Generally flat inflationary potentials

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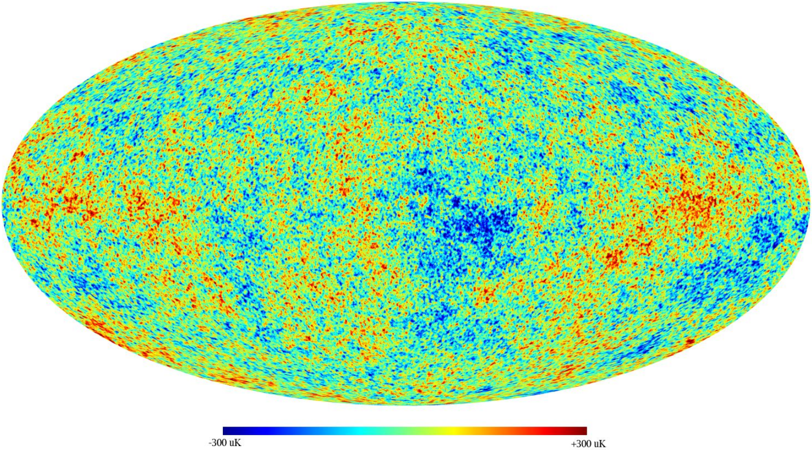
(with Z. Lalak, M. Lewicki and J. Rubio)

Cosmic microwave background



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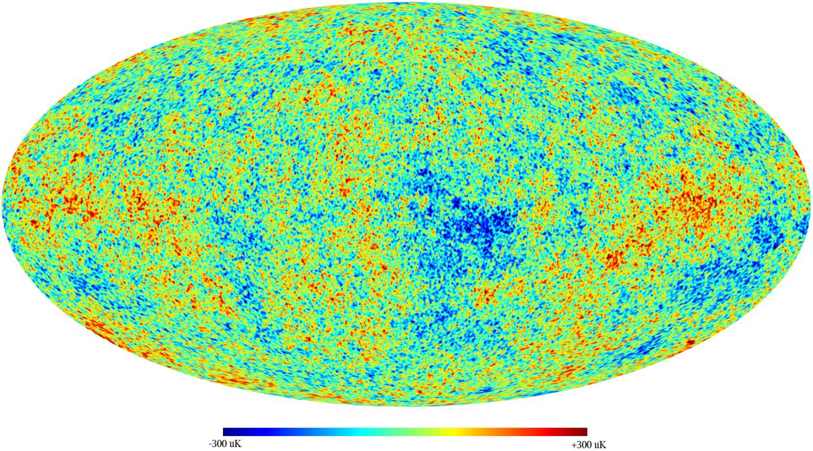

Convention: $8\pi G = 1 = M_p^{-2}$, where $M_p \simeq 2.5 \times 10^{18} GeV$

Introduction to cosmic inflation

Let us assume, that the flat FRW Universe with the metric tensor

$$ds^2 = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2) ,$$

is filled with a homogeneous scalar field $\phi(t)$ with potential $V(\phi)$. The $a(t)$ is the scale factor. Then Einstein equations are following

$$3H^2 = \rho = \frac{1}{2}\dot{\phi}^2 + V , \qquad 2\dot{H} = -(\rho + P) = -\dot{\phi}^2 , \qquad (1)$$

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where $H = \frac{\dot{a}}{a}$ is a Hubble parameter. Let us note that

$$\frac{\dot{H}}{H^2} = -\frac{3\dot{\phi}^2}{\dot{\phi}^2 + 2V} \quad \Rightarrow \quad \dot{H} \ll H^2 \ \text{ for } \ \dot{\phi}^2 \ll V \ . \qquad (2)$$

When $H \sim const$ one obtains $a \sim e^{Ht} \rightarrow$ exponential expansion of the Universe! This is an example of the cosmic inflation.

Primordial inhomogeneities

What we observe are anisotropies of the CMB radiation. We know how to relate them to primordial curvature perturbations $\mathcal{R}$ generated during inflation. We define the power spectrum of $\mathcal{R}$ by

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2}|\mathcal{R}_k|^2 \, . \tag{3}$$

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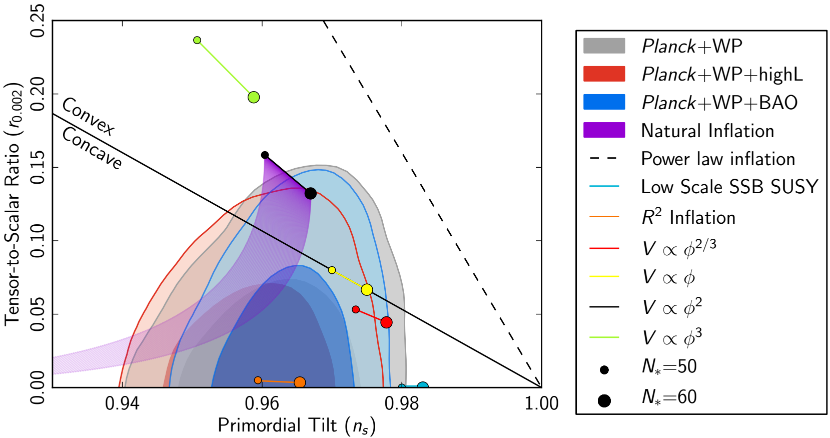
- Tensor-to scalar ratio $r = \mathcal{P}_h/\mathcal{P}_{\mathcal{R}} \simeq 8\left(\frac{V'}{V}\right)^2$

Comparison to the data



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- What could be responsible for such a local flatness? A stationary point! The saddle point doesn't fit the data (too small n_s), so we need higher order stationary points.
- Why asking for little? Let's look for a general potential, which would be as flat as it's possible!

Flat scalar potentials!

Let's start from a general scalar theory with minimal coupling to gravity

$$S = \int d^4\sqrt{-g}\left[\frac{1}{2}R + \frac{1}{2}(\partial\phi)^2 - V(f(\phi))\right] , \tag{4}$$

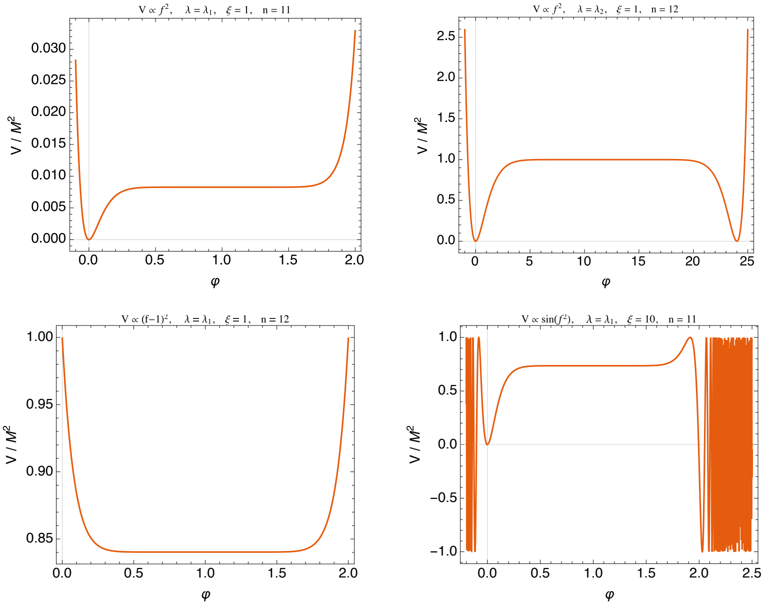
where

$$f(\phi) = \xi\sum_{k=1}^{n}\lambda_k\,\phi^k\,, \tag{5}$$

In general such a potential does not need to be flat anywhere and therefore it is not suitable for inflation. We want to assume that V (and therefore $f(\phi)$) is at least locally flat $\Rightarrow$ has a stationary point at some ϕ_s. The maximal order of ϕ_s is $n-1$, which gives

$$f(\phi) = \frac{\xi}{n}(n\,\lambda_n)^{\frac{-1}{n-1}}\left(1 - \left(1 - (n\,\lambda_n)^{\frac{1}{n-1}}\,\phi\right)^n\right)\,. \tag{6}$$

Flat potentials for finite n



Starobinsky inflation as a maximally flat theory

What would happen if we require $n \to \infty$, i.e. infinitely flat potential around the stationary point? For general form of λ the $f(\phi)$ does not converge. But for

$$\lambda_n = \frac{1}{\xi}\left(\frac{\xi}{n}\right)^n \tag{7}$$

one finds in the $n \to \infty$ limit

$$f(\phi) = 1 - e^{-\xi\phi} \tag{8}$$

so for $V \propto f^2$ one obtains

$$V \propto (1 - e^{-\xi\phi})^2\,, \tag{9}$$

which is the generalised Einstein frame Starobinsky potential.

Flat potentials and α-attractors

α-attractors - theory with kinetic term with a pole

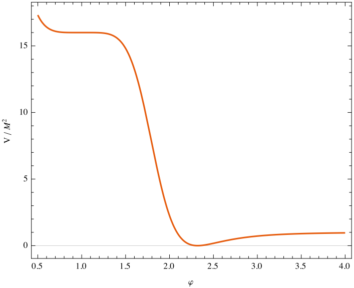
$$\mathcal{L} = \sqrt{-g}\left(\frac{(\partial\psi)^2}{\left(1-\frac{\psi^2}{6\alpha^2}\right)^2} - V(\psi)\right) . \tag{10}$$

If you'd re-define the field to obtain a canonical kinetic term it would appear, that any potential $V(\psi)$ is stretched around the pole, just like for $V(f(\varphi))$. In fact you can use $f(\varphi)$ as a scalar field and find a direct relation between f and ψ.

$$\psi(f) = \frac{\sqrt{6\alpha}\left((6\alpha(1-f)\xi)^{\sqrt{\frac{2}{3\alpha}}\xi} - 1\right)}{(6\alpha(1-f)\xi)^{\sqrt{\frac{2}{3\alpha}}\xi} + 1} . \tag{11}$$

Application to a scalar-tensor theory

$$S = \int d^4\sqrt{-g}\left[\frac{1}{2}f(\varphi)R + \frac{1}{2}(\partial\varphi)^2 - M^2(f(\varphi)-1)^2\right], \qquad (12)$$



Advantages? Very low scale of inflation and pre-inflationary era, which may be a solution to the problem of initial conditions for inflation

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- Higgs/Starobinsky inflation are examples of the flattest potentials
- You can do the same for $f(R)$ theory and the flattest possible potential will also be Starobinsky-like
- If you implement this idea to the Scalar-tensor theory you will get two flat regions - multi phase inflation, possible solution to problem of initial conditions