The Phase Diagram of QCD Matter

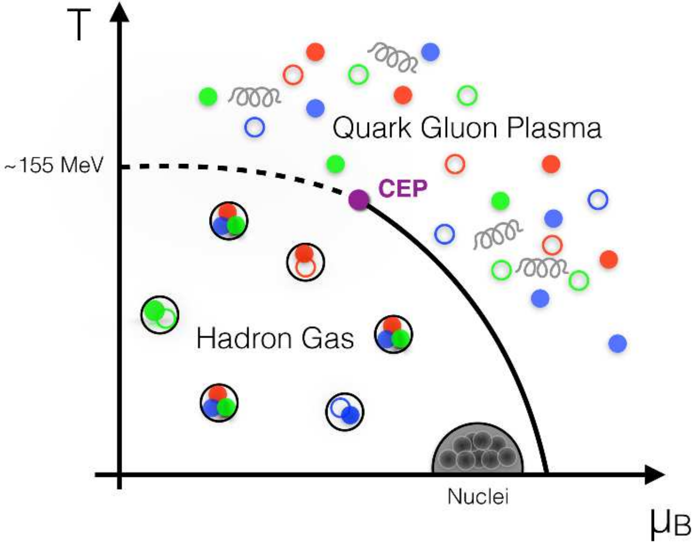
N. G. Antoniou

University of Athens,
Department of Physics,
Sector of Nuclear and Elementary Particle Physics

August 31th - September 12th, 2016, Corfu, Greece

1. Crossover transition at $\mu_b = 0$
2. Second-order transition at the critical end point ($\mu_b \approx 200$ MeV)
3. Fluctuations at the QCD critical point
4. Experimental Observations
5. Dynamical aspects of the QCD critical point
6. Concluding remarks and expectations

Phase diagram of QCD - crossover



From QCD Lagrangian to QCD Thermodynamics

$$\mathcal{L} = \frac{1}{2g^2}\mathrm{Tr}(F_{\mu\nu}F_{\mu\nu}) + \bar{\Psi}\gamma_\mu(\partial_\mu + A_\mu + m)\Psi$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]$$

(Euclidean Lagrangian with gauge coupling g)

$$S = \int d^4x\mathcal{L} \quad ; \quad Z = \sum_{\text{field configurations}} \exp(-S)$$

The ambiguities in Lattice studies are removed:

- Using physical quark masses
- Extrapolating to vanishing lattice spacing

Chiral susceptibility: χ_{ch}

Susceptibility is obtained from the second derivative

of the Gibbs free energy G with respect to the ordering field H:

$$G = -T \ln Z \quad , \quad M = -\frac{\partial G}{\partial H} \quad , \quad \chi = \frac{1}{V}\frac{\partial M}{\partial H} \Rightarrow \chi = \frac{\partial^2}{\partial H^2}\left(\frac{T}{V}\ln Z\right)$$

For chiral susceptibility the ordering field is the mass of light quarks $m_{u,d}$:

$$\chi_{ch} = \frac{\partial^2}{\partial m_{u,d}^2}\left(\frac{T}{V}\ln Z\right) \qquad ; \qquad \chi_{ch}(N_s, N_t)$$

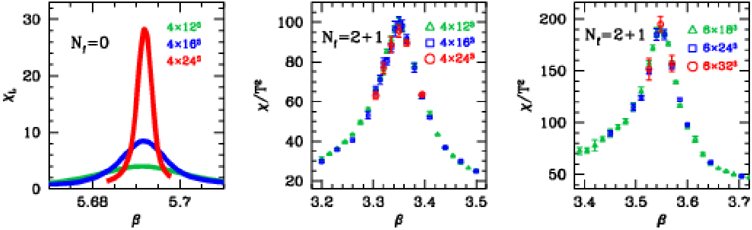
where N_s (N_t) is the spatial (Euclidean time) extension.

Finite-Size Scaling and the susceptibility peak

Near T_c and for finite but large volume $V = L^d$ we have a peak in the susceptibility with the following volume dependence:

- 1st order transition : $\chi_T^{max} \sim V$
- 2nd order transition : $\chi_T^{max} \sim V^{\gamma/d\nu}$ (γ, ν critical exponents)
- crossover transition : $\chi_T^{max} \sim \mathrm{const}$

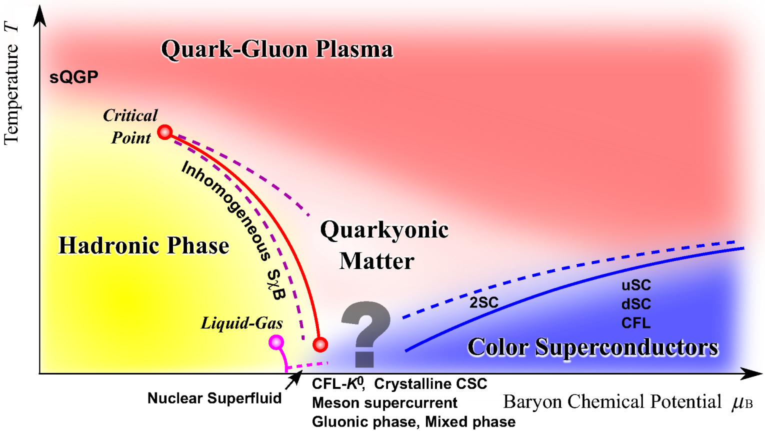
Proof of crossover transition in Lattice QCD



[Y. Aoki, G. Endrödi, Z. Fodor, S. D. Katz and K. K. Szabo, Nature **443**, 675 (2006)]
[Z. Fodor and S. D. Katz, Landolt-Börnstein Group I: Elementary Particles, Nuclei and Atoms **23**, Springer 2009; arXiv: 09083341v1 [hep-ph]]

Phase diagram of QCD - Critical Point

- Objective: Detection / existence of the QCD Critical Point (CP)



K. Fukushima, T. Hatsuda, Rept. Prog. Phys. 74:014001 (2011)

- Look for observables tailored for the CP; Scan phase diagram by varying energy and size of collision system.

Effective action at the critical point

Quark field Ψ forms a "hot" condensate (chiral)

$$\sigma(\mathbf{x}) = \langle \overline{\Psi}(\mathbf{x}) \Psi(\mathbf{x}) \rangle$$

at $T_c \approx 170$ MeV, $\mu_{b,c} \approx 200$ MeV (critical point).

Effective action for the condensate*:

$$S_c[\sigma] = T_c^{-1} \int d^3x \left[\frac{1}{2} (\nabla \sigma)^2 + G T_c^4 \left(T_c^{-1} \sigma \right)^{\delta+1} \right]$$

Universality class: 3D Ising

$$\Downarrow$$

$G \approx 2$, $\delta \approx 5$ (isothermal critical exponent), dim$[\sigma]$=(length)$^{-1}$

*[M.M. Tsypin, Phys. Rev. Lett. **73**, 2015 (1994); J. Berges, N. Tetradis and C. Wetterich, Phys. Rep. **363**, 139 (2004)]

Clusters with fractal geometry

- Generation of fractal clusters in $2D$ transverse space:

$$\langle n_a(\vec{x}_\perp) n_a(0) \rangle \sim |\vec{x}_\perp|^{d_{F,a}-2} \quad ; \quad d_{F,a} = \frac{2(\delta - s_a)}{\delta + 1}$$

$a = \sigma,\ B$ with $s_\sigma = 1$, $s_B = 0$ since $n_\sigma \sim \langle \overline{\Psi}\Psi \rangle^2$ and $\delta n_B \sim \delta\sigma$

- Fractality leads to critical opalescence in transverse momentum space (via Fourier transform*):

$$\langle \tilde{n}_a(\Delta\vec{p}_T) \tilde{n}_a(\vec{0}) \rangle \sim |\Delta\vec{p}_T|^{\tilde{d}_{F,a}-2}$$

with $\tilde{d}_{F,a} = 2 - d_{F,a}$

*[N. G. Antoniou, N. Davis, and F. K. Diakonos, Phys. Rev. **C 93**, 014908 (2016)]

Critical Observables; the Order Parameter (OP)

CP observables

Event-by-event (global) fluctuations:
Variance, skewness, kurtosis –
sensitive to experimental acceptance

Local:
density fluctuations of OP
in transverse space
(stochastic fractal)

Order parameter

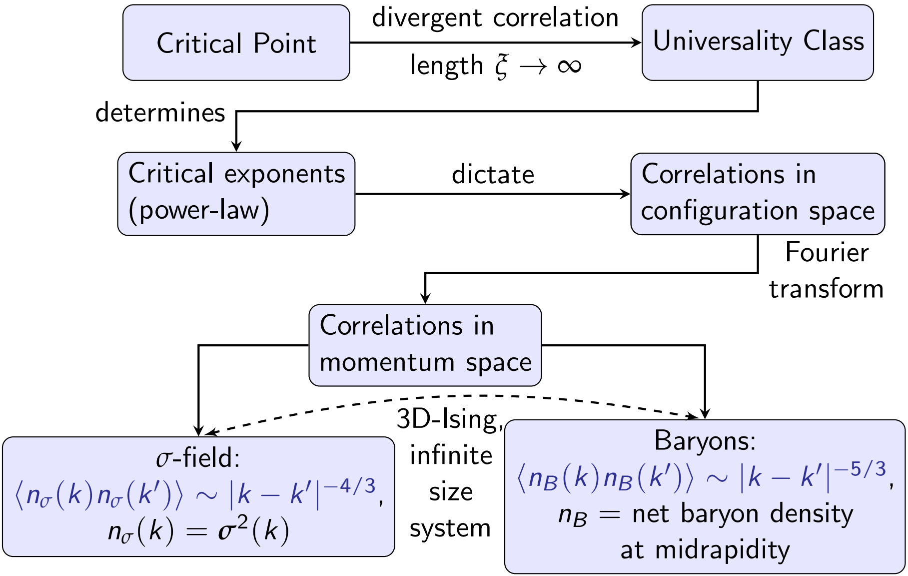
Chiral condensate
$\sigma(\boldsymbol{x}) = \langle \overline{\Psi}(\boldsymbol{x})\Psi(\boldsymbol{x}) \rangle$

coupling ⇕ induced critical fluctuations*

Net proton density
$n_B(\boldsymbol{x})$

*[Y. Hatta and M. A. Stephanov, PRL**91**, 102003 (2003)]

Self-similar density fluctuations near the CP



Observing power-law fluctuations

Experimental observation of local, power-law distributed fluctuations

$\Downarrow$

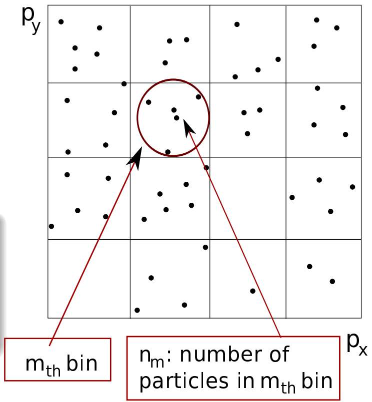
Intermittency in transverse momentum space (net protons at mid-rapidity)

(Critical opalescence in ion collisions*)

- Transverse momentum space is partitioned into M^2 cells
- Calculate second factorial moments $F_2(M)$ as a function of cell size $\Leftrightarrow$ number of cells M:

$$F_2(M) \equiv \frac{\sum_m \langle n_m(n_m-1)\rangle}{\sum_m \langle n_m\rangle^2},$$

where $\langle \ldots \rangle$ denotes averaging over events.



*[F.K. Diakonos, N.G. Antoniou and G. Mavromanolakis, PoS (CPOD2006) 010, Florence]

Scaling of factorial moments – Correlator

In the presence of strong background use the correlator $\Delta F_2(M)$ approximated by:

$$\Delta F_2^{(e)}(M) = F_2^{\mathrm{data}}(M) - F_2^{\mathrm{mix}}(M)$$

For a critical system, ΔF_2 scales with cell size (number of cells, M) as:

$$\Delta F_2(M) \sim \left(M^2\right)^{\varphi_2}$$

where φ_2 is the intermittency index.

Theoretical predictions for φ_2

universality class, effective actions

$$\varphi_{2,cr}^{(\sigma)} = \frac{2}{3} \; (0.66\ldots)$$

sigmas (neutral isoscalar dipions)

[N. G. Antoniou **et al**, Nucl. Phys. A **693**, 799 (2001)]

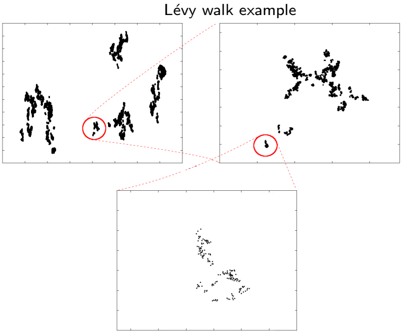
$$\varphi_{2,cr}^{(p)} = \frac{5}{6} \; (0.833\ldots)$$

net baryons (protons)

[N. G. Antoniou, F. K. Diakonos, A. S. Kapoyannis, K. S. Kousouris, Phys. Rev. Lett. **97**, 032002 (2006)]

Critical Monte Carlo (CMC) algorithm for baryons

- Simplified version of CMC* code:
  - Only protons produced
  - One cluster per event, produced by random Lévy walk: $\tilde{d}_F^{(B,2)} = 1/3 \Rightarrow \phi_2 = 5/6$
  - Lower / upper bounds of Lévy walks $p_{min,max}$ plugged in.
  - Cluster center exponential in p_T, slope adjusted by T_c parameter.
  - Poissonian proton multiplicity distribution.



Input parameters

Parameter	p_{min} (MeV)	p_{max} (MeV)	λ_{Poisson}	T_c (MeV)
Value	$0.1 \to 1$	$800 \to 1200$	$\langle p \rangle_{\text{non-empty}}$	163

* *[Antoniou, Diakonos, Kapoyannis and Kousouris, Phys. Rev. Lett. 97, 032002 (2006).]*

NA49 analysed data sets & cuts

- Published in [T. Anticic *et al.*, Eur. Phys. J. C 75:587 (2015), arXiv:1208.5292v5]

A	"C"+C*	"Si"+Si*	Pb+Pb
# Bootstrap Samples	1000		
Rapidity range	$-0.75 \leq y_{CM} \leq 0.75$		
# lattice positions	11 $(2 \times 5 +$ central)		
Lattice range (GeV)	$[-1.529, 1.471] \rightarrow [-1.471, 1.529]$		
Beam Energy ($\sqrt{s_{NN}}$)	158 A GeV (17.3 GeV)		
Centrality range	$0 \rightarrow 12\%$		$0 \rightarrow 10\%$
Proton purity	$> 80\%$		$> 90\%$
# events	148 060	165 941	329 789
$\langle p_{data} \rangle$ (after cuts)	1.6 ± 0.9	3.1 ± 1.7	9.12 ± 3.15

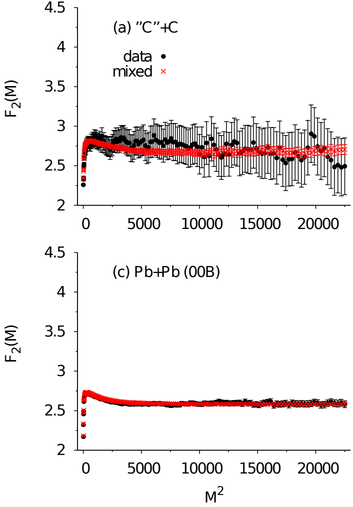
* Beam Components: "C" = C,N, "Si" = Si,Al,P

- Standard NA49 event/track cuts [T. Anticic *et al*, PR**C 81**, 149 (2010)].
- q_{inv} cut to remove split tracks, F-D effects and Coulomb repulsion
- Mid-rapidity selected because of approximately constant proton density in rapidity in this region

[N.G. Antoniou, F.K. Diakonos, A.S. Kapoyannis and K.S. Kousouris, PRL.**97**, 032002 (2006)]

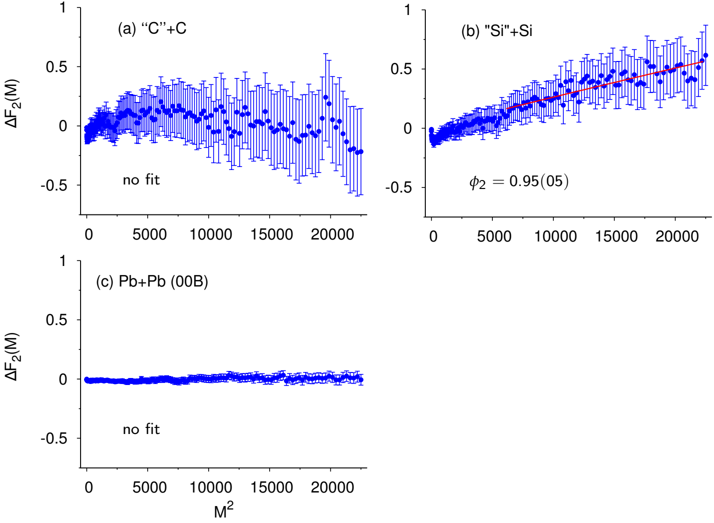
Analysis results - $F_2(M)$ for protons

- Evidence for intermittent behaviour in "Si"+Si – but large statistical errors.



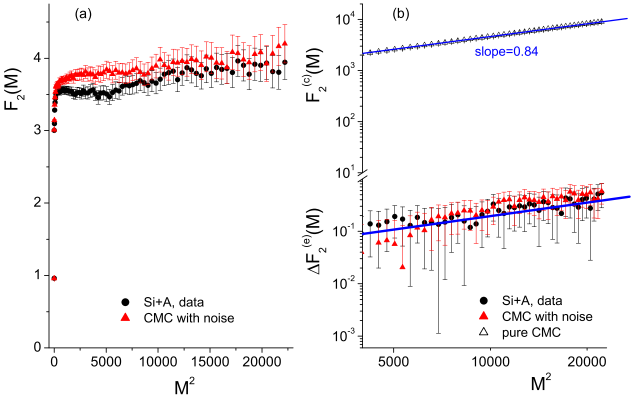
Analysis results - $\Delta F_2(M)$ for protons

- Fit with $\Delta F_2^{(e)}(M\ ;\ \mathcal{C}, \phi_2) = e^{\mathcal{C}} \cdot \left(M^2\right)^{\phi_2}$, for $M^2 \geq 6000$



Noisy CMC (baryons) – estimating the level of background

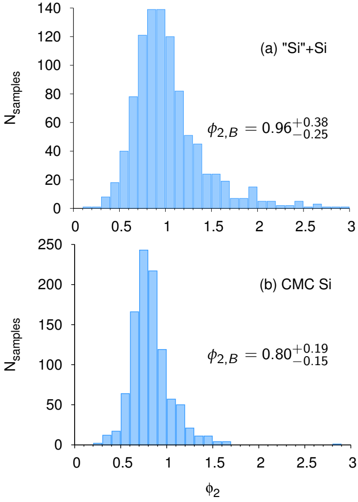
- $F_2(M)$ of noisy CMC approximates "Si"+Si for $\lambda \approx 0.99$
- Correlator $\Delta F_2^{(e)}(M)$ has slope $\phi_2 = 0.80^{+0.19}_{-0.15}$, very close to $\phi_2 = 0.84$ of pure $F_2^{(c)}(M)$



- $\Delta F_2^{(e)}(M)$ reproduces critical behaviour of pure CMC, even though their moments differ by orders of magnitude!
- Noisy CMC results show our approximation is reasonable for dominant background.

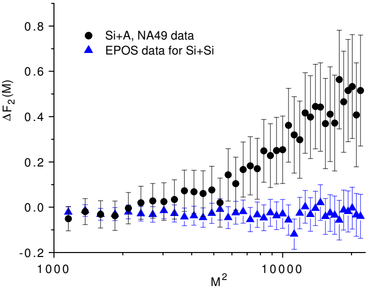
Analysis results - φ_2 bootstrap distribution

- Distributions are highly asymmetric due to closeness of $F_2^{(d)}(M)$ to $F_2^{(m)}(M)$.



- CMC model with a dominant background can reproduce the spread of ϕ_2 values observed in the "Si"+Si dataset
- The spread is partly artificial due to pathological fits (negative $\Delta F_2(M)$ values in some bootstrap samples)

Can jets "fake" intermittency effect?



*[K. Werner, F. Liu, and T. Pierog, Phys. Rev. C 74, 044902 (2006)]

- EPOS event generator* includes high-p_T jets $\Rightarrow$ possible spurious intermittency by non-critical protons.
- We simulate 630K Si+Si EPOS events:
  1. Z=14, A=28, for both beam and target
  2. $b_{max} = 2.6$ fm (12% most central)
  3. $\sqrt{s_{NN}} = 17.3$ GeV
  4. Rapidity cuts as in NA49 data
- Intermittency analysis (data & mixed events) repeated for EPOS.
- EPOS clearly cannot account for intermittency presence $\Rightarrow \Delta F_2(M)$ fluctuates around zero.

Dynamics near the QCD Critical point

1. Net-baryons are associated with the **slow component** of the order parameter (n_b: baryon-number density)
2. In a process out of equilibrium, net baryons relax to a **3d Ising** system in equilibrium
3. The basic mechanism: **thermal diffusion** and **sound waves**
4. Viscosity of net-baryon system near the QCD critical point:

$$\eta = \left(T, v_s, \xi; \frac{C_P}{C_V} \right) \quad ; \quad \zeta = \left(\rho, v_s, \xi; \frac{C_P}{C_V} \right)$$

Shear and Bulk Viscosity

From dimensional considerations:

$$[\textbf{viscosity}] = [\textbf{energy density}] \times [\textbf{time}]$$

$$\frac{\eta}{s} = \frac{K_B T v_s^{-1}}{\xi^2 s} F^{(s)}\left(\frac{C_P}{C_V}\right) \;\; ; \;\; \frac{\zeta}{s} = \frac{\rho v_s \xi}{s} F^{(b)}\left(\frac{C_P}{C_V}\right)$$

- v_s: velocity of sound waves
- ξ: correlation length
- ρ: mass density in the bulk, $\rho = \frac{\varepsilon + P}{c^2}$ (enthalpy density)
- s: entropy density ;
- $F^{(i)}\left(\frac{C_P}{C_V}\right) = f^{(i)} \frac{C_P}{C_V}$*

(With the choice $k_B = c = \hbar = 1$ the ratios $\frac{\eta}{s}$, $\frac{\zeta}{s}$ are dimensionless)

*[L.P.Kadanoff, J.Swift, Phys.Rev. **165** (1968) 310; Phys.Rev. **166** (1968) 89]

Thermodynamics & Universality Class

Basic thermodynamics:

$$C_P - C_V = T k_T \left(\frac{\partial P}{\partial T}\right)_V^2 \quad , \quad \frac{C_P}{C_V} = \frac{k_T}{k_s} \quad , \quad v_s^2 = (\rho k_s)^{-1}$$

$$s = \frac{\varepsilon + P}{T} - \frac{\mu_b n_b}{T}$$

k_T, k_s: isothermal, isoentropic (adiabatic) compresibility

Universality:

$$C_V = A_\pm |t|^{-\alpha} \quad , \quad k_T = \Gamma_\pm |t|^{-\gamma} \quad , \quad \xi = \xi_\pm |t|^{-\nu} \qquad \left(t \equiv \frac{T - T_c}{T_c}\right)$$

The critical exponents (α, γ, ν) and the amplitudes:

$$\frac{A_+}{A_-} \quad , \quad \frac{\Gamma_+}{\Gamma_-} \quad , \quad \frac{\xi_+}{\xi_-}$$

are fixed within the universality class*

*[K.Huang, "Statistical Mechanics", Wiley, New York (1987); P.M.Chaikin and T.C.Lubensky, "Principles of condensed matter physics" Cambridge University Press (1995)]

Boundary Condition

In the quark-matter phase $(T > T_c)$ and in a distance from the critical point $(T \geq 2T_c)$ we assume that the thermodynamic quantities are determined by the equation of state of a **massless, classical, ideal gas**:

$$\varepsilon = 3P \ , \ P = n_b T \ ,$$

$$h = 4 n_b T \ (enthalpy\ density) \ ,$$

$$C_V = 3n_b \ , \ C_P = 4n_b \ , \ k_T = (n_b T)^{-1} \ ,$$

$$s = \left(4 - \frac{\mu_b}{T}\right) n_b \ , \ v_s = \frac{1}{3}$$

The solution

Shear Viscosity

$$\left(\frac{\eta}{s}\right)_{\pm} = f^{(s)} M_{\pm} \left(1 + \Lambda_{\pm} |t|^{\gamma-\alpha}\right)^{1/2} |t|^{-\gamma+2\nu+\frac{\alpha}{2}}$$

Bulk Viscosity

$$\left(\frac{\zeta}{s}\right)_{\pm} = f^{(b)} N_{\pm} \left(1 + \Lambda_{\pm} |t|^{\gamma-\alpha}\right)^{3/2} |t|^{-\gamma-\nu+\frac{3\alpha}{2}}$$

The constants $M_{\pm}$, $N_{\pm}$, $\Lambda_{\pm}$, **are fixed by the critical state** (T_c, μ_c, n_c):
A typical choice consistent with NA49 measurements:

$$T_c \simeq 160 \text{ MeV}, \; \mu_c \simeq 220 \text{ MeV}, \; n_c \simeq 0.13 \text{ fm}^{-3} *$$

*[Latest NA49 results: https:\\edms.cern.ch\ui\file\1075059\1\na49_compil.pdf and references therein

N.G.Antoniou, F.K.Diakonos, A.S.Kapoyannis, Phys.Rev. **C81** (2010) 011901 (R)]

Final Constraints (Beyond SM)

KSS bound:

$$\left(\frac{\eta}{s}\right)_{\min} = \frac{1}{4\pi} \ ;$$

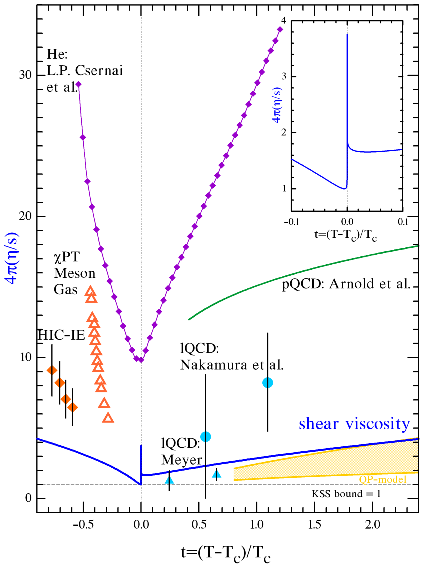
$$\left\langle \left(\frac{\zeta}{s}\right)_{+} \right\rangle \simeq \frac{1}{8\pi}\left(\frac{1}{3} - \langle v_s^2 \rangle\right) \ : \frac{3T_c}{2} \leq T \leq 2T_c \ ;$$

$$v_s^2 = \frac{t^{\alpha}}{4}\left(t^{\gamma-\alpha} + \frac{1}{3}\right) \ \Rightarrow \ \langle v_s^2 \rangle \simeq 0.27$$

[P.K.Kovtun, D.T.Son, A.O.Starinets, Phys.Rev.Lett. **94** (2005) 111601; J.Noronha-Hostler, G.S.Denicol, J.Noronha, R.P.G.Andrade, F.Grassi, Phys.Rev. **C88** (2013) 044916; A.Buchel, Phys.Lett. **B663** (2008) 286]

$$f^{(s)} \simeq 4\times10^{-2}, \ f^{(b)} \simeq 3\times10^{-3}$$

Shear Viscosity



blue line: [Our solution]

empty orange triangles: [M.Prakash *et al.*, Phys.Rept. **227** (1993) 321; J.-W.Chen *et al.*, Phys.Rev. **D76** (2007) 114011]

solid orange rectangles: [P.Danielewicz *et al.*, AIP Conf.Proc. **1128** (2009) 104-111; W.Schmidt *et al.*,Phys.Rev. **C47** (1993) 2782]

solid blue circles: [A.Nakamura *et al.*, Phys.Rev.Lett. **94** (2005) 072305]

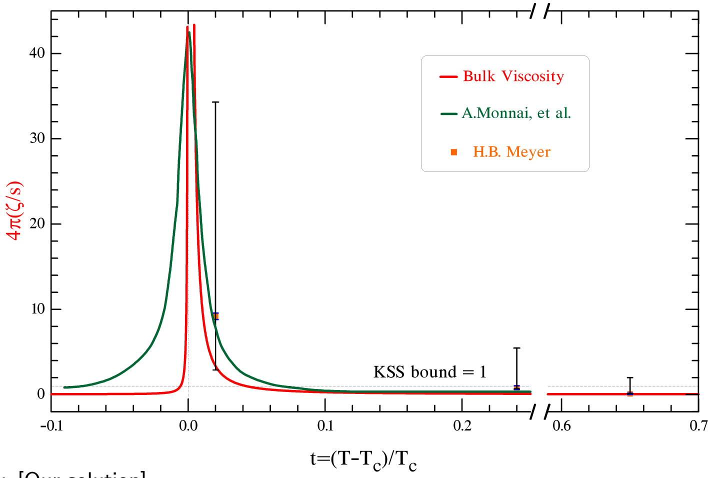
solid blue triangles: [H.B.Meyer, Phys. Rev. **D76** (2007) 101701(R)]

green line: [P.B.Arnold *et al.*, JHEP **0305** (2003) 051]

purple line and rectangles: [L.P.Csernai *et al.*, Phys.Rev.Lett. **97** (2006) 152303]

yellow band: [S.Plumari *et al.* Rhys.Rev. **D84** (2011) 111601]

Bulk Viscosity



red line: [Our solution]

green line: [A.Monnai *et al.*, arXiv: 1606.00771 v1 [nucl-th] (2016)]

solid orange rectangles: [H.B.Meyer, Phys.Rev.Lett. **100** (2008) 162001, with systematic (black) and statistical (blue) uncertainties]

Concluding remarks

1. Exact result from first principles (lQCD): crossover transition for $\mu_b = 0$
2. Critical fluctuations consistent with Ising-QCD universality class, have been observed at SPS energies ($Si + Si$ at 158 AGeV) suggesting, for the location of the QCD critical point, the neighbourhood of the state: $\mu_b \simeq 220$ MeV, $T_c \simeq 160$ MeV
3. The dynamics of the QCD critical point requires a study of net baryon systems produced in nuclear collisions. The size of shear viscosity in the hadronic phase is expected in the region $1 \leq 4\pi\frac{\eta}{s} \leq 3$ for $\frac{T_c}{2} \leq T < T_c$.

Expectations

1. Lattice QCD calculations at non-zero baryon-number density ($\mu_b \neq 0$), search for the critical point, calculation of transport properties (viscosity) with fully dynamical, light quarks.
2. Precision measurements (NA61 experiment) in a search for critical fluctuations in collisions of light nuclei (Be + Be, Ar + Sc, Xe + La) at SPS energies. Also at RHIC in the Beam Energy Scan Program.
3. Precision measurements of shear viscosity in net-baryon systems (elliptic flow of net protons + hydrodynamics) at SPS energies, close to the critical point.

Final remark

All these questions are discussed every year in the Workshop CPOD (Critical Point, Onset of Deconfinement)

1. CPOD 2016 : Wroclaw
2. CPOD 2017 : Stony Brook
3. CPOD 2018 : Corfu!! (hopefully)

Thank you!