3 S_0 + \xi \bar{\rho} S_1 - \frac{2}{3} \frac{\left(1 - \bar{\rho}^2 \Lambda_{fv} V_{fv}\right)^{3/2} - 1}{\Lambda_{fv}^2 V_{fv}} + \frac{2}{3} \frac{\left(1 - \bar{\rho}^2 \Lambda_{tv} V_{tv}\right)^{3/2} - 1}{\Lambda_{tv}^2 V_{tv}} \right)$$

where Λ_{fv} and Λ_{tv} are constant field values modified by ξ, S_0 and S_1 are the bubble-wall results without and with gravity, respectively.

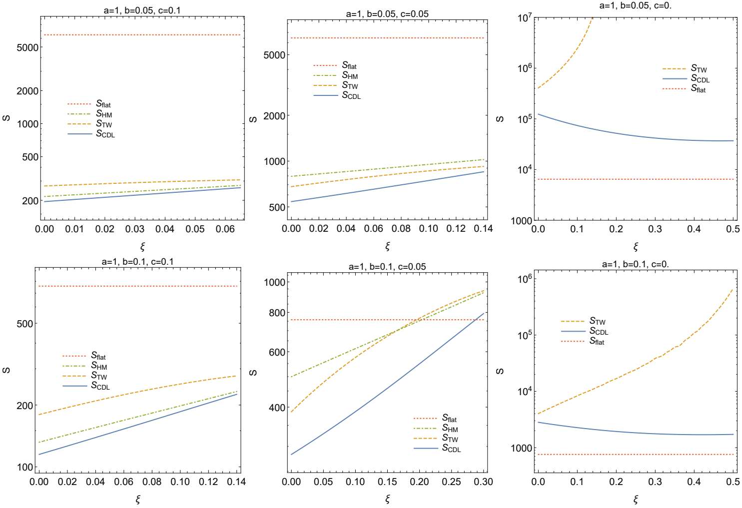
Hawking-Moss solution

Simpler HM solution S.W. Hawking, I.G. Moss, Phys.Lett. B 110 (1982) describes the probability for a whole spacetime volume to transition simultaneously to the top of the barrier (max) and continue by a classical roll-down:

$$S_{HM} = S_{\max} - S_{\mathrm{fv}} =$$
$$= -\frac{24\pi^2(1-\kappa\xi\phi_{\max}^2)^2}{\kappa^2 V_{\max}} + \frac{24\pi^2(1-\kappa\xi\phi_{\mathrm{fv}}^2)^2}{\kappa^2 V_{\mathrm{fv}}}$$

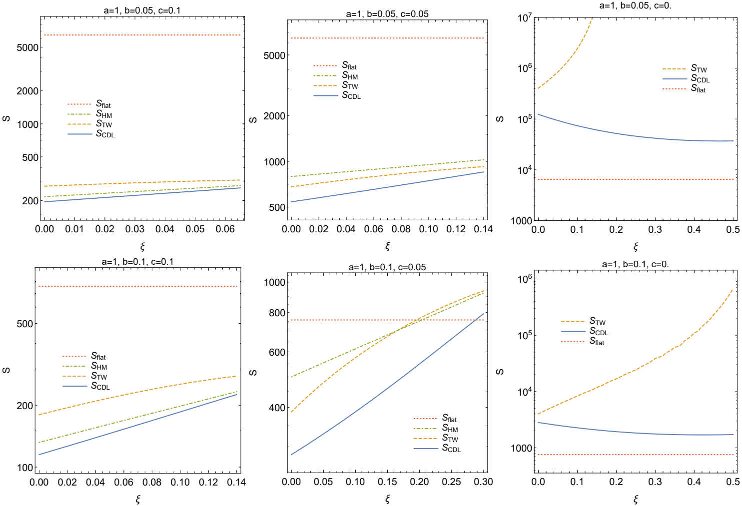


Comparison of the results



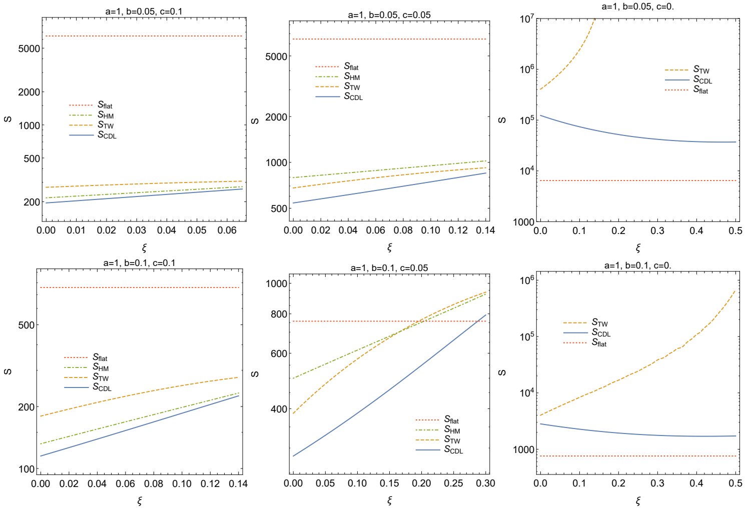
Tunnelling action as a function of non-minimal coupling obtained using four different methods.

Comparison of the results



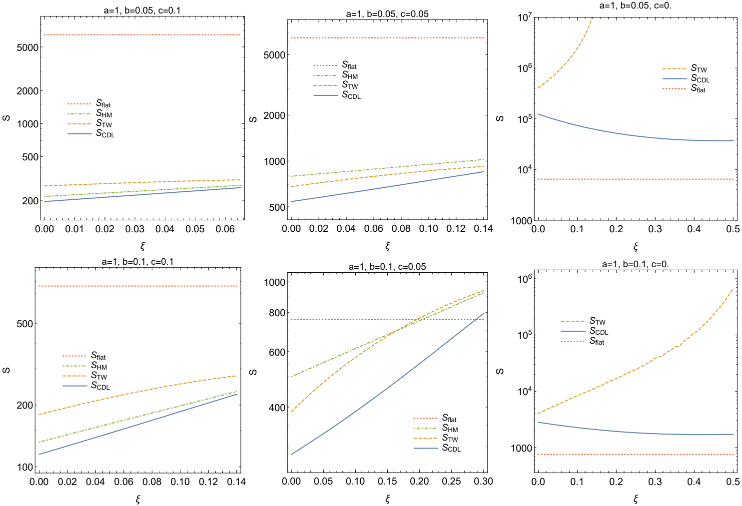
1) both approximations, TW and HM, always overestimate the action

Comparison of the results



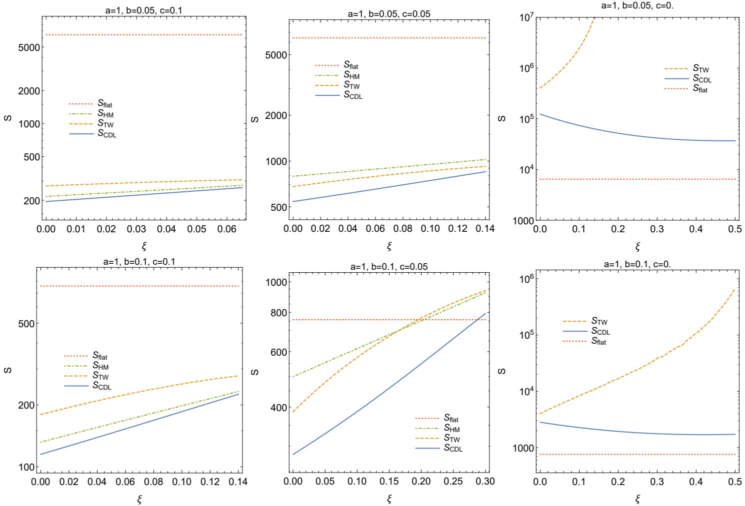
2) for large vacuum energies S_{HM}, for smaller vacuum energy S_{TW} are the best approximations

Comparison of the results



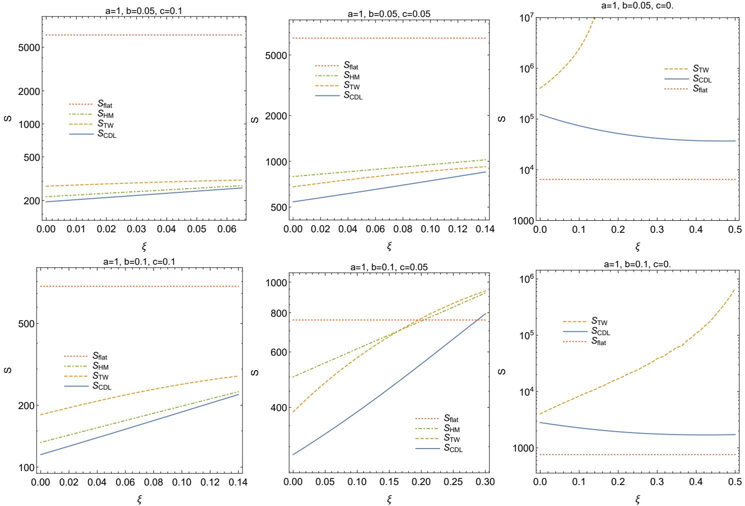
3) but: both approximations become less accurate as the vacuum energy decreases

Comparison of the results



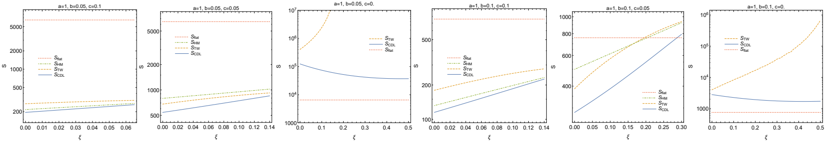
4) action quickly decreases as the false vacuum energy increases

Comparison of the results



5) in Minkowski case ($c = 0$) HM solution does not exist - S_{HM} would be infinite

Conclusions



The influence of non-minimal coupling to gravity is very different in cases of Minkowski and dS vacua:

- dS - the decay probability quickly decreases as the coupling grows, vacuum can be made absolutely stable
- Minkowski - effect is much weaker, the decay rate increases for small values of ξ

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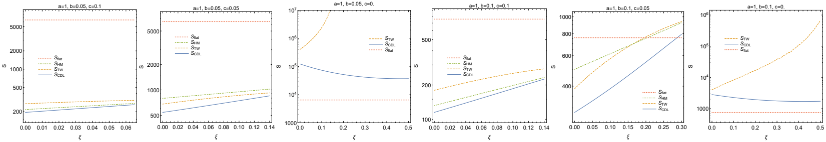

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Thank you for your attention.

Back-up slides

Conclusions

- both approximations, TW and HM, always overestimate the action
- for large vacuum energies S_{HM}, for smaller vacuum energy S_{TW} are the best approximations
- but: both approximations become less accurate as the vacuum energy decreases
- action quickly decreases as the false vacuum energy increases
- both approximations become less accurate as the vacuum energy decreases, in Minkowski case ($c = 0$):
  - the gravitational effects suppress vacuum decay by increasing the action
  - HM solution does not exist - S_{HM} would be infinite
  - TW severely overestimates the modification due to non zero coupling ξ
- the influence of non-minimal coupling to gravity is very different in cases of Minkowski and dS vacua:
  - dS - the decay probability quickly decreases as the coupling grows, vacuum can be made absolutely stable
  - Minkowski - effect is much weaker, the decay rate increases for small values of ξ

Boundary conditions

Bounce - a solution of the Euclidean field equations obeying appropriate boundary conditions.

Without loss of generality we chose boundary conditions for CDL

$$\dot{\phi}(0) = \dot{\phi}(\tau_{max}) = 0 \qquad \rho(\tau_{max}) = 0 \quad (\text{ fv = dS})$$

$$\rho(0) = 0 \qquad \rho(\tau_{max}) = \rho_{max} \neq 0 \quad (\text{fv = Minkowski})$$

Thin-Wall

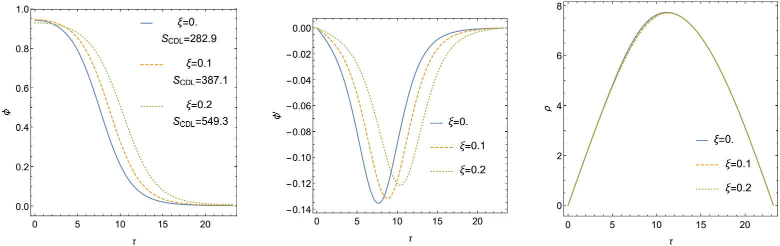
For Minkowski background ($V_{\text{fv}} = 0$)

$$S_{TW} = 2\pi^2 \left(\bar{\rho}^3 S_0 + \xi \bar{\rho} S_1 + \frac{\bar{\rho}^2}{\Lambda_{\text{fv}}} + \frac{2}{3} \frac{\left(1 - \bar{\rho}^2 \Lambda_{\text{tv}} V_{\text{tv}}\right)^{3/2} - 1}{\Lambda_{\text{tv}}^2 V_{\text{tv}}} \right)$$

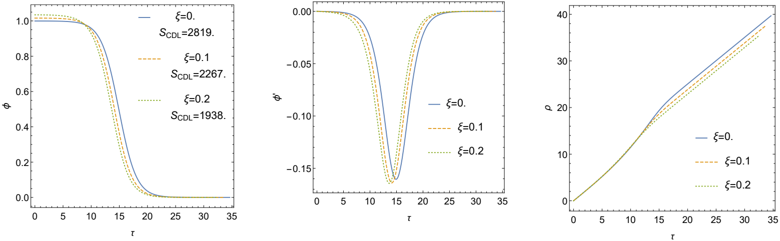
Numerical calculation of the CDL bounce

We solve the field EOMs and two Friedman equations with given boundary conditions (we find the field value ϕ_0 by undershoot/overshoot method known from the flat setup ZL, ML, PO: 1402.3826).

$c = 0.05$



$c = 0$



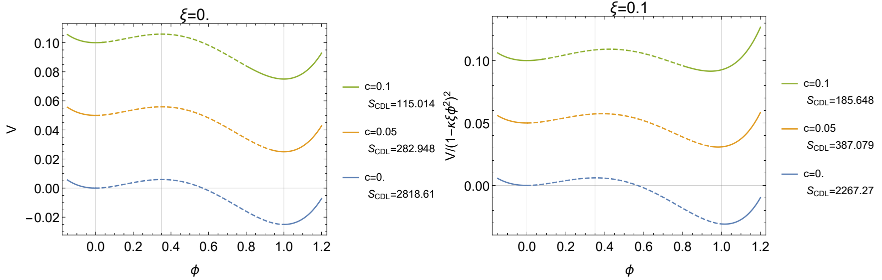
Bubble profiles and their modification due to the non-minimal coupling for $b = 1/10$.

Probed range

Action quickly decreases as the false vacuum energy increases

- reason: in this regime temperature effects coming from an effective temperature are induced by our compact spacetime A.R. Brown, E.J. Weinberg, 0706.1573
- our bounce solutions do not have to reach the false vacuum but only pass the bubble wall

For a fixed positive vacuum energy (given c) increasing ξ also results in more flat potential which means the bounce probes only values closer to the top of the barrier making them more similar to the HM solutions. Also when value of ξ is too large the potential becomes too flat and as a result the CDL bounces cease to exist.



Part of the potential actually probed by the tunnelling solution is dashed.

Equations

From the euclidean action we obtain the equation of motion of the scalar field,

$$\ddot{\phi} + 3\frac{\dot{\rho}}{\rho}\dot{\phi} - \xi\phi R = \frac{\partial V}{\partial\phi} \,,$$

the second Friedman equation,

$$\ddot{\rho} = \frac{\kappa\rho}{3\left(1-\kappa\xi\phi^2\right)}\left(-\dot{\phi}^2 - V + 3\xi\left(\dot{\phi}^2 + \ddot{\phi}\phi + \dot{\phi}\phi\frac{\dot{\rho}}{\rho}\right)\right) \,,$$

and the first Friedman equation

$$\dot{\rho}^2 = 1 + \frac{\kappa\rho^2}{3(1-\kappa\xi\phi^2)}\left(\frac{1}{2}\dot{\phi}^2 - V + 6\xi\dot{\phi}\phi\frac{\dot{\rho}}{\rho}\right) \,,$$

where $R = -6\frac{\ddot{\rho}\rho+\dot{\rho}^2-1}{\rho^2} \approx \frac{6}{\rho^2}$.

Order of ξ

We also checked that expanding to the second order in ξ does not improve our results. In general this correction only slightly increases the action. As we will see later on, this method overestimates the correct result, and so we can say that the error of this approximation comes from our assumption on the shape of the bounce rather than from expanding in the non-minimal coupling ξ.

Boundary term in the action

It is important here to point out that including the boundary term in the action is crucial when the false vacuum has a vanishing energy. In this case ρ asymptotes to a linear function instead of crossing zero again at τ_{max} and the boundary term is sizeable.