

SUPERSYMMETRIC HYBRID INFLATION

Non-“SUSY” version: The most important disadvantage of “inflationary” scenarios such as ‘new’ or ‘chaotic’ is that they require ‘extremely’ small couplings in order to reproduce the results of “COBE” on CMB radiation.

This difficulty was overcome by “Linde” who proposed the ‘hybrid’ inflationary scenario in the context of non-“SUSY” GUTs

The basic idea was to use “two” real scalar fields χ and σ instead of one that was normally used.

χ provides the vacuum energy density which “drives” inflation, and σ is the slowly varying field during inflation.

This splitting of “roles” allows us to reproduce “COBE” results with “natural” (not too small) values of the relevant coupling constant.

The scalar potential is

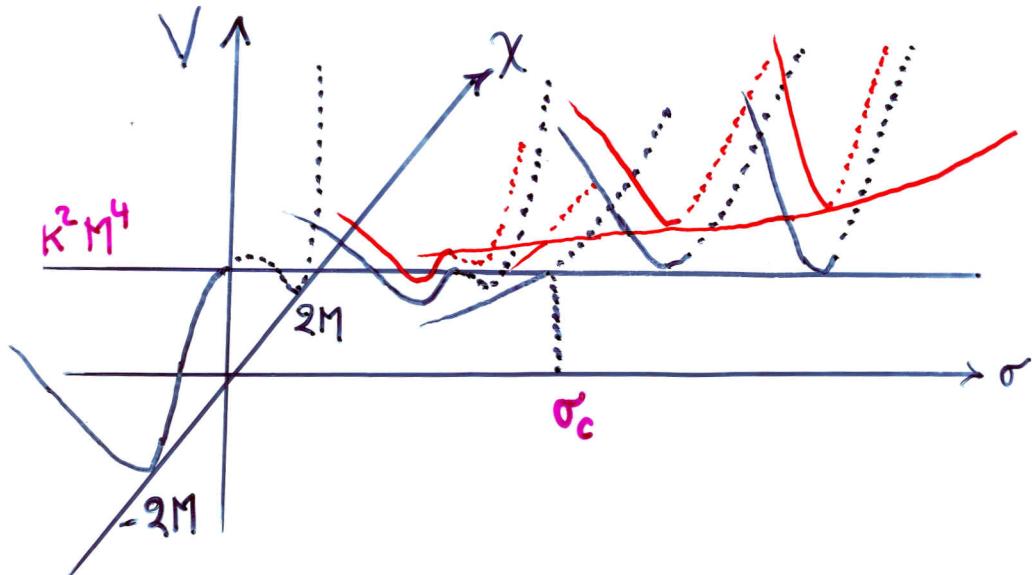
$$V(\chi, \sigma) = \kappa^2 \left(M^2 - \frac{1}{4} \chi^2 \right)^2 + \frac{1}{4} \lambda^2 \chi^2 \sigma^2 + \frac{1}{2} m^2 \sigma^2$$

where κ, λ “dimensionless” positive coupling constants and M, m mass parameters.

The vacua lie at $\langle \chi \rangle = \pm 2M$ and $\langle \sigma \rangle = 0$.

Let us put $m=0$ and observe that V possesses an exactly flat direction at $\chi=0, \forall \sigma$, with $V(\chi=0, \sigma) = \kappa^2 M^4$.

The mass² of χ along this "flat" direction is $m_\chi^2 = -\kappa^2 M^2 + \lambda \sigma^2 / 2$. So, for $|\sigma| \gg \sigma_c = \sqrt{2} \kappa M / \lambda$, we obtain a flat "valley" of minima.



Reintroducing $m \neq 0 \rightarrow$ The flat valley acquires a "slope" and the system can "inflate" as it rolls down this valley.

This scenario is called "hybrid" since:

(i) Vacuum energy density $= \kappa^2 M^4$ is provided by χ

(ii) Slowly rolling field (Inflation) is σ .

ϵ, η criteria \rightarrow Inflation continues till $\sigma = \sigma_c$ (for values below)

At $\sigma = \sigma_c$, "Inflation" terminates "abruptly" followed by a "Waterfall" regime \rightarrow Copious production of "topological defects" (magnetic monopoles, domain walls, ...)

So, if the underlying 'GUT' gauge symmetry breaking (by $\langle \chi \rangle$) predicts the existence of such "defects" \rightarrow "Cosmological Catastrophe".

Onset of "hybrid" inflation requires a region in the universe with size $\gtrsim H^{-1}$ (H =the "Hubble" constant during inflation)

where χ, σ happen to be almost "uniform" with negligible kinetic energy and values close to the "valley".

Such a "region", at $t_p = M_p^{-1}$, would have been much larger than $\ell_p = M_p^{-1}$ and it is difficult to imagine how it could emerge so "homogeneous" → Problem of "initial conditions" for hybrid inflation.

The "quadrupole anisotropy" of CMBR is calculated to be

$$\left(\frac{\delta T}{T}\right)_Q \approx \left(\frac{16\pi}{45}\right)^{1/2} \frac{\lambda k^2 M^5}{M_p^3 m^2} .$$

COBE ($(\delta T/T)_Q \approx 6.6 \times 10^{-6}$) → $m = \kappa \lambda^{1/2} \cdot 1.3 \times 10^{15} \text{ GeV}$
for $M \approx 2.86 \times 10^{16} \text{ GeV}$ (= SUSY GUT scale).

For $\kappa, \lambda \sim 10^{-2}$, $m \sim 10^{12} \text{ GeV}$.

SUSY version: "Hybrid" inflation turns out to be "tailor made" for "SUSY GUTs" except that an "intermediate" scale mass for σ cannot be obtained there (Actually, all scalar masses are expected to be $\sim m_{3/2}$ = the gravitino mass $\sim 1 \text{ TeV}$).

Consider the renormalizable superpotential

$$W = \kappa S (\bar{\phi} \phi - M^2) , \quad \kappa, M > 0$$

S is a gauge singlet LH superfield

$\bar{\phi}, \phi$ are the SM singlet components of a conjugate pair LH superfields belonging to non-trivial representations of G . Their "vets" reduce the rank of G .

$F_S = 0 \rightarrow \langle \bar{\phi} \rangle \langle \phi \rangle = M^2$ in the "SUSY" vacuum.

Vanishing of "D-term" → $|\langle \bar{\phi} \rangle| = |\langle \phi \rangle|$.

→ $\langle \bar{\phi}^* \rangle = \langle \phi \rangle = \pm M$. So, W leads to spontaneous GUT symmetry breaking.

The same superpotential W leads to "hybrid inflation".

W gives rise to the scalar potential

$$V(\bar{\phi}, \phi, S) = \kappa^2 |S|^2 (|\bar{\phi}|^2 + |\phi|^2) + \kappa^2 |\bar{\phi}\phi - M^2|^2 + D\text{-terms}$$

"D-flatness" $\rightarrow \bar{\phi}^* = e^{i\theta} \phi$, which contains the "SUSY" vacua for $\theta=0$. We will "concentrate" on this "D-flat" direction.

Note that W possesses a $U(1)_R$ R-symmetry:

$$S \rightarrow e^{i\alpha} S, \quad \bar{\phi}\phi \rightarrow \bar{\phi}\phi, \quad W \rightarrow e^{i\alpha} W$$

Actually, W is the most general renormalizable superpotential allowed $U(1)_R$ and G .

By G, R -transformations \Rightarrow

$$S = \frac{\sigma}{\sqrt{2}}, \quad \bar{\phi} = \phi = \frac{\chi}{2} \quad (\sigma, \chi = \text{"normalized" real scalars})$$

" V " then takes Linde's form with $\kappa = \lambda$ (from SUSY) but with no mass term for σ (Soft SUSY breaking $\rightarrow m_\sigma \sim m_{3/2}$, which is too small).

One way to generate the necessary "slope" along the "inflationary" trajectory is by "Radiative Corrections".

On the "flat valley", "SUSY" is broken since $V = \kappa^2 M^4 \neq 0$.

\Rightarrow "Fermion-Boson" mass splitting in $\bar{\phi}, \phi$ supermultiplets:

$$\bar{\phi}, \phi \text{ Fermions : } m^2 = \kappa^2 |S|^2 \text{ (4 degrees of freedom)}$$

$$\bar{\phi}, \phi \text{ Bosons : } m^2 = \kappa^2 |S|^2 \pm \kappa^2 M^2 (g+g \parallel \parallel)$$

The "one-loop" "Radiative Corrections" to " V " along the inflationary

trajectory can be found from "Coleman-Weinberg":

$$\Delta V = \frac{1}{64\pi^2} \sum_i (-)^{F_i} M_i^4 \ln\left(\frac{M_i^2}{\Lambda^2}\right),$$

We 'sum' over all "helicities" i , F_i = fermion \otimes , Λ = renormalization scale. We find that

$$\Delta V(|S|) = \kappa^2 M^4 \frac{\kappa^2 N}{32\pi^2} \left[2 \ln\left(\frac{\kappa^2 |S|^2}{\Lambda^2}\right) + (z+1)^2 \ln(1+z) + (z-1)^2 \ln(1-z) \right],$$

where $z = x^2 = |S|^2/M^2$, $N = \otimes$ of $\bar{\Phi}, \phi$ superfields.

For $z \gg 1$ ($|S| \gg S_c = M$, the 'instability' point on the 'inflationary traj.') the 'effective' potential on this 'trajectory' is expanded as

$$V_{\text{eff}}(|S|) = \kappa^2 M^4 \left[1 + \frac{\kappa^2 N}{16\pi^2} \left(\ln \frac{\kappa^2 |S|^2}{\Lambda^2} + \frac{3}{2} - \frac{1}{12z^2} + \dots \right) \right].$$

The "RC" provide the 'slope' driving the "inflaton" to SUSY vacua. Note that this 'slope' is Λ -independent.

The 'quadrupole' anisotropy is

$$\left(\frac{\delta T}{T}\right)_Q \approx \frac{8\pi}{\sqrt{N}} \left(\frac{N_Q}{4S}\right)^{1/2} \left(\frac{M}{M_p}\right)^2 x_Q^{-1} y_Q^{-1} \Lambda'(x_Q)$$

with

$$\Lambda(z) = (z+1) \ln(1+z) + (z-1) \ln(1-z) ,$$

$$y_Q^2 = \int_1^{x_Q^2} \frac{dz}{z} \Lambda'(z) , \quad y_Q \geq 0 .$$

Here, $N_Q = \otimes$ of 'e-foldings' of our horizon during 'inflation', $x_Q = |S_Q|/M$, S_Q being the value of S when our horizon size "crossed outside" the inflationary horizon.

$$\text{For } |S| \gg S_c , \quad y_Q \approx x_Q \left(1 - 7/12 x_Q^2 + \dots \right).$$

The coupling κ is

$$\kappa \approx \frac{8\pi^{3/2}}{\sqrt{N}} \frac{1}{\sqrt{N_Q}} Y_Q \frac{M}{M_P}.$$

The "slow roll" conditions for inflation are

$$\epsilon, |\eta| \ll 1$$

where

$$\epsilon = \left(\frac{\kappa^2 M_P}{16\pi^2 M} \right)^2 \frac{N^2 x^2}{8\pi} \Lambda^2(x^2),$$

$$\eta = \left(\frac{\kappa M_P}{4\pi M} \right)^2 \frac{N}{8\pi} \left[(3z+1) \ln(1+z) + (3z-1) \ln(1-z) \right], z=x^2.$$

Note that $\eta \rightarrow -\infty$ as $x \rightarrow 1^+$, but for most relevant values of parameters ($\kappa \ll 1$) the slow roll conditions are violated only "infinitesimally" close to the "critical point" at $x=1$ ($S=S_c$). So, inflation continues "practically" till the 'instability' point, where the "waterfall" occurs.

Using "COBE" result $(\delta T/T)_Q \approx 6.6 \times 10^{-6}$ and eliminating x_Q between the Eqs. for $(\delta T/T)_Q$ and $\kappa \longrightarrow$ the scale M as a function of κ .

For $x_Q \gg 1$, $y_Q \rightarrow x_Q$, $x_Q y_Q \wedge (x_Q^2) \rightarrow 1^-$

\longrightarrow "maximal" $M \approx 10^{16}$ GeV (for $N=8$, $N_Q \approx 55$)
This is "close" to (but a bit smaller than) the SUSY GUT scale.

With bigger N 's we can get this scale too.

As a numerical example, take $\kappa = 4 \times 10^{-3} \longrightarrow$
 $M \approx 9.57 \times 10^{15}$ GeV, $x_Q \approx 2,633$, $y_Q \approx 2,42$.
The "slow roll" conditions are violated at $x=1 \approx 7.23 \times 10^{-5}$
where $\eta = -1$ ($\epsilon \approx 8.17 \times 10^{-8}$ at $x=1$). The spectral "index"
of density perturbations $n = 1 - 6\epsilon + 2\eta \approx 0.985$.

SUSY "hybrid" inflation is "natural":

- (i) No need of "tiny" couplings ($\kappa \approx 10^{-3} - 10^{-2}$)
- (ii) W has the most general "renormalizable" form allowed by 'gauge' and R symmetries.

Coexistence of S and $S\bar{\phi}\phi$ terms in $W \implies$

$\bar{\phi}\phi$ is "neutral" under all symmetries \implies All "non-renormalizable" $S(\bar{\phi}\phi)^n$ terms ($n \geq 2$) are "necessarily" present.

The "leading" of these couplings $S(\bar{\phi}\phi)^2$, if its dimensionless coefficient is ~ 1 , can be comparable to $S\bar{\phi}\phi$ (remember $\kappa \sim 10^{-3}$) and play a role in inflation (see below).

All the higher order terms of this "type" with $n \geq 3$ give negligible contributions to the scheme.

Note that the R symmetry is "extremely" important since it guarantees the "linearity" of W in S to all orders excluding terms like $S^2 \rightarrow$ 'inflaton' mass $\gtrsim H$, thereby ruining inflation.

(iii) SUSY \rightarrow "Radiative Corrections" do not invalidate inflation, but rather provide the 'necessary' slope along the inflationary trajectory.

(iv) SUGRA corrections are under control so as to leave inflation intact.

The scalar potential in SUGRA is

$$V = \exp(K/m_p^2) \left[(K^{-1})_i^j F^i F_j - 3 |W|^2/m_p^2 \right],$$

$F^i = W^i + K^i W/m_p^2$, upper (lower) indices denote differentiation

w.r.t. $\phi_i (\phi^{i*})$ and K is the "Kähler" potential expanded as

$$K = \underbrace{|S|^2 + |\bar{\Phi}|^2 + |\Phi|^2}_{\text{"minimal" Kähler}} + \alpha |S|^4/m_p^2 + \dots$$

The "minimal" term $|S|^2$, with coefficient "necessarily" normalized to "unity", could generate a mass² for S along the inflationary trajectory $\sim \kappa^2 M^4/m_p^2 \sim H^2$, which could ruin inflation by violating the "slow roll" conditions.

Fortunately, with this form of W (including all "higher order" terms) this mass² is "exactly" cancelled in V . Crucial for this cancellation is the "linearity" of W in S , guaranteed to "all orders" by R symmetry. This is an "important" property of the model.

The $|S|^4$ term in K also \rightarrow mass² of S via

$$\frac{\partial^2 K}{\partial S \partial S^*} = 1 + \frac{4\alpha |S|^2}{m_p^2} + \dots$$

This is not cancelled, and we must assume $\alpha \lesssim 10^{-3}$ in order to avoid ruining inflation.

All other higher terms in " K " are harmless since they give suppressed contributions ($|S| \ll m_p$ on the trajectory).

So, we see that a "mild" tuning of 'just one' parameter is adequate to control SUGRA corrections.

We consider this as a "great" advantage of this model since in other cases tuning of 'infinitely' many parameters is required.

(Note that with 'special' K 's even this tuning may be avoided.)

This property persists even in the "extensions" of the model we will consider later.

In summary, for all the reasons mentioned so far, we consider SUSY hybrid inflation (and extensions) the most "natural" scenario.

We will now try to "embed" SUSY hybrid inflation in more concrete SUSY GUTs.

Let us start by considering a "moderate" extension of MSSM based on the "Left-Right" symmetric gauge group

$$G_{LR} = \text{SU}(3)_c \times \text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)_{B-L}$$

The breaking of G_{LR} to G_S is achieved via a conjugate pair of $\text{SU}(2)_R \times \text{U}(1)_{B-L}$ doublet superfields $\ell^c, \bar{\ell}^c$ which acquire VEVs along their v_H^c, \bar{v}_H^c directions.

v_H^c, \bar{v}_H^c correspond to $\phi, \bar{\phi}$ in our previous discussion.

The "renormalizable" superpotential for the breaking of G_{LR} is

$$W = \kappa S (\ell^c \bar{\ell}^c - M^2), \quad \kappa, M > 0$$

This $W \rightarrow$ hybrid inflation "exactly" as before. In the formulas for $(\delta T/T)_Q$ and κ we must put $N=2$ since $\ell^c, \bar{\ell}^c$ have two components each.

An important "shortcoming" of MSSM is the lack of understanding how the "SUSY" μ term, with the right magnitude of $\mu \sim 10^2 - 10^3$ GeV, arises.

One way to solve this " μ problem" is via a Peccei-Quinn (PQ) symmetry, which also solves the strong CP problem. The axion decay constant f_a is \sim of an 'intermediate' scale ($10^8 - 10^{12}$ GeV) and, thus, $\mu \sim f_a^2/m_p$.

f_a itself can be obtained as $\sim (m_p m_{3/2})^{1/2}$, where $m_{3/2} \sim 1$ TeV is the "gravitino" mass.

To this end, introduce a pair of gauge singlet superfields N, \bar{N} with PQ charges $-1, 1$ and the "non-renormalizable"

couplings $\lambda_1 N^2 h^2 / m_p$, $\lambda_2 N^2 \bar{N}^2 / m_p$ in the superpotential. Here, $h = (h^{(1)}, h^{(2)})$ is the EW Higgs superfield, which is a bidoublet under $SU(2)_L \times SU(2)_R$.

After SUSY breaking, the $N^2 \bar{N}^2$ term leads to the potential:

$$V_{PQ} = \left(m_{3/2}^2 + 4\lambda_2^2 \left| \frac{N \bar{N}}{m_p} \right|^2 \right) \left[(|N| - |\bar{N}|)^2 + 2|N||\bar{N}| \right] + 2|A|m_{3/2}\lambda_2 \frac{|N \bar{N}|^2}{m_p} \cos(\epsilon + 2\theta + 2\bar{\theta}),$$

where A is the dimensionless coefficient of the "soft" SUSY breaking term corresponding to $N^2 \bar{N}^2$, and $\epsilon, \theta, \bar{\theta}$ the phases of A, N, \bar{N} .

Minimization of V_{PQ} requires $|N| = |\bar{N}|$, $\epsilon + 2\theta + 2\bar{\theta} = \pi \implies$

$$V_{PQ} = 2|N|^2 m_{3/2}^2 \left(4\lambda_2^2 \frac{|N|^4}{m_{3/2}^2 m_p^2} - |A|\lambda_2 \frac{|N|^2}{m_{3/2} m_p} + 1 \right)$$

In the case $|A| > 4$, the absolute minimum lies at

$$|\langle N \rangle| = |\langle \bar{N} \rangle| = (m_{3/2} m_p)^{1/2} \left(\frac{|A| + (|A|^2 - 12)^{1/2}}{12\lambda_2} \right)^{1/2} \sim (m_{3/2} m_p)^{1/2} \sim 10^{11} \text{ GeV}.$$

The 'μ term' is generated from $N^2 h^2$ with $\mu = 2\lambda_1 \langle N \rangle^2 / m_p$, which is of the right magnitude.

V_{PQ} possesses also a "local" minimum at $N = \bar{N} = 0$, which is separated from the 'global' PQ minimum by a sizable potential "barrier", which prevents a "successful" transition from the 'trivial' to the PQ vacuum.

This situation persists at all T 's after the 'reheating' which follows 'hybrid' inflation, as shown by considering the one-loop T corrections to the potential.

We are, thus, obliged to assume, after "inflation", the system emerges in the PQ vacuum since, otherwise, it will be stuck "for ever" in the trivial vacuum.

G_{LR} → presence of ν_i^c , which form $SU(2)_R$ doublets
 $L_i^c = (\nu_i^c, e_i^c)$ with the 'L.H' charged antilepton superfields e_i^c
(i is the "family" index).

To give "intermediate" scale masses to ν^c we introduce the superpotential couplings $\gamma_i \bar{\ell}^c \ell^c L_i^c \bar{L}_i^c / m_p$ (in a basis where γ 's are "diagonal" and $\gamma_i > 0$).

The ν^c masses are $M_i = 2 \gamma_i M^2 / m_p$.

Light ν masses are generated via the 'seesaw' mechanism and are "naturally" hierarchical → No role as 'HDM' in the universe.

'Hierarchical' ν masses are more appropriate for a universe with $\Lambda \neq 0$, where the presence of 'HDM' is not "necessary".

The same "non-renormalizable" coupling is responsible for the 'inflaton' decay (see below).

The W of the model must then contain the following "extra" couplings:

$h Q Q^c, h L L^c, h^2 N^2, N^2 \bar{N}^2, \bar{\ell}^c \ell^c L^c \bar{L}^c,$

where $Q_i = SU(2)_L$ doublet quark superfields

$Q_i^c = (u_i^c, d_i^c) = SU(2)_R$ doublet antiquarks

$L_i = SU(2)_L$ doublet lepton superfields

The continuous global symmetries of W are $U(1)_B$ ($\rightarrow U(1)_L$) under which $S, \ell^c, \bar{\ell}^c, N, \bar{N}$ are "neutral" and an 'anomalous' $U(1)_{PQ}$ and a non-anomalous $U(1)_R$. The charges are

PQ: $S, \ell^c, \bar{\ell}^c, Q^c, L^c(0), h, \bar{N}(1), Q, L, N(-1)$

R: $h, \ell^c, \bar{\ell}^c, \bar{N}(0), S(1), Q, Q^c, L, L^c, N(1/2)$

Note that $U(1)_B$ ($\rightarrow U(1)_L$) is "automatically" implied by $U(1)_R$ even if all "non-renormalizable" terms are included, since the R charges of any three color (anti)triplets exceed unity and cannot be compensated (no negative R charges are available).

To avoid "undesirable" mixing of L 's with $h^{(2)}$ or \bar{L}^c via the allowed couplings $N\bar{N}Lh\ell^c$, $N\bar{N}L^c\bar{\ell}^c$ we impose an extra Z_2 symmetry ("lepton parity") under which L, L^c change sign. This is equivalent to Z_2 "matter parity" under which L, L^c, Q, Q^c charge sign since $U(1)_B$ is also present and contains "baryon parity" under which Q, Q^c change sign.

The only W terms permitted by $U(1)_R$, $U(1)_{PQ}$ and "matter parity" are the ones already included and $LL\ell^c\ell^c\bar{N}^2\ell^c\bar{\ell}^c$, $LL\ell^c\ell^chh$ modulo multiplications by $\ell^c\bar{\ell}^c$.

The 'vers' of $\ell^c, \bar{\ell}^c$ and N, \bar{N} leave unbroken only the symmetries G_S , $U(1)_B$ and "matter parity".

A complete "inflationary scenario" should be followed by a successful "reheating" which satisfies the "gravitino constraint" on the 'reheat temperature' T_r ($\lesssim 10^9$ GeV) and can generate the observed BAU.

After the end of "inflation" the system falls towards the SUSY vacua and performs "damped oscillations" about them.

The "inflaton" (oscillating system) consists of two complex scalar fields $S, \Theta = (\delta v_H^c + \delta \bar{v}_H^c)/\sqrt{2}$ with common mass $m_{infl} = \sqrt{2} \kappa M$.

S and Θ decay into a pair of sneutrinos (ν_i^c) and

neutrinos ($\Psi_{\nu_i^c}$) respectively via the W couplings $\bar{\ell}^c \ell^c L^c \bar{L}^c$ and $S \bar{L}^c \bar{\ell}^c$.

The relevant Lagrangian terms are:

$$\mathcal{L}_{\text{decay}}^S = -\sqrt{2} \gamma_i \frac{M}{m_p} S^* \nu_i^c \nu_i^c m_{\text{infl}} + \text{h.c.}$$

$$\mathcal{L}_{\text{decay}}^\Theta = -\sqrt{2} \gamma_i \frac{M}{m_p} \Theta \Psi_{\nu_i^c} \Psi_{\nu_i^c} + \text{h.c.}$$

and the "common" (as it turns out) "decay width" is

$$\Gamma_{S \rightarrow \nu_i^c \nu_i^c} = \Gamma_{\Theta \rightarrow \Psi_{\nu_i^c} \Psi_{\nu_i^c}} = \frac{1}{8\pi} \left(\frac{M_i}{M} \right)^2 m_{\text{infl}}$$

provided the mass of the 'relevant' ν_i^c , $M_i = 2 \gamma_i M^2 / m_p < m_{\text{infl}}/2$.

To minimize the ~~X~~ of small parameters, we assume that $M_2 < m_{\text{infl}}/2 \leq M_3 = 2M^2/m_p$ ($\gamma_3=1$) so that the "inflaton" decays into the 2nd heaviest ν_2^c with mass M_2 . The 2nd inequality $\rightarrow \gamma_Q \leq \sqrt{2N_Q}/\pi \approx 3.34$ (for $N_Q \approx 55$) $\rightarrow X_Q \leq 3.5$.

As an example, choose $X_Q \approx 1.05$ (bigger values do not give adequate $n_L(s)$) $\rightarrow \gamma_Q \approx 0.28$. COBE $\rightarrow M \approx 4.06 \times 10^{15} \text{ GeV}$, $K \approx 4 \times 10^{-4}$, $m_{\text{infl}} \approx 2.3 \times 10^{12} \text{ GeV}$, $M_3 \approx 1.35 \times 10^{13} \text{ GeV}$.

The 'reheat' temperature, for MSSM spectrum, is

$$T_r \approx \frac{1}{\pi} (\Gamma M_p)^{1/2},$$

and must satisfy the "gravitino constraint" $T_r \lesssim 10^9 \text{ GeV}$, for gravity-mediated SUSY breaking with universal boundary conditions.

To maximize "naturalness", we take the "maximal" $M_2 \approx 2.7 \times 10^{10} \text{ GeV}$ ($\gamma_2 \approx 2 \times 10^{-3}$) allowed by the "gravitino constraint" ($M_2 < m_{\text{infl}}/2$).

In 'hybrid' inflationary models, it is "generally" not so convenient to generate the BAU, as usually, via the decay of color $3, \bar{3}$'s

Some of the "reasons" are:

(i) $B-\cancel{X}$ is "practically" conserved in most models (in some cases as a consequence of 'baryon parity'). In the model considered $B-\cancel{X}$ is "exactly" conserved, as a consequence of the R symmetry.

(ii) The 'gravitino constraint' on $T_r (\lesssim 10^9 \text{ GeV})$ would require that the mass of these (anti)triplets $\leq 10^{10} \text{ GeV} \rightarrow$ strong deviations from the MSSM gauge unification and possibly problems with 'proton decay'.

It is generally more "preferable" to produce first a 'primordial' lepton asymmetry which is then partly converted into 'baryon asymmetry' by the non-perturbative 'sphaleron' effects of the electroweak sector.

In the G_{LR} model under consideration (and in many others) this is the 'only way' since the 'inflaton' decays to ν^c 's. Their subsequent decay to $L(E)$ and electroweak 'Higgs' superfields can produce a 'primordial' lepton asymmetry.

It is important that this "lepton asymmetry" is not erased by lepton \cancel{X} violating $2 \rightarrow 2$ scatterings such as $LL \rightarrow h^{(1)*} h^{(1)*}$ or $Lh^{(1)} \rightarrow Lh^{(1)*}$, at all T 's between T_r and $\sim 100 \text{ GeV}$.

This requirement is "automatically" satisfied since the lepton asymmetry is protected by SUSY from T_r until about $T = 10^7 \text{ GeV}$ (Ibanez et al) whereas between this temperature and 100 GeV these $2 \rightarrow 2$ scatterings are "out of equilibrium".

For MSSM spectrum, $n_B/s = (-28/79) n_L/s$.

The 'primordial' "lepton asymmetry" is produced via the decay of the superfield ψ_2^c (= decay product of "inflaton") to EW Higgses and (anti) leptons.

The relevant diagrams are of both the 'vertex' and 'self-energy' type with ψ_3^c exchange. We obtain, for $M_2 \ll M_3$,

$$\frac{n_L}{s} \approx 1.33 \frac{9 T_F}{16\pi M_{\text{infl}}} \frac{M_2}{M_3} \frac{c^2 s^2 \sin 2\delta (m_3^D)^2}{|\langle h^{(1)} \rangle|^2 (m_3^D s^2 + m_2^D c^2)},$$

where $|\langle h^{(1)} \rangle| \approx 174 \text{ GeV}$, $m_{2,3}^D$ the 'Dirac' ν masses taken diagonal, and $c = \cos \theta$, $s = \sin \theta$ with θ and δ being the 'rotation angle' and 'phase' which diagonalize the 'Majorana' mass matrix of ψ^c 's with eigenvalues $M_{2,3}$.

Here, we considered only the two heaviest families ($i=2,3$) and ignored the 1st one. We were able to do this since CHOOZ \rightarrow 'Solar' and 'Atmospheric' ν oscillations decouple.

The light ν mass matrix is given by 'seesaw'

$$m_\nu = - \tilde{m}^D M^{-1} m^D,$$

where m^D is the 'Dirac' mass matrix and M the 'Majorana' one.

The 'determinant' and 'trace' invariance of m_ν implies the two constraints:

$$m_2 m_3 = \frac{(m_2^D m_3^D)^2}{M_2 M_3},$$

$$m_2^2 + m_3^2 = \frac{(m_2^D c^2 + m_3^D s^2)^2}{M_2^2} + \frac{(m_3^D c^2 + m_2^D s^2)^2}{M_3^2} + \frac{2(m_3^D - m_2^D)^2 c^2 s^2 \cos 2\delta}{M_2 M_3},$$

where $m_2 = m_{\nu_\mu}$, $m_3 = m_{\nu_\tau}$ are the eigenvalues of m_ν .

The $\nu_\mu - \nu_\tau$ mixing angle Θ_{23} ($= \Theta_{\mu\tau}$) lies in the range:

$$|\varphi - \Theta^D| \leq \theta_{23} \leq \varphi + \Theta^D$$

where φ is the 'rotation angle' which diagonalizes m_ν , and Θ^D is the 'Dirac' (unphysical) mixing angle in the absence of 'Majorana' ν masses.

From the small angle MSW resolution of the 'solar' ν puzzle
 $\rightarrow m_{\nu_\mu} (=m_2) \approx 2.6 \times 10^{-3}$ eV (central value)

From "SuperK" data $\rightarrow m_{\nu_\tau} (=m_3) \approx 7 \times 10^{-2}$ eV (central value)

We choose $\delta \approx \pi/4$ to 'maximize' n_L/s .

Further, assume that Θ^D is negligible, so that 'maximal' mixing (from SuperK) corresponds to $\varphi \approx \pi/4$.

From the 'determinant' and 'trace' constraints and the diagonalization of m_ν , we can determine the value of m_3^D corresponding to 'maximal' $\nu_\mu - \nu_\tau$ mixing ($\varphi \approx \pi/4$) for a given K .

We find that such a m_3^D exists provided $M_2 \leq 0.037 M_3$.

For the numerical example we discussed ($x_Q \approx 1.05$)
 $\rightarrow m_3^D \approx 8.3$ GeV $\rightarrow m_2^D \approx 0.98$ GeV.

The "lepton asymmetry" $n_L/s \approx 2.23 \times 10^{-10}$ and the "baryogenesis" constraint is also satisfied.

We see that with not so 'unnatural' values of K ($\approx 4 \times 10^{-4}$) and the other relevant parameters ($\gamma_2 \approx 2 \times 10^{-3}$, $\gamma_3 = 1$) not only COBE is reproduced but we also have "successful" reheating satisfying the "gravitino" and "baryogenesis" (via leptogenesis) constraints together with the requirements from 'SuperK' and 'solar' ν oscillations.

In trying to extend SUSY "hybrid inflation" to higher GUT gauge groups which predict the existence of "superheavy" "Magnetic Monopoles", we encounter the following problem:

Inflation is terminated 'abruptly' as the system reaches the 'instability' point on the inflationary trajectory and is followed by the 'waterfall' regime during which $\phi, \bar{\phi}$ develop their vevs starting from "zero", and the "spontaneous" breaking of the GUT gauge group occurs.

$\phi, \bar{\phi}$ can end up at any point of the vacuum manifold with equal probability and, thus, 'monopoles' are "copiously" produced via the "Kibble" mechanism leading to "cosmological catastrophe".

One of the simplest GUT models predicting monopoles is the 'Pati-Salam' model based on the gauge group

$$G_{PS} = SU(4)_c \times SU(2)_L \times SU(2)_R.$$

The monopoles carry "two" units of 'Dirac' magnetic charge.

We will discuss a solution of the monopole problem within the SUSY PS model, although our mechanism can be extended to other 'semisimple' groups such as $SU(3)_c \times SU(3)_L \times SU(3)_R$ (the "trinification" group emerging from string theory) and possibly simple groups such as $SO(10)$.

The idea is to use the 'leading' non-renormalizable term in the standard W for "hybrid" inflation, which, as explained, is "necessarily" present. If its "dimensionless" coefficient is ~ 1 , this term can be comparable with the "standard" trilinear coupling (whose coefficient $\kappa \sim 10^{-3}$).

The coexistence of these terms reveals a completely "new" picture. There appears a "non-trivial" classically flat direction along which G_{PS} is spontaneously broken ($\phi, \bar{\phi} \neq 0$, 'constant').

“Hybrid inflation” can take place along this “non-trivial” trajectory with the ‘inflaton’ driven again by Radiative Corrections.

Inflation is again followed by a ‘waterfall’ but no “monopoles” are produced since G_{PS} is already broken during ‘inflation’.

The breaking of G_{PS} to G_S is achieved via the ‘vevs’ of a conjugate pair of Higgs superfields

$$H^c = (\bar{q}, 1, 2) = \begin{pmatrix} u_H^c & u_H^c & u_H^c & v_H^c \\ d_H^c & d_H^c & d_H^c & e_H^c \end{pmatrix}, \quad \bar{H}^c = (4, 1, 2) = \begin{pmatrix} \bar{u}_H^c & \bar{u}_H^c & \bar{u}_H^c & \bar{v}_H^c \\ \bar{d}_H^c & \bar{d}_H^c & \bar{d}_H^c & \bar{e}_H^c \end{pmatrix}.$$

These ‘vevs’ are in the v_H^c, \bar{v}_H^c directions.

The relevant part of the superpotential, including the leading “non-renormalizable” term is

$$\delta W = \kappa S(\bar{H}^c H^c - M^2) - \beta \frac{S(\bar{H}^c H^c)^2}{M_S^2},$$

where $M_S \approx 5 \times 10^{17}$ GeV is a ‘string’ scale (choose $\beta > 0$ for simplicity).

“D-flatness” $\rightarrow |\bar{H}^c| = |H^c| \rightarrow H^c = e^{i\theta} \bar{H}^c *$. We restrict ourselves to the direction $\theta=0$ ($H^c = \bar{H}^c *$) which contains an “non-trivial” flat direction for ‘inflation’ along which the V derived from δW takes the form

$$V = \left[\kappa (|H^c|^2 - M^2) - \beta \frac{|H^c|^4}{M_S^2} \right]^2 + 2\kappa^2 |S|^2 |H^c|^2 \left[1 - \frac{2\beta}{\kappa M_S^2} |H^c|^2 \right]^2.$$

Defining ‘dimensionless variables’ $w = |S|/M$, $y = |H^c|/M$,

$$\tilde{V} = \frac{V}{\kappa^2 M^4} = (y^2 - 1 - \frac{2}{3} y^4)^2 + 2w^2 y^2 (1 - 2\frac{2}{3} y^2)^2,$$

where $\zeta = \beta M^2 / \kappa M_S^2$.

This potential is a “simple” extension of the ‘standard’ potential for SUSY hybrid inflation (obtained at $\zeta=0$) and is quite generic for models with the “non-renormalizable” term included.

For constant $w(|S|)$, \tilde{V} possesses the extrema

$$y_1 = 0 , \quad y_2 = \frac{1}{\sqrt{2\zeta}} ,$$

$$y_{3\pm} = \frac{1}{\sqrt{2\zeta}} \left[(1 - 6\zeta w^2) \pm \left((1 - 6\zeta w^2)^2 - 4\zeta (1 - w^2) \right)^{1/2} \right]^{1/2}$$

The first two extrema (y_1, y_2) are $|S|$ -independent and, thus, correspond to 'classically' flat directions, the 'trivial' ($y_1=0$) with $\tilde{V}_1=1$; and the 'non-trivial' ($y_2 = 1/\sqrt{2\zeta} = \text{const.}$) with $\tilde{V}_2 = (1/4\zeta - 1)^2$, which we will use as our 'inflationary' trajectory.

The 'trivial' trajectory is a valley of 'minima' for $w > 1$, while the 'non-trivial' for $w > w_0 = (1/8\zeta - 1/2)^{1/2}$, which is its instability point.

We take $\zeta < 1/4$, so that $w_0 > 0$ and the 'non-trivial' trajectory is destabilized before w reaches zero (the destabilization is in the chosen direction $H^c = \bar{H}^c$).

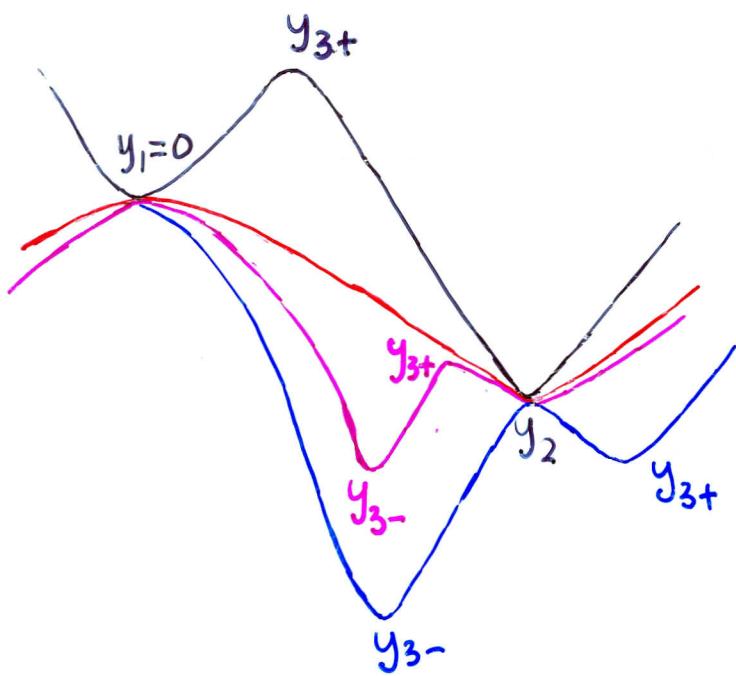
The $y_{3\pm}$ extrema (which are w -dependent and not 'flat') do not exist for $\forall w, \zeta$, since the 'expressions' under the "square roots" are not always 'non-negative'. These two extrema develop, at $w=0$, into SUSY vacua.

The relevant SUSY vacuum (see below) is $y_{3-}(w=0)$ and, thus, the common "vev" v_0 of H^c, \bar{H}^c is given by

$$\left(\frac{v_0}{M}\right)^2 = \frac{1}{2\zeta} \left(1 - \sqrt{1 - 4\zeta}\right) .$$

We will now discuss the 'structure' of \tilde{V} and the corresponding 'inflationary' history in the most interesting range of ζ only, which is $1/4 > \zeta > 1/6$:

For fixed $w > 1$, there exist two local minima at $y_1=0$ and $y_2 = 1/\sqrt{2\zeta}$ (lies lower) and a local maximum at y_{3+} between them.



As $w < 1$, y_1 turns into a local "maximum" and the y_{3+} extremum disappears. The system can "freely" fall into the 'non-trivial' trajectory even if it started at $y_1=0$.

As we further decrease w below $(2 - \sqrt{363-5})^{1/2} / 3\sqrt{23}$, a pair of 'new' extrema appears: a local "maximum" at y_{3+} and a "minimum" at y_{3-} .

As w crosses $(1/83 - 1/2)^{1/2}$, the local maximum at y_{3+} crosses over y_2 becoming a local 'minimum', and y_2 turns into 'maximum'. At this point 'inflation' (along the 'non-trivial' trajectory) is terminated and the system falls into the minimum at y_{3-} , which develops into the SUSY minimum at $w=0$.

We see that, no matter where the system starts from, it "always" passes from the 'non-trivial' trajectory (where the 'relevant' part of 'inflation' takes place driven by Radiative Corrections) before falling into the SUSY vacuum. So, G_{ps} is already broken during "inflation" and no 'Monopoles' are produced at this transition (waterfall).

It should be noted that, after 'inflation', the system could fall into the y_{3+} minimum instead of the y_{3-} one. This does not happen since in the last 'e-folding' or so the 'barrier' between the minima at y_{3-} and y_2 is considerably reduced and the decay of the "false vacuum" at y_2 to the minimum at y_{3-} is completed within a "fraction" of 'e-folding' before the y_{3+} minimum even comes into existence. This transition is further accelerated by the 'inflationary' density perturbations.

The mass '**spectrum**' of the model along the '**non-trivial**' trajectory can be evaluated. We find that the only mass '**splitting**' in supermultiplets occurs in the $\nu_H^c, \bar{\nu}_H^c$ sector. Namely, we get one '**Majorana**' fermion with $m^2 = 4K^2|S|^2$ corresponding to the direction $(\nu_H^c + \bar{\nu}_H^c)/\sqrt{2}$ and two '**normalized**' real scalars $\text{Re}(\delta\nu_H^c + \delta\bar{\nu}_H^c)$ and $\text{Im}(\delta\nu_H^c + \delta\bar{\nu}_H^c)$ with $m_{\pm}^2 = 4K^2|S|^2 \mp 2K^2m^2$ (with $m = M(1/4\zeta - 1)^{1/2}$). Here, $\delta\nu_H^c = \nu_H^c - v$, $\delta\bar{\nu}_H^c = \bar{\nu}_H^c - \bar{v}$ with $v = (K M_S^2 / 2\beta)^{1/2}$ being the common vev of H^c, \bar{H}^c on the trajectory.

The Radiative corrections on the '**non-trivial**' trajectory can then be found from "**Coleman-Weinberg**" and the formulas for $(\delta T/T)_Q$ and K can be constructed.

We find the same formulas as in the previous model with $N=2$ in the $(\delta T/T)_Q$ formula, $N=4$ in the K formula and M generally replaced by $m = M(1/4\zeta - 1)^{1/2}$.

COBE can be reproduced, for instance, with $K \approx 4 \times 10^3$, which corresponds to $\zeta = 1/5$, $v_0 \approx 1.7 \times 10^{16}$ GeV (for $\beta = 1$). The scales $M \approx 1.45 \times 10^{16}$ GeV, $m \approx 7.23 \times 10^{15}$ GeV and the '**inflationary**' scale (characterizing the inflationary "**vacuum**" energy density) $V_{\text{infl}} = K^{1/2}m \approx 4.57 \times 10^{14}$ GeV. The spectral index $n \approx 0.954$.

The model can be completed by adding (as before) the W terms

$$N^2 \bar{N}^2, N^2 h^2, F^c F h, \bar{H}^c \bar{H}^c F^c F^c, G H^c H^c, G \bar{H}^c \bar{H}^c.$$

The first '**two**' are for **PQ** symmetry and the μ problem, the third is the Yukawa term with $F = (4, 2, 1)$, $F^c = (\bar{4}, 1, 2)$ being the '**matter**' superfields, and the fourth is for ν^c, ν masses and the '**inflaton**' decay. The last two are '**new**' with $G = (6, 1, 1)$ introduced in order to give masses to d_H^c, \bar{d}_H^c in H^c, \bar{H}^c .

The model possesses again an R and a PQ symmetry with charges:

R: $H^c, \bar{H}^c, \bar{N}, h(0), F, F^c, N(1/2), S, G(1)$

PQ: $H^c, \bar{H}^c, S, G, F^c(0), \bar{N}, h(1), F, N(-1)$

Further imposing Z_2^{mp} (matter parity) with F, F^c changing sign, we find that

$FFH^cH^c\bar{N}^2, FFH^cH^chh, FF\bar{H}^c\bar{H}^c\bar{N}^2, FF\bar{H}^c\bar{H}^chh, F^cF^cH^cH^c$

are also allowed by the symmetries. All W terms can be multiplied by arbitrary powers of $H^c\bar{H}^c, (H^c)^4, (\bar{H}^c)^4$. After $N, \bar{N}, H^c, \bar{H}^c$ acquire their vevs, only $G_{SM} \times Z_2^{mp}$ remain unbroken.

B-~~**~~ (and L-~~**~~) is violated by the last three red terms above (and the combinations $(H^c)^4, (\bar{H}^c)^4$) \rightarrow couplings like $u^c d^c \bar{d}_H^c \nu_H^c, u^c d^c u_H^c e_H^c, u d \bar{d}_H^c \bar{\nu}_H^c, u d \bar{u}_H^c \bar{e}_H^c$. Also, the terms $G H^c H^c, G \bar{H}^c \bar{H}^c$ give B and L violating couplings. This is in contrast to the previous case.

Proton can decay via 'effective' dim 5 operators from 1-loop graphs with two of the u_H^c, d_H^c or one of the u_H^c, d_H^c and one of the ν_H^c, e_H^c in the loop. The 'lifetime' is long enough to make proton practically stable.

The 'reheating' is similar to the previous model. The 'inflaton' system ($S, \theta = (\delta S_H + \delta \bar{S}_H)/\sqrt{2}$) with

$$m_{\text{infl}}^2 = 2 k^2 v_0^2 \left(1 - \frac{2 \zeta v_0^2}{M^2}\right)^2$$

again decays into ν_2^c superfields with $M_2 = 2 \gamma_2 v_0^2 / M_S$, which is taken so that the "gravitino" constraint is 'saturated'.

The BAU is again generated via a primordial leptogenesis from the subsequent decay of ν_2^c .

The 'gravitino' and 'leptogenesis' ($1.8 \times 10^{-10} \lesssim -n_L/s \lesssim 2.3 \times 10^{-10}$ corresponding to $0.017 \lesssim S_B h^2 \lesssim 0.021$ from the 'low' deuterium measurements) constraints can be satisfied with 'natural' values of the relevant couplings and in accord with the ν oscillation data.

A typical solution, for $K \approx 4 \times 10^{-3}$ ($m_{\text{infl}} \approx 4.1 \times 10^{13} \text{ GeV}$), is $M_2 \approx 5.9 \times 10^{10} \text{ GeV}$, $M_3 \approx 1.1 \times 10^{15} \text{ GeV}$ ($\gamma_3 = 1/2$), $m_{\nu_\mu} = 7.6 \times 10^{-3} \text{ eV}$, $m_{\nu_\tau} \approx 8 \times 10^{-2} \text{ eV}$, $m_3^D \approx 120 \text{ GeV}$, $m_2^D \approx 1.2 \text{ GeV}$, $\sin^2 2\theta \approx 0.87$, $n_L/s \approx -1.8 \times 10^{-10}$ ($\theta \approx 0.0016$, $\delta \approx -\pi/3$).

Note that m_3^D is consistent with the "asymptotic" Yukawa unification implied by $SU(4)_c$, after renormalization effects for large $\tan\beta$ and MSSM spectrum.

Also, m_{ν_μ} is "typically" consistent with the large $(3.6-13) \times 10^{-3} \text{ eV}$ rather than the small $(2-3.2) \times 10^{-3} \text{ eV}$ angle MSW resolution of the solar ν puzzle and $m_2^D \sim 1 \text{ GeV}$ (not too far from $SU(4)_c$).

Finally, another 'inflationary' scenario which solves the "monopole" problem can be obtained from the above PS model by imposing the extra discrete Z_2 symmetry: $H^c \bar{H}^c \rightarrow -H^c \bar{H}^c$ (say $H^c \rightarrow -H^c$).

Nothing changes w.r.t. allowed W terms, symmetries, proton decay, etc. except that only 'even' powers of $H^c \bar{H}^c$ are allowed.

The "inflationary" superpotential becomes

$$\delta W = S \left(-\mu^2 + \frac{(H^c \bar{H}^c)^2}{M_S^2} \right),$$

where we absorbed the "dimensionless" couplings K, β in μ, M_S .

The resulting "inflationary" V is given by

$$\tilde{V} = \frac{V}{\mu^4} = (1 - \tilde{\chi}^4)^2 + 16 \tilde{\sigma}^2 \tilde{\chi}^2,$$

where we used "dimensionless" fields $\tilde{\chi} = \chi / 2(\mu M_S)^{1/2}$, $\tilde{\sigma} = \sigma / 2(\mu M_S)^{1/2}$.

with χ, σ being "normalized" real scalar fields defined by $S = \sigma/\sqrt{2}$, $\nu_H^c = \bar{\nu}_H^c = \chi/2$ (after rotating $S, \nu_H^c, \bar{\nu}_H^c$ to the real axis).

The emerging "picture" is completely different. The flat direction

at $\tilde{\chi} = 0$ is now 'always' a 'local' maximum in the $\tilde{\chi}$ direction, and 'two' symmetric 'valleys' of minima appear at the value $\tilde{\chi}^2 = 6\tilde{\sigma}^2 \left[\left(1 + \frac{1}{36\tilde{\sigma}^4}\right)^{1/2} - 1 \right]$. At $\tilde{\sigma} = 0$,

they lead to the SUSY vacua at $\tilde{\chi} = \pm 1$. These "valleys" are not classically flat, they possess 'already' a classical

slope which can drive the 'inflaton' and there is no 'need' of radiative corrections. The potential along these "trajectories" is

$$\begin{aligned} \tilde{V} &= 48\tilde{\sigma}^4 \left[72\tilde{\sigma}^4 \left(1 + \frac{1}{36\tilde{\sigma}^4}\right) \left(\left(1 + \frac{1}{36\tilde{\sigma}^4}\right)^{1/2} - 1\right) - 1 \right] \\ &\approx 1 - \frac{1}{216\tilde{\sigma}^4} + \dots, \text{ for } \tilde{\sigma} \gg 1. \end{aligned}$$

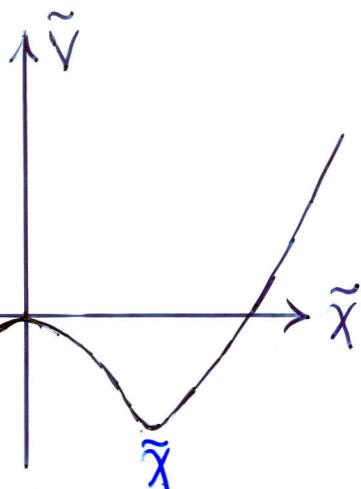
The system follows from the beginning a particular trajectory and, thus, ends up at a particular point of the "vacuum manifold" leading to no production of monopoles.

The $\tilde{\sigma}$ at which 'inflation' is terminated is found from the ϵ, η criteria with the use of the derivatives:

$$\frac{d\tilde{V}}{d\tilde{\sigma}} = 192\tilde{\sigma}^3 \left[\left(144\tilde{\sigma}^4 + 1\right) \left(\left(1 + \frac{1}{36\tilde{\sigma}^4}\right)^{1/2} - 1\right) - 2 \right],$$

$$\frac{d^2\tilde{V}}{d\tilde{\sigma}^2} = \frac{16}{3\tilde{\sigma}^2} \left\{ \left(504\tilde{\sigma}^4 + 1\right) \left[72\tilde{\sigma}^4 \left(\left(1 + \frac{1}{36\tilde{\sigma}^4}\right)^{1/2} - 1\right) - 1 \right] - \left(252\tilde{\sigma}^4 + 1\right) \left[\left(1 + \frac{1}{36\tilde{\sigma}^4}\right)^{-1/2} - 1\right] \right\}.$$

The 'quadrupole' anisotropy of CMB radiation and the \propto of 'e-foldings' of our horizon (N_Q) can be found from the standard formulas using the first derivative of \tilde{V} .



One advantage of this scenario is the H^c, \bar{H}^c VEV $= (\mu M_S)^{1/2}$ is not so rigid. It can be adjusted to become = SUSY GUT scale ($M_G \approx 2.86 \times 10^{16}$ GeV). We then get the 'approximate' formula

$$\left(\frac{S_T}{T}\right)_Q \approx \frac{1}{\sqrt{5}} \left(\frac{6}{\pi}\right)^{1/3} N_Q^{5/6} M_G^{10/3} M_P^{-4/3} M_S^{-2}$$

'COBE' with $N_Q \approx 57 \rightarrow M_S \approx 7.89 \times 10^{17}$ GeV, $\mu \approx 1.04 \times 10^{15}$ GeV, which are quite 'natural' values. The relevant part of inflation occurs from $\sigma_Q \approx (9N_Q/2)^{1/6} \sigma_0$ and $\sigma_0 \approx (2M_P/9\sqrt{\pi} M_G)^{1/3} M_G$, which are 2.72×10^{17} GeV and 1.08×10^{17} GeV respectively for the above values.

The "inflaton" has mass $m_{\text{infl}} = 2\sqrt{2}(\mu/M_S)^{1/2}$ and decays again into γ^c 's. With our numbers, $m_{\text{infl}} \approx 1.07 \times 10^{14}$ GeV.

One can show again that the "gravitino" and "Leptogenesis" constraints can be satisfied with 'natural' parameters together with the requirement from 'solar' and 'atmospheric' ν oscillations and $SU(4)_c$ -invariance. Only, a little higher T_R 's (up to 10^{10} GeV) should be allowed, which is perfectly acceptable for somewhat high $m_{3/2}$'s and low (BR) of the 'LSP' to γ 's.

Conclusion: Hybrid Inflation arises 'naturally' in many SUSY GUTs in the sense that no 'tiny' couplings are needed, W is restricted only by symmetries, and can be protected against SUGRA corrections.

It can readily be 'incorporated' in concrete models together with other requirements such as the solution of CP and μ problems (via a PQ symmetry), ν masses and oscillations.

Inflation is followed by "successful" reheating satisfying the gravitino and 'baryogenesis' (via Leptogenesis) requirements together with 'solar' and 'atmospheric' ν oscillation data.

'Natural' extensions of the 'basic scheme' can also solve the problem of overproduction of Monopoles at the end of inflation.