Asymptotic Symmetry Algebra of Conformal Gravity

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Introduction and Motivation

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- ⇒ importance of the boundary conditions

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- Solutions of CG can be classified according to the ASA they form - we have classified BH and geon solutions
- Geon = pp wave solutions

Conformal Gravity

$$S = \alpha \int d^4x C^{\mu}{}_{\nu\sigma\rho} C_{\mu}{}^{\nu\sigma\rho}$$

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$$C_{\mu\nu\rho\sigma} = R_{\mu\nu\rho\sigma} - \frac{2}{n-2}\left(g_{\mu[\rho}R_{\sigma]\nu} - g_{\nu[\rho}R_{\sigma]\mu}\right) + \frac{2}{(n-1)(n-2)} R g_{\mu[\rho}g_{\sigma]\nu}$$

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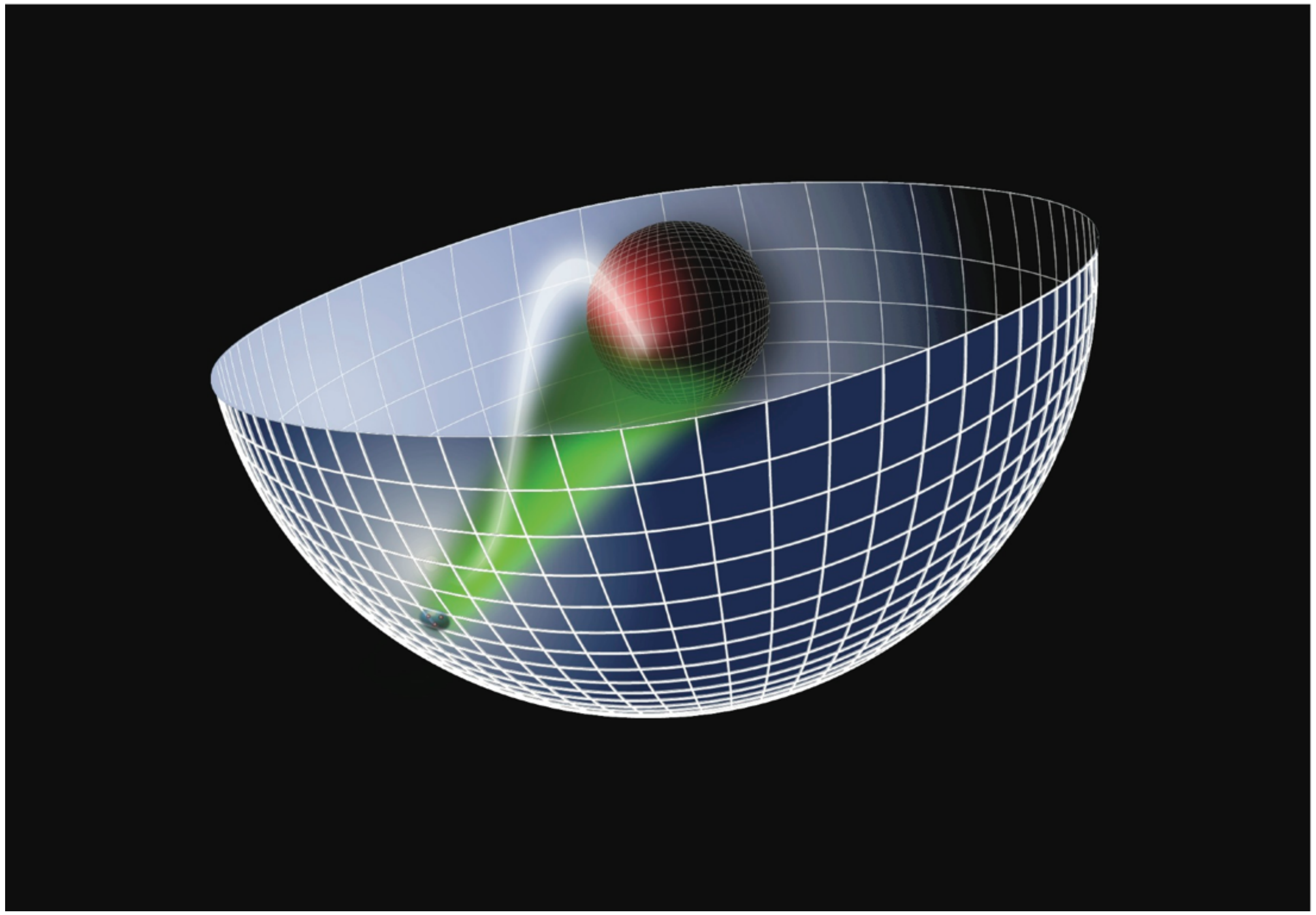
Weyl rescaling

$$g_{\mu\nu} \to e^{2\omega} g_{\mu\nu}$$

$$\left(\nabla^{\rho}\nabla_{\sigma} + \frac{1}{2} R^{\rho}{}_{\sigma}\right) C^{\sigma}{}_{\mu\rho\nu} = 0$$

Boundary Conditions



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$$x^\mu \to x^\mu + \xi^\mu$$

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$$ds^2 = \frac{\ell^2}{\rho^2}\left(-\sigma d\rho^2 + \gamma_{ij} dx^i dx^j\right)$$

$$\gamma_{ij} = \gamma_{ij}^{(0)} + \rho\gamma_{ij}^{(1)} + \frac{1}{2}\rho^2\gamma_{ij}^{(2)} + \ldots$$

$$\delta g_{\rho\rho} = 0$$

$$\delta g_{\rho i} = 0$$

Boundary Conditions

$$\mathcal{D}_i \xi_j^{(0)} + \mathcal{D}_j \xi_i^{(0)} = \frac{2}{3} \gamma_{ij}^{(0)} \mathcal{D}_k \xi^{(0)k} \qquad (1)$$

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$$\pounds_{\xi^{(0)}} \gamma_{ij}^{(1)} = \frac{1}{3} \mathcal{D}_k \xi_{(0)}^k \gamma_{ij}^{(1)} \qquad (2)$$

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translations

Boundary Conditions

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$$\xi^{(0)} = \partial_t, \qquad \xi^{(1)} = \partial_x, \qquad \xi^{(2)} = \partial_y$$

$$L_{ij} = (x_i\partial_j - x_j\partial_i)$$

Lorentz rotations

Boundary Conditions

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$$\xi^{(6)} = t\partial_t + x\partial_x + y\partial_y$$

dilatations

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$$L_{ij} = (x_i\partial_j - x_j\partial_i)$$

$$\xi^{(6)} = t\partial_t + x\partial_x + y\partial_y$$

special conformal trafos

$$\xi^{(7)} = tx\partial_t + \frac{t^2 + x^2 - y^2}{2}\partial_x + xy\partial_y$$

$$\xi^{(8)} = ty\partial_t + xy\partial_x + \frac{t^2 + y^2 - x^2}{2}\partial_y$$

$$\xi^{(9)} = \frac{t^2 + x^2 + y^2}{2}\partial_t + tx\partial_x + ty\partial_y$$

Boundary Conditions

- conformal algebra o(3,2)

$$[\xi^d, \xi^t_j] = -\xi^t_j \qquad [\xi^d, \xi^{sct}_j] = \xi^{sct}_j$$

$$[\xi^t_l, L_{ij}] = (\eta_{li}\xi^t_j - \eta_{lj}\xi^t_i) \qquad [\xi^{sct}_l, L_{ij}] = -(\eta_{li}\xi^{sct}_j - \eta_{lj}\xi^{sct}_i)$$

$$[\xi^{sct}_i, \xi^t_j] = -(\eta_{ij}\xi^d - L_{ij})$$

$$[L_{ij}, L_{mj}] = -L_{im}$$

Boundary Conditions

$$\mathcal{D}_i \xi_j^{(0)} + \mathcal{D}_j \xi_i^{(0)} = \frac{2}{3} \gamma_{ij}^{(0)} \mathcal{D}_k \xi^{(0)k} \qquad (1)$$

$$\pounds_{\xi^{(0)}} \gamma_{ij}^{(1)} = \frac{1}{3} \mathcal{D}_k \xi_{(0)}^k \gamma_{ij}^{(1)} \qquad (2)$$

(1) $\Longrightarrow$ $\gamma_{ij}^{(0)} = \eta_{ij} = diag(-1,1,1)$ (t,x,y)

$\Longrightarrow$ $\xi_i^{(0)}$ $\Longrightarrow$ Killing vectors (KVs) define conformal algebra

Boundary Conditions

(2) $\Longrightarrow$ $\gamma_{ij}^{(1)}$

1. subalgebra of o(3,2) allows (or not) to find $\gamma_{ij}^{(1)}$

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1. subalgebra of o(3,2) allows (or not) to find $\gamma_{ij}^{(1)}$
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$\Longrightarrow$ here, we focus on 1.

$\gamma_{ij}^{(1)}$ can depend on all the coordinates on the boundary

$\Longrightarrow$ we can consider from the simplest constant ones, to those that depend on three coordinates of the boundary

$\Longrightarrow$ symmetric behaviour

Boundary Conditions

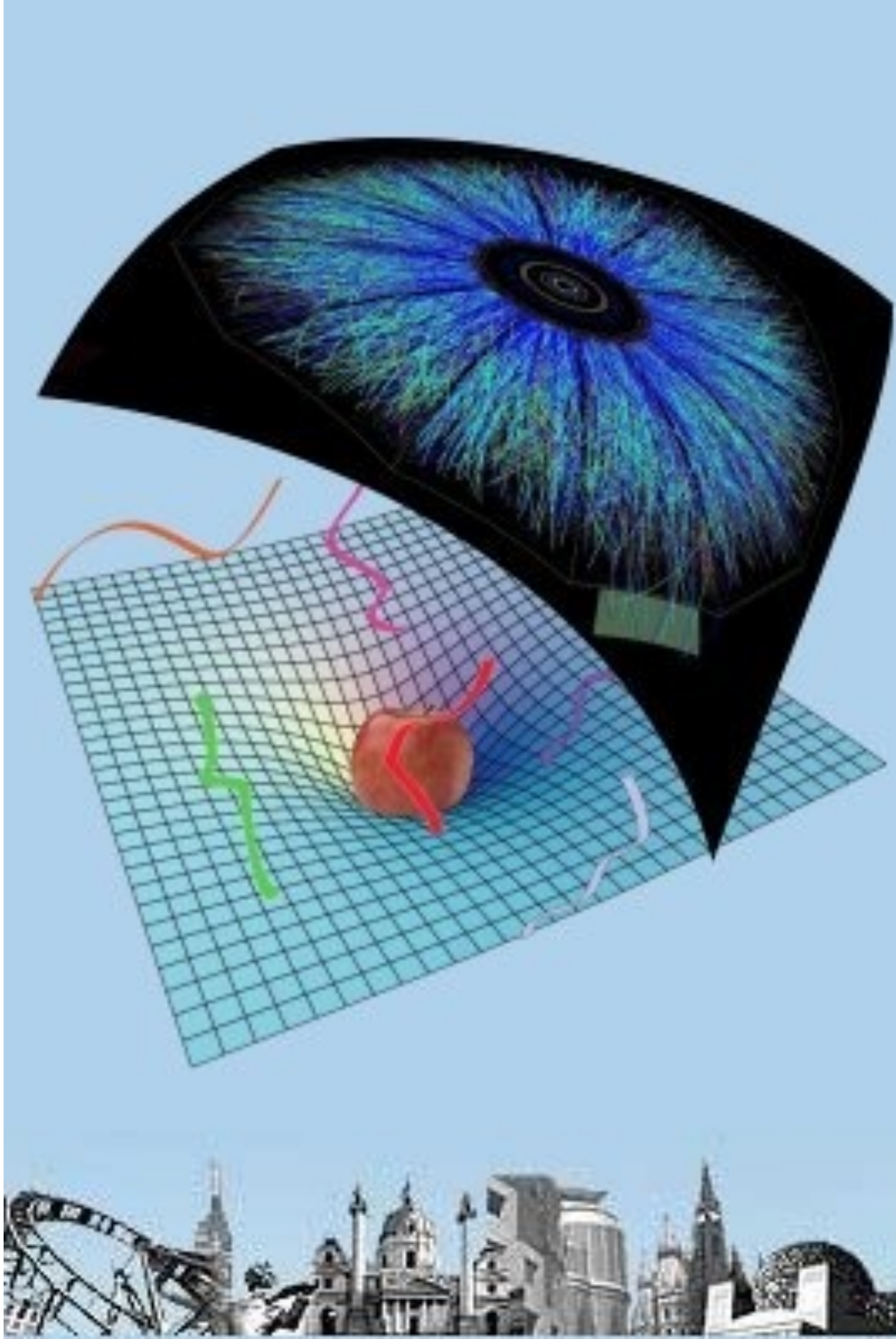
(2) $\Longrightarrow$ $\gamma_{ij}^{(1)}$

1. subalgebra of o(3,2) allows (or not) to find $\gamma_{ij}^{(1)}$
2. restriction on $\gamma_{ij}^{(1)}$ defines subalgebra

Patera et al. classification $\Longrightarrow$ linearly combined KVs

$$\xi^{lc} = a_0\xi^{(0)} + a_1\xi^{(1)} + a_2\xi^{(2)} + a_3\xi^{(3)} + a_4\xi^{(4)} + a_5\xi^{(5)} + a_6\xi^{(6)} + a_7\xi^{(7)} + a_8\xi^{(8)} + a_9\xi^{(9)}$$

Asymptotic Symmetry Algebra (ASA) of CG



Asymptotic Symmetry Algebra (ASA) of CG

- (2) $\Longrightarrow$ $\gamma_{ij}^{(1)}$
- classification

allowed boundary conditions (realizations of linear term) can be set in one of the subalgebras

Interesting are the ones with the largest number of KVs

$\Longrightarrow$ **5 and 4 dimensional subalgebras**

Asymptotic Symmetry Algebra (ASA) of CG

- (2) $\Longrightarrow$ $\gamma_{ij}^{(1)}$
- classification

allowed boundary conditions (realizations of linear term) can be set in one of the subalgebras

1. similitude algebra sim(2,1)
2. optical algebra opt(2,1)
3. maximal compact subalgebra $o(3) \otimes o(2)$
4. $o(2) \otimes o(2,1)$
5. o(2,2)
6. Lorentz algebra o(3,1)
7. irreducible subalgebra o(2,1)

Asymptotic Symmetry Algebra (ASA) of CG

1. similitude algebra sim(2,1) $\Longrightarrow$

Highest number of KVs is 7 however, boundary conditions allow as largest 5 dimensional ASA

$$P_0 = -\xi^{(0)}, P_1 = \xi^{(1)}, P_2 = \xi^{(2)}, F = \xi^{(6)},$$
$$K_1 = \xi^{(3)}, K_2 = \xi^{(4)}, L_3 = \xi^{(5)}$$

$$[\xi^d, \xi^t_j] = -\xi^t_j$$
$$[\xi^t_l, L_{ij}] = -(\eta_{li}\xi^t_j - \eta_{lj}\xi^t_i)$$
$$[L_{ij}, L_{mj}] = L_{im}$$

Asymptotic Symmetry Algebra (ASA) of CG

2. optical algebra opt(2,1) $\Longrightarrow$ Highest subalgebra: 7 KVs
ASA: 5, 4 KVs

$$W = -\frac{\xi^{(6)} + \xi^{(4)}}{2}$$

$$K_1 = \frac{\xi^{(6)} - \xi^{(4)}}{2}$$

$$K_2 = \frac{1}{2}\left[\xi^{(0)} - \xi^{(2)} + \frac{(\xi^{(8)} - \xi^{(9)})}{2}\right]$$

$$L_3 = \frac{1}{2}\left[\xi^{(0)} - \xi^{(2)} - \frac{(\xi^{(8)} - \xi^{(9)})}{2}\right]$$

$$M = -\sqrt{2}\xi^{(1)}$$

$$Q = \frac{\xi^{(5)} - \xi^{(3)}}{2\sqrt{2}}$$

$$N = -(\xi^{(0)} + \xi^{(2)})$$

Asymptotic Symmetry Algebra (ASA) of CG

$$[K_1, K_2] = -L_3, \quad [L_3, K_1] = K_2, \quad [L_3, K_2] = -K_1, \quad [M, Q] = -N,$$
$$[K_1, M] = -\frac{1}{2}M, \quad [K_1, Q] = \frac{1}{2}Q, \quad [K_1, N] = 0, \quad [K_2, M] = \frac{1}{2}Q,$$
$$[K_2, Q] = \frac{1}{2}M, \quad [K_2, N] = 0 \quad [L_3, M] = -\frac{1}{2}Q, \quad [L_3, Q] = \frac{1}{2}M,$$
$$[L_3, N] = 0 \quad [W, M] = \frac{1}{2}M, \quad [W, Q] = \frac{1}{2}Q, \quad [W, N] = \frac{1}{2}N$$

Asymptotic Symmetry Algebra (ASA) of CG

5. o(2,2) $\Longrightarrow$ Highest subalgebra: 6 KVs
ASA: 4 KVs

$$A_1 = -\frac{1}{2}\left[\frac{\xi^{(9)}+\xi^{(8)}}{2} - (\xi^{(0)}+\xi^{(2)})\right], \quad A_2 = \frac{1}{2}(\xi^{(6)}+\xi^{(4)}),$$

$$A_3 = \frac{1}{2}\left[-\frac{\xi^{(9)}+\xi^{(8)}}{2} - (\xi^{(0)}+\xi^{(2)})\right], \quad B_1 = -\frac{1}{2}\left[\frac{-\xi^{(9)}+\xi^{(8)}}{2} + (\xi^{(0)}-\xi^{(2)})\right],$$

$$B_2 = \frac{1}{2}(\xi^{(6)}-\xi^{(4)}), \quad B_3 = \frac{1}{2}\left[\frac{\xi^{(9)}-\xi^{(8)}}{2} + (\xi^{(0)}-\xi^{(2)})\right]$$

$$[A_1, A_2] = -A_3, \quad [A_3, A_1] = A2, \quad [A_2, A_3] = A_1$$

$$[B_1, B_2] = -B_3, \quad [B_3, B_1] = B_2, \quad [B_2, B_3] = B_1$$

$$[A_i, B_k] = 0 \qquad (i, k = 1, 2, 3)$$

Asymptotic Symmetry Algebra (ASA) of CG

6. o(3,1) $\Longrightarrow$ Highest subalgebra: 6 KVs
ASA: 4 KVs

$$L_1 = \xi^{(7)} + \frac{\xi^{(2)}}{2}, \qquad L_2 = \xi^{(5)}, \qquad L_3 = \xi^{(8)} + \frac{1}{2}\xi^{(1)},$$

$$K_1 = \xi^{(8)} - \frac{1}{2}\xi^{(1)}, \qquad K_2 = \xi^{(6)}, \qquad K_3 = -\xi^{(7)} + \frac{1}{2}\xi^{(2)}$$

$$[L_i, L_j] = \epsilon_{ijk} L_k,$$
$$[L_i, K_j] = \epsilon_{ijk} K_k,$$
$$[K_i, K_j] = -\epsilon_{ijk} L_k$$

Asymptotic Symmetry Algebra (ASA) of CG

3. $o(3) \otimes o(2)$, 4. $o(2) \otimes o(2,1)$, 7. o(2,1)

$$\Longrightarrow$$

Highest subalgebra: 7 KVs

ASA: $<$ 4 KVs

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Highest subalgebra: 7 KVs
ASA: < 4 KVs

- Allowed ASAs - not straightforwardly extended to global solution

Global solutions

$$\gamma_{ij}^{(1)} = \begin{pmatrix} c & c & 0 \\ c & c & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Global solutions

$$ds^2 = dr^2 + (-1 + cf(r))dx_i^2 + 2cf(r)dx_i dx_j + (1 + cf(r))dx_j^2 + dx_k^2$$

$$x_i = t, x_j = y, x_k = y$$

$$f(r) = c_1 + c_2 r + c_3 r^2 + c_4 r^3$$

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$$\tau_{ij} = \begin{pmatrix} -cc_4 & -cc_4 & 0 \\ -cc_4 & -cc_4 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad P_{ij} = \begin{pmatrix} cc_3 & cc_3 & 0 \\ cc_3 & cc_3 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Global solutions

$$\gamma_{ij}^{(1)} = \begin{pmatrix} -c \cdot b(t-y) & 0 & c \cdot b(t-y) \\ 0 & 0 & 0 \\ c \cdot b(t-y) & 0 & -c \cdot b(t-y) \end{pmatrix}$$

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conserves

$$\xi_2^{(n1)} = -P_0 + P_2, \xi_2^{(n2)} = P_1, \xi_2^{(n3)} = P_1, \xi_2^{(n3)} = L_3 - K_1$$

Global solutions

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solving for 4th KV, one obtains global solution with the function b of the form

4th KV	b(t-y)	4th KV	b(t-y)
F	$\frac{b}{t-y}$	$F - K_2$	$\frac{b}{(t-y)^{3/2}}$
$F + K_2 + \epsilon(-P_0 - P_2)$	$b \cdot e^{\frac{t-1}{2\epsilon}}$	K_2	$\frac{b}{(t-y)^2}$
$P_0 - P_2$	$b(t-1)$	$F + cK_2$	$b \cdot (t-y)^{\frac{1-2c}{-1+c}}$

Conclusion and Outline

- There is a number of boundary conditions that can be imposed to CG depending on the linear term in the FG expansion.

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- There is a number of boundary conditions that can be imposed to CG depending on the linear term in the FG expansion.
- They are classified according to subalgebras of o(3,2)
- Clever choice can lead to global solution
- Global solutions can be classified according to the allowed subalgebras

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- Largest subalgebra belongs to sim(2,1) and opt(2,1), contains 5 KVs and defines pp wave solution

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- o(2,2) and o(3,1) algebras, define ASAs with maximally 4 KVs
- There are more global solutions, not discussed here

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- Largest subalgebra belongs to sim(2,1) and opt(2,1), contains 5 KVs and defines pp wave solution

- o(2,2) and o(3,1) algebras, define ASAs with maximally 4 KVs

- There are more global solutions, not discussed here

- Further research - further global solutions: black holes, black branes, black strings

Thank you for the attention!

Realised subalgebras, sim(2,1)

Patera name	generators	Realisation	Name
$a_{5,4}^a$ $a \neq 0, \pm 1$	$F + \frac{1}{2}K_2, -K_1 + L_3,$ P_0, P_1, P_2	$\begin{pmatrix} c & c & 0 \\ c & c & 0 \\ 0 & 0 & 0 \end{pmatrix}$	
$a_{4,1} = b_{4,6}$	$P_1 \oplus \{K_2, P_0, P_2\}$	$\begin{pmatrix} \frac{c_1}{2} & 0 & 0 \\ 0 & c_1 & 0 \\ 0 & 0 & -\frac{c_1}{2} \end{pmatrix}$	$\mathcal{R} \oplus o(3)$
$a_{4,2}$	$P_0 - P_2 \oplus \{F - K_2; P_0 + P_2, P_1\}$	$\begin{pmatrix} -c_1 & 0 & 0 \\ 0 & c_1 & 0 \\ 0 & 0 & -2c_1 \end{pmatrix}$	
$a_{4,3}$	$P_0 \oplus \{L_3, P_1, P_2\}$	$\begin{pmatrix} 2c_1 & 0 & 0 \\ 0 & c_1 & 0 \\ 0 & 0 & c_1 \end{pmatrix}$	MKR $\mathcal{R} \oplus o(3)$
$a_{4,4}$	$F \oplus \{K_1, K_2, L_3\}$	$\begin{pmatrix} 2f(t) & 0 & 0 \\ 0 & f(t) & 0 \\ 0 & 0 & f(t) \end{pmatrix}$	
$a_{4,5}$	$F\{K_2; P_0 - P_2\}\oplus$ $\{F - K_2, P_1\}$	$\begin{pmatrix} 0 & \frac{c}{t-y} & 0 \\ \frac{c}{t-y} & 0 & \frac{c}{y-t} \\ 0 & \frac{c}{y-t} & 0 \end{pmatrix}$	
$a_{4,6} = b_{4,9}$	$\{F + K_2, P_0 - P_2\} \oplus$ $\{F - K_2, P_0 + P_2\}$	$\begin{pmatrix} \frac{c}{x} & 0 & 0 \\ 0 & \frac{2c}{x} & 0 \\ 0 & 0 & -\frac{c}{x} \end{pmatrix}$	
$a_{4,7}$	$L_3 - K_1, P_0 + P_2;$ $P_0 - P_2, P_1$	leads to 5 KV algebra	
$a_{4,10}^b = b_{4,13}$ $b > 0, \neq 1$	$\{F - bK_2, P_0, P_1, P_2\}$	$\begin{pmatrix} c & 0 & c \\ 0 & 0 & 0 \\ c & 0 & c \end{pmatrix}, \begin{pmatrix} 0 & c & 0 \\ c & 0 & -c \\ 0 & -c & 0 \end{pmatrix}, \begin{pmatrix} 0 & c & 0 \\ c & 0 & c \\ 0 & c & 0 \end{pmatrix}$	
$a_{4,11}^b = b_{4,13}$ $b > 0, [b \neq 0]$	$\{F + bL_3, P_0, P_1, P_2\}$	$\begin{pmatrix} 0 & c & ic \\ c & 0 & 0 \\ ic & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & c & -ic \\ c & 0 & 0 \\ -ic & 0 & 0 \end{pmatrix}$	