DOUBLE HYBRID INFLATION AND GRAVITY WAVES

1 Introduction

- Recent results of BICEP2 on the B-mode in the polarization of the CMBR at degree angular scales indicate that inflationary scenarios may have to face a new challenge.
- They should accommodate appreciable values of the tensor-to-scalar ratio r, since a B-mode could be due to the production of gravitational waves during inflation.
- Although r seems to be smaller than initially claimed due to possible underestimation of the foreground from Galactic polarized-dust emission, $r \sim 0.01$ cannot be excluded.
- The most recent joint analysis of the Planck and BICEP2 data yields $r \lesssim 0.12$ at 95% c.l.
- SUSY Hybrid Inflation is a promising inflation scenario.
- In its simplest realization, though, it suffers from some problems.
- The GUT gauge symmetry is spontaneously broken only at the end of inflation and, thus, if magnetic monopoles are predicted, they are copiously produced, leading to a cosmological catastrophe.
- Also, although accurate measurements imply that the scalar spectral index $n_{\rm s}$ is clearly lower than unity, this scenario gives values very close to unity or even larger within minimal SUGRA.
- These problems are solved within a two stage Hybrid Inflation with minimal SUGRA: the standard-smooth hybrid inflation scenario.
- The cosmological scales exit the horizon during the first stage of inflation, which is of the standard hybrid type.

- It occurs on a 'trivial' path on which the gauge group is unbroken.
- Restricting the number of e-foldings during this stage, we can achieve adequately low $n_{\rm s}$'s.
- The extra e-foldings for solving the horizon and flatness problems of big bang are generated by a second inflation along a classically non-flat valley of minima, where the gauge group is broken.
- Consequently, monopoles are produced only at the end of the first stage, but are adequately diluted by the second one.
- This scenario was realized within an extended SUSY PS particle physics GUT model with only renormalizable interactions, which was constructed for a very differed reason.
- Namely, the simplest SUSY PS model predicts Yukawa unification and, with universal boundary conditions, yields unacceptable m_b's.
- In the extended model, Yukawa unification is naturally and moderately violated and this problem is solved.
- Here, we will show that a reduced version of this extended model can also yield a two stage inflationary scenario which can predict r's up to about 0.05 together with acceptable $n_{\rm s}$'s.
- Larger r's would require unacceptably large running of $n_{\rm s}$.
- The first stage occur along the trivial path, stabilized by SUGRA, and our present horizon undergoes a limited number of e-foldings.
- The obtained r's can be appreciable thanks to strong radiative and relatively mild SUGRA corrections to the inflationary potential.
- The second stage occurs on the so-called semi-shifted path, where $U(1)_{B-L}$ is unbroken, and generates the extra e-foldings required.

- This is possible since the SUGRA corrections on the semi-shifted path are also mild and this path, being orthogonal to the trivial one, is not affected by the strong radiative corrections on it.
- We take units where $m_{\rm P} = 1$.

2 The model in global SUSY

- The reduced version of the extended SUSY PS model is based on the left-right symmetric gauge group $G_{\rm LR} = SU(3)_c \times SU(2)_{\rm L} \times SU(2)_{\rm R} \times U(1)_{B-L}$, subgroup of the PS group.
- The superfields relevant for inflation are the following:
- A conjugate pair of Higgses H, $\bar{H}$ in the $(1,1,2)_1$ and $(1,1,2)_{-1}$ representations of $G_{\rm LR}$ causing the breaking of $G_{\rm LR}$ to $G_{\rm SM}$.
- A gauge singlet S triggering this breaking, and a conjugate pair Φ, $\bar{\Phi} \in (1,1,3)_0$. The $\langle\Phi\rangle$ breaks $G_{\rm LR} \to G_{\rm SM} \times U(1)_{B-L}$.
- The superpotential relevant for inflation is

$$W = \kappa S\left(M^2 - \Phi^2\right) - \gamma S H\bar{H} + m\Phi\bar{\Phi} - \lambda\bar{\Phi}H\bar{H}.$$

- M, m are superheavy masses and κ, γ, λ dimensionless constants.
- All these parameters but one can be made real and positive by rephasing the superfields. For definiteness, we choose the remaining complex parameter to be real and positive too.
- The resulting F–term scalar potential is

$$\begin{aligned} V_F^0 \; = \; & |\kappa(M^2 - \Phi^2) - \gamma H\bar{H}|^2 + |m\bar{\Phi} - 2\kappa S\Phi|^2 \\ & + |m\Phi - \lambda H\bar{H}|^2 + |\gamma S + \lambda\bar{\Phi}\,|^2\left(|H|^2 + |\bar{H}|^2\right). \end{aligned}$$

- From V_F^0 and the vanishing of the D–terms, implying $\bar{H}^* = e^{i\theta}H$, one finds two distinct continua of SUSY vacua:

$$\Phi = \Phi_+, \quad \bar{H}^* = H, \quad |H| = \sqrt{\frac{m\Phi_+}{\lambda}} \quad (\theta = 0),$$
$$\Phi = \Phi_-, \quad \bar{H}^* = -H, \quad |H| = \sqrt{\frac{-m\Phi_-}{\lambda}} \quad (\theta = \pi)$$

with $\bar{\Phi} = S = 0$, where

$$\Phi_\pm \equiv \pm M\sqrt{1 + \left(\frac{\gamma m}{2\kappa\lambda M}\right)^2} - \frac{\gamma m}{2\kappa\lambda}.$$

- The model generally possesses three flat directions:
- The usual trivial path at $\Phi = \bar{\Phi} = H = \bar{H} = 0$ with $V_F^0 = V_{\textsf{tr}} \equiv \kappa^2 M^4$, where $G_{\rm LR}$ is unbroken.
- The new shifted path at

$$\Phi = -\frac{\gamma m}{2\kappa\lambda}, \quad \bar{\Phi} = -\frac{\gamma}{\lambda}S, \quad H\bar{H} = \frac{\kappa\gamma(M^2 - \Phi^2) + \lambda m\Phi}{\gamma^2 + \lambda^2}$$
$$\text{with} \quad V_F^0 = V_{\textsf{nsh}} \equiv \kappa^2 M^4 \left(\frac{\lambda^2}{\gamma^2 + \lambda^2}\right)\left(1 + \frac{\gamma^2 m^2}{4\kappa^2\lambda^2 M^2}\right)^2.$$

  This path supports new shifted hybrid inflation with $G_{\rm LR} \to G_{\rm SM}$.

- The semi-shifted path, which exists only for $M^2 > m^2/2\kappa^2$, at

$$\Phi = \pm M\sqrt{1 - \frac{m^2}{2\kappa^2 M^2}}, \quad \bar{\Phi} = \frac{2\kappa\Phi}{m}S, \quad H = \bar{H} = 0$$
$$\text{with} \quad V = V_{\textsf{ssh}} \equiv m^2 M^2 \left(1 - \frac{m^2}{4\kappa^2 M^2}\right).$$

  It yields semi-shifted hybrid inflation with $U(1)_{B-L}$ unbroken.

- We take $M^2 > m^2/2\kappa^2$ and, thus, the semi-shifted path exists and always lies lower than the trivial and the new shifted one.

- We also take $\kappa \sim 1$, $\gamma \ll \lambda \ll \kappa$, and $m \ll M$, so that the new shifted path (for $|S| < 1$) essentially coincides with the trivial one and, thus, plays no independent role in our scheme.

3 The first stage of inflation

- The first stage of inflation takes place along the trivial path, which, for large $|S|$'s, is stabilized by SUGRA corrections.
- Although the number of e-foldings is limited, all the cosmological scales exit the horizon during this stage.
- Strong radiative and relatively mild SUGRA corrections to the potential then guarantee an appreciable r with an acceptable $n_{\rm s}$.
- We adopt the Kähler potential

$$\begin{aligned} K \;=\; & -\ln\left(1-|S|^2\right) - \ln\left(1-|\bar{\Phi}|^2\right) + |\Phi|^2 + |H|^2 + |\bar{H}|^2 \\ & -2\ln\left(-\ln|Z_1|^2\right) + |Z_2|^2. \end{aligned}$$

- The two extra $G_{\rm LR}$ singlets Z_1 and Z_2 do not enter W.
- The F–term potential in SUGRA is then

$$V_F = \left[\sum_i |W_{X_i} + K_{X_i} W|^2 K^{-1}_{X_i X_i^*} - 3|W|^2\right] e^K,$$

  where the sum is over all the fields S, $\bar{\Phi}$, Φ, H, $\bar{H}$, Z_1, Z_2 and a subscript X_i denotes derivation w.r.t. to X_i .
- The values of Z_1 and Z_2 are fixed by anomalous D–terms.
- S, $\bar{\Phi}$, Z_1 have no-scale type Kähler potentials which, in view of

$$|K_{Z_1}|^2 K^{-1}_{Z_1 Z_1^*} = 2,$$

guarantee the exact flatness of the potential along the trivial path and its approximate flatness on the semi-shifted one for $Z_2 = 0$.

- The relation then

$$|K_{Z_2}|^2 K^{-1}_{Z_2 Z_2{}^*} = |Z_2|^2 \equiv \beta$$

implies that the complex inflatons S and $\bar{\Phi}$ for the two paths, respectively, acquire $m^2 \propto \beta$ when Z_2 becomes non-zero.

- Using the symmetries, we can rotate S and H on the real axis. The fields $\bar{\Phi}$, Φ, $\bar{H}$ remain in general complex.

- For simplicity, we restrict $\bar{\Phi}$, Φ, $\bar{H}$ on the real axis too.

- The canonically normalized real scalar fields σ, $\bar{\phi}$, ϕ, h, $\bar{h}$ corresponding to K are given by

$$S = \tanh\frac{\sigma}{\sqrt{2}}, \quad \bar{\Phi} = \tanh\frac{\bar{\phi}}{\sqrt{2}},$$

$$\Phi = \frac{\phi}{\sqrt{2}}, \quad H = \frac{h}{\sqrt{2}}, \quad \bar{H} = \frac{\bar{h}}{\sqrt{2}}.$$

- We evaluate V_F with the factor $\exp\left[-2\ln\left(-\ln|Z_1|^2\right) + |Z_2|^2\right]$ absorbed into redefined parameters κ, γ, m, and λ.

- We find

$$\begin{aligned} V_F \;=\; & \left[A_1^2\cosh^2\frac{\bar{\phi}}{\sqrt{2}} - A_2^2\sinh^2\frac{\bar{\phi}}{\sqrt{2}} + \beta A_3^2 + A_4^2 + A_5^2\right. \\ & + \frac{1}{2}\left(h^2 + \bar{h}^2\right)A_6^2 + \frac{1}{2}\left(\phi^2 + h^2 + \bar{h}^2\right)A_3^2 \\ & \left. + \left(\sqrt{2}\phi A_5 - 2h\bar{h}A_6\right)A_3\right] e^{\frac{1}{2}\left(\phi^2 + h^2 + \bar{h}^2\right)}. \end{aligned}$$

- Here

$$A_1 = \kappa\left(M^2 - \frac{\phi^2}{2}\right) - \frac{\gamma}{2}h\bar{h}, \quad A_2 = m\frac{\phi}{\sqrt{2}} - \frac{\lambda}{2}h\bar{h},$$

$$A_3 = A_1 \sinh\frac{\sigma}{\sqrt{2}}\cosh\frac{\bar{\phi}}{\sqrt{2}} + A_2\cosh\frac{\sigma}{\sqrt{2}}\sinh\frac{\bar{\phi}}{\sqrt{2}},$$

$$A_4 = A_1 \sinh\frac{\sigma}{\sqrt{2}}\sinh\frac{\bar{\phi}}{\sqrt{2}} + A_2\cosh\frac{\sigma}{\sqrt{2}}\cosh\frac{\bar{\phi}}{\sqrt{2}},$$

$$A_5 = m\cosh\frac{\sigma}{\sqrt{2}}\sinh\frac{\bar{\phi}}{\sqrt{2}} - \sqrt{2}\kappa\phi\sinh\frac{\sigma}{\sqrt{2}}\cosh\frac{\bar{\phi}}{\sqrt{2}},$$

$$A_6 = \gamma\sinh\frac{\sigma}{\sqrt{2}}\cosh\frac{\bar{\phi}}{\sqrt{2}} + \lambda\cosh\frac{\sigma}{\sqrt{2}}\sinh\frac{\bar{\phi}}{\sqrt{2}}.$$

- On the trivial path ($\bar{\phi},\ \phi,\ h,\ \bar{h} = 0$), V_F becomes

$$V_F = \kappa^2 M^4\left[1 + \beta\sinh^2\frac{\sigma}{\sqrt{2}}\right].$$

- The m^2 eigenvalues in the directions perpendicular to this path for $\sinh^2\left(\sigma/\sqrt{2}\right) \gg M^2/2$ are

$$m_\phi^2 \simeq 4\kappa^2\sinh^2\frac{\sigma}{\sqrt{2}}, \quad m_{\bar{\phi}}^2 \simeq \kappa^2 M^4\left[1 + (1+\beta)\sinh^2\frac{\sigma}{\sqrt{2}}\right],$$

$$m_{\chi_1,\chi_2}^2 = (\kappa M^2 \mp \gamma)\left[\kappa M^2 + \left((1+\beta)\kappa M^2 \mp \gamma\right)\sinh^2\frac{\sigma}{\sqrt{2}}\right],$$

where $\chi_{1,2} = (h \pm \bar{h})/\sqrt{2}$ and their m^2 formulas hold for any σ.

- Thus, for $\gamma < \kappa M^2$, the trivial path is stable for large $|\sigma|$'s.
- However, as $|\sigma|$ decreases, the eigenvalues and eigenstates of the $\phi - \bar{\phi}$ system change.

- When $\sinh^2\left(\sigma/\sqrt{2}\right) \simeq M^2/2 + m^2/2\kappa^2 M^2$, one of the eigenvalues vanishes with $\bar{\phi}$ dominating the corresponding eigenstate.
- As $\sinh^2\left(\sigma/\sqrt{2}\right) \to M^2/2$, the eigenvalues become opposite to each other with ϕ, $\bar{\phi}$ contributing equally to both the eigenstates.
- A further decrease of $\sinh^2\left(\sigma/\sqrt{2}\right)$ leads to the domination of the unstable eigenstate by ϕ.
- Since ϕ must become nonzero to cancel the energy density $\kappa^2 M^4$ on the trivial path, we say that this path is destabilized at $\sigma_{\rm c}$ with

$$\sinh^2\frac{\sigma_{\rm c}}{\sqrt{2}} = \frac{M^2}{2}.$$

- To V_F on the trivial path we add the dominant one-loop radiative corrections from the N_ϕ-dimensional supermultiplet Φ ($N_\phi = 3$):

$$V_r^\phi = \kappa^2 M^4 \left(\frac{N_\phi \kappa^2}{8\pi^2}\right) \ln\frac{2\tanh^2\frac{\sigma}{\sqrt{2}}}{M^2}.$$

- Note that the renormalization scale in these corrections is chosen such that V_r^ϕ vanishes at $|\sigma| = |\sigma_{\rm c}|$.
- The full inflationary potential V and its derivatives w.r.t. σ are:

$$\frac{V}{\kappa^2 M^4} = 1 + \beta\sinh^2\frac{\sigma}{\sqrt{2}} + \frac{\delta_\phi}{4}\ln\frac{2\tanh^2\frac{\sigma}{\sqrt{2}}}{M^2} \equiv C(\sigma),$$

$$\frac{V'}{\kappa^2 M^4} = \frac{1}{\sqrt{2}}\sinh(\sqrt{2}\sigma)\left(\beta + \frac{\delta_\phi}{\sinh^2(\sqrt{2}\sigma)}\right),$$

$$\frac{V''}{\kappa^2 M^4} = \cosh(\sqrt{2}\sigma)\left(\beta - \frac{\delta_\phi}{\sinh^2(\sqrt{2}\sigma)}\right),$$

$$\frac{V'''}{\kappa^2 M^4} = \sqrt{2}\sinh(\sqrt{2}\sigma)\left(\beta - \frac{\delta_\phi}{\sinh^2(\sqrt{2}\sigma)}\right) + \frac{2\sqrt{2}\delta_\phi}{\tanh^2(\sqrt{2}\sigma)\sinh(\sqrt{2}\sigma)}$$

$$\text{with} \quad \delta_\phi = \frac{N_\phi \kappa^2}{2\pi^2}.$$

- The usual slow-roll parameters for inflation are then

$$\epsilon = \frac{1}{2}\left(\frac{V'}{\kappa^2 M^4}\right)^2 \frac{1}{C^2(\sigma)},$$

$$\eta = \left(\frac{V''}{\kappa^2 M^4}\right)\frac{1}{C(\sigma)},$$

$$\xi = \left(\frac{V'}{\kappa^2 M^4}\right)\left(\frac{V'''}{\kappa^2 M^4}\right)\frac{1}{C^2(\sigma)} = 2\tanh(\sqrt{2}\sigma)\eta\sqrt{\epsilon} + \frac{4\delta_\phi\sqrt{\epsilon}}{C(\sigma)\tanh^2(\sqrt{2}\sigma)\sinh(\sqrt{2}\sigma)}.$$

- From these expressions, we evaluate n_s, its running α_s, r, and V:

$$n_\mathrm{s} = 1 + 2\eta - 6\epsilon, \quad \alpha_\mathrm{s} = 16\eta\epsilon - 24\epsilon^2 - 2\xi, \quad r = 16\epsilon,$$

$$V = \frac{3\pi^2}{2}A_\mathrm{s} r.$$

- As a numerical example, take $\sigma_* = 1.45$ at horizon exit of the pivot scale $k_* = 0.05\ \mathrm{Mpc}^{-1}$, $\kappa = 1.7$, $\beta = 0.022$, and the scalar power spectrum amplitude $A_\mathrm{s} = 2.215 \times 10^{-9}$ at the same k_*.
- We then find $M = 3.493 \times 10^{-3}$, $C(\sigma_*) = 2.2941$, $\epsilon = 0.00188$, $\eta = -0.01389$, $n_\mathrm{s} = 0.9609$, $r = 0.0301$, and $\alpha_\mathrm{s} = -0.01674$.

- So we can not only be consistent with the latest Planck data, but also accommodate large values of $r \sim \mathrm{few} \times 10^{-2}$.
- Note that large r's require relatively large ϵ's, which reduce $n_{\rm s}$ below unity, but not enough to make it compatible with the data.
- So large negative η's are needed, which requires that the parenthesis in the formula for V'' is dominated by the second term.
- A similar parenthesis appears in the formula for V' too, but with the two terms added.
- So both these terms have to be appreciable with the second one being larger, which is possible only for large κ's controlling the radiative corrections on the trivial path.
- Inflation ends before the system reaches $\sigma_{\rm c}$ by violating the slow-roll conditions and the obtained number of e-foldings is limited due to the large ϵ's and the fact that $\sigma_* \sim 1$.

4 The second inflationary stage

- We choose, for the rest of the parameters, $m = 1.827 \times 10^{-5}$, $\lambda = 0.1$, and $\gamma = 10^{-6}$ and include the D–terms from H, $\bar{H}$.
- Numerically, we find that there are initial conditions for which, after the first stage of inflation, the energy density approaches $m^2 M^2$, $\phi^2 \simeq 2M^2$, h, $\bar{h} \simeq 0$, and $A_5 \simeq 0$ with $\sigma^2 \ll 1$.
- So the system reaches the semi-shifted path, where V_F becomes

$$V_F \underset{M^2 \ll \beta}{\simeq} m^2 M^2 \left[1 + \beta \sinh^2 \frac{\bar{\phi}}{\sqrt{2}}\right].$$

- Notice the striking similarity with V_F on the trivial path. So the SUGRA corrections remain relatively mild on this path too.

- From $A_5 \simeq 0$, the combination of S, $\bar{\Phi}$ which is the complex inflaton in the second stage is

$$\frac{mS + 2\kappa < \Phi > \bar{\Phi}}{\sqrt{m^2 + 4\kappa^2 M^2}} \simeq \bar{\Phi},$$

since $\bar{\Phi}$ contributes here $2\kappa M/m \simeq 650$ times more than S.

- The mass eigenstates for the $h-\bar{h}$ system during the second stage of inflation are $\chi_{1,2} = (h \pm \bar{h})/\sqrt{2}$ with masses-squared

$$m^2_{\chi_1,\chi_2} = (\lambda \mp mM)\left[(\lambda \mp (1+\beta)mM)\sinh^2\frac{\bar{\phi}}{\sqrt{2}} \mp mM\right].$$

- χ_1 develops an instability terminating the semi-shifted valley.

- The critical value of the real canonically normalized inflaton $\bar{\phi}$:

$$\sinh^2\frac{\bar{\phi}_{\rm c}}{\sqrt{2}} = \frac{mM}{\lambda}.$$

- During the second stage, the dominant radiative corrections from the N_h-dimensional superfields H, $\bar{H}$ ($N_h = 2$) have to be added

$$V_r^h \simeq m^2 M^2 \left(\frac{N_h \lambda^2}{16\pi^2}\right) \ln \frac{\lambda \tanh^2 \frac{\bar{\phi}}{\sqrt{2}}}{mM}.$$

- The renormalization scale is chosen so that $V_r^h = 0$ at $|\bar{\phi}| = |\bar{\phi}_{\rm c}|$.

- The radiative corrections from Φ are neglected being relatively very small.

- This is because Φ couples to the complex inflaton only through S and the contribution of S to this inflaton is severely suppressed.

- This is an important property of the model resulting from the fact that, for the parameters chosen, the semi-shifted path is almost orthogonal to the trivial one.

- So the very strong radiative corrections on the trivial path, needed for accommodating appreciable r's, do not affect the second stage.

- This is crucial since otherwise the semi-shifted path would be too steep to generate the extra e-foldings required.

- The number of e-foldings during the second stage between $\bar{\phi}_{\rm in}$ and $\bar{\phi}_{\rm f}$ is $N(\bar{\phi}_{\rm in}) - N(\bar{\phi}_{\rm f})$, where

$$N(\bar{\phi}) \simeq \frac{1}{2\beta\sqrt{1-(\delta_h/\beta)}} \ln \frac{\cosh(\sqrt{2}\bar{\phi}) - \sqrt{1-(\delta_h/\beta)}}{\cosh(\sqrt{2}\bar{\phi}) + \sqrt{1-(\delta_h/\beta)}}$$

with $\delta_h = N_h \lambda^2/4\pi^2$.

- The termination of inflation is due to the radiative corrections and occurs at $\bar{\phi}_{\rm f}$ ($|\bar{\phi}_{\rm f}| \gg |\bar{\phi}_{\rm c}|$):

$$\cosh(\sqrt{2}\bar{\phi}_{\rm f}) \simeq \frac{\delta_h}{2} + \sqrt{1 + \frac{\delta_h^2}{4}}.$$

- Numerically, we find that, with the chosen values, the first stage gives rise to 13 e-foldings.

- So another 38-39 e-foldings must be provided by the second stage, which requires $|\bar{\phi}_{\rm in}| \simeq 0.23$ at the onset of this stage.

- This requirement can indeed be fulfilled in our numerical example as we have shown by extensive numerical studies.

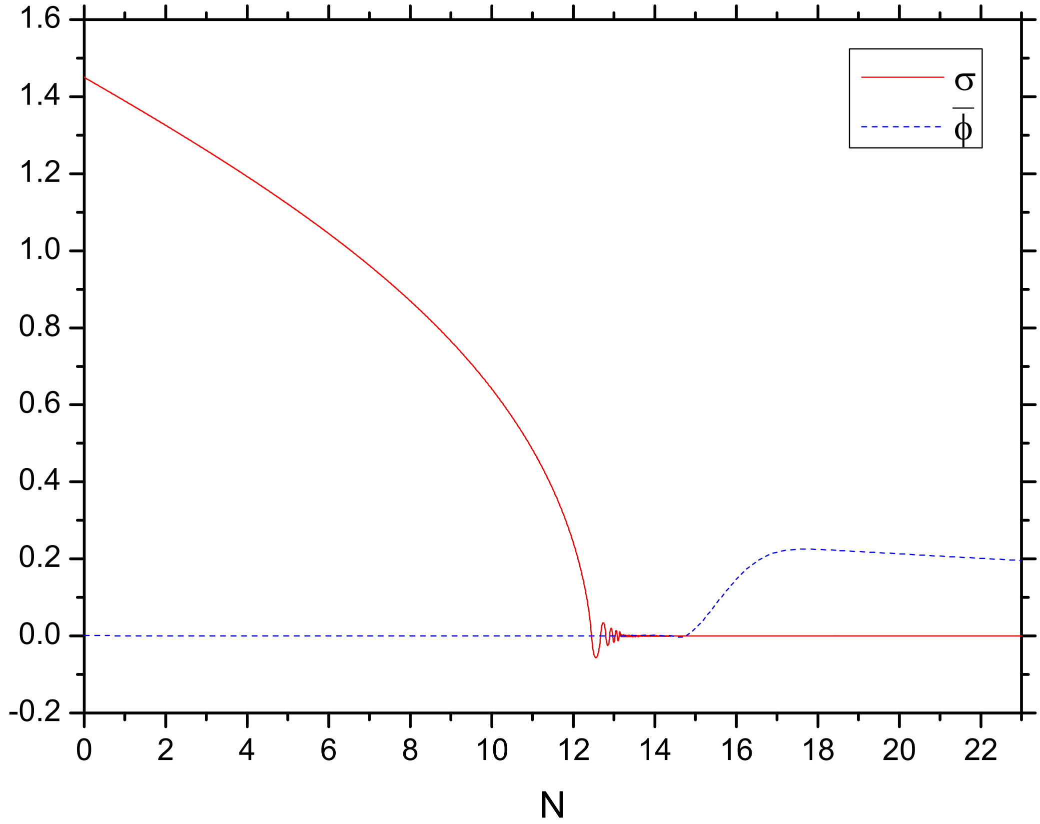

Figure 1: σ and $\bar{\phi}$ for the case with $r = 0.0301$ versus N. We take the initial conditions $\sigma = 1.45$, $\bar{\phi} = 10^{-3}$, $\phi = 10^{-8}$, $h = 10^{-4}$, $\bar{h} = 1.01 \times 10^{-4}$, and $d\sigma/dt = -1.1074 \times 10^{-6}$.

- In Fig. 1, we depict the evolution of σ and $\bar{\phi}$ versus the number of e-foldings N from the horizon exit k_*.
- We take as initial conditions $\sigma = 1.45$, $\bar{\phi} = 10^{-3}$, $\phi = 10^{-8}$, $h = 10^{-4}$, and $\bar{h} = 1.01 \times 10^{-4}$.
- All the fields are given zero initial velocity except for σ which is given its actual velocity on the trivial path, -1.1074×10^{-6}.
- σ remains above its critical value for about 13 e-foldings.
- Near the end of the first stage, σ oscillates around zero four times.
- When the amplitude of the oscillations falls below the critical value of σ, ϕ moves to the semi-shifted path and $\bar{\phi}$ starts performing slow oscillations with variable amplitudes of order M.

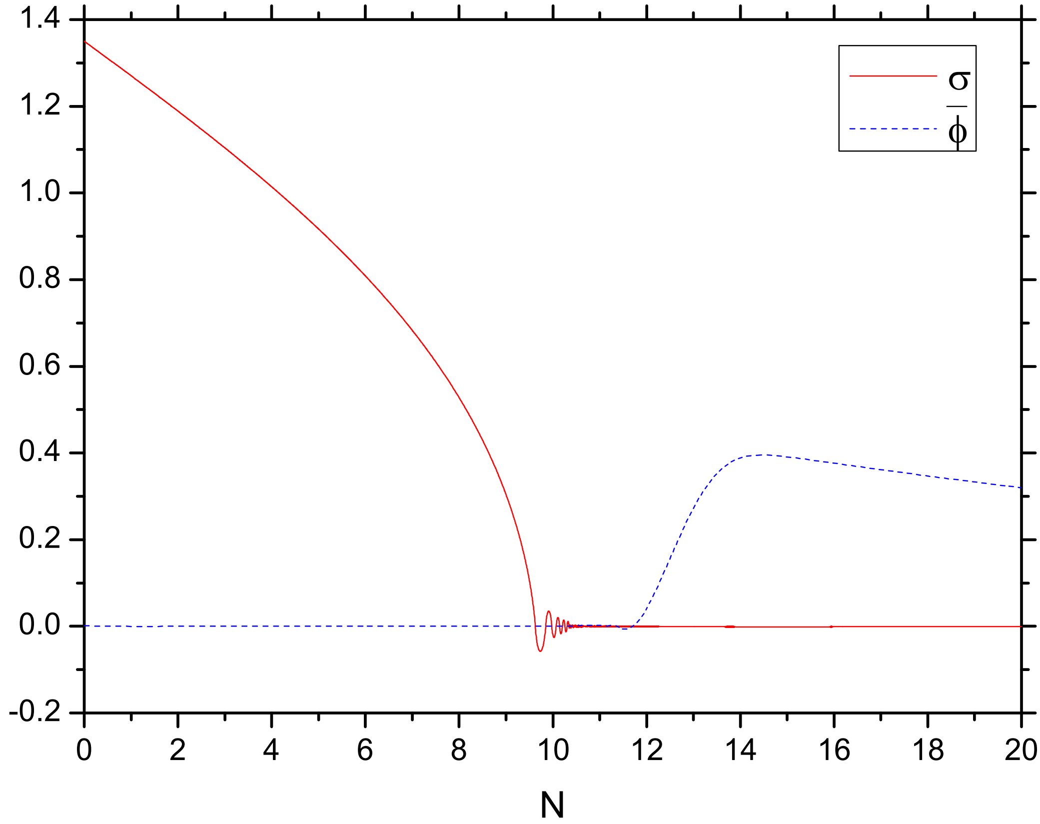

Figure 2: σ and $\bar{\phi}$ for the case with $r = 0.0502$ versus N. We take the initial conditions $\sigma = 1.35$, $\bar{\phi} = 10^{-3}$, $\phi = 10^{-8}$, $h = 9 \times 10^{-4}$, $\bar{h} = 9.01 \times 10^{-4}$, and $d\sigma/dt = -1.8523 \times 10^{-6}$.

- The size of $\bar{\phi}$ remains small for about 1.7 e-foldings before starting its growth and acquires its largest value $\simeq 0.225$ at $N \simeq 17.7$.
- Allow for a stronger running of $n_{\rm s}$, we may obtain larger r's.
- For example, taking $\sigma_* = 1.35$, $\kappa = 1.75$, and $\beta = 0.037$, we find $M = 3.891 \times 10^{-3}$, $C(\sigma_*) = 2.3479$, $\epsilon = 0.00314$, $\eta = -0.00844$, $n_{\rm s} = 0.9643$, $\alpha_{\rm s} = -0.03007$, and $r = 0.0502$.
- In addition, we choose $m = 3.891 \times 10^{-5}$, $\lambda = 0.1$, and $\gamma = 10^{-6}$.
- k_* suffers about 10 e-foldings during the first stage and, thus, another $41 - 42$ e-foldings must be provided by the second stage.
- This implies that $|\bar{\phi}_{\rm in}|$ lies in the range $0.38 - 0.40$.
- We verified that the fulfillment of this requirement is feasible.

- In Fig. 2, we depict the evolution of σ and $\bar{\phi}$ as functions of N for a particular choice of initial conditions.

5 Monopoles and cosmic strings

- After the first inflation stage, the system reaches the semi-shifted path, $SU(2)_{\rm R} \to U(1)$ by $\Phi \neq 0$ and monopoles are formed.
- The mean monopoles-antimonopoles distance is $p\,(2\kappa M)^{-1}$ as determined by the Higgs boson mass ($p \sim 1$ is a geometric factor.)
- In the matter dominated era between the two inflationary stages, this distance increases by a factor $\sim (\kappa^2 M^4/m^2 M^2)^{1/3}$.
- Here $\kappa^2 M^4$ and $m^2 M^2$ are the classical potential energy densities on the trivial and the semi-shifted paths.
- The second inflationary stage stretches this distance by a factor $\exp N_2$, where N_2 is the number of e-foldings during this stage.
- During damped inflaton oscillations, this distance increases by a factor $\sim (m^2 M^2/c(T_{\rm r})T_{\rm r}^4)^{1/3}$.
- $T_{\rm r} \simeq 10^9$ GeV is the reheat temperature and $c(T) = \pi^2 g(T)/30$ ($g(T)$=effective number of massless degrees of freedom).
- In the radiation dominated period which follows, the monopole-antimonopole distance is multiplied by a factor

$$\sim T_{\rm r}/T \sim (4c(T)/3)^{1/4} T_{\rm r}\sqrt{t}.$$

- So this distance at t in the radiation dominated period is

$$\sim \left(\frac{4}{3}\right)^{\frac{1}{4}} c(T_{\rm r})^{-\frac{1}{3}} c(T)^{\frac{1}{4}} p\,(2\kappa M)^{-1} e^{N_2} \left(\frac{\kappa^2 M^4}{T_{\rm r}^4}\right)^{\frac{1}{3}} T_{\rm r} t^{\frac{1}{2}}.$$

- Equating this distance with the post-inflationary horizon $\sim 2t$, we find the time $t_{\rm H}$ at which the monopoles enter this horizon:

$$t_{\rm H} \sim \frac{p^2}{8\sqrt{3}}\, c(T_{\rm r})^{-\frac{2}{3}} c(T_{\rm H})^{\frac{1}{2}} e^{2N_2} \left(\frac{M}{\kappa\, T_{\rm r}}\right)^{\frac{2}{3}}, \tag{1}$$

  where $T_{\rm H}$ is the cosmic temperature at $t_{\rm H}$.

- After the end of the second inflationary stage, the system settles in one of the two distinct continua of SUSY vacua.
- A linear combination of $U(1)_{B-L}$ and the unbroken $U(1)$ subgroup of $SU(2)_{\rm R}$ breaks and local cosmic strings are generated.
- These strings, had they survived after recombination, could give a small contribution to the CMBR power spectrum.
- This contribution is parametrized by the dimensionless string tension $G\mu_{\rm s}$ where G is Newton's constant and

$$\mu_{\rm s} = 4\pi |\langle H \rangle|^2$$

  is the energy per unit length of the string.

- In our case, however, the strings decay well before recombination and, thus, do not affect the CMBR.
- The reason is that they are open connecting (anti)monopoles.
- Indeed, the breaking $SU(2)_{\rm R} \times U(1)_{B-L} \to U(1)_Y$ by $\langle H \rangle$ and $\langle \bar{H} \rangle$ is similar to the breaking of the electroweak gauge group.
- Thus, no topologically stable monopoles or strings can appear.
- We can only have topologically unstable dumbbell configurations of a monopole and an antimonopole connected by an open string.
- Actually, these strings are like random walks with step $\sim$ the particle horizon connecting monopoles to antimonopoles.

- Before the entrance of the monopoles into the horizon, there is about one string segment per horizon.
- So, the ratio of the energy density $\rho_{\rm s}(t)$ of the string network to the total energy density $\rho_{\rm tot}(t)$ remains practically constant.
- At $t_{\rm H}$, we have about one monopole, antimonopole pair per horizon volume connected by a string of the size of the horizon.
- Thus, at $t_{\rm H}$, the energy density of the strings $\rho_{\rm s}(t_{\rm H}) \sim 3G\mu_{\rm s}/2t_{\rm H}^2$.
- After this time, more and more string segments enter the horizon, but the length of each segment remains constant.
- Consequently, the strings behave like pressureless matter and the 'relative string energy density' is ($\rho_\gamma(t)=$ photon energy density)

$$\frac{\rho_{\rm s}(t)}{\rho_\gamma(t)} \sim 2G\mu_{\rm s}\left(\frac{t}{t_{\rm H}}\right)^{\frac{1}{2}}.$$

- This density increases with t until the final decay of the strings at

$$t_{\rm d} \sim \frac{1}{\Gamma G\mu_{\rm s}}2t_{\rm H}, \quad \Gamma \sim 50.$$

- The energy density of the emitting gravitational waves is given by

$$\frac{\rho_{\rm gw}(t_{\rm d})}{\rho_\gamma(t_{\rm d})} \sim 2\left(\frac{2}{\Gamma}\right)^{\frac{1}{2}}(G\mu_{\rm s})^{\frac{1}{2}}.$$

- This formula also gives the maximal relative string energy density.
- Taking the lowest value of N_2 and $p=2$, we find that, for our two numerical examples, respectively,

$$t_{\rm H} \sim 4.76\times 10^{-7}~{\rm sec} \quad {\rm and} \quad 1.04\times 10^{-4}~{\rm sec}.$$

- Here we took $g(T_{\rm r}) = 228.75$ from MSSM, and $g(T_{\rm H}) = 40.75$ and 10.75 in the two examples consistently with the obtained $T_{\rm H}$.
- So the strings enter the horizon well before nucleosynthesis.
- Their decay time is $t_{\rm d} \sim 5.97 \times 10^{-2}~{\rm sec}$ and $5.49~{\rm sec}$, in the two cases, as found from the dimensionless string tensions

$$G\mu_{\rm s} = \frac{|\langle H\rangle|^2}{2} \simeq \frac{mM}{2\lambda} \simeq 3.19 \times 10^{-7} \quad {\rm and} \quad 7.57 \times 10^{-7}.$$

- So the strings decay around nucleosynthesis and, thus, well before recombination which takes place at $t \sim 10^{13}~{\rm sec}$.
- As a consequence, they do not affect the CMBR.
- Their maximal relative energy density in the universe is $\sim 2.26 \times 10^{-4}$ and 3.48×10^{-4} for our two cases.
- They are always subdominant and do not disturb nucleosynthesis.
- Had the strings survived until now, we would have to assume that

$$G\mu_{\rm s} \lesssim 3.2 \times 10^{-7},$$

  to keep their imprint on the CMBR at an acceptable level.
- In our first case, $G\mu_{\rm s}$ saturates this bound, but violates the recent more stringent bound from pulsar timing arrays:

$$G\mu_{\rm s} \lesssim 3.3 \times 10^{-8},$$

  which also holds for strings surviving until now.
- Our second example violates both these bounds and, thus, both examples are only possible because the strings decay early enough.

- The ratio of the energy density of the gravity waves produced by the strings to that of the photons at the present time t_0 is

$$\frac{\rho_{\rm gw}(t_0)}{\rho_\gamma(t_0)} \sim 2\left(\frac{2}{\Gamma}\right)^{\frac{1}{2}} (G\mu_{\rm s})^{\frac{1}{2}} \left(\frac{3.9}{10.75}\right)^{\frac{4}{3}}.$$

- The present abundance of these gravity waves is then

$$\Omega_{\rm gw}h^2(t_0) \sim \left(\frac{\rho_{\rm gw}(t_0)}{\rho_\gamma(t_0)}\right)\left(\frac{\rho_\gamma(t_0)}{\rho_{\rm c}(t_0)}\right) h_0^2 \, .$$

- $\rho_{\rm c}(t_0)$ is the present critical energy density of the universe and $h_0 \simeq 0.7$ the Hubble constant in units of $100 \ {\rm km\ sec^{-1}\ Mpc^{-1}}$.
- For our two examples, $\Omega_{\rm gw}h^2(t_0) \sim 2.18 \times 10^{-9}$ and 3.35×10^{-9}.
- The frequency $f(t_{\rm d})$ of these waves at production must be $\sim t_{\rm H}^{-1}$ since the length of the decaying strings is $\sim 2t_{\rm H}$.
- The present value of this frequency is then

$$f(t_0) \sim t_{\rm H}^{-1}\left(\frac{t_d}{t_{\rm eq}}\right)^{\frac{1}{2}}\left(\frac{t_{\rm eq}}{t_0}\right)^{\frac{2}{3}},$$

  where $t_{\rm eq}$ is the equidensity time after which matter dominates.
- For the two examples, $f(t_0) \sim 1.06 \times 10^{-4}$ Hz and 4.68×10^{-6} Hz.
- These frequencies are too high to yield any restriction from CMBR.
- They are also well above the range probed by the pulsar timing arrays and the relevant stringent bound does not apply.
- The frequency in our first example lies marginally within the range of the future space-based observatories such as eLISA/NGO expected to be able to detect $\Omega_{\rm gw}h^2(t_0)$'s as low as 4×10^{-10}.

- So the monopole-string net disappears causing no trouble.
- However, the gravity waves generated may be probed by future space-based laser interferometer observations.

6 Conclusions

- We considered a reduced version of the extended SUSY PS model which was initially constructed for solving the m_b problem of the simplest SUSY PS model with universal boundary conditions.
- We find that this model can yield a two stage hybrid inflation scenario predicting $r \sim \mathrm{few} \times 10^{-2}$.
- The model in global SUSY possesses two classically flat directions: the trivial and the semi-shifted one.
- SUGRA stabilizes the trivial path which can then support a first stage of inflation with a limited number of e-foldings.
- r can be appreciable as a result of mild SUGRA corrections combined with strong radiative corrections, while $n_{\rm s}$ is acceptable.
- The extra e-foldings required are generated by a second stage of inflation along the semi-shifted path, where $U(1)_{B-L}$ is unbroken.
- This is possible since the SURGA corrections on the semi-shifted path remain mild and this path is almost orthogonal to the trivial one and, thus, is not affected by the strong radiative corrections.
- At the end of the first inflationary stage, monopoles are formed which after the second stage get connected by open strings.
- Later the monopoles enter the horizon and the string-monopole system decays into gravity waves with no trace in CMBR.
- These gravity waves, however, may be measurable in the future.