Reduction of Couplings in Finite Thories and in the MSSM

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Corfu, September 2016

- What happens as we approach the Planck scale? or just as we go up in energy...
- How do we go from a fundamental theory to field theory as we know it?
- How are the gauge, Yukawa and Higgs sectors related at a more fundamental level?
- How do particles get their very different masses?
- What is the nature of the Higgs?
- Is there one or many? How this affects all the above?
- Where is the new physics??

Search for understanding relations between parameters

addition of symmetries.

$N = 1$ SUSY GUTs.

Complementary approach: look for RGI relations among couplings at GUT scale $\longrightarrow$ Planck scale

$\Rightarrow$ **reduction of couplings**

resulting theory: less free parameters $\therefore$ more predictive

Gauge Yukawa Unification – GYU

Remarkable: reduction of couplings provides a way to relate two previously unrelated sectors

gauge and Yukawa couplings

Reduction of couplings in third generation provides predictions for quark masses (top and bottom)

Including soft breaking terms gives Higgs masses and SUSY spectrum

Kapetanakis, M.M., Zoupanos (1993), Kubo, M.M., Olechowski, Tracas, Zoupanos (1995,1996,1997); Oehme (1995); Kobayashi, Kubo, Raby, Zhang (2005); Gogoladze, Mimura, Nandi (2003,2004); Gogoladze, Li, Senoguz, Shafi, Khalid, Raza (2006,2011); M.M., Tracas, Zoupanos (2014)

Gauge Yukawa Unification in Finite Theories

Dimensionless sector of all-loop finite $SU(5)$ model

$M_{top} \sim 178$ **GeV**

large $\tan\beta$, heavy SUSY spectrum

Kapetanakis, M.M., Zoupanos, Z.f.Physik (1993)

$M_{top}^{exp} 176 \pm 18$ GeV	found in 1995
$M_{top}^{th} \sim 172.5$	2007
$M_{top}^{exp} 173.1 \pm .09$ GeV	2013

Very promising, a more detailed analysis was clearly needed

Higgs mass $\sim 121 - 126$ GeV

Heinemeyer M.M., Zoupanos, JHEP, 2007; Phys.Lett.B (2013)

$M_H^{exp} 126 \pm 1$ GeV 2013

Gauge Yukawa Unification in the MSSM

- Possible to have a reduced system in the third generation compatible with quark masses

large $\tan\beta$, heavy SUSY spectrum

- Higgs mass $\sim 123 - 126$ GeV

M.M., Tracas, Zoupanos, Phys.Lett.B 2014

Reduction of Couplings

A RGI relation among couplings $\Phi(g_1, \ldots, g_N) = 0$ satisfies

$$\mu \, d\Phi/d\mu \; = \; \sum_{i=1}^{N} \beta_i \, \partial\Phi/\partial g_i \; = \; 0.$$

g_i = **coupling,** β_i **its** β **function**

Finding the $(N-1)$ independent Φ's is equivalent to solve the reduction equations (RE)

$$\beta_g \, (dg_i/dg) = \beta_i \; ,$$

$i = 1, \cdots, N$

- Reduced theory: only one independent coupling and its β function
- complete reduction: power series solution of RE

$$g_a = \sum_{n=0} \rho_a^{(n)} \, g^{2n+1}$$

- uniqueness of the solution can be investigated at one-loop
  valid at all loops Zimmermann, Oehme, Sibold (1984,1985)
- The complete reduction might be too restrictive, one may use fewer Φ's as RGI constraints
- Reduction of couplings is essential for finiteness

finiteness: absence of ∞ renormalizations

$$\Rightarrow \quad \beta^N = 0$$

- SUSY no-renormalization theorems
  - $\Rightarrow$ only study one and two-loops
  - guarantee that is gauge and reparameterization invariant to all loops

Reduction of couplings: example without symmetry

One Dirac spinor ψ and one pseudoscalar B in renormalizable interaction

$$\mathcal{L}_{int} = g\bar{\psi}B\gamma_5\psi - \frac{\lambda}{4!}B^4$$

β functions at one-loop order are

$$\beta_{g^2} = \frac{1}{16\pi^2}(5g^4 + ...)$$

$$\beta_\lambda = \frac{1}{16\pi^2}(\frac{3}{2}\lambda^2 + 4\lambda g^2 - 24g^4 ...)$$

We solve the reduction equations

$$\beta_{g^2} \frac{d\lambda}{dg^2} = \beta_\lambda$$

with a power series solution

$$\lambda = g^2(\rho^{(0)} + \rho^{(1)} g^2)$$

The only possible solution for small g is

$$\lambda = \frac{1}{3}(1 + \sqrt{145}) g^2 + \rho^{(1)} g^4 + ...$$

The two coupling model has been reduced to a one coupling renormalizable one.

Quartic coupling constant determined uniquely by Yukawa coupling

No symmetries are known for this system $\Rightarrow$ symmetries need not be the only way to reduce couplings consistently

Reduction of couplings: example with symmetry

Consider an $SU(N)$ gauge theory with the following matter content:

$\phi^i(\mathbf{N})$ and $\hat{\phi}_i(\overline{\mathbf{N}})$ are complex scalars
$\psi^i(\mathbf{N})$ and $\hat{\psi}_i(\overline{\mathbf{N}})$ are left-handed Weyl spinors
$\lambda^a(a = 1, \ldots, N^2 - 1)$ is right-handed Weyl spinor in the adjoint representation of $SU(N)$.

The Lagrangian is:

$$
\begin{aligned}
\mathcal{L} &= -\frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} + i\sqrt{2}\{\, g_Y \overline{\psi} \lambda^a T^a \phi - \hat{g}_Y \overline{\hat{\psi}} \lambda^a T^a \hat{\phi} + \text{h.c.}\,\} \\
&\quad - V(\phi, \overline{\phi}), \\
V(\phi, \overline{\phi}) &= \frac{1}{4} \lambda_1 (\phi^i \phi^*_i)^2 + \frac{1}{4} \lambda_2 (\hat{\phi}_i \hat{\phi}^{*\,i})^2 \\
&\quad + \lambda_3 (\phi^i \phi^*_i)(\hat{\phi}_j \hat{\phi}^{*\,j}) + \lambda_4 (\phi^i \phi^*_j)(\hat{\phi}_i \hat{\phi}^{*\,j}),
\end{aligned}
$$

Applying the REs method and searching for a power solution we get:

$$
\begin{aligned}
g_Y &= \hat{g}_Y = g \, , \\
\lambda_1 &= \lambda_2 = \frac{N-1}{N} g^2 \, , \\
\lambda_3 &= \frac{1}{2N} g^2 \, , \; \lambda_4 = -\frac{1}{2} g^2 \, .
\end{aligned}
$$

which corresponds to an $N = 1$ supersymmetric gauge theory
SUSY solutions appear in many cases when the RE method is applied

Reduction of couplings: the Standard Model

It is possible to make a reduced system in the Standard Model in the matter sector:
solve the REs, reduce the Yukawa and Higgs in favour of α_S gives

$$\alpha_t/\alpha_s = \frac{2}{9}\,; \quad \alpha_\lambda/\alpha_s = \frac{\sqrt{689}-25}{18} \simeq 0.0694$$

border line in RG surface, Pendleton-Ross infrared fixed line
But including the corrections due to non-vanishing gauge couplings up to two-loops, changes these relations and gives

$$M_t = 98.6 \pm 9.2 GeV$$

and

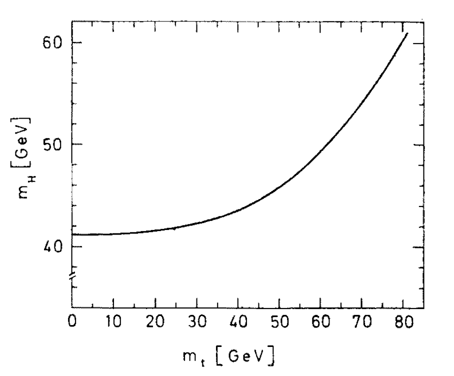
$$M_h = 64.5 \pm 1.5 GeV$$

Both out of the experimental range

Kubo, Sibold and Zimmermann, 1984, 1985

General asymptotic reduction in SM

More parameters (couplings):
if top and Higgs heavy $\Rightarrow$ new physics heavy



Kubo, Sibold and Zimmermann, 1985; Sibold and Zimmermann, 1987

Finiteness

A chiral, anomaly free, $N=1$ globally supersymmetric gauge theory based on a group G with gauge coupling constant g has a superpotential

$$W = \frac{1}{2}\, m^{ij}\, \Phi_i\, \Phi_j + \frac{1}{6}\, C^{ijk}\, \Phi_i\, \Phi_j\, \Phi_k \ ,$$

Requiring one-loop finiteness $\beta_g^{(1)} = 0 = \gamma_i^{j(1)}$ gives the following conditions:

$$\sum_i T(R_i) = 3C_2(G)\,, \qquad \frac{1}{2} C_{ipq} C^{jpq} = 2\delta_i^j g^2 C_2(R_i)\,.$$

$C_2(G)$ quadratic Casimir invariant, $T(R_i)$ Dynkin index of R_i, C_{ijk} Yukawa coup., g gauge coup.

- **restricts the particle content of the models**
- **relates the gauge and Yukawa sectors**

- One-loop finiteness $\Rightarrow$ two-loop finiteness

Jones, Mezincescu and Yao (1984,1985)

- One-loop finiteness restricts the choice of irreps R_i, as well as the Yukawa couplings
- Cannot be applied to the susy Standard Model (SSM): $C_2[U(1)] = 0$
- The finiteness conditions allow only SSB terms

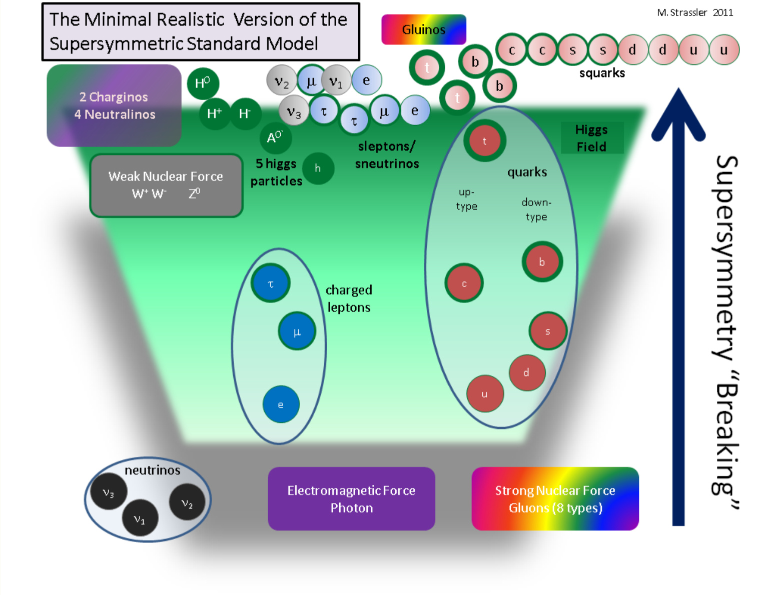
It is possible to achieve all-loop finiteness $\beta^n = 0$:

Lucchesi, Piguet, Sibold

1. One-loop finiteness conditions must be satisfied
2. The Yukawa couplings must be a formal power series in g, which is solution (isolated and non-degenerate) to the reduction equations

SUSY breaking soft terms

Introduce over 100 new free parameters ☹



RGI in the Soft Supersymmetry Breaking Sector

Supersymmetry is essential. It has to be broken, though...

$$-\mathcal{L}_{\rm SB} = \frac{1}{6}\, h^{ijk}\, \phi_i\phi_j\phi_k + \frac{1}{2}\, b^{ij}\, \phi_i\phi_j + \frac{1}{2}\, (m^2)^j_i\, \phi^{*\,i}\phi_j + \frac{1}{2}\, M\, \lambda\lambda + \text{H.c.}$$

h trilinear couplings (A), b^{ij} bilinear couplings, m^2 squared scalar masses, M unified gaugino mass

The RGI method has been extended to the SSB of these theories.

- One- and two-loop finiteness conditions for SSB have been known for some time

  Jack, Jones, et al.

- It is also possible to have all-loop RGI relations in the finite and non-finite cases

  Kazakov; Jack, Jones, Pickering

SSB terms depend only on g and the unified gaugino mass M
universality conditions

$$h = -MC, \qquad m^2 \propto M^2, \qquad b \propto M\mu$$

Very appealing! But too restrictive

it leads to phenomenological problems:

- Charge and colour breaking vacua
- Incompatible with radiative electroweak breaking
- The lightest susy particle (LSP) is charged

Possible to relax the universality condition to a sum-rule for the soft scalar masses

$\Rightarrow$ better phenomenology.

Kobayashi, Kubo, M.M., Zoupanos

All-loop RGI relations in the soft sector

From reduction equations

$$\frac{dC^{ijk}}{dg} = \frac{\beta_C^{ijk}}{\beta_g}$$

we assume the existence of a RGI surface on which

$$h^{ijk} = -M \frac{dC(g)^{ijl}}{d \ln g}$$

holds too in all-orders. Then one can prove, that the following relations are RGI to all-loops

$$\begin{aligned} M &= M_0 \ \frac{\beta_g}{g}, \\ h^{ijk} &= -M_0 \ \beta_C^{ijk}, \\ b^{ij} &= -M_0 \ \beta_\mu^{ij}, \\ (m^2)^i{}_j &= \frac{1}{2} \ |M_0|^2 \ \mu \frac{d\gamma^i{}_j}{d\mu}, \end{aligned} \tag{1}$$

where M_0 is an arbitrary reference mass scale. If $M_0 = m_{3/2}$ we get exactly the anomaly mediated breaking terms

Soft scalar sum-rule for the finite case

Finiteness implies

$$C^{ijk} = g \sum_{n=0} \rho^{ijk}_{(n)} g^{2n} \Rightarrow h^{ijk} = -MC^{ijk} + \cdots = -M\rho^{ijk}_{(0)}\, g + O(g^5)$$

If lowest order coefficients $\rho^{ijk}_{(0)}$ and $(m^2)^i_j$ satisfy diagonality relations

$$\rho_{ipq(0)}\rho^{jpq}_{(0)} \propto \delta^j_i\ , \qquad (m^2)^i_j = m^2_j \delta^i_j$$

for all p and q.

We find the the following soft scalar-mass sum rule, also to all-loops

for $i,\ j,\ k$ with $\rho^{ijk}_{(0)} \neq 0$, where $\Delta^{(1)}$ is the two-loop correction =0 for universal choice

$$(\ m^2_i + m^2_j + m^2_k\ )/MM^\dagger = 1 + \frac{g^2}{16\pi^2}\Delta^{(2)} + O(g^4)$$

Kazakov et al; Jack, Jones et al; Yamada; Hisano, Shifman; Kobayashi, Kubo, Zoupanos

Also satisfied in certain class of orbifold models, where massive states are organized into $N = 4$ supermultiples

Several aspects of Finite Models have been studied

- $SU(5)$ Finite Models studied extensively

  Rabi et al; Kazakov et al; López-Mercader, Quirós et al; M.M, Kapetanakis, Zoupanos; etc
- One of the above coincides with a non-standard Calabi-Yau $SU(5) \times E_8$ Greene et al; Kapetanakis, M.M., Zoupanos
- Finite theory from compactified string model also exists (albeit not good phenomenology) Ibáñez
- Criteria for getting finite theories from branes Hanany, Strassler, Uranga
- $N = 2$ finiteness Frere, Mezincescu and Yao
- Models involving three generations Babu, Enkhbat, Gogoladze
- Some models with $SU(N)^k$ **finite** $\Longleftrightarrow$ **3 generations,** good phenomenology with $SU(3)^3$ Ma, M.M, Zoupanos
- Relation between commutative field theories and finiteness studied Jack and Jones
- Proof of conformal invariance in finite theories Kazakov

$SU(5)$ Finite Models

We study two models with $SU(5)$ gauge group. The matter content is

$$3\,\overline{\mathbf{5}} + 3\,\mathbf{10} + 4\,\{\mathbf{5} + \overline{\mathbf{5}}\} + \mathbf{24}$$

The models are finite to all-loops in the dimensionful and dimensionless sector. In addition:

- The soft scalar masses obey a sum rule
- At the M_{GUT} scale the gauge symmetry is broken and we are left with the MSSM
- At the same time finiteness is broken
- The two Higgs doublets of the MSSM should mostly be made out of a pair of Higgs $\{\mathbf{5} + \overline{\mathbf{5}}\}$ which couple to the third generation

The difference between the two models is the way the Higgses couple to the $\mathbf{24}$

Kapetanakis, Mondragón, Zoupanos; Kazakov et al.

The superpotential which describes the two models takes the form

$$W = \sum_{i=1}^{3} \left[\frac{1}{2} g_i^u \, \mathbf{10}_i \mathbf{10}_i H_i + g_i^d \, \mathbf{10}_i \overline{\mathbf{5}}_i \, \overline{H}_i \right] + g_{23}^u \, \mathbf{10}_2 \mathbf{10}_3 H_4$$
$$+ g_{23}^d \, \mathbf{10}_2 \overline{\mathbf{5}}_3 \, \overline{H}_4 + g_{32}^d \, \mathbf{10}_3 \overline{\mathbf{5}}_2 \, \overline{H}_4 + \sum_{a=1}^{4} g_a^f \, H_a \, \mathbf{24} \, \overline{H}_a + \frac{g^\lambda}{3} \, (\mathbf{24})^3$$

find isolated and non-degenerate solution to the finiteness conditions

The unique solution implies discrete symmetries
We will do a partial reduction, only third generation

The finiteness relations give at the M_{GUT} scale

Model A

- $g_t^2 = \frac{8}{5}\, g^2$
- $g_{b,\tau}^2 = \frac{6}{5}\, g^2$
- $m_{H_u}^2 + 2m_{\mathbf{10}}^2 = M^2$
- $m_{H_d}^2 + m_{\mathbf{\bar{5}}}^2 + m_{\mathbf{10}}^2 = M^2$

- **3 free parameters:**
  M, $m_{\mathbf{\bar{5}}}^2$ and $m_{\mathbf{10}}^2$

Model B

- $g_t^2 = \frac{4}{5}\, g^2$
- $g_{b,\tau}^2 = \frac{3}{5}\, g^2$
- $m_{H_u}^2 + 2m_{\mathbf{10}}^2 = M^2$
- $m_{H_d}^2 - 2m_{\mathbf{10}}^2 = -\frac{M^2}{3}$
- $m_{\mathbf{\bar{5}}}^2 + 3m_{\mathbf{10}}^2 = \frac{4M^2}{3}$

- **2 free parameters:**
  M, $m_{\mathbf{\bar{5}}}^2$

Phenomenology

The gauge symmetry is broken below M_{GUT}, and what remains are boundary conditions of the form $C_i = \kappa_i g$, $h = -MC$ and the sum rule at M_{GUT}, below that is the MSSM.

- Fix the value of $m_\tau \Rightarrow \tan\beta \Rightarrow M_{top}$ and m_{bot}
- We assume a unique susy breaking scale
- The LSP is neutral
- The solutions should be compatible with radiative electroweak breaking
- No fast proton decay

We also

- Allow 5% variation of the Yukawa couplings at GUT scale due to threshold corrections
- Include radiative corrections to bottom and tau, plus resummation **(very important!)**
- Estimate theoretical uncertainties

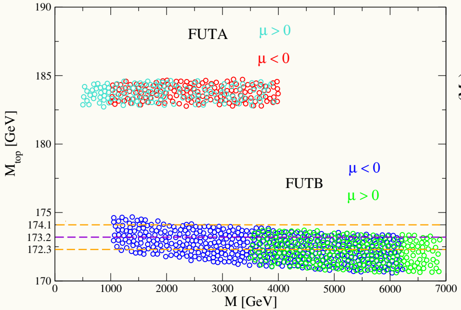
We look for the solutions that satisfy the following constraints:

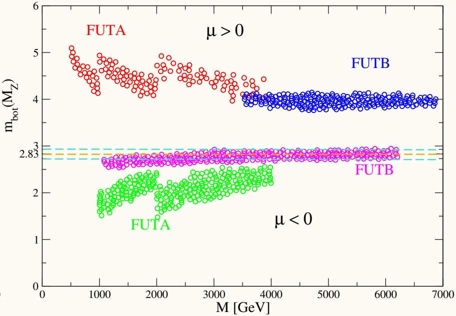
- Right masses for top and bottom
  fact of life FeynHiggs
- B physics observables
  fact of life

The lightest MSSM Higgs boson mass
The SUSY spectrum

FeynHiggs, FUT

TOP AND BOTTOM MASS





FUTA: $M_{top} \sim 182 \sim 185\ GeV$ FUTB: $M_{top} \sim 172 \sim 174\ GeV$

Theoretical uncertainties $\sim 4\%$

Δb and $\Delta\tau$ included, resummation done

FUTB $\mu < 0$ favoured

Experimental data

- We use the experimental values of M_H to compare with our previous results ($M_H =\sim 121 - 126$ GeV, 2007) and put extra constraints $M_H^{exp} = 126 \pm 2 \pm 1$ and $M_H^{exp} = 125.1 \pm 3.1$ 2 GeV theoretical, 1 GeV experimental
- We also use the BPO constraints

$$\mathrm{BR}(b \to s\gamma) SM/MSSM : |BRbsg - 1.089| < 0.27$$

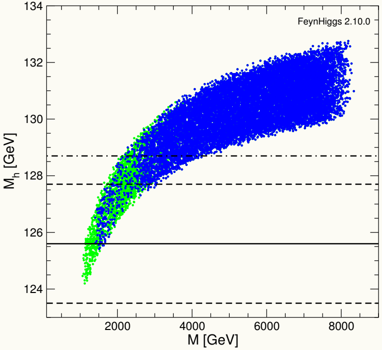
$$\mathrm{BR}(B_u \to \tau\nu) SM/MSSM : |BRbtn - 1.39| < 0.69$$

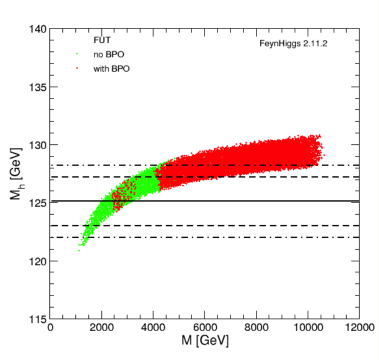
$$\Delta M_{B_s} SM/MSSM : 0.97 \pm 20$$

$$\mathrm{BR}(B_s \to \mu^+\mu^-) = (2.9 \pm 1.4) \times 10^{-9}$$

- We can now restrict (partly) our boundary conditions on M

Higgs mass





FUTB

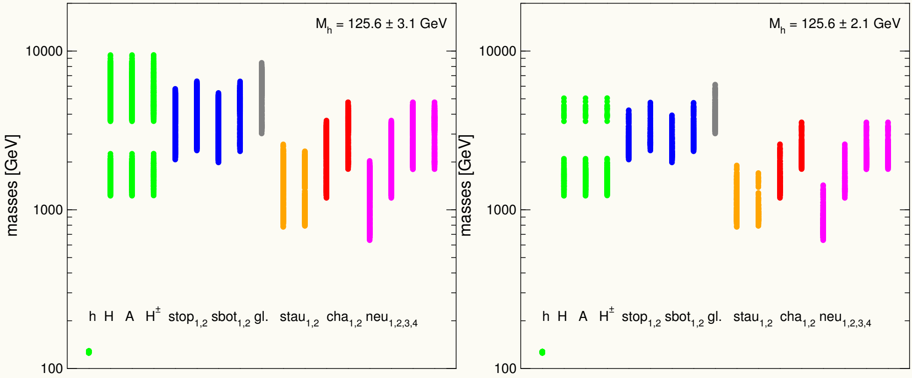
constrained by $M_{Higgs} \sim 126 \pm 3$ (2013) and 125.1 ± 3 GeV (2015)

blue and red points satisfy B Physics constraints 2013 and 2016, 2016 with 2-loop m_{top} corrections

Uncertainties ± 3 GeV (FeynHiggs)

Heinemeyer, M.M., Tracas, Zoupanos (2013) (2016)

S-SPECTRUM



SUSY spectrum with B physics constraints

Challenging for LHC

Heinemeyer, M.M., Zoupanos (2014)

Results

When confronted with low-energy precision data

only FUTB $\mu < 0$ survives

- $M_{top} \sim 173\ GeV$ 4% $\quad M^{exp}_{top}\mathrm{exp} = (173.2 \pm 0.9)\mathrm{GeV}$
- $m_{bot}(M_Z) \sim 2.8\ GeV$ 8 % $\quad m^{exp}_{bot}(M_Z) = (2.83 \pm 0.10)\mathrm{GeV}$
- $M_{Higgs} \sim 125\ GeV(\pm 3 GeV)$ $\quad M^{exp}_{Higgs} = 125.1 \pm 1$
- $\tan\beta \sim 44 - 46$
- s-spectrum $\gtrsim 500\ GeV$ $\quad$ consistent with exp bounds

In progress

- 3 families with discrete symmetry $\quad$ under way
- neutrino masses via $\not{R}$

Reduction of couplings in the MSSM

The superpotential

$$W = Y_t H_2 Q t^c + Y_b H_1 Q b^c + Y_\tau H_1 L \tau^c + \mu H_1 H_2$$

with soft breaking terms,

$$-\mathcal{L}_{SSB} = \sum_\phi m_\phi^2 \phi^* \phi + \left[m_3^2 H_1 H_2 + \sum_{i=1}^{3} \frac{1}{2} M_i \lambda_i \lambda_i + \mathrm{h.c} \right]$$
$$+ \left[h_t H_2 Q t^c + h_b H_1 Q b^c + h_\tau H_1 L \tau^c + \mathrm{h.c.} \right],$$

then, reduction of couplings implies

$$\beta_{Y_{t,b,\tau}} = \beta_{g_3} \frac{dY_{t,b,\tau}}{dg_3}$$

Boundary conditions at the unification scale

$$\frac{Y_t^2}{4\pi} = c_1 \frac{g_3^2}{4\pi} + c_2 \left(\frac{g_3^2}{4\pi}\right)^2 \tag{2}$$

$$\frac{Y_b^2}{4\pi} = p_1 \frac{g_3^2}{4\pi} + p_2 \left(\frac{g_3^2}{4\pi}\right)^2 \tag{3}$$

are given by

$$c_1 = \frac{1}{3} + \frac{71}{525}\frac{\alpha_1}{\alpha_3} + \frac{3}{7}\frac{\alpha_2}{\alpha_3} + \frac{1}{35}\frac{Y_\tau^2/4\pi}{\alpha_3},$$

$$p_1 = \frac{1}{3} + \frac{29}{525}\frac{\alpha_1}{\alpha_3} + \frac{3}{7}\frac{\alpha_2}{\alpha_3} - \frac{6}{35}\frac{Y_\tau^2/4\pi}{\alpha_3}$$

Y_τ not reduced, its reduction gives imaginary values

c_2 and p_2 also found (long expressions not shown)

Soft breaking terms

The reduction of couplings in the SSB sector gives the following boundary conditions at the unification scale

$$Y_t^2 = c_1 g_3^2 + c_2 g_3^4/(4\pi) \quad \text{and} \quad Y_b^2 = p_1 g_3^2 + p_2 g_3^4/(4\pi)$$
$$h_{t,b} = -c_{t,b} M Y_{t,b},$$

At the unification scale $c_t = c_b = 1$ and we recover

$$m_{H_2}^2 + m_Q^2 + m_{t^c}^2 = M^2,$$
$$m_{H_1}^2 + m_Q^2 + m_{b^c}^2 = M^2,$$

M is unified gaugino mass

coincide with anomaly mediated susy breaking

Corrections

- Same as with Yukawas the τ coupling cannot be reduced
- Possible to calculate the corrections coming from $\alpha_1, \alpha_2, g_\tau$ to sum rule

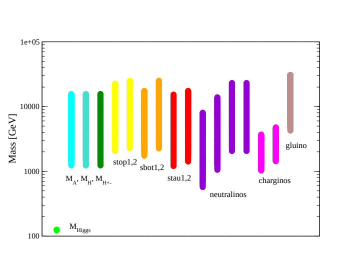
$$c_t = \frac{N_t}{D} \qquad c_b = \frac{N_b}{D}$$

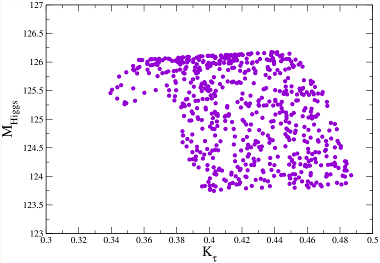
$$N_t = 240 + 400p_1 + 39\rho_1^2 + 71p_1\rho_1^2 + 135\rho_2^2 + 225p_1\rho_2^2 - 15p_1\sigma_h\sqrt{\rho_\tau}$$

$$N_b = 240 + 400c_1 + 21\rho_1^2 + 29c_1\rho_1^2 + 135\rho_2^2 + 225c_1\rho_2^2 + 45\sigma_h\sqrt{\rho_\tau} + 90c_1\sigma_h\sqrt{\rho_\tau}$$

$$D = 15(9 + 18p_1 + 18c_1 + 35c_1p_1)$$

Before corrections to the sum rule (2013,2014)





SUSY spectrum and the Higgs mass

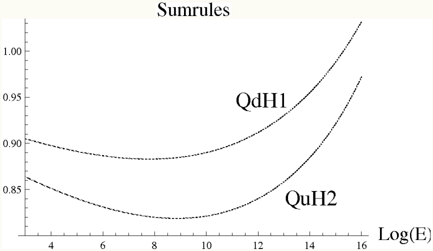
Reduction of the third generation of quark masses at M_{GUT}, before corrections to the sum rule and with one-loop corrections from m_t to the Higgs mass

Corrections to sum rule

Corrections to the sum-rule are scale dependent.

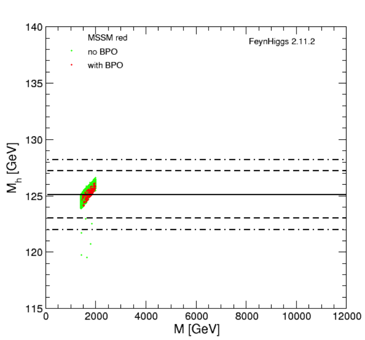
$$m_{H_2}^2 + m_Q^2 + m_{t^c}^2 = M_3^2 \frac{N_{SR1}}{D_{SR}},$$

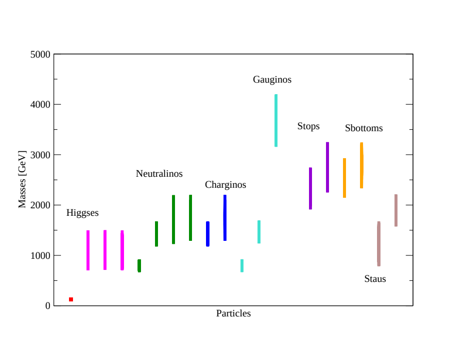
$$m_{H_1}^2 + m_Q^2 + m_{b^c}^2 = M_3^2 \frac{N_{SR2}}{D_{SR}},$$



At the unification scale they are $< 3-5\%$, at other scales they can be larger ($< 17\%$)

Example, preliminary – not complete





Example with sume rule at $3.0 \times 10^{13}\ GeV$**, no corrections to sum-rule, satisfying phenomenological constraints**

Results in Reduced MSSM

- Possible to have reduction of couplings in MSSM
- Up to now only attempted in SM or in GUTs
- Reduced system further constrained by phenomenology: compatible with quark masses with $\mu < 0$
- SUSY spectrum, large $\tan\beta$
- Higgs mass $\sim 123 \sim 127$ *GeV*
- Heavy susy spectrum 500 *GeV* $\lesssim M_{LSP}$

Conclusions

- Reduction of couplings: powerful principle implies Gauge Yukawa Unification
- **Finiteness, interesting and predictive principle**
  $\Rightarrow$ reduces greatly the number of free parameters
- completely finite theories
  i.e. including the SSB terms, that satisfy the sum rule
- Confronting the $SU(5)$ FUT models with low-energy precision data **does** distinguish among models
  FUTB favoured
- **Possible to have reduction of couplings in MSSM (RMSSM)**
- Heavy SUSY spectrum
- only solutions for $\mu < 0$ compatible with quark masses
- large $\tan\beta$
- s-spectrum starts above ~ 500 GeV
- Higgs mass $M_h \sim 125$ GeV
- corrections to Yukawa couplings and sum rule calculated, depend on scale
- Detailed study of SUSY masses, Higgs decays, and three generations in progress
- Similar global results for FUT and RMSSM, but details differ